1. 가 $(c, d)◎(a, b)=(ac+bd, bc+ad)$

$(a, b)◎(c, d)=(c, d)+(a, b)$ ∴교환법칙 성립

나 $(a, b)◎(c, d)=(ac+bd, ad+bc)=(a, b)$

$c=1$ $d=0$ $(1,0)$존재

다 $(a, b)◎(x, y)=(1,0)$

$ax+by=1$---①

$bx+ay=0$---②

①과②를 연립하여 풀면 $x=\frac{a}{a^2-b^2}$ 이 된다.

따라서, $a=b$ or $a=-b$ 일 때 역원이 존재하지 않는다.

∴ 답 ②

2. $m=2a$ $n=2b$ (a, b는 자연수)

$m◎n=2a◎2b=4ab-2a-2b=2(2ab-a-b)$ →짝수

그러나, $a=1$, $b=1$ 일때 0이 될 수도 있다.

∴ 성립하지 않는다.

이; $m=2a-1$ $n=2b-1$ (a, b는 자연수)

$m◎n=(2a-1)◎(2b-1)$

$=(2a-1)(2b-1)-(2a-1)-(2b-1)$

$=4ab-4a-4b+3$

$=2(2ab-2a-2b+1)+1$ →홀수

그러나 $a=1$, $b=3$ 일때 -1이다.

∴ 성립하지 않는다.

∴답⑤

3. $ax^4+bx^3+cx^2-3x=(x^2+4)(ax^2-\frac{3}{4}x)$

$bx^3=-\frac{3}{4}x^3$ $\therefore b=-\frac{3}{4}$

$cx^2=4ax^2$ $\therefore a=\frac{c}{4}$

근의 합 $-\frac{b}{a}=-\frac{-\frac{3}{4}}{\frac{c}{4}}=\frac{3}{c}$

∴답③

4. α : 공통근

$(1-\sqrt{2}i)+(1+\sqrt{2}i)+\alpha=-a$---①

$x^2+ax+2=0$ 의 두 근 α, β

$\alpha+\beta=-a$----②

①과② 연립 $2+\alpha=\alpha+\beta$ $\beta=2$

2를 대입하면, $2a+6=0$ $a=-3$

$x^2-3x+2=0$ $x=1$ *or* 2 $\therefore\alpha=1$

$-a=(1-\sqrt{2}i)+(1+\sqrt{2}i)+1=3$ $a=-3$

$b=(1-\sqrt{2}i)(1+\sqrt{2}i)+(1-\sqrt{2}i)+(1+\sqrt{2}i)=5$

$-c=(1-\sqrt{2}i)(1+\sqrt{2}i)\cdot 1=3$ $c=-3$

$\therefore abc=45$

∴답④

5. $-6x^2+2^y=4^x$

$2^y=(2^x)^2+6\cdot 2^x$

$-10\cdot 2^x+2^y=(2^x)^2-4(2^x)$

$=(2^x-2)^2-4$ $\therefore$ $x=1$, $y=4$

$\therefore x+y=5$

∴답 ⑤

6. $f(x)=\frac{1}{2}\log(x+1)$ $\therefore x>-1$, $x\neq 1$

∴답 ③

7. $ab(c+1)+b(c+1)+a(c+1)+c+1=30$

$(c+1)(ab+a+b+1)=30$

$(c+1)(a+1)(b+1)=30=2\times 3\times 5$

$\therefore a+b+c=1+2+4=7$

∴답③

8. i) $x=\alpha$ 일 때

$\alpha^2+\alpha+1=0$

$(\alpha+1)(\alpha^2+\alpha+1)=0$

$\therefore\alpha^3=-1$

$x^{10}+3x^3-2=(x^2-x+1)Q(x)+P(x)$

$\alpha^{10}+3\alpha^3-2=(\alpha^2-\alpha+1)Q(\alpha)+P(\alpha)$

$(\alpha^3)^3 \cdot \alpha + 3(\alpha^3) - 2 = P(\alpha)$

$P(\alpha) = -\alpha - 5$

ii) $x^{10} + 3x^3 - 2 = (x^2 - x + 1)Q(x) + ax + b$

$P(x) = ax + b$

$x^2 - x + 1 = 0$ 의 허근 w 라고하면

$w^2 - w + 1 = 0 \quad \therefore w^3 = -1$

준식에 w 를 대입하면, $w^{10} + 3w^3 - 2 = aw + b$

$-w - 5 = aw + b$

$\therefore a = -1,\ b = -5$ 이므로 $p(x) = -x - 5$

∴답 ④

9. $a, b \in R$

가, $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & a+b \\ 0 & 1 \end{pmatrix}$

나, $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & a+b \\ 0 & 2 \end{pmatrix} \neq \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$

다, $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -a \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ →항등원 따라서 $\begin{pmatrix} 1 & -a \\ 0 & 1 \end{pmatrix}$은 역원이다.

$D = 1 - 0 \neq 0$ 이므로 항상 역원이 존재한다.

∴ 가, 다

∴답 ④

10. $B = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} \quad B^2 = \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 8 & 1 \end{pmatrix}$

$B^3 = \begin{pmatrix} 1 & 0 \\ 8 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 12 & 1 \end{pmatrix} \cdots\cdots B^n = \begin{pmatrix} 1 & 0 \\ 4n & 1 \end{pmatrix}$

$A_0 = E \quad A_1 = B^1 \quad A_2 = B^2 \cdot B^1$

$\therefore A_n = B^{1+2+3+\cdots+n} = B^{\frac{n(n+1)}{2}}$

$\begin{pmatrix} a_n & b_n \\ c_n & d_n \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2n(n+1) & 1 \end{pmatrix}$

따라서, $a_n + b_n + c_n + d_n = 2 + 2n(n+1) \leq 2004$

$n(n+1) \leq 1001$

1) $n = 31$ 일 때, $n(n+1) = 992 \leq 1001$

2) $n = 32$ 일 때, $n(n+1) = 1056 \geq 1001$

따라서, $n = 31$.

∴답③

11. $\sum_{k=1}^{2004} a_k = \log \frac{\sqrt{2}}{\sqrt{1}} \cdot \frac{\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{4}}{\sqrt{3}} \cdots \frac{\sqrt{2001}}{\sqrt{2004}} = \log\sqrt{2005}$

$\sqrt{2005} = 44.xxx\cdots \qquad \therefore \log\sqrt{2005} = 1.xx\cdots$

∴ 답②

12. $f(1)+f(2)+f(3)+\cdots+f(n-1) = (n^2-1)f(n) = (n-1)^2 f(n-1)$

$(n^2-1)f(n) = (n-1)^2 f(n-1)$

$(n+1)f(n) = (n-1)f(n-1)$

$f(n) = \frac{n-1}{n+1} f(n-1) \quad (n \geq 2)$

$f(1)=1 \quad f(2)=1\cdot\frac{1}{3} \qquad f(3)=1\cdot\frac{1}{3}\cdot\frac{2}{4} \qquad f(4)=1\cdot\frac{1}{3}\cdot\frac{2}{4}\cdot\frac{3}{5}$

$\therefore f(2003) = 1\cdot\frac{1}{3}\cdot\frac{2}{4}\cdot\frac{3}{5}\cdots\frac{2001}{2003}\cdot\frac{2002}{2004}$

$= \frac{2}{2003\cdot 2004} = \frac{1}{2003\cdot 1002}$

∴ 답④

13. $\sum_{n=1}^{\infty} (n+1)^2 a_{n+1} - (n+1)^2 a_n$

$= \sum_{n=1}^{\infty} (n+1)^2 a_n - n^2 a_n - \sum_{n=1}^{\infty} 2na_n + a_n$

$= -1\cdot a_1 - 2\sum_{n=1}^{\infty} na_n - \sum_{n=1}^{\infty} a_n$

$= -1 - 2B - A$

∴ 답①

14. 가, $m=n=0$

$f(f(0)) = f(f(0)) + f(0) \qquad \therefore f(0)=0$

다, $m=0$ 이면 $f(f(n)) = f(f(0)) + f(n) = f(n)$

나, 다에 의해서 성립한다.

∴ 가,나,다

∴ 답⑤

15. $y = \frac{a(x+c)-ac+b}{x+c} = a + \frac{-ac+b}{x+c}$

$\therefore (-c, a) = (2, 1)$

$(3,3)$을 대입 $\quad 3 = 1 + \frac{2+b}{3-2} \qquad b=0$

$\therefore y = 1 + \frac{2}{x-2}$

$3M-m=3f(-1)-f(1)=1-(-1)=2$

∴ 답②

16.

$S=3\sqrt{2}\cdot 2\sqrt{2}\cdot \frac{1}{2}=6$

∴ 답③

17. $\frac{y_2-y_1}{x_2-x_1}=1$

$y_2-y_1=x_2-x_1$

$(x_2{}^2+2x_2)-(x_1{}^2+2x_1)=x_2-x_1$

$x_2{}^2-x_1{}^2+(x_2-x_1)=0$

$(x_2-x_1)(x_2+x_1+1)=0$

$x_2\neq x_1$ 이므로 $x_1+x_2=-1$

∴ 답②

18. 로피탈의 정리로 구할 수 있다. ∴ 답⑤

19. $f(1)=1$, $f'(1)=1$, $f(x)=x^3+x^2+ax+b$

$f'(x)=3x^2+2x+a$

$f'(1)=5+a=1 \quad a=-4$

$f(1)1+1+(-4)+b=1 \quad b=3$

$f(2)=8+4-8+3=7$

∴ 답③

20. $f'(x)=3ax^2+2bx+2$

$\tan\theta+\cot\theta=-\frac{2b}{3a}$

$\tan\theta\cdot\cot\theta=\frac{2}{3a}=1 \quad a=\frac{2}{3}$

b가 양수, $\tan\theta, \cot\theta$ → 음수

$-\tan\theta-\cot\theta=\frac{2b}{3\cdot\frac{2}{3}}=b\geq 2\sqrt{(-\tan\theta)(-\cot\theta)}$

∴ 답①

21. $V=\pi\int_0^h x^2dx=\pi\int_0^h y\,dy$

$=\pi\left[\frac{1}{2}y^2\right]_0^h=\pi\cdot\frac{1}{2}h^2$

$\frac{dV}{dt}=\pi h\cdot\frac{dh}{dt}=2$

$t=4$ 일때 $V=8=\pi\cdot\frac{1}{2}\cdot h^2$ $\therefore h=\frac{4}{\sqrt{\pi}}$

$\therefore \frac{dh}{dt}=\frac{2}{\pi}\cdot\frac{4}{\sqrt{\pi}}=\frac{1}{2\sqrt{\pi}}$

∴ 답①

22.

$\int_1^2 f(x)dx+\int_1^{10} g(x)dx=20-1=19$

∴ 답②

23. $f(x)=x^3-3x^2+\int_0^2 g(t)dt=x^3-3x^2+a$

$g(x)=3x^2+2+\int_{-1}^1 f(t)dt=3x^2+2+b$

$\int_{-1}^1 x^3-3x^2+a\,dx=b$

$2[-x^3+ax]_0^1=b$

$-2+2a=b \rightarrow 2a-b=2$ −−−−−−−①

$\int_0^2 3x^2+2+b\,dx=a$

$[x^3+2x+bx]_0^2=a$

$12+2b=a \rightarrow a-2b=12$ −−−−②

①-②를 계산하면, $a+b=-10$

$\therefore g(x)+f(x)=x^3+2-10=x^3-8$

∴ 답①

24. $\frac{h}{\tan a} - \frac{h}{\tan 3a} = L$

$h(\frac{\cos a}{\sin a} - \frac{\cos 3a}{\sin 3a}) = h(\frac{\sin 3a \cdot \cos a - \cos 3a \cdot \sin a}{\sin a \cdot \sin 3a})$

$= h \cdot \frac{\sin 2a}{\sin a \cdot \sin 3a} = L$

$\therefore h = \frac{\sin a \cdot \sin 3a}{\sin 2a} \cdot L$

∴ 답⑤

25. $N(x) = X, \quad N(y) = Y$

$|X| + |Y| = 2 \qquad (1+X) \cdot (Y-1) = 0$

$(X, Y) = (1, 1), (-1, 1), (-1, -1)$

$X = 1,\ 10 \le x < 100 \qquad X = -1,\ 0.1 \le x < 1$

$\therefore 90 \cdot 90 + 90 \cdot 0.9 + 0.9 \cdot 0.9 = 8100 + 81 + 0.81 = 8181.81$

∴ 답③