A study of pricing pattern under the network effect

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I. Introduction

It is well known that a higher introductory price is practised for a new good by a seller when a seller has no information about the distribution of reservation price of consumers or when both a seller and an individual buyer do not know a total demand but the total output supplied is fixed (see Png, 1991 and Lazear, 1986). However, a lower introductory is frequently applied if consumers can not make sure about quality of product. We can also find this lower introductory when a network effect occurs as consumers’ demand increases. Since consumers’ reservation price depend on the total amount already sold or expected demand in case of network effect economy, a firm may try to charge a lower introductory price to sell a certain amount of product which guarantees some level of network effect and increases the value of product at next period (see Becker, 1991). Hence, if an intertemporal price discrimination is possible, a firm strategically charges a lower price at
initial stage, which induces a certain amount of product. After a certain amount of sale firm can increase a price since a network effect established by the first sale shifts up the value of product.

The purpose of this paper is to show the pricing pattern when a firm practise price discrimination over time against consumers whose demand for some good depends on the amount already demanded by other consumers, that is, the level of realized network effect. We will also study to investigate which market structure is more effeective under this type of pricing scheme. Our main findings are: first, the price discrimination with lower introductory price is possible only when the network effect is beyond a certain level. Second a monopoly firm is more aggressive than an oligopoly firm in practising price discrimination, and monopoly is more effeective than oligopoly in terms of total output produced.

The model is kept as simple as possible to focus most clearly on the effects of network externalities and the basic assumptions is as follows:
(1) There are two time periods, t=1,2.
(2) Each consumer buys one unit of product in his life time if his reservation price is lower than a market price. And we assume that a discount rate is zero for simplicity.
(3) A seller commits that a price at t=1 (P_1) is not greater than a price at t=2 (P_2), which gets rid of waiting problem caused by consumers.\(^1\)
(4) The reservation price at t=1 is uniformly distributed over [0,1] and is shifted up at t=2 if the amount purchased at t=1 is positive. (Note that we assume there is no existing demand at the beginning of t=1.
(5) The consumers are homogeneous in the degree of shiftness of reservation price caused by network effect.

\(^1\) Consumer may concern about the expected future value of good, which will affect the decision at t=1 (see Katz and Shapiro, 1986). However, we assume that consumers make a decision based on the current value of product at purchasing point since it is hard for him to obtain an information about final sale, and the seller is assumed to commit that P_1 is lower than P_2. Actually the commitement of lower P_1 may induce at t=1, the competition among the consumers whose reservation price is lower than P_1. This contrasts with assumption 2. In this paper we assume that consumer has no information about firm’s sale strategy over two period.
(6) The marginal consumer at \( t=1 \) is decided by

\[
P = 1 - N ; \quad N \in [0, 1]
\]

where \( P \) is the seller's price, and \( N \) is the \( N^{th} \) ranked reservation price. Hence, \( N (=1-P) \) is the number of consumers who will buy a product at price \( P \) at initial stage \( (t=1) \).

If the realized demand at \( t=1 \) is \( N_1 \), the marginal consumer at \( t=2 \) is assumed to be decided by

\[
P = (1 + bN_1) - N
\]

where \( b \) is constant and positive. That is, the reservation price of each consumer is shift up by \( bN_1 \) at beginning of period 2 regardless of the size of original reservation price in this analysis.

### II. Analysis

Based on the demand curve described above, we, first, analyse the pricing strategy of monopoly firm. For simplicity, fixed cost is assumed to be zero and marginal cost constant with unit cost \( c \) which is less than \( 1 \) because a maximum reservation price at \( t=1 \) is less than \( 1 \) by assumption (6). Then the first period profit becomes as follow:

\[
\Pi_1 = (P_1-c)N_1 = (P_1-c)(1-P_1)
\]

where \( P_1 < 1 \)

If the price of first period is \( P_1 \), that is, \( N_1 (=1-P_1) \) is sold at \( t=1 \), the inverse demand of second period becomes by assumption (6)

\[
P = (1+bN_1) - N = (1+b(1-P_1)) - N. \quad \text{(or } N = 1+b(1-P_1)-P)\]

However, the net demand for the second period is

\[
N = 1 + b(1-P_1) - P - (1-P_1)
\]

since the consumers whose reservation price is over \([ P_1, 1] \) already
purchased a good at $t=1$ by assumption 2. For the graphical exposition see Figure 1 below: the solid line on Figure 1 represents the net demand curve for the second period.

![Figure 1](image)

Therefore, the second period profit is

$$\Pi_2 = (P_2 - c)(1 + b(1 - P_1)) - (1 - P_1).$$

Hence, total profit $\Pi$ is

$$\Pi = \Pi_1 + \Pi_2 = (P_1 - c)(1 - P_1) + (P_2 - c)(1 + (b - 1)(1 - P_1) - P_2). \quad (4)$$

Thus the optimal prices for the monopoly are decided by

$$\text{Max } \Pi \quad \Pi_1, P_2$$

The optimization problem above gives the solutions for optimal prices and quantities as follows:

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2) Maximization problem should be executed in sequence rather than simultaneously: maximize $\Pi_2$ w.r.t. $P_2$ at given $P_1$ and substitute $P_2^*$ into $\Pi$, and take a derivative w.r.t. $P_1$. However, both methods yield same result. Of course, we will follow a sequence method in the case of oligopoly.
\[ P_1 = \frac{2+c-b}{3-b} \quad P_2 = \frac{1+2c-bc}{3-b} \]

\[ N_1 = \frac{1-c}{3-b} \quad N_2 = \frac{1-c}{3-b} \]

These results show that \( P_1(P_2) \) decreases (increases) as \( b \) increases. And \( N_1 \) is always equal to \( N_2 \). This finding says that a monopoly seller keep an output constant but charge different prices across periods. However, note that \( b \) should be greater than 1 to satisfy the assumption (3) that \( P_1 \) should be less than \( P_2 \). Furthermore, \( b \) should be less than 3 for the positive output \( N \). These restrictions for \( b \) decide the range for the feasible price discrimination as follows:

\[ 1 < b < 3 \]

Thus, if a network effect is weak such that \( b \) is less than 1, a monopoly firm has to practice uniform pricing with considering network effect. The profit function for uniform price is obtained form (4) as follow:

\[ \Pi = \Pi_1 + \Pi_2 = (P-c)(1-P) + (P-c)(1+(b-1)(1-P)-P) = (1+b)(P-c)(1-P) \]

First derivative of this profit function with respect to \( P \) gives optimal price,

\[ P = \frac{1+c}{2}, \quad \text{that is, } P_1=P_2=\frac{1+c}{2}. \]

Hence, the prices for a monopoly is

\[ P_1=P_2=\frac{1+c}{2}, \quad \text{and } N_1=N_2=\frac{1+c}{2} \quad \text{if } 0 < b < 1 \]

\[ P_1=\frac{2+c-b}{3-b}, \quad P_2=\frac{1+2c-bc}{3-b} \quad \text{and } N_1=N_2=\frac{1-c}{3-b} \quad \text{if } 1< b < 3 \]

Based on the results above, we establish Table 1 to see the compound effect of \( b \) and \( c \) on prices and outputs in detail. To see the results more clearly, we establish Table 1 without considering the condition of \( b \) required for \( P_1 < P_2 \).
Table 1

<table>
<thead>
<tr>
<th></th>
<th>b=0</th>
<th>b=1</th>
<th>b=3/2</th>
<th>b=2</th>
<th>b=5/2</th>
</tr>
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<tbody>
<tr>
<td>P_1</td>
<td>(2+c)/3</td>
<td>(1+c)/2</td>
<td>(1+2c)/3</td>
<td>c</td>
<td>2c-1</td>
</tr>
<tr>
<td>P_2</td>
<td>(1+2c)/3</td>
<td>(1+c)/2</td>
<td>(2+c)/3</td>
<td>1</td>
<td>2-c</td>
</tr>
<tr>
<td>N_1</td>
<td>(1-c)/3</td>
<td>(1-c)/2</td>
<td>2(1-c)/3</td>
<td>1-c</td>
<td>2(1-c)</td>
</tr>
<tr>
<td>N_2</td>
<td>(1-c)/3</td>
<td>(1-c)/2</td>
<td>2(1-c)/3</td>
<td>1-c</td>
<td>2(1-c)</td>
</tr>
<tr>
<td>N_1+N_2</td>
<td>2(1-c)/3</td>
<td>1-c</td>
<td>4(1-c)/3</td>
<td>2(1-c)</td>
<td>4(1-c)</td>
</tr>
<tr>
<td>Π</td>
<td>(1-c)^2/3</td>
<td>(1-c)^2/2</td>
<td>2(1-c)^2/3</td>
<td>(1-c)^2</td>
<td>2(1-c)^2</td>
</tr>
</tbody>
</table>

As shown on Table 1, constant marginal cost c does make a role at given value of b in analysing the impact of network effect (a size of b) on price. Utilizing Table 1, we can find proposition as follow:

Proposition 1: For the monopoly, the degree of price difference between P_1 and P_2 becomes larger as c decreases while b increases.

Since a higher marginal cost decreases an output, a monopoly gets a lower degree of freedom in utilizing network effect through the amount of output at period one. On the other hands, a higher value of b (that is, higher network effect) gives a monopoly a higher degree of freedom for the intertemporal price discrimination.

For a simplicity, let us assume that c is zero and compare the case with c being not zero, which allows us to analyse the further role of c and b in investigating a pricing pattern of monopoly. Prices and outputs with c=0 are as follows:

\[
P_1 = (2-b)/(3-b) \quad P_2 = 1/(3-b)
\]
\[
N_1 = 1/(3-b) \quad N_2 = 1/(3-b)
\]

Based on these prices, we can find:

1. If b=0, that is, there is no network effect, P_1=2/3, P_2=1/3 (and N_1=1/3, N_2=1/3). This is a traditional result of two period price discrimination.
2. If b=1, P_1=P_2=1/2. Hence, there exist no price discrimination.
3. If b=2, P_1=0, and P_2=1. This case satisfies the assumption (3) that P_1
should be greater than $P_2$. But free disposal occurs at period one: note that $P_1$ becomes marginal cost (not free disposal) if $c$ is not zero (see Table 1). In other words a monopoly will seek zero profit at period one if network effect is greater such as $b=2$

(4) If $b=5/2$, $P_1=-1$ and $P_2=2$.

These result shows that if $b$ is greater than 1, $P_1$ becomes less than $P_2$ and negative price with $c=0$ but positive price with $1/2<c<1$ (see Table 1). And Table 1 shows that price difference between two periods becomes greater as the size of $c$ decreases and the value of $b$ moves away from 1.

In sum, a seller will not practise price discrimination if a network effect is not great such that $b$ is between 0 and 1. But a seller will charge a different price over two periods if a network effect is large such that $b$ is between 1 and 3. And the degree of price discrimination becomes deeper as marginal cost is smaller. Again the negative relationship between degree of price discrimination and size of marginal cost can be explained such that the smaller output caused by a higher marginal cost itself decreases a network effect which highly depends on the size of output. This reduced network effect decreases difference of prices. The pricing patterns reflecting the result described so far is graphically shown on Figure 2.
Now let us move into the case in which two firms compete each other and compares with the case of monopoly. For a simplicity, we consider two Cournot firms which produce identical good and compete for quantity over two periods. Here each firm is assumed to compete for quantity only and market decides a price. Consumers buy one unit of product at their life time as long as their reservation price is less than market price with comparing two prices $P_1$, $P_2$. Hence each firm should decide its output for each period considering a rival firm’s output and expected resulting market price for each period to avoid the waiting problem of consumers. Like the monopoly case we, at first, derive a profit maximizing output and check out whether the resulting equilibrium market prices satisfy the condition that $P_1$ should be less than $P_2$. Then we analyse the condition under which the price discrimination is possible avoiding the waiting problem of consumers. This is different from our basic model which considers the waiting problem of consumer from the beginning. However this does not matter because this is a matter of methodology which does not change the result at a great level ex post and our main point is to compare the pricing pattern with monopoly when a network effect is most important.

Since $N^i = N^i_1 + N^i_2$ (where superscript $i$ represents period ($i=1,2$) and subscripts 1,2 mean firm 1 and firm 2 respectively, that is, $N^1_2$ = the output of firm 2 at period 1), the demand curve for each period is

$$P^1 = 1 - N^1_1 - N^1_2 \quad \text{for period 1}$$ (5)

and

$$P^2 = 1 + b \left( N^1_1 + N^1_2 \right) - (N^1_1 + N^1_2) - (N^2_1 + N^2_2) = 1 + \left( b - 1 \right) \left( N^1_1 + N^1_2 \right) - (N^1_1 + N^2_2) \quad \text{for period 2}$$ (6)

since the consumers whose reservation price is over $[N^1_1 + N^1_2, 1]$ already obtain a good at period 1. Then the profits for firm 1 and firm 2 are
\[ \Pi_1 = \Pi_1^1 + \Pi_1^2 = (P^1 - c) N_1^1 + (P^2 - c) N_1^2 \\
= (1 - N_1^1 - N_2^1 - c) N_1^1 + [1 + (b - 1)(N_1^1 + N_2^1) - (N_1^2 + N_2^2) - c] N_1^2 \]  
\] (7)

\[ \Pi_2 = \Pi_2^1 + \Pi_2^2 = (P^1 - c) N_2^1 + (P^2 - c) N_2^2 \\
= (1 - N_1^1 - N_2^2 - c) N_2^1 + [1 + (b - 1)(N_1^2 + N_2^2) - (N_1^1 + N_2^2) - c] N_2^2 \]  
\] (8)

For the subgame perfect Nash equilibrium, we assume at first that each firm decides optimal outputs for period 2 at given outputs of period 1 with basic Cournot assumption that rival firm’s output is given. The conditions for optimal outputs for period 2 are

\[ \frac{\partial \Pi}{\partial N_1^2} = -N_1^2 + 1 + (b-1)(N_1^1 + N_2^1) - (N_1^1 + N_2^2) - c = 0 \]  
\] (9)

\[ \frac{\partial \Pi}{\partial N_2^2} = -N_2^2 + 1 + (b-1)(N_1^2 + N_2^1) - (N_1^2 + N_2^2) - c = 0 \]  
\] (10)

Thus, from (9) and (10)

\[ N_1^2 = \frac{1}{2} [1 - N_2^2 - c + (b-1)(N_1^1 + N_2^1)] \]  
\] (9')

\[ N_2^2 = \frac{1}{2} [1 - N_1^2 - c + (b-1)(N_1^1 + N_2^1)] \]  
\] (10')

From (9)' and (10)' , we obtain \((N_1^2)’\) and \((N_2^2)’\), which are the equilibrium outputs for period 2 at given outputs of period 1, as follow:

\[ (N_1^2)' = \frac{1}{3} + \frac{1}{3} (b-1)(N_1^1 + N_2^1) - \frac{1}{3} c \]  
\] (11)

\[ (N_2^2)' = \frac{1}{3} + \frac{1}{3} (b-1)(N_1^2 + N_2^1) - \frac{1}{3} c . \]  
\] (12)

Now substitute \((N_1^2)’\) and \((N_2^2)’\) into total \(\Pi\) functions, (9), (10) and take derivative w.r.t. \(N_1^1\) and \(N_2^1\) respectively to obtain optimal outputs for period
one (actually reaction functions at period 1), \((N_1^1)\)' and \((N_2^1)\)' , which is subgame perfect reaction function. The procedure is as follows:

from (7), (11), and (12),

\[
\Pi_1 = (1 - N_1^1 - N_2^1 - c) N_1^1 + \frac{1}{9} (1 + (b - 1)(N_1^1 + N_2^1) - c)^2
\]

and (8), (11), and (12)

\[
\Pi_2 = (1 - N_1^1 - N_2^1 - c) N_2^1 + \frac{1}{9} (1 + (b - 1)(N_1^1 + N_2^1) - c)^2
\]

the profit functions above consist of outputs of period one only. Now we take a first derivative of these functions w.r.t \(N_1^1\) and \(N_2^1\) and let them zero to obtain subgame perfect reaction functions for period one:

\[
\frac{\partial \Pi_1}{\partial N_1^1} = 0 \text{ yields } N_1^1 = \frac{(1 - c)(1 + \frac{2}{9}(b - 1))}{2\left(1 - \frac{1}{9}(b - 1)^2\right)} - \frac{1 - \frac{2}{9}(b - 1)^2}{2\left(1 - \frac{1}{9}(b - 1)^2\right)} N_2^1 \quad (A)
\]

\[
\frac{\partial \Pi_1}{\partial N_2^1} = 0 \text{ yields } N_2^1 = \frac{(1 - c)(1 + \frac{2}{9}(b - 1))}{2\left(1 - \frac{1}{9}(b - 1)^2\right)} - \frac{1 - \frac{2}{9}(b - 1)^2}{2\left(1 - \frac{1}{9}(b - 1)^2\right)} N_1^1 \quad (B)
\]

These are reaction functions of period 1 with considering the impact of \(N_1^1\) and \(N_2^1\) on \(N_1^2\) and \(N_2^2\), which bring about the final solution of outputs of first period.

From A and B,

\[
(N_1^1)^* = \frac{(1 - c)(1 + \frac{2}{9}(b - 1))}{3 - \frac{4}{9}(b - 1)^2}
\]

\[
(N_2^1)^* = \frac{(1 - c)(1 + \frac{2}{9}(b - 1))}{3 - \frac{4}{9}(b - 1)^2}
\]

- 72 -
These are optimal outputs of each firm for period one. To obtain optimal outputs for period 2, substitute (13) and (14) into (11) and (12).

Then,

\[
(N_1^*) = \frac{1}{3} + \frac{2}{3} (b-1) (N_1^*) \quad (11)'
\]

\[
(N_2^*) = \frac{1}{3} + \frac{2}{3} (b-1) (N_2^*) \quad (12)'
\]

That is, the final solutions of outputs for each period are

\[
(N_1^*) = (N_2^*) = \frac{(1-c)(1+\frac{2}{3} (b-1))}{3-\frac{4}{3} (b-1)^2}
\]

\[
(N_1^*) = (N_2^*) = \frac{1}{3} + \frac{2}{3} (b-1) (N_1^*)
\]

By substituting these outputs above into demand function, we obtain final prices.

\[
P_1 = 1 - 2(N_1^*) = \frac{27 - 4 (b-1)^2 - 2(1-c)(7+2b)}{27 - 4 (b-1)^2}
\]

\[
P_2 = 1 + (b-1)2(N_1^*) - 2(N_2^*)
\]

From (13), (14), (11)', (12)', \Pi and demand curve, we can establish Table 2 below.

<table>
<thead>
<tr>
<th></th>
<th>(b=0)</th>
<th>(b=1)</th>
<th>(b=3/2)</th>
<th>(b=2)</th>
<th>(b=5/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_1^*)</td>
<td>(\frac{7}{23} (1-c))</td>
<td>(\frac{1}{3} (1-c))</td>
<td>(\frac{5}{13} (1-c))</td>
<td>(\frac{11}{23} (1-c))</td>
<td>(\frac{2}{3} (1-c))</td>
</tr>
<tr>
<td>(N_2^*)</td>
<td>(\frac{7}{23} (1-c))</td>
<td>(\frac{1}{3} (1-c))</td>
<td>(\frac{5}{13} (1-c))</td>
<td>(\frac{11}{23} (1-c))</td>
<td>(\frac{2}{3} (1-c))</td>
</tr>
<tr>
<td>(N_1^*)</td>
<td>(\frac{23 - 14 (1-c)}{69})</td>
<td>(\frac{1}{3} (1-c))</td>
<td>(\frac{13 + 5 (1-c)}{39})</td>
<td>(\frac{23 + 22 (1-c)}{69})</td>
<td>(1 + 2 (1-c))</td>
</tr>
<tr>
<td>(N_2^*)</td>
<td>(\frac{23 - 14 (1-c)}{69})</td>
<td>(\frac{1}{3} (1-c))</td>
<td>(\frac{13 + 5 (1-c)}{39})</td>
<td>(\frac{23 + 22 (1-c)}{69})</td>
<td>(1 + 2 (1-c))</td>
</tr>
<tr>
<td>(P_1)</td>
<td>(\frac{23 - 14 (1-c)}{23})</td>
<td>(\frac{1}{3} (1+2c))</td>
<td>(\frac{13 - 10 (1-c)}{13})</td>
<td>(\frac{23 - 22 (1-c)}{23})</td>
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<td>(\frac{23 - 14 (1-c)}{69})</td>
<td>(\frac{1}{3} (1+2c))</td>
<td>(\frac{13 + 5 (1-c)}{39})</td>
<td>(\frac{23 + 22 (1-c)}{69})</td>
<td>(1 + 2 (1-c))</td>
</tr>
<tr>
<td>(N_1 + N_2)</td>
<td>(\frac{46 + 14 (1-c)}{69})</td>
<td>(\frac{4}{3} (1-c))</td>
<td>(\frac{26 + 40 (1-c)}{39})</td>
<td>(\frac{46 + 110 (1-c)}{69})</td>
<td>(2 + 8 (1-c))</td>
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- 73 -
As shown on Table 2, \( P_1 \) is greater (less) than \( P_2 \) when \( b \) is less (greater) than 1, which implies that price discrimination is possible with greater network effect \((b>1)\) if consumer rationally anticipate the price of period 2, that is, adopt waiting strategy as indicated by assumption 3. Note that this is also valid in case of monopoly. After all, we can conclude that price discrimination is impossible regardless of type of market structure if \( b \) is less than 1, that is, network effect is smaller. And the price difference between two periods become larger as marginal cost \( c \) decreases while network effect \((=b)\) increases, which is same as the case of monopoly.

Comparing with Table 2 with Table 1 shows that

1. As \( b \) increases, the price difference between two periods is greater in case of monopoly than in case of oligopoly. This implies that a monopoly is more aggressive than oligopoly.
2. Each oligopoly produce less at period one than at period 2 if \( b \) is greater than 1, which is different from the case of monopoly keeping output constant over two periods. This phenomenon arises due to the fact that each oligopoly firm competes for output but tries to get the advantage of network effect caused by the rival firm’s output if \( b \) is greater than 1, that is, network effect is greater. So, the optimal strategy is to produce less in relative sense but hope for the rival to produce more at period 1. This strategy becomes stronger as the size of \( b \) increases. This type of strategy of oligopoly brings about the result that the total output under the oligopoly is less than that under the monopoly if \( b \) (network effect) is great such that \( b \) is equal to 5/2 and marginal cost \( c \) is less than \( 1/2^3 \).

Proposition 2: The total output under the oligopoly is less than that under the monopoly is greater if network effect is greater and marginal cost is low.

Hence, an oligopoly is less effective than a monopoly in terms of total

3) Note that if marginal cost \( c \) is less than 1/2, \( P_1 \) of monopoly is negative. We may find a negative price in real world, for example, a free sample with additional gift.
output produced if network effect is greater. This is greatly different from the results obtained under the traditional model without network effect.

III. Conclusion

Based on a simple linear demand curves, we investigate the pricing patterns under the different market structure when network effect prevails. Our main finding is that price discrimination across periods is possible only when the degree of network effect is beyond a certain level due to the waiting strategy of consumers. The degree of price difference across periods becomes greater as network effect is greater while marginal cost is lower.

And we find that monopoly is more aggressive in practicing price discrimination than oligopoly. A monopoly firm adopt even a negative price when network effect is highly great. However, oligopoly is less aggressive because each oligopoly tries to strategically utilize the rival firm’s output for the next period. This characteristics of oligopoly firm yield lower total output than monopoly when the network effect is great. This is different from the results of traditional studies.

Reference


E.P.Lazear, “Retail Pricing and Clearance Sales”, AER, Vol 76.(1986), 14-32