※ *Fourier transform Pairs*

(α, τ, σ, w_o, W be real const.)

Table of Fourier transform pairs, we define	
▶ $u(\zeta)=\begin{cases}1 & \zeta>0\\0 & \zeta<0\end{cases}$	▶ $Sa(\zeta)=\dfrac{\sin(\zeta)}{\zeta}$
▶ $rect(\zeta)=\begin{cases}1 & \lvert\zeta\rvert<1/2\\0 & \lvert\zeta\rvert>1/2\end{cases}$	▶ $tri(\zeta)=\begin{cases}1-\lvert\zeta\rvert & \lvert\zeta\rvert<1\\0 & \lvert\zeta\rvert>1\end{cases}$

x(t)	*X(w)*	
▶ $\alpha\delta(t)$	α	
▶ $\dfrac{\alpha}{2\pi}$	$\alpha\delta(w)$	
▶ $u(t)$	$\pi\delta(w)+\dfrac{1}{jW}$	
▶ $\dfrac{1}{2}\delta(t)-\dfrac{1}{j2\pi t}$	$u(w)$	
▶ $rect\,(t/\tau)$	$\tau\,Sa\,(w\tau/2)$	$\tau>0$
▶ $tri\,(t/\tau)$	$\tau\,Sa^2(w\tau/2)$	$\tau>0$
▶ $(\dfrac{W}{\pi})Sa(Wt)$	$rect\,(\dfrac{w}{2W})$	$W>0$
▶ $(\dfrac{W}{\pi})Sa^2(Wt)$	$tri\,(\dfrac{w}{2W})$	$W>0$
▶ e^{jw_0t}	$2\pi\delta(w-w_0)$	
▶ $\delta(t-\tau)$	$e^{-jw\tau}$	
▶ $\cos(w_0t)$	$\pi[\delta(w-w_0)+\delta(w+w_0)]$	
▶ $\sin(w_0t)$	$-j\pi[\delta(w-w_0)-\delta(w+w_0)]$	
▶ $u(t)\cos(w_0t)$	$\dfrac{\pi}{2}[\delta(w-w_0)+\delta(w+w_0)]+\dfrac{jw}{w_0^2-w^2}$	
▶ $u(t)\sin(w_0t)$	$-j\dfrac{\pi}{2}[\delta(w-w_0)-\delta(w+w_0)]+\dfrac{w_0}{w_0^2-w^2}$	
▶ $u(t)e^{-\alpha t}$	$\dfrac{1}{\alpha+jw}$	$\alpha>0$
▶ $u(t)te^{-\alpha t}$	$\dfrac{1}{(\alpha+jw)^2}$	$\alpha>0$
▶ $u(t)t^2e^{-\alpha t}$	$\dfrac{2}{(\alpha+jw)^3}$	$\alpha>0$
▶ $u(t)t^3e^{-\alpha t}$	$\dfrac{6}{(\alpha+jw)^4}$	$\alpha>0$
▶ $e^{-\alpha\lvert t\rvert}$	$\dfrac{2\alpha}{\alpha^2+w^2}$	$\alpha>0$
▶ $e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}\,e^{-\sigma^2w^2/2}$	$\sigma>0$

설　명	공　　식
변환의 정의 역변환	$X(w)=\mathcal{F}\{x(t)\}=\int_{-\infty}^{\infty}x(t)e^{-jwt}dt$ $x(t)=\mathcal{F}^{-1}\{X(w)\}=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(w)e^{jwt}dw$
선형성	$x(t)=\sum_{n=1}^{N}\alpha_n x_n(t)\leftrightarrow\sum_{n=1}^{N}\alpha_n X_n(w)=X(w)$
시간 이동 주파수 이동	$x(t-t_0)\leftrightarrow X(w)e^{-jwt_0}$ $x(t)e^{jw_0t}\leftrightarrow X(w-w_0)$
scale change	$x(\alpha t)\leftrightarrow\frac{1}{\lvert\alpha\rvert}X\left(\frac{w}{\alpha}\right)$
Duality	$X(t)\leftrightarrow 2\pi x(-w)$, $X(t)\leftrightarrow x(-f)$
미　분	$\frac{d^n x(t)}{dt^n}\leftrightarrow(jw)^n X(w)$ $(-jt)^n x(t)\leftrightarrow\frac{d^n X(w)}{dw^n}$
적　분	$\int_{-\infty}^{t}x(\tau)d\tau\leftrightarrow\pi X(0)\delta(w)+\frac{X(w)}{jw}$ $\pi x(0)\delta(t)-\frac{x(t)}{jt}\leftrightarrow\int_{-\infty}^{w}X(\zeta)d\zeta$
Conjugation	$x^*(t)\leftrightarrow X^*(-w)$ $x^*(-t)\leftrightarrow X^*(w)$
Convolution	$x(t)=\int_{-\infty}^{\infty}x_1(\tau)x_2(t-\tau)d\tau$ $\leftrightarrow X_1(w)X_2(w)=X(w)$ $x(t)=x_1(t)x_2(t)$ $\leftrightarrow\frac{1}{2\pi}\int_{-\infty}^{\infty}X_1(\zeta)X_2(w-\zeta)d\zeta=X(w)$
Corrlation	$x(t)=\int_{-\infty}^{\infty}x_1^*(\tau)x_2(\tau+t)d\tau$ $\leftrightarrow X_1^*(w)X_2(w)=X(w)$ $x(t)=x_1^*(t)x_2(t)$ $\leftrightarrow\frac{1}{2\pi}\int_{-\infty}^{\infty}X_1^*(\zeta)X_2(\zeta+w)d\zeta=X(w)$

※ *Some Fourier Transform Theorems*

Operation	Function ⇔ Fourier Transform
Linearity	$a_1 w_1(t) + a_2 w_2(t) \Leftrightarrow a_1 W_1(f) + a_2 W_2(f)$
Time Delay	$w(t - T_d) \Leftrightarrow W(f) e^{-jwT_d}$
scale change	$w(at) \Leftrightarrow \frac{1}{\lvert a \rvert} W\left(\frac{f}{a}\right)$
Conjugation	$w^*(t) \Leftrightarrow W^*(-f)$
Duality	$W(t) \Leftrightarrow w(-f)$
Real Signal [w(t) is real]	$w(t)\cos(w_c t + \Theta) \Leftrightarrow \frac{1}{2}[e^{j\Theta} W(f - f_c) + e^{-j\Theta} W(f + f_c)]$
Complex signal	$w(t) e^{jw_c t} \Leftrightarrow W(f - f_c)$
Bandpass Signal	$Re\{g(t) e^{jw_c t}\} \Leftrightarrow \frac{1}{2}[G(f - f_c) + G^*(-f - f_c)]$
Differentiation	$\frac{d^n w(t)}{dt^n} \Leftrightarrow (j2\pi f)^n W(f)$
Integration	$\int_{-\infty}^{t} w(\lambda) d\lambda \Leftrightarrow (j2\pi f)^{-1} W(f) + \frac{1}{2} W(0)\delta(f)$
Convolution	$w_1(t) * w_2(t) = \int_{-\infty}^{\infty} w_1(\lambda) \cdot w_2(t - \lambda) d\lambda$ $\Leftrightarrow W_1(f) W_2(f)$
Multiplication	$w_1(t) w_2(t)$ $\Leftrightarrow W_1(f) * W_2(f) = \int_{-\infty}^{\infty} W_1(\lambda) \cdot W_2(t - \lambda) d\lambda$
Multiplication by t^n	$t^* w(t) \Leftrightarrow (-j2\pi)^{-n} \frac{d^n W(f)}{df^n}$
Parseval's Theorem	$\int_{-\infty}^{\infty} x_1^*(\tau) x_2(\tau) d\tau = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1^*(w) X_2(w) dw$ when $x_1(t) = x_2(t) = x(t)$ $\Rightarrow \int_{-\infty}^{\infty} \lvert x(t) \rvert^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert X(w) \rvert^2 dw$
Corrlation	$x(t) = \int_{-\infty}^{\infty} x_1^*(\tau) x_2(\tau + t) d\tau$ $\leftrightarrow X_1^*(w) X_2(w) = X(w)$ $x(t) = x_1^*(t) x_2(t)$ $\leftrightarrow \frac{1}{2\pi} \int_{-\infty}^{\infty} X_1^*(\zeta) X_2(\zeta + w) d\zeta = X(w)$

※ *Some Fourier Transform Pairs*

Function	Time Waveform $w(t)$ $\Leftrightarrow$ Spectrum $W(f)$
Rectangular	$\Pi\left(\frac{t}{T}\right) \Leftrightarrow T[Sa(\pi f T)]$
Triangular	$\Lambda\left(\frac{t}{T}\right) \Leftrightarrow T[Sa(\pi f T)]^2$
Unit Step	$u(t) \triangleq \begin{cases} +1, & t>0 \\ 0, & t<0 \end{cases} \Leftrightarrow \frac{1}{2}\delta(f) + \frac{1}{j2\pi f}$
Signum	$sgn(t) \triangleq \begin{cases} +1, & t>0 \\ -1, & t<0 \end{cases} \Leftrightarrow \frac{1}{j\pi f}$
Constant	$1 \Leftrightarrow \delta(f)$
Impulse at $t=t_0$	$\delta(t-t_0) \Leftrightarrow e^{-j2\pi f t_0}$
Sinc	$Sa(2\pi W t) \Leftrightarrow \frac{1}{2W}\Pi\left(\frac{f}{2W}\right)$
Phasor	$e^{j(w_0 t+\phi)} \Leftrightarrow e^{j\phi}\delta(f-f_0)$
Sinusoid	$\cos(w_c t+\Theta) \Leftrightarrow \frac{1}{2}e^{j\phi}\delta(f-f_c)+\frac{1}{2}e^{-j\phi}\delta(f+f_c)$
Gaussian	$e^{-\pi(t/t_0)^2} \Leftrightarrow t_0 e^{-\pi(f t_0)^2}$
Exponential, one-sided	$\begin{cases} e^{-t/T}, & t>0 \\ 0, & t<0 \end{cases} \Leftrightarrow \frac{T}{1+j2\pi f T}$
Exponential, two-sided	$e^{-\lvert t \rvert/T} \Leftrightarrow \frac{2T}{1+(2\pi f T)^2}$
Impulse train	$\sum_{k=-\infty}^{k=\infty}\delta(t-kT) \Leftrightarrow f_0\sum_{n=-\infty}^{n=\infty}\delta(f-nf_0)$, *where* $f_0=1/T$