A recent article by Morse\(^1\) described interesting electrostatics experiments using an MBL charge sensor. In this application, the charge sensor has a large capacitance compared to the charged test object, so nearly all charges can be transferred to the sensor capacitor from the capacitor to be measured. However, the typical capacitance of commercial charge sensors is 10 nF, which is quite small compared to general capacitances for electric circuit experiments. In this paper, we will describe how to use the commercial charge sensor to measure a large capacitance.

Our basic idea is multiple measurements. Because a large charge can be transferred into a test capacitor having a large capacitance, using a commercial MBL charge sensor\(^2\) to determine the capacitance of the test capacitor, we can measure charge not once but many times. In reality, two measurements could adequately determine capacitance and initial charge; however, continual measurements would increase accuracy as long as the charge sensor is able to detect charge remaining in the test capacitor. Figure 1 shows the theoretical background for multiple measurements. The test capacitor, whose capacitance \(C_{\text{test}}\) is unknown, is charged by an external battery. The charge sensor should be disconnected from the test capacitor while it is charging. If the charge sensor remains connected to the test capacitor, more charges will be transferred because the two capacitors would be connected in parallel. Once the test capacitor has finished charging, the battery is disconnected. The charging time is normally very short because there is almost no resistance. \(C_{\text{sensor}}\) denotes the sensor capacitor, known to be 10 nF. It appears invisible because it is molded in an MBL black box. When measuring charge in the test capacitor, we connect the sensor to the test capacitor\(^3\) using sensor leads.

The sequence for multiple measurements is as follows: A discharged sensor that is connected to the test capacitor measures charges from the test capacitor. Once charges are measured, the sensor is disconnected and discharged. Whenever the discharged sensor is connected to the test capacitor, charges from the test capacitor will be transferred to the sensor capacitor until potential differences in both capacitors are equal. The sensor capacitor should be completely discharged before measurement is attempted again. To discharge the sensor capacitor, simply connect the line to a ground potential or push the reset switch. If charges remain in the test capacitor once measurement has been completed, part of this charge will again be transferred to the empty sensor capacitor for the next measurement. After many repetitions of the detection procedure, the charge of the test capacitor will be virtually nil.

An external battery, voltage \(V^{(0)}\), charges the test capacitor, whose initial charge can be represented by

\[
Q^{(0)} = C_{\text{test}} V^{(0)}.
\] (1)

After the external battery is disconnected, for the first measurement, the sensor is connected to the test capacitor. Charge transfer will occur until the voltages \(V^{(1)}\) of the two capacitors are equal.\(^3\) The respective charges in each capacitor can be represented by

\[
Q_{\text{test}}^{(1)} = C_{\text{test}} V^{(1)}
\] (2)

Fig. 1. Experimental setup for multiple measurements.

Fig. 2. Typical data for multiple measurements \(C_{\text{test}}\): 10 nf.
In general, the nth time measurement of sensor charge can be written as

$$Q^{(n)}_{\text{sensor}} = \frac{C_{\text{sensor}}}{C_{\text{test}} + C_{\text{sensor}}} Q^{(n-1)}_{\text{test}}.$$  \hspace{1cm} (9)

Equation (8) can be generalized to

$$C_{\text{test}} = C_{\text{sensor}} \left( \frac{Q^{(n)}_{\text{test}}}{Q^{(n-1)}_{\text{test}} - Q^{(n)}_{\text{test}}} \right),$$  \hspace{1cm} (10)

where $Q^{(n)}$ is the sensor charge at the nth step and $Q^{(n-1)}$ at the previous step.

Figure 2 shows typical data obtained from multiple measurements. A 5-V external battery charged a $C_{\text{test}}$ of 10 nF ($C_{\text{test}}$ is equal to $C_{\text{sensor}}$ in this case). The first measurement was 0.025 μC. The second measurement was taken after discharging the sensor capacitor by pushing the reset switch. The connecting wire between $C_{\text{test}}$ and $C_{\text{sensor}}$ was disconnected during discharging. In this manner, several repeated measurements were taken to obtain the results shown in Fig. 2.

We then took multiple measurements for three different capacitors: 1 nF, 10 nF, and 100 nF, with the same charging voltage. As shown in Table I, larger capacitance retained more charge than smaller capacitance after the first measurement. Also, a larger capacitor allowed more measurements, while a smaller capacitor reached nearly zero potential after measurements. For a 1-nF capacitor, a one-time measurement would be sufficient. This concept can be explained by Eqs. (5) and (6). If the $C_{\text{test}}$ is very small compared to the $C_{\text{sensor}}$, the charge $Q_{\text{test}}^{(1)}$ in the test capacitor after the first measurement will become nearly zero. However, for a 10-nF capacitor, seven measurements can be taken.

After using three different voltages with a capacitor of 10 nF, we normalized the charge by dividing each number with the first number $Q^{(1)}_{\text{sensor}}$ as shown in Fig. 3. All three cases fell into the same curve. We can calculate the capacitance of the test capacitor by inserting values from the first two points into Eqs. (8) or (10). The capacitance works out to about 10.4 nF, which is exceedingly close to the actual value written on the capacitor.

Now, it is possible to use an unknown capacitor. We can charge it with a known voltage, and then measure charges multiple times to decide capacitance and the first stored charge accurately. Also, this idea can be extended to various connections of capacitors like parallel, serial, or mixed.

### Table I. Charges ($\mu$C) measured in each step during multiple measurements for three different capacitors.

<table>
<thead>
<tr>
<th>Order</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 nf</td>
<td>0.0046</td>
<td>0.0004</td>
<td>0.0001</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>10 nf</td>
<td>0.025</td>
<td>0.013</td>
<td>0.0065</td>
<td>0.0034</td>
<td>0.0015</td>
<td>0.0008</td>
<td>0.0003</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 nf</td>
<td>0.0445</td>
<td>0.0403</td>
<td>0.0366</td>
<td>0.0332</td>
<td>0.0302</td>
<td>0.0275</td>
<td>0.025</td>
<td>0.023</td>
<td>0.0208</td>
<td>0.0189</td>
<td>0.0171</td>
<td>0.0157</td>
<td>0.014</td>
</tr>
</tbody>
</table>

![Fig. 3. Charges normalized after multiple measurements with three different charging voltages to the 10-nf capacitor.](image-url)

Using Eqs. (5) and (7), $C_{\text{test}}$ can be given as

$$C_{\text{test}} = C_{\text{sensor}} \left( \frac{Q^{(2)}_{\text{sensor}}}{Q^{(1)}_{\text{sensor}} - Q^{(2)}_{\text{sensor}}} \right).$$  \hspace{1cm} (8)
References
2. We used a PASCO CA-6783 charge sensor.

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