Basic Forms

- $\int u\, dv = vu - \int v\, du$
- $\int x^n\, dx = \frac{1}{n+1} x^{n+1} + C \; (n \neq -1)$
- $\int \frac{1}{x}\, dx = \ln|x| + C$
- $\int e^x\, dx = e^x + C$
- $\int a^x\, dx = \frac{1}{\ln a} a^x + C$
- $\int \sin x\, dx = -\cos x + C$
- $\int \cos x\, dx = \sin x + C$
- $\int \sec^2 x\, dx = \tan x + C$
- $\int \csc^2 x\, dx = -\cot x + C$
- $\int \sec x \tan x\, dx = \sec x + C$
- $\int \csc x \cot x\, dx = -\csc x + C$
- $\int \tan x\, dx = \ln|\sec x| + C$
- $\int \cot x\, dx = \ln|\sin x| + C$
- $\int \sec x\, dx = \ln|\sec x + \tan x| + C$
- $\int \csc x\, dx = \ln|\csc x - \cot x| + C$
- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$
- $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} + C$
- $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\frac{x}{a} + C$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left|\frac{x+a}{x-a}\right| + C$
- $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + C$

Forms Involving $\sqrt{a^2+x^2}$, $a>0$

- $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln(x+\sqrt{a^2+x^2}) + C$
- $\int x^2\sqrt{a^2+x^2}\,dx = \frac{x}{8}(a^2+2x^2)\sqrt{a^2+x^2} - \frac{a^4}{8}\ln(x+\sqrt{a^2+x^2}) + C$
- $\int \frac{\sqrt{a^2+x^2}}{x}\,dx = \sqrt{a^2+x^2} - a\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right| + C$
- $\int \frac{\sqrt{a^2+x^2}}{x^2}\,dx = -\frac{\sqrt{a^2+x^2}}{x} + \ln(x+\sqrt{a^2+x^2}) + C$
- $\int \frac{dx}{\sqrt{a^2+x^2}} = \ln(x+\sqrt{a^2+x^2}) + C$
- $\int \frac{x^2}{\sqrt{a^2+x^2}}\,dx = \frac{x}{2}\sqrt{a^2+x^2} - \frac{a^2}{2}\ln(x+\sqrt{a^2+x^2}) + C$
- $\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2+x^2}+a}{x}\right| + C$
- $\int \frac{dx}{x^2\sqrt{a^2+x^2}} = -\frac{\sqrt{a^2+x^2}}{a^2x} + C$
- $\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} + C$

Forms Involving $\sqrt{a^2-x^2}$, $a>0$

- $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
- $\int x^2\sqrt{a^2-x^2}\,dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2} + \frac{a^4}{8}\sin^{-1}\frac{x}{a} + C$
- $\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$
- $\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\frac{1}{x}\sqrt{a^2-x^2} - \sin^{-1}\frac{x}{a} + C$
- $\int \frac{x^2dx}{\sqrt{a^2-x^2}} = -\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
- $\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$
- $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{1}{a^2x}\sqrt{a^2-x^2} + C$
- $\int (a^2-x^2)^{3/2}\,dx = -\frac{x}{8}(2x^2-5a^2)\sqrt{a^2-x^2} + \frac{3a^4}{8}\sin^{-1}\frac{x}{a} + C$
- $\int \frac{dx}{(a^2-x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2-x^2}} + C$

Form Involving $\sqrt{x^2-a^2}$, $a>0$

- $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$
- $\int x^2\sqrt{x^2-a^2}\,dx = \frac{x}{8}(2x^2-a^2)\sqrt{x^2-a^2} - \frac{a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right| + C$
- $\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} - a\cos^{-1}\frac{a}{x} + C$
- $\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = -\frac{1}{x}\sqrt{x^2-a^2} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$
- $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x+\sqrt{x^2-a^2}\right| + C$
- $\int \frac{x^2\,dx}{\sqrt{x^2-a^2}} = \frac{x}{2}\sqrt{x^2-a^2} + \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$
- $\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x} + C$
- $\int \frac{dx}{(x^2-a^2)^{3/2}} = -\frac{x}{a^2\sqrt{x^2-a^2}} + C$

Inverse Trigonometric Forms

- $\int \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1-x^2} + C$
- $\int \cos^{-1}x\,dx = x\cos^{-1}x - \sqrt{1-x^2} + C$
- $\int \tan^{-1}x\,dx = x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + C$
- $\int x\sin^{-1}x\,dx = \frac{2x^2-1}{4}\sin^{-1}x + \frac{x\sqrt{1-x^2}}{4} + C$
- $\int x\cos^{-1}x\,dx = \frac{2x^2-1}{4}\cos^{-1}x - \frac{x\sqrt{1-x^2}}{4} + C$
- $\int x\tan^{-1}x\,dx = \frac{x^2+1}{2}\tan^{-1}x - \frac{x}{2} + C$
- $\int x^n\sin^{-1}x\,dx = \frac{1}{n+1}\left[x^{n+1}\sin^{-1}x - \int \frac{x^{n+1}dx}{\sqrt{1-x^2}}\right]$, $n \neq -1$
- $\int x^n\cos^{-1}x\,dx = \frac{1}{n+1}\left[x^{n+1}\cos^{-1}x + \int \frac{x^{n+1}dx}{\sqrt{1-x^2}}\right]$, $n \neq -1$
- $\int x^n\tan^{-1}x\,dx = \frac{1}{n+1}\left[x^{n+1}\tan^{-1}x - \int \frac{x^{n+1}dx}{1+x^2}\right]$, $n \neq -1$

Forms Involving a+bx

- $\displaystyle\int \frac{x\,dx}{a+bx} = \frac{1}{b^2}(a+bx-a\ln|a+bx|)+C$
- $\displaystyle\int \frac{x^2dx}{a+bx} = \frac{1}{2b^3}[(a+bx)^2-4a(a+bx)+2a^2\ln|a+bx|]+C$
- $\displaystyle\int \frac{dx}{x(a+bx)} = \frac{1}{a}\ln\left|\frac{x}{a+bx}\right|+C$
- $\displaystyle\int \frac{dx}{x^2(a+bx)} = -\frac{1}{ax}+\frac{b}{a^2}\ln\left|\frac{a+bx}{x}\right|+C$
- $\displaystyle\int \frac{x\,dx}{(a+bx)^2} = \frac{a}{b^2(a+bx)}+\frac{1}{b^2}\ln|a+bx|+C$
- $\displaystyle\int \frac{dx}{x(a+bx)^2} = \frac{1}{a(a+bx)}-\frac{1}{a^2}\ln\left|\frac{a+bx}{x}\right|+C$
- $\displaystyle\int \frac{x^2dx}{(a+bx)^2} = \frac{1}{b^3}\left(a+bx-\frac{a^2}{a+bx}-2a\ln|a+bx|\right)+C$
- $\displaystyle\int x\sqrt{a+bx}\,dx = \frac{2}{15b^2}(3bx-2a)(a+bx)^{3/2}+C$
- $\displaystyle\int \frac{x\,dx}{\sqrt{a+bx}} = \frac{2}{3b^2}(bx-2a)\sqrt{a+bx}+C$
- $\displaystyle\int \frac{x^2dx}{\sqrt{a+bx}} = \frac{2}{15b^3}(8a^2+3b^2x^2-4abx)\sqrt{a+bx}+C$
- $\displaystyle\int \frac{dx}{x\sqrt{a+bx}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+bx}+\sqrt{a}}\right|+C\,,\quad \text{if } a>0$

  $\displaystyle\qquad = \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bx}{-a}}+C\,,\quad \text{if } a<0$
- $\displaystyle\int \frac{\sqrt{a+bx}}{x}dx = 2\sqrt{a+bx}+a\int\frac{dx}{x\sqrt{a+bx}}$
- $\displaystyle\int \frac{\sqrt{a+bx}}{x^2}dx = -\frac{\sqrt{a+bx}}{x}+\frac{b}{2}\int\frac{dx}{x\sqrt{a+bx}}$
- $\displaystyle\int x^n\sqrt{a+bx}\,dx = \frac{2}{b(2n+3)}\left[x^n(a+bx)^{3/2}-na\int x^{n-1}\sqrt{a+bx}\,dx\right]$
- $\displaystyle\int \frac{x^ndx}{\sqrt{a+bx}} = \frac{2}{b(2n+1)}\left[x^n\sqrt{a+bx}-na\int\frac{x^{n-1}dx}{\sqrt{a+bx}}\right]$
- $\displaystyle\int \frac{dx}{x^n\sqrt{a+bx}} = -\frac{\sqrt{a+bx}}{a(n-1)x^{n-1}}-\frac{b(2n-3)}{2a(n-1)}\int\frac{dx}{x^{n-1}\sqrt{a+bx}}$

Trigonometric Forms

- $\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$
- $\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$
- $\int \tan^2 x\,dx = \tan x - x + C$
- $\int \cot^2 x\,dx = -\cot x - x + C$
- $\int \sin^3 x\,dx = -\frac{1}{3}(2 + \sin^2 x)\cos x + C$
- $\int \cos^3 x\,dx = \frac{1}{3}(2 + \cos^2 x)\sin x + C$
- $\int \tan^3 x\,dx = \frac{1}{2}\tan^2 x + \ln|\cos x| + C$
- $\int \cot^3 x\,dx = -\frac{1}{2}\cot^2 x - \ln|\sin x| + C$
- $\int \sec^3 x\,dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$
- $\int \csc^3 x\,dx = -\frac{1}{2}\csc x\cot x + \frac{1}{2}\ln|\csc x - \cot x| + C$
- $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx$
- $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$
- $\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1}x - \int \tan^{n-2}x\,dx$
- $\int \cot^n x\,dx = -\frac{1}{n-1}\cos^{n-1}x - \int \cot^{n-2}x\,dx$
- $\int \sec^n x\,dx = \frac{1}{n-1}\tan x\sec^{n-2}x + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$
- $\int \csc^n x\,dx = -\frac{1}{n-1}\cot x\csc^{n-2}x + \frac{n-2}{n-1}\int \csc^{n-2}x\,dx$
- $\int \sin ax\sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C$
- $\int \cos ax\cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C$
- $\int \sin ax\cos bx\,dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + C$
- $\int x\sin x\,dx = \sin x - x\cos x + C$
- $\int x\cos x\,dx = \cos x + x\sin x + C$
- $\int x^n\sin x\,dx = -x^n\cos x + n\int x^{n-1}\cos x\,dx$
- $\int x^n\cos x\,dx = x^n\sin x - n\int x^{n-1}\sin x\,dx$
- $$\int \sin^n x\cos^m x\,dx = -\frac{\sin^{n-1}x\cos^{m+1}x}{n+m} + \frac{n-1}{n+m}\int \sin^{n-2}x\cos^m x\,dx$$
  $$= \frac{\sin^{n+1}x\cos^{m-1}x}{n+m} + \frac{m-1}{n+m}\int \sin^n x\cos^{m-2}x\,dx$$

Exponential And Logarithmic Forms

- $\int xe^{ax}dx = \frac{1}{a^2}(ax-1)e^{ax} + C$
- $\int x^n e^{ax}dx = \frac{1}{a}x^n e^{ax} - \frac{n}{a}\int x^{n-1}e^{ax}dx$
- $\int e^{ax}\sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx) + C$
- $\int e^{ax}\cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx) + C$
- $\int \ln x\, dx = x\ln x - x + C$
- $\int x^n \ln x\, dx = \frac{x^{n+1}}{(n+1)^2}[(n+1)\ln x - 1] + C$
- $\int \frac{1}{x\ln x}dx = \ln|\ln x| + C$
- $\int_0^\infty x^{2n}e^{-ax^2}dx = \frac{1\cdot 3\cdot 5\cdots(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}}$

Hyperbolic Forms

- $\int \sinh x\, dx = \cosh x + C$
- $\int \cosh x\, dx = \sinh x + C$
- $\int \tanh x\, dx = \ln\cosh x + C$
- $\int \coth x\, dx = \ln|\sinh x| + C$
- $\int \mathrm{sec\,h}\, x\, dx = \tan^{-1}|\sinh x| + C$
- $\int \mathrm{csc\,h}\, x\, dx = \ln\left|\tanh\frac{1}{2}x\right| + C$
- $\int \mathrm{sec\,h}^2 x\, dx = \tanh x + C$
- $\int \mathrm{csc\,h}^2 x\, dx = -\coth x + C$
- $\int \mathrm{sec\,h}\, x\tanh x\, dx = -\mathrm{sec\,h}\, x + C$
- $\int \mathrm{csc\,h}\, x\coth x\, dx = -\mathrm{csc\,h}\, x + C$

Forms Involving $\sqrt{2ax-x^2}$

- $\int \sqrt{2ax-x^2}\,dx = \frac{x-a}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-x}{a}\right) + C$
- $\int x\sqrt{2ax-x^2}\,dx = \frac{2x^2-ax-3a^2}{6}\sqrt{2ax-x^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a-x}{a}\right) + C$
- $\int \frac{\sqrt{2ax-x^2}}{x}\,dx = \sqrt{2ax-x^2} + a\cos^{-1}\left(\frac{a-x}{a}\right) + C$
- $\int \frac{\sqrt{2ax-x^2}}{x^2}\,dx = -\frac{2\sqrt{2ax-x^2}}{x} - \cos^{-1}\left(\frac{a-x}{a}\right) + C$
- $\int \frac{dx}{\sqrt{2ax-x^2}} = \cos^{-1}\left(\frac{a-x}{a}\right) + C$
- $\int \frac{x\,dx}{\sqrt{2ax-x^2}} = -\sqrt{2ax-x^2} + a\cos^{-1}\left(\frac{a-x}{a}\right) + C$
- $\int \frac{x^2dx}{\sqrt{2ax-x^2}} = -\frac{(x+3a)}{2}\sqrt{2ax-x^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a-x}{a}\right) + C$
- $\int \frac{dx}{x\sqrt{2ax-x^2}} = -\frac{\sqrt{2ax-x^2}}{ax} + C$

Rational Algebraic Functions

- $\int (a+bx)^n\,dx = \frac{(a+bx)^{n+1}}{b(n+1)} \qquad 0<n$
- $\int \frac{dx}{a+bx} = \frac{1}{b}\ln|a+bx|$
- $\int \frac{dx}{(a+bx)^n} = \frac{-1}{(n-1)b(a+bx)^{n-1}} \qquad 1<n$
- $\int \frac{dx}{c+bx+ax^2} = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) \qquad b^2<4ac$

  $= \frac{1}{\sqrt{b^2-4ac}}\ln\left|\frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}}\right| \qquad b^2>4ac$

  $= \frac{-2}{2ax+b} \qquad b^2=4ac$
- $\int \frac{x}{c+bx+ax^2}\,dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{2a}\int \frac{dx}{c+bx+ax^2}$
- $\int \frac{dx}{a^2+b^2x^2} = \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$
- $\int \frac{x}{a^2+x^2}\,dx = \frac{1}{2}\ln(a^2+x^2)$
- $\int \frac{x^2}{a^2+x^2}\,dx = x - a\tan^{-1}\left(\frac{x}{a}\right)$

Rational Algebraic Functions

- $$\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3}\tan^{-1}\left(\frac{x}{a}\right)$$
- $$\int \frac{xdx}{(a^2+x^2)^2} = \frac{-1}{2(a^2+x^2)}$$
- $$\int \frac{x^2dx}{(a^2+x^2)^2} = \frac{-x}{2(a^2+x^2)} + \frac{1}{2a}\tan^{-1}\left(\frac{x}{a}\right)$$
- $$\int \frac{dx}{(a^2+x^2)^3} = \frac{x}{4a^2(a^2+x^2)^2} + \frac{3x}{8a^4(a^2+x^2)} + \frac{3}{8a^5}\tan^{-1}\left(\frac{x}{a}\right)$$
- $$\int \frac{x^2dx}{(a^2+x^2)^3} = \frac{-x}{4(a^2+x^2)^2} + \frac{x}{8a^2(a^2+x^2)} + \frac{1}{8a^3}\tan^{-1}\left(\frac{x}{a}\right)$$
- $$\int \frac{x^4dx}{(a^2+x^2)^3} = \frac{a^2x}{4(a^2+x^2)^2} - \frac{5x}{8(a^2+x^2)} + \frac{3}{8a}\tan^{-1}\left(\frac{x}{a}\right)$$
- $$\int \frac{dx}{(a^2+x^2)^4} = \frac{x}{6a^2(a^2+x^2)^3} + \frac{5x}{24a^4(a^2+x^2)^2} + \frac{5x}{16a^6(a^2+x^2)} + \frac{5}{16a^7}\tan^{-1}\left(\frac{x}{a}\right)$$
- $$\int \frac{x^2dx}{(a^2+x^2)^4} = \frac{-x}{6(a^2+x^2)^3} + \frac{x}{24a^2(a^2+x^2)^2} + \frac{x}{16a^4(a^2+x^2)} + \frac{1}{16a^5}\tan^{-1}\left(\frac{x}{a}\right)$$
- $$\int \frac{x^4dx}{(a^2+x^2)^4} = \frac{a^2x}{6(a^2+x^2)^3} - \frac{7x}{24(a^2+x^2)^2} + \frac{x}{16a^2(a^2+x^2)} + \frac{1}{16a^3}\tan^{-1}\left(\frac{x}{a}\right)$$
- $$\int \frac{dx}{a^4+x^4} = \frac{1}{4a^3\sqrt{2}}\ln\left(\frac{x^2+ax\sqrt{2}+a^2}{x^2-ax\sqrt{2}+a^2}\right) + \frac{1}{2a^3\sqrt{2}}\tan^{-1}\left(\frac{ax\sqrt{2}}{a^2-x^2}\right)$$
- $$\int \frac{x^2dx}{a^4+x^4} = -\frac{1}{4a\sqrt{2}}\ln\left(\frac{x^2+ax\sqrt{2}+a^2}{x^2-ax\sqrt{2}+a^2}\right) + \frac{1}{2a\sqrt{2}}\tan^{-1}\left(\frac{ax\sqrt{2}}{a^2-x^2}\right)$$

Definite Integrals

- $$\int_{-\infty}^{\infty} e^{-a^2x^2+bx}dx = \frac{\sqrt{\pi}}{a}e^{b^2/(4a^2)} \qquad a>0$$
- $$\int_0^{\infty} x^2e^{-x^2}dx = \frac{\sqrt{\pi}}{4}$$
- $$\int_0^{\infty} Sa(x)dx = \int_0^{\infty}\frac{\sin(x)}{x}dx = \frac{\pi}{2}$$
- $$\int_0^{\infty} Sa^2(x)dx = \frac{\pi}{2}$$