Chapter 5

5.1. (a) 478
   (b) 743
   (c) 2025
   (d) 41567
   (e) 61680

5.2. (a) 478
   (b) −280
   (c) −1

5.3. (a) 478
   (b) −281
   (c) −2

5.4. The numbers are represented as follows:

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Sign and Magnitude</th>
<th>1’s Complement</th>
<th>2’s Complement</th>
</tr>
</thead>
<tbody>
<tr>
<td>73</td>
<td>0000010100101</td>
<td>0000010010010</td>
<td>0000010010010</td>
</tr>
<tr>
<td>1906</td>
<td>0111011100100</td>
<td>0111011100100</td>
<td>0111011100100</td>
</tr>
<tr>
<td>−95</td>
<td>1000010111111</td>
<td>111110100000</td>
<td>111110100000</td>
</tr>
<tr>
<td>−1630</td>
<td>1110010111110</td>
<td>1001101000010</td>
<td>1001101000010</td>
</tr>
</tbody>
</table>

5.5. The results of the operations are:

(a): 001110110 54

(b): 01110101 117

(c): 11011111  (−33)

(d): 001101110 54

(e): 01110101  (117)

(f): 11010011  (−45)

Arithmetic overflow occurs in example e; note that the pattern 10011111 represents −97 rather than +159.

5-1
5.6. The associativity of the XOR operation can be shown as follows:

\[
\begin{align*}
\text{\( x \oplus (y \oplus z) \)} &= \text{\( x \oplus (yz + \overline{y}) \)} \\
&= \text{\( \overline{x}yz + x(\overline{yz} + y) \)} \\
&= \text{\( \overline{x} \cdot \overline{yz} + \overline{x}y \cdot \overline{z} + x \cdot \overline{z} \)}
\end{align*}
\]

\[
\begin{align*}
\text{\((x \oplus y) \oplus z \)} &= \text{\( (x \cdot y + \overline{x} \cdot \overline{y}) \oplus z \)} \\
&= \text{\( (x \cdot y + \overline{x} \cdot \overline{y})z + (x \cdot y + \overline{x} \cdot \overline{y}) \overline{z} \)} \\
&= \text{\( \overline{x} \cdot \overline{yz} + x \cdot \overline{yz} + \overline{x}y \cdot \overline{z} \)}
\end{align*}
\]

The two SOP expressions are the same.

5.7. In the circuit of Figure 5.5b, we have:

\[
\text{s}_i = (x_i \oplus y_i) \oplus c_i
\]

\[
= x_i \oplus y_i \oplus c_i
\]

\[
\text{c}_{i+1} = (x_i \oplus y_i)c_i + x_iy_i
\]

\[
= (x_i \cdot y_i + \overline{x}_i \cdot \overline{y}_i)c_i + x_iy_i
\]

\[
= x_iy_i c_i + x_i \overline{y}_i c_i + x_i y_i
\]

\[
= y_i c_i + x_i c_i + x_i y_i
\]

The expressions for \( s_i \) and \( c_{i+1} \) are the same as those derived in Figure 5.4b.

5.8. We will give a descriptive proof for ease of understanding. The 2’s complement of a given number can be found by adding 1 to the 1’s complement of the number. Suppose that the number has \( k \) 0s in the least-significant bit positions, \( b_{k-1} \ldots b_0 \), and it has \( b_k = 1 \). When this number is converted to its 1’s complement, each of these \( k \) bits has the value 1. Adding 1 to this string of 1s produces \( b_k b_{k-1} \ldots b_{k-2} b_0 = 100 \ldots 0 \). This result is equivalent to copying the \( k \) 0s and the first 1 (in bit position \( b_k \)) encountered when the number is scanned from right to left. Suppose that the most-significant \( n-k \) bits, \( b_{n-1} b_{n-2} \ldots b_k \), have some pattern of 0s and 1s, but \( b_k = 1 \). In the 1’s complement this pattern will be complemented in each bit position, which will include \( b_k = 0 \). Now, adding 1 to the entire \( n \)-bit number will make \( b_k = 1 \), but no further carries will be generated; therefore, the complemented bits in positions \( b_{n-1} b_{n-2} \ldots b_{k+1} \) will remain unchanged.

5.9. Construct the truth table

<table>
<thead>
<tr>
<th>( x_{n-1} )</th>
<th>( y_{n-1} )</th>
<th>( c_{n-1} )</th>
<th>( c_n )</th>
<th>( s_{n-1} ) (sign bit)</th>
<th>Overflow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Note that overflow cannot occur when two numbers with opposite signs are added. From the truth table the overflow expression is

\[
\text{Overflow} = \overline{c_n} c_{n-1} + c_n \overline{c}_{n-1} = c_n \oplus c_{n-1}
\]
5.10. Since \( s_k = x_k \oplus y_k \oplus c_k \), it follows that
\[
\begin{align*}
x_k \oplus y_k \oplus s_k &= (x_k \oplus y_k) \oplus (x_k \oplus y_k) \oplus c_k \\
&= (x_k \oplus y_k) \oplus (x_k \oplus y_k) \oplus c_k \\
&= 0 \oplus c_k \\
&= c_k
\end{align*}
\]

5.11. Yes, it works. The NOT gate that produces \( c_i \) is not needed in stages where \( i > 0 \). The drawback is “poor” propagation of \( \tau_i = 1 \) through the topmost NMOS transistor. The positive aspect is fewer transistors needed to produce \( \tau_{i+1} \).

5.12. From Expression 5.4, each \( c_i \) requires \( i \) AND gates and one OR gate. Therefore, to determine all \( c_i \) signals we need \( \sum_{i=1}^{n}(i + 1) = (n^2 + 3n)/2 \) gates. In addition to this, we need \( 3n \) gates to generate all \( g, p, \) and \( s \) functions. Therefore, a total of \( (n^2 + 9n)/2 \) gates are needed.

5.13. 84 gates.

5.14. The circuit for a 4-bit version of the adder based on the hierarchical structure in Figure 5.18 is constructed as follows:

![Circuit Diagram]

Blocks 0 and 1 have the structure similar to the circuit in Figure 5.16. The overall circuit is given by the expressions
\[
\begin{align*}
p_i &= x_i + y_i \\
g_i &= x_iy_i \\
P_0 &= p_1p_0 \\
G_0 &= g_1 + p_1g_0
\end{align*}
\]
\[ P_1 = p_3 p_2 \]
\[ G_1 = g_3 + p_3 g_2 \]
\[ c_2 = G_0 + P_0 c_0 \]
\[ c_4 = G_1 + P_1 G_0 + P_1 P_0 c_0 \]

5.15. The longest path, which causes the critical delay, is from the inputs \( m_0 \) and \( m_1 \) to the output \( p_7 \), indicated by the dashed path in the following copy of Figure 5.33a:

Propagation through the block \( A \) involves one gate delay in the AND gate shown in Figure 5.33b and two gate delays to generate the carry-out in the full-adder. Then, in each of the blocks \( B, C, D, E, F, G, \) and \( H \), two more gate delays are needed to generate the carry-out signals in the circuits depicted by Figure 5.33c. Therefore, the total delay along the critical path is 17 gate delays.

5.16. (a) LIBRARY ieee ;
USE ieee.std_logic_1164.all ;

ENTITY row0 IS
PORT ( q0, q1, cin, mk, mkp1 : IN STD_LOGIC ;
  s, cout : OUT STD_LOGIC )
END row0;

ARCHITECTURE LogicFunc OF row0 IS
SIGNAL a0, a1 : STD_LOGIC ;
BEGIN
  a0 <= q0 AND mkp1 ;
a1 <= q1 AND mk ;
s <= cin XOR a0 XOR a1 ;
cout <= (cin AND a0) OR (cin AND a1) OR (a0 AND a1) ;
END LogicFunc ;
(b) LIBRARY ieee;
USE ieee.std_logic_1164.all;
ENTITY row1 IS
PORT ( qj, cin, mk, BitPPi : IN STD_LOGIC;
       s, cout : OUT STD_LOGIC);
END row1;
ARCHITECTURE LogicFunc OF row1 IS
SIGNAL a0 : STD_LOGIC;
BEGIN
a0 <= qj AND mk;
s <= cin XOR a0 XOR BitPPi;
cout <= (cin AND a0) OR (cin AND BitPPi) OR (a0 AND BitPPi);
END LogicFunc;

(c) LIBRARY ieee;
USE ieee.std_logic_1164.all;
ENTITY mult4x4 IS
PORT ( cin : IN STD_LOGIC;
       M, Q : IN STD_LOGIC_VECTOR(3 DOWNTO 0);
       P : OUT STD_LOGIC_VECTOR(7 DOWNTO 0) );
END mult4x4;
ARCHITECTURE Structure OF mult4x4 IS
COMPONENT row0
PORT ( q0, q1, cin, mk, mkp1 : IN STD_LOGIC;
       s, cout : OUT STD_LOGIC);
END COMPONENT;
COMPONENT row1
PORT ( qj, cin, mk, BitPPi : IN STD_LOGIC;
       s, cout : OUT STD_LOGIC);
END COMPONENT;
SIGNAL PP1 : STD_LOGIC_VECTOR(5 DOWNTO 2);
SIGNAL PP2 : STD_LOGIC_VECTOR(6 DOWNTO 3);
SIGNAL Crow0, Crow1, Crow2 : STD_LOGIC_VECTOR(1 TO 3);
BEGIN
P(0) <= Q(0) AND M(0);
row0_1: row0 PORT MAP ( Q(0), Q(1), cin, M(0), M(1), P(1), Crow0(1) );
row0_2: row0 PORT MAP ( Q(0), Q(1), Crow0(1), M(1), M(2), PP1(2), Crow0(2) );
row0_3: row0 PORT MAP ( Q(0), Q(1), Crow0(2), M(2), M(3), PP1(3), Crow0(3) );
row0_4: row0 PORT MAP ( Q(0), Q(1), Crow0(3), M(3), cin, PP1(4), PP1(5) );
row1_2: row1 PORT MAP ( Q(2), cin, M(0), PP1(2), P(2), Crow1(1) );
row1_3: row1 PORT MAP ( Q(2), Crow1(1), M(1), PP1(3), PP2(3), Crow1(2) );
row1_4: row1 PORT MAP ( Q(2), Crow1(2), M(2), PP1(4), PP2(4), Crow1(3) );
row1_5: row1 PORT MAP ( Q(2), Crow1(3), M(3), PP1(5), PP2(5), PP2(6) );
row2_3: row1 PORT MAP ( Q(3), cin, M(0), PP2(3), P(3), Crow2(1) );
row2_4: row1 PORT MAP ( Q(3), Crow2(1), M(1), PP2(4), P(4), Crow2(2) );
row2_5: row1 PORT MAP ( Q(3), Crow2(2), M(2), PP2(5), P(5), Crow2(3) );
row2_6: row1 PORT MAP ( Q(3), Crow2(3), M(3), PP2(6), P(6), P(7) );
END Structure;
5.17. The code in Figure P5.2 represents a multiplier. It multiplies the lower two bits of Input by the upper two bits of Input, producing the four-bit Output. The style of code is poor, because it is not readily apparent what is being described.

5.18. Let \( Y = y_3 y_2 y_1 y_0 \) be the 9’s complement of the BCD digit \( X = x_3 x_2 x_1 x_0 \). Then, \( Y \) is defined by the truth table

<table>
<thead>
<tr>
<th>( x_3 )</th>
<th>( x_2 )</th>
<th>( x_1 )</th>
<th>( x_0 )</th>
<th>( y_3 )</th>
<th>( y_2 )</th>
<th>( y_1 )</th>
<th>( y_0 )</th>
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<tbody>
<tr>
<td>0</td>
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</tr>
</tbody>
</table>

This gives

\[
\begin{align*}
y_0 &= \overline{x}_0 \\
y_1 &= x_1 \\
y_2 &= \overline{x}_3 x_1 + x_2 \overline{x}_1 \\
y_3 &= \overline{x}_3 \overline{x}_2 \overline{x}_1
\end{align*}
\]

5.19. BCD subtraction can be performed using 10’s complement representation, using an approach that is similar to 2’s complement subtraction. Note that 10’s and 2’s complements are the radix complements in number systems where the radices are 10 and 2, respectively. Let \( X \) and \( Y \) be BCD numbers given in 10’s complement representation, such that the sign (left-most) BCD digit is 0 for positive numbers and 9 for negative numbers. Then, the subtraction operation \( S = X - Y \) is performed by finding the 10’s complement of \( Y \) and adding it to \( X \), ignoring any carry-out from the sign-digit position.

For example, let \( X = 068 \) and \( Y = 043 \). Then, the 10’s complement of \( Y \) is 957, and \( S' = 068 + 957 = 1025 \). Dropping the carry-out of 1 from the sign-digit position gives \( S = 025 \).

As another example, let \( X = 032 \) and \( Y = 043 \). Then, \( S = 032 + 957 = 989 \), which represents \(-1111_{16}\). The 10’s complement of \( Y \) can be formed by adding 1 to the 9’s complement of \( Y \). Therefore, a circuit that can add and subtract BCD operands can be designed as follows:
For the 9’s completer one can use the circuit designed in problem 5.18. The BCD adder is a circuit based on the approach illustrated in Figure 5.40.

5.21. A full-adder circuit can be used, such that two of the bits of the number are connected as inputs \( x \) and \( y \), while the third bit is connected as the carry-in. Then, the carry-out and sum bits will indicate how many input bits are equal to 1.
5.22. Using the approach explained in the solution to problem 5.21, the desired circuit can be built as follows:

5.23. Using the approach explained in the solutions to problems 5.21 and 5.22, the desired circuit can be built as follows:
5.24. The graphical representation is

For example, the addition $-3 + (+5) = 2$ involves starting at 997 ($= -3$) and going clockwise 5 numbers, which gives the result 002 ($= +2$). Similarly, the subtraction $4 - (+8) = -4$ involves starting at 004 ($= +4$) and going counterclockwise 8 numbers, which gives the result 996 ($= -4$).

5.25. The ternary half-adder in Figure P5.3 can be defined using binary-encoded signals as follows:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>Carry</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$c_{out}$</td>
<td>$s_1$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
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<td>0</td>
<td>1</td>
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<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The remaining 7 (out of 16) valuations, where either $a_1 = a_0 = 1$, or $b_1 = b_0 = 1$, can be treated as don’t care conditions. Then, the minimum cost expressions are:

\[
\begin{align*}
c_{out} &= a_1b_1 + a_1b_0 + a_0b_0 \\
s_1 &= a_0b_1 + \overline{a_1a_0}b_1 + a_1b_1b_0 \\
s_0 &= a_1b_1 + \overline{a_1a_0}b_0 + a_0b_1b_0
\end{align*}
\]
5.26. Ternary full-adder is defined by the truth table:

<table>
<thead>
<tr>
<th>$c_{in}$</th>
<th>A</th>
<th>B</th>
<th>$c_{out}$</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0 0 1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0 0 2</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>0 1 0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0 1 1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
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<td>0 1 2</td>
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<td>0 2 0</td>
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</tr>
<tr>
<td>0 2 1</td>
<td>1</td>
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<td>1</td>
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</tr>
<tr>
<td>0 2 2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1 0 0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1 0 1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1 0 2</td>
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<td>1</td>
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<td>0</td>
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<tr>
<td>1 1 0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>1 1 1</td>
<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>1 1 2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1 2 0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1 2 1</td>
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<tr>
<td>1 2 2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Using binary-encoded signals for this full-adder gives the following truth table:

<table>
<thead>
<tr>
<th>$c_{in}$</th>
<th>$a_1$</th>
<th>$a_0$</th>
<th>$b_1$</th>
<th>$b_0$</th>
<th>$c_{out}$</th>
<th>$s_1$</th>
<th>$s_0$</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0 0 1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0 0 2</td>
<td>0</td>
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<td>0</td>
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</tr>
<tr>
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<td>0</td>
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<td>1</td>
<td>2</td>
</tr>
<tr>
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Treating the 14 (out of 32) valuations where either $a_1 = a_0 = 1$ or $b_1 = b_0 = 1$ as don’t care conditions, leads to the minimum cost expressions

$$c_{out} = a_0b_1 + a_1b_0 + a_1c_{in} + b_1c_{in} + a_0b_0c_{in}$$

$$s_1 = a_0b_0c_{in} + a_1b_0c_{in} + a_1b_1c_{in} + a_0b_1c_{in} + a_0b_0c_{in} + a_0b_1c_{in}$$

$$s_0 = a_1b_1c_{in} + a_1b_0c_{in} + a_0b_0c_{in} + a_0b_1c_{in} + a_0b_1c_{in} + a_0b_0c_{in}$$

5.27. The subtractions $26 - 27 = 99$ and $18 - 34 = 84$ make sense if the two-digit numbers 00 to 99 are interpreted so that the numbers 00 to 49 are positive integers from 0 to +49, while the numbers 50 to 99 are negative integers from −50 to −1. This scheme can be illustrated graphically as follows: