◉ disjoint : P(A∩B) = P(A|B) = 0

◉ Baye′stheorem : $P(B|A)=\dfrac{P(B\cap A)}{P(A)}=\dfrac{P(A|B)\ P(B)}{P(A)}$

◉ Independent : P(A∩B) = P(A)P(B)

◉ Bernoulli Trials : only two possible outcomes at any trial

$$P_n=\binom{n}{k}p^k g^{n-k}=\frac{n!}{k!(n-k)!}\,p^k g^{n-k}$$

◉ **Cumulative Distribution Function (cdf)** : $F_X(x)\triangleq P(X\le x)$

◉ cdf′s Properties : ① $0\le F_X(x)\le 1$ for $all\ x\in R$

② $F_X(-\infty)=0$, $F_X(+\infty)=1$

③ $F_X(x)$ is x의 단조증가함수

④ $F_X(x)$ is continuous from the right

⑤ $P(x_1 < X\le x_2)=F_X(x_2)-F_X(x_1)$

◉ For discrete r.v′.s : $F_X(x)=\sum_{i=1}^{n}P(X=x_i)u(x-x_i)$ ← $P(X=x_i)$: 계단의 높이

where $u(x)$ is a unit-step ftn. $\triangleq \begin{cases}1 & x\ge 0\\ 0 & x<0\end{cases}$

◉ **Probability Density Function (pdf)** : $f_X(x)\triangleq\dfrac{dF_X(x)}{dx}$

◉ pdf′s Properties : ① $f_X(x)\ge 0$ for $all\ x\in R$

② $F_X(x)=\int_{-x}^{x}f_X(x')dx'$

★ ③ $\int_{-\infty}^{\infty}f_X(x)dx=1$

④ $P(x_1<X\le x_2)=\int_{x_1}^{x_2}f_X(x)dx$

⑤ $f_x(x)dx=P(x-dx<X\le x)$ for $dx\to 0^+$

◉ For discrete r.v′.s : $f_X(x)=\sum_{i=1}^{n}P(X=x_i)\delta(x-x_i)$ $\leftarrow\delta(x)=\dfrac{du(x)}{dx}$

where $\delta(x)$ is $\triangleq \begin{cases}0 & x\ne 0\\ unbounded & x=0\end{cases}$

◉ Gaussian r.v. pdf : $f_X(x)=\dfrac{1}{\sqrt{2\pi\sigma_x^2}}\,e^{-\frac{(x-m_x)^2}{2\sigma_x^2}}$

◉ Gaussian r.v. cdf : $F_X(x)=\dfrac{1}{\sqrt{2\pi\sigma_x^2}}\int_{-\infty}^{x}e^{-\frac{(x'-m_x)^2}{2\sigma_x^2}}dx'$

◉ **Binomial** (← Bernoulli) discrete : $F_X(x)=\sum_{k=0}^{n}\binom{n}{k}p^k g^{n-k}u(x-k)$

$$f_X(x)=\sum_{k=0}^{n}\binom{n}{k}p^k g^{n-k}\delta(x-k)$$

◉ **Poisson** : $F_X(x) = \sum_{k=0}^{n} e^{-b}\frac{b^k}{k!}u(x-k)$

$$f_X(x) = \sum_{k=0}^{n} e^{-b}\frac{b^k}{k!}\delta(x-k)$$

◉ **Uniform** (continuous) : $F_X(x) = \begin{cases} \frac{x-a}{b-a} & , a \le x < b \\ 0 \quad x < a, & 1 \quad x > b \end{cases}$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & , a \le x \le b \\ 0 & , otherwise \end{cases}$$

◉ **Exponential** (continuous) : $F_X(x) = \begin{cases} 1 - e^{-\frac{(x-a)}{b}} & , x > a \\ 0 & , x < a \end{cases}$

$$f_X(x) = \begin{cases} \frac{1}{b}e^{-\frac{(x-a)}{b}} & , x > a \\ 0 & , x < a \end{cases}$$

◉ **Rayleigh** (continuous) : $F_X(x) = \begin{cases} 1 - e^{-\frac{(x-a)^2}{b}} & , x \ge a \\ 0 & , x < a \end{cases}$

$$f_X(x) = \begin{cases} \frac{2}{b}(x-a)e^{-\frac{(x-a)^2}{b}} & , x \ge a \\ 0 & , x < a \end{cases}$$

◉ ***Conditional cdf's*** : $F_X(x|B) \triangleq P(X \le x|B) = \frac{P(X \le x \cap B)}{P(B)}$

◉ Properties Conditional cdf : ① $F_X(-\infty|B) = 0$, $F_X(\infty|B) = 1$

② $0 \le F_X(x|B) \le 1$

③ $F_X(x_1|B) \le F_X(x_2|B)$ if $x_1 < x_2$

④ $P(x_1 < X \le x_2|B) = F_X(x_2|B) - F_X(x_1|B)$

⑤ $F_X(x^+|B) = F_X(x|B)$

◉ ***Conditional pdf's*** : $f_X(x|B) \triangleq \frac{dF_X(x|B)}{dx}$

◉ Properties Conditional pdf : ① $f_X(x|B) \ge 0$

② $\int_{-\infty}^{\infty} f_x(x|B)dx = 1$

③ $F_X(x|B) = \int_{-\infty}^{x} f_X(\delta|B)\, d\delta$

④ $P(x_1 < X \le x_2|B) = \int_{x_1}^{x_2} f_X(x|B)dx$

◉ **Joint cdf** : $F_{XY}(x,y) \triangleq P(X \le x, Y \le y) = P(A \cap B)$

- discrete r.v.'s

$$F_{XY}(x,y) = \sum_{i=1}^{n}\sum_{j=1}^{m} P(X = x_i, Y = y_j)u(x-x_i)u(y-y_j)$$

◉ Properties Joint cdf : ❶ $F_{XY}(-\infty,-\infty)=F_{XY}(-\infty,y)=F_{XY}(x,-\infty)=0$

❷ $F_{XY}(\infty,\infty)=1$

③ $0\le F_{XY}(x,y)\le 1$ for $all\ x,y\in R$

④ $F_{XY}(x,y)$ is X & Y 의 단조증가함수

⑤ $P(x_1<X\le x_2,\ Y\le y)=F_{XY}(x_2,y)-F_{XY}(x_1,y)$

⑥ $P(x_1<X\le x_2,\ y_1<Y\le y_2)=F_{XY}(x_2,y_2)+F_{XY}(x_1,y_1)$
$-F_{XY}(x_1,y_2)-F_{XY}(x_2,y_1)$

★ ⑦ $F_{XY}(x,\infty)=F_X(x)$, $F_{XY}(\infty,y)=F_Y(y)$

◉ **Joint pdf** : $f_{XY}(x,y)\triangleq\dfrac{\partial^2 F_{XY}(x,y)}{\partial x\partial y}$

- discrete r.v.'s

$$f_{XY}(x,y)=\sum_{i=1}^{n}\sum_{j=1}^{m}P(X=x_i,\ Y=y_j)\delta(x-x_i)\delta(y-y_j)$$

◉ Properties of Joint pdf : ① $f_{XY}(x,y)\ge 0$ for $all\ x,y\in R$

② $F_{XY}(x,y)=\int_{-\infty}^{y}\int_{-\infty}^{x}f_{XY}(x',y')dx'dy'$

★ ③ $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f_{XY}(x',y')dx'dy'=1$

④ $P(x_1<X\le x_2,y_1<Y\le y_2)=\int_{y_1}^{y_2}\int_{x_1}^{x_2}f_{XY}(x',y')dx'dy'$

◉ ***Marginal cdf*** : $F_X(x)=F_{XY}(x,\infty)=\int_{-\infty}^{x}\int_{-\infty}^{\infty}f_{XY}(x',y')dy'dx'$

$$F_Y(y)=F_{XY}(\infty,y)=\int_{-\infty}^{y}\int_{-\infty}^{\infty}f_{XY}(x',y')dx'dy'$$

◉ ***Marginal pdf*** : $f_X(x)=\dfrac{dF_X(x)}{dx}=\int_{-\infty}^{\infty}f_{XY}(x,y')dy'$

$$f_Y(y)=\frac{dF_Y(y)}{dy}=\int_{-\infty}^{\infty}f_{XY}(x',y)dx'$$

◉ conditional cdf : $F_X(x|y)=\dfrac{\int_{-\infty}^{x}f_{XY}(x',y)dx'}{f_Y(y)}$

$$F_Y(y|x)=\frac{\int_{-\infty}^{y}f_{XY}(x,y')dy'}{f_X(x)}$$

◉ conditional cdf : $f_X(x|y)=\dfrac{dF_X(x|y)}{dx}=\dfrac{f_{XY}(x,y)}{f_Y(y)}$

$$f_Y(y|x)=\frac{dF_Y(y|x)}{dy}=\frac{f_{XY}(x,y)}{f_X(x)}$$

- distribution cdf & pdf of a *sum of r.v.'s*
  -for two indep. r.v'.s X&Y W≙X+Y
  - cdf : $F_W(w) = \int_{-\infty}^{\infty} f_Y(y') \int_{-\infty}^{w-y'} f_X(x') dx' dy'$
  - pdf : $f_W(w) = \dfrac{dF_W(w)}{dw} = \int_{-\infty}^{\infty} f_Y(y') f_X(w-y') dy'$ => convolution
- central limit theorem independent r.v'.s
  - pdf : $f_Y(y)$ *of* $Y = \sum_{i=1}^{n} X_i$ *is* $f_Y(y) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y-m)^2}{2\sigma^2}}$: gaussian pdf
- For discrete r.v. : $E(X) = \sum_{i=1}^{n} x_i \cdot P(X = x_i) = \overline{X}$
- $E(x) = \int_{-\infty}^{\infty} x \cdot f_X(x) dx = X$ if a pdf is symmetric about x=a, then E(X)=a
- For discrete r.v. : $f_X(x) = \sum_{i=1}^{n} P(X = x_i)\delta(x - x_i)$

$$E(X) = \int_{-\infty}^{\infty} \sum_{i=1}^{n} P(X = x_i) x\delta(x - x_i) dx$$

$$= \int_{-\infty}^{\infty} \sum_{i=1}^{n} P(X = x_i) \int_{-\infty}^{\infty} x\delta(x - x_i) dx$$

- Expectation of ftn of an r.v. X : Y=g(X) --> $E(Y) = \int_{-\infty}^{\infty} g(x) \cdot f_X(x) dx$
- conditional expectation : $E(X|B) = \int_{-\infty}^{\infty} x \cdot f_X(x|B) dx$

  if B={X<=b} $E(X|B) = E(X|X \le b) = \dfrac{\int_{-\infty}^{b} x \cdot f_X(x) dx}{\int_{-\infty}^{b} f_X(x) dx}$
- Moment about origin (m_n) : $g(X)=X^n$ (n=0,1,2,3,...)

$$m_n = E(g(x)) = E(X^n) = E((X-0)^n) = \int_{-\infty}^{\infty} x^n \cdot f_X(x) dx$$

$$\Rightarrow m_0 = 1 \quad , \quad m_1 = E(X) = X$$

- Moments about mean (μ_n) : $g(X)=(X-X)^n$ (n=0,1,2,3,...)

$$\mu_n = E((X-X)^n) = \int_{-\infty}^{\infty} (x-X)^n f_X(x)\, dx$$

$$\Rightarrow \mu_0 = 1 \quad , \quad \mu_1 = 0$$

- Variance (분산) : 평균구했을 경우 쉬움!

$$\mu_2 = E((X-X)^2) = \int_{-\infty}^{\infty} (x-X)^2 f_X(x)\, dx \triangleq \sigma_X^2 = E(X^2) - E(X)^2 = m_2 - m_1^2$$

- skew : $\mu_3 = E((X-X)^3) = \int_{-\infty}^{\infty} (x-X)^3 \cdot f_X(x) dx$

◉ *Characteristic* function of an r.v. X

$$\Phi_X(w) \triangleq E(e^{jwx}) = \int_{-\infty}^{\infty} e^{jwx} f_X(x)\,dx$$

Fourier transform of fx(x) with the sign of w reversed (− → +)

$$f_X(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_X(w) e^{-jwx}\,dw$$

1) $m_n = E(X^n) = (-j)^n \left.\frac{d^n \Phi_X(w)}{dw^n}\right|_{w=0}$ (n=0,1,2,...)

2) $|\Phi_X(w)| \le \Phi_X(0) = 1$ for *all* w

3) for discrete r.v. : $\Phi_X(w) = \sum_{i=1}^{n} e^{jwx_i} P(X = x_i)$

4) If Y=aX+b : $\Phi_Y(w) = e^{jbw}\Phi_X(aw)$