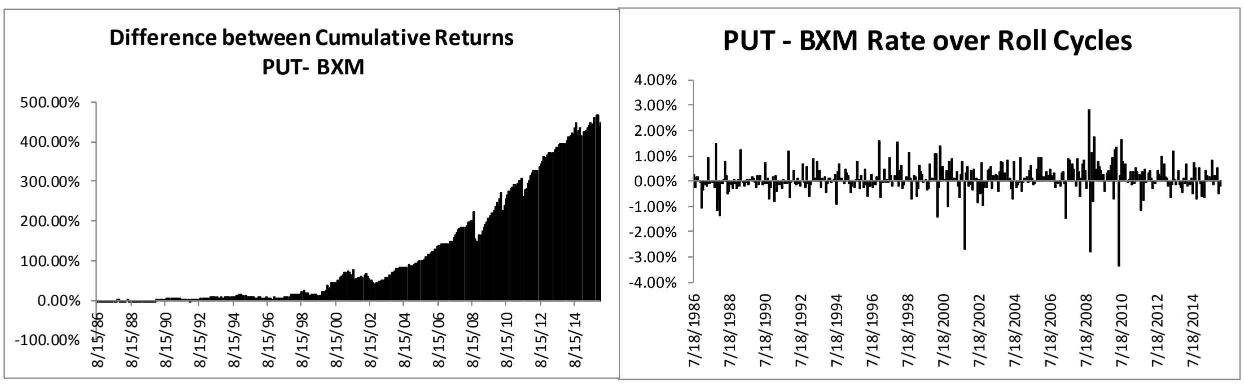
The BXM and PUT Conundrum, Catherine Shalen

Introduction

As shown in Figure 1, the cumulative rate of return of the CBOE S&P 500 PutWrite Index (PUT^{SM} index) has exceeded the rate of the CBOE S&P 500 BuyWrite Index (BXM^{SM} index) since June 1986, their common inception date. The gap between the rates progressively widened after 1992. From 1986 to 1992 the PUT earned .65% more than the BXM annually. The difference increased to .78% between 1992 and 2004, and to 2.11% from 2004 to 2014.

Figure 1. Difference between Cumulative and Roll Cycle Rates of Return of PUT and BXM



Source: CBOE

The difference between the PUT and BXM never ceases to surprise investors. Intuitively, the PUT and BXM should have the same return because their underlying strategies look equivalent. The BXM index writes an at-the-money call over the S&P 500, while the PUT collateralizes an at-the-money put with Treasury bills. So why don't they? That is the BXM and PUT Conundrum, a puzzle we will try to unravel in this paper. One telling detail is the widening of the difference in late 1992. As it happens, in November 1992, the final settlement of SPX options shifted from the close to the open. Since then, the options have been settled to a Special Opening Quotation (SOQ) of the S&P 500 instead of its closing value. Hence the rates of return of the PUT and BXM have come to depend on the SOQ, but in different ways. The PUT is long the SOQ while the BXM is short the SOQ. This could be the solution to the puzzle because the SOQ is often higher than nearby S&P 500 values on option expiration dates. The reason is that index arbitrageurs are typically short stock and have to buy stocks to unwind positions at expiration.

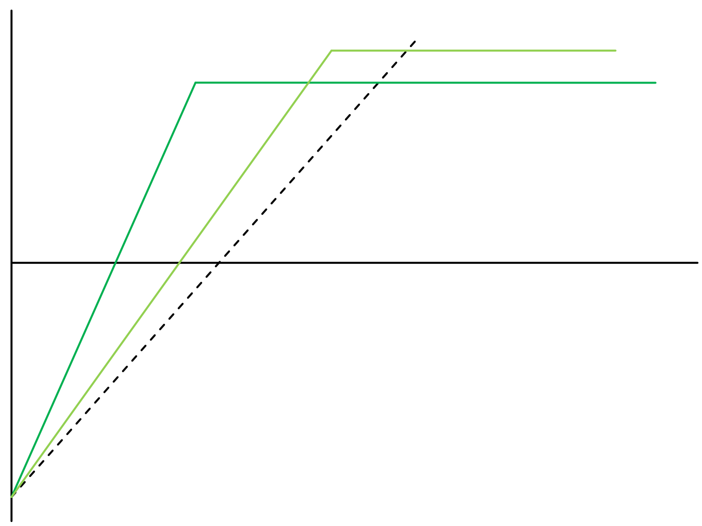
There could be more nuances to the story. One is that the PUT has greater leverage, or beta than the BXM. The PUT therefore tends to do better than the BXM in down markets and vice versa. This is why the PUT rate is not systematically higher than the BXM rate, as shown in right panel of Figure 1. We now explore and tests all the different possible factors of the conundrum.

Distinct Features of PUT and BXM

What are all the possible factors that could account for the difference between PUT and BXM returns:

1. **Different Leverage**: Both PUT and BXM options are theoretically at-the-money, but if there is no listed strike at-the-money, the closest out-of-the-money strike is selected. Hence the strike of the put in the PUT is smaller than the BXM strike. Figure 3 is a stylized representation which shows that the rate of return of the PUT has greater leverage than the BXM up to its strike (see Appendix. In other words, its slope relative to the S&P 500 is steeper than the slope of the BXM and the PUT rate of return is greater up their crossing point. Past the crossing point, it is the reverse. The difference between PUT and BXM should therefore vary inversely with the S&P 500 rate of return.

Figure 2. **Rates of Return of S&P 500, PUT and BXM**



In formula terms, define the roll cycle as the period between two successive times at which the options are sold. Ignoring dividends, the stylized cycle rates of the PUT and BXM from 1986 to 1992 were:

1986 - 1992

$$1+R_{BXM} = \frac{S_T - \max[0, S_T - K_{BXM}]}{S_0 - C_{bid}}, \ 1+R_{PUT} = \frac{K_{PUT} - \max[0, K_{PUT} - S_T]}{K_{PUT}/(1+R) - P_{bid}}$$

K is the strike, S0, ST are the closing values of the S&P 500 at the start and end of the roll cycle, R is the Treasury bill rate, C_{bid} and P_{bid} the bid quotes of the call and put at which the options are deemed sold.

The PUT and BXM depend on the rate of return of the S&P 500 when the put settles in-the-money and the call settles out-of-the-money:

$$1+R_{BXM} = \frac{S_T}{S_0 - C_{bid}} = \frac{1+R_S}{1-\frac{C_{bid}}{S_0}}, \quad 1+R_{PUT} = \frac{S_T}{K_{PUT}/(1+R) - P_{bid}} = \frac{1+R_S}{\frac{K_{PUT}}{S_0(1+R)} - \frac{P_{bid}}{S_0}}$$

The leverage of the call is $L_{BXM} = \frac{1}{1-\frac{C_{bid}}{S_0}}$. The leverage of the put is $L_{PUT} = \frac{1}{\frac{K_{PUT}}{S_0(1+R)} - \frac{P_{bid}}{S_0}}$

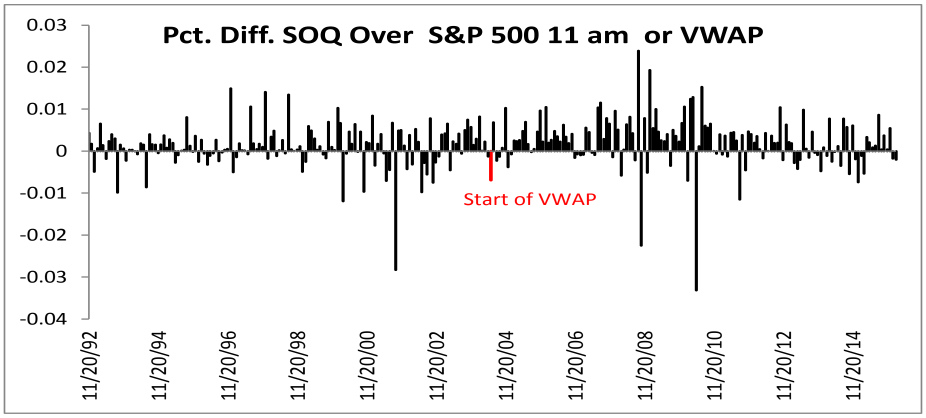
and $L_{BXM} < L_{PUT}$ because $C_{bid} = C_{bid(K_{put})} - \varepsilon$, $\varepsilon > 0$, and, assuming put call parity at the put strike, this implies

$$L_{BXM} = \frac{1}{1-\frac{1}{S_0}(P_{bid} + S_0 - \frac{K_{PUT}}{(1+R)} - \varepsilon)} = \frac{1}{\frac{K_{PUT}}{S_0(1+R)} - \frac{P_{bid}}{S_0} + \frac{\varepsilon}{S_0}}$$

2. **Different Rates of Interest:** The bulk of the Treasury bill position in the PUT is invested at a 3-month Treasury bill rate, whereas the risk free rate implicit in the BXM is a one-month rate. The difference between the two rates was negligible until 2008.

3. **The Role of SOQ in Final Settlement:** As described earlier, in November 1992, the settlement of SPX options shifted from the close to the open and the options have since been settled to the SOQ. As illustrated in Figure 3, the SOQ is often at a premium relative to other intra-day values of the S&P 500 on expiration dates.

Figure 3 Percent Difference between SOQ and S&P 500 11:00 am or VWAP Values.



Source: CBOE

Until May 2004 the BXM rate of return on roll dates was compounded from two rates: the rate from the last close to settlement, and the rate from settlement to the close. The rate from settlement to the close depends on the ratio of the closing value of the S&P 500 to the SOQ. Hence, a relatively high SOQ value decreases this second rate.

Since the June 2004 expiration, the BXM is deemed settled at 11:00 am ET, and the next call is sold two hours later at its volume-weighted price (VWAP). The BXM rate is compounded from three rates: from previous close to settlement of the SPX option, from settlement to sale of next call at VWAP, and from sale to close.

The stylized cycle rates of the BXM and PUT over the two periods are:

1992-2004

$$1 + R_{BXM} = \frac{S_{11} - \max[0, SOQ - K_{BXM}]}{S_0 - C_{bid}},$$

$$1 + R_{PUT} = \frac{K_{PUT} - \max[0, K_{PUT} - SOQ]}{K_{PUT} /(1 + R) - P_{bid}}$$

S_t is the closing value of the S&P 500 at date t.

C_{bid} and P_{bid} are the bid quotes of the call and put.

K_{BXM} and K_{PUT} are the strikes of the call and put.

R is the effective risk-free rate from 0 to T.

2004-Present

$$1+R_{BXM} = \frac{SOQ - \max[0, SOQ - K_{BXM}]}{S_{0,VWAP} - C_{VWAP}} \frac{S_{T,VWAP}}{SOQ},$$

$$1+R_{PUT} = \frac{K_{PUT} - \max[0, K_{PUT} - SOQ]}{K_{PUT}/(1+R) - P_{VWAP}}$$

where VWAP subscript indicates the volume weighted average price.

The BXM is short the SOQ when the call is in-the-money and independent of the SOQ otherwise. The PUT is long the SOQ when in-the-money and independent of the SOQ otherwise. Thus the PUT tends to have an edge over the BXM.

4. High Put Premia

If puts are priced "richly", as several articles have proposed[1], it could increase the rate of return of the PUT relative to the BXM.

Testing the Different Hypotheses

To gauge the importance of the four possible sources of the PUT-BXM disparity, we run regressions where the dependent variable is the percentage difference between PUT and BXM rates over successive roll cycles, and the four independent variables are 1. the percentage difference between three and one month Treasury bill rates, 2. the S&P 500 rate of return, a proxy for the effect of differences in leverage, 3. the difference in percent option premia, and 4. the percentage difference between the SOQ and either the 11 am value of the S&P 500 (to May 2004) or the VWAP value of the S&P 500 (since June 2004) (not available before November 1992). This regression is run separately over three periods: July 1986- October 1992, November 1992-May 2004, June 2004- August 2014.

Figure 4. Regression Summary

[1] See O.Bondarenko, "Why Are Put Options so Expensive?" http://www.investps.com/images/Why_Are_Put_Options_So_Expensive.pdf

86 to 92	PM Settlement		92 to 04	AM-Settlement No V		04 to 14	AM Settlement VWAP	
Regression Statistics			*Regression Statistics*			*Regression Statistics*		
Multiple R	0.61		Multiple R	0.82		Multiple R	0.97	
R Square	0.37		R Square	0.68		R Square	0.94	
PUT-BXM	Coefficient	t Stat		Coefficient	t Stat		Coefficient	t Stat
Intercept	0.00	0.94	Intercept	0.00	0.57	Intercept	0.00	-4.37
3 - 1 MT-Bill Diff	0.00	0.31	3 - 1 MT-Bill Diff	0.00	1.42	3 - 1 MT-Bill Diff	0.00	-0.40
S&P 500 Return	-0.13	**-6.18**	S&P 500 Return	-0.08	**-10.14**	S&P 500 Return	-0.03	**-10.85**
Option Premium D	0.00	0.29	Option Premium D	-0.04	-0.88	Option Premium	0.00	**3.26**
			SOQ to 10 am	0.96	**14.49**	SOQ to VWAP	0.98	**41.79**

The summary results of the three regressions shown in Figure 4 indicate that the independent variables explain the variation of the spread between PUT and BXM cycle rates increasingly better from period to period. The t-statistics associated with the coefficients of the independent variables convey their statistical significance. Over the three periods, a decrease in the S&P 500 rate of return decreased the PUT-BXM spread. This is consistent with the leverage hypothesis, which predicts that other things equal, the PUT has a lower return than the BXM when the S&P 500 increases. The difference between the SOQ and either the S&P 500 value at 11:00 AM ET or the S&P 500 VWAP is the most significant factor, especially in the third period. As this difference increases, so does the PUT-BXM spread. The difference in put and call option premia becomes statistically significant in the third period, but is not as strong.

In conclusion, we find that two factors explain the PUT-BXM spread. First, the difference in leverage between the PUT and BXM caused by their different strikes, and second the different exposures of the PUT and BXM to the SOQ. The first factor by itself would not impart an upward bias to the PUT-BXM spread because the S&P 500 returns are positive on average. This leaves the “SOQ factor” as the most likely systematic source of the positive spread between the PUT and BXM.