I. Introduction

The upstream monopolist (UM) of intermediate product has an incentive to integrate forward into downstream industries (DIs) as a means of achieving implicit third-degree price discrimination.\(^1\) While raising the market price of the input to non-integrated downstream industries, the upstream monopolist can implicitly lower the internal price to the integrated downstream industries by expanding production of its final good. This type of problem has been studied under the situations in which DIs are perfectly competitive and, in particular, are characterized by free entry. This assumption constrains the set of DIs into which the UM can integrate and successfully practise implicit price discrimination to those DIs with relatively elastic derived demands. Otherwise, the practice of price discrimination through integration would call for decreased input prices for non-integrated firms. This would allow entry to the newly integrated sector and would eliminate any gains from forward integration associated with the attempt to practise implicit price discrimination. Moreover, in the

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cases where the UM practises implicit price discrimination through forward integration, the effect on social welfare is ambiguous in general.\textsuperscript{2)}

As well known, the UM has an incentive to integrate vertically with a downstream monopolist (DM).\textsuperscript{3)} This type of integration increases joint profits by eliminating the inefficiency associated with successive monopolies. Since the optimal prices selected for the newly integrated DI falls as a direct consequence of the integration, the social welfare effect is definitely positive.

Recently, it is also examined vertical integration when the UM serves a set of monopolized DIs.\textsuperscript{4)} In this case, the UM has an dual incentive to integrate forward. There are gains to be made through the elimination of successive monopoly and through the practice of implicit price discrimination is can integrate and successfully practise implicit price discrimination is un constrained. And the impacts on social welfare depend on which downstream firm the UM will integrate with. If the UM integrate with a DM possessing a relatively inelastic demand for the UM’s product, final output prices fall in both the integrated and non-integrated sectors. In this case, there is a Pareto improvement. However, if the UM integrates with a DM possessing a relatively elastic demand for the UM’s product, then out price falls in the integrated sector but output price rises in the non-integrated firm. Here, the effect on social welfare is ambiguous. Hence, the welfare results of this type of model show that preventing vertical integration might be a costly mistake when DIs are monopolized; therefore this type of model has important implications for anti-trust law.

The present paper will examines the welfare effect of vertical integration when the UM serves two separated downstream industries where one DI is monopoly and the other DI is oligopoly. As the case of set of monopolized DIs, UM also has a dual incentive to integrate forward. there are gains to be made through the practice of implicit price discrimination and through the elimination of imperfect market distortion caused by monopoly or oligopoly market structure. Comparing with the

\textsuperscript{2)} See Schmalensee (1981).
\textsuperscript{3)} See McChlup and Taber (1960) and Warren-Boulton (1977).
\textsuperscript{4)} See Romano (1988) and Lee (1992).
previous case of a set of monopolized DIs, the only difference is that one of downstream industries is not monopoly but oligopoly type. The results show that the impact on social welfare depends on which downstream firm (that is, market structure) the UM will integrate with. In particular, one of results shows that the third degree price discrimination through forward integration with downstream oligopoly rather than monopoly brings about the greater welfare improvement and even Pareto improvement at a certain condition. This is a new finding against the concept of previous studies that forward integration with monopoly rather than oligopoly can eliminate the greater amount of market distortion to bring about welfare improvement and even Pareto improvement since monopoly is less competitive than oligopoly. Thus, the results of this paper has an implication for anti-trust policy since price discrimination associated with vertical integration has been a concern in anti-trust proceedings. And this paper shows that the type of market structure of downstream industry becomes an important factor in analysing the effect of vertical integration if the type of market structure of DIs is imperfectly competitive and heterogeneous.

II. The Model

A UM produces good \( z \) at constant average (and marginal) cost \( c \). Good \( z \) is an input in two DIs. The two DIs have different market structure: the one is a monopolized downstream industry (\( \text{DI}_m \)) and the other is an oligopoly downstream industry (\( \text{DI}_o \)) consisted of \( n \) identical firms. These DIs produce an homogeneous good \( x \). The industry demand for good \( x \) at each DI is identical and is denoted \( x(p) \) (with inverse demand \( p(x) \)); and \( p'(x) < 0 \). Here we assume that \( n \) identical downstream firms are Cournot type oligopolist in retailing good \( x \) and price takers in obtaining input \( z \). For simplicity, assume that two DIs are completely separated and that a resale is impossible. To focus the analysis, assume that \( z \) is used in fixed proportion to produce \( x \), thereby eliminating the
incentive to integrate deriving from downstream technologies having variable proportions.\textsuperscript{5)} Also, to focus the analysis, assume that the supplies of other inputs used in the production of \( x \) are perfectly elastic, thus avoiding monopsony incentives to integrate.\textsuperscript{6)} These assumptions permit us to employ the following production function: \( x = az_i (i = m, o) \) where \( z_i \) is the employment of input \( z \) in \( DI_i \). That is, both downstream industries produce an identical good \( x \) at identical production technology.

Since one of \( DI \)s is assumed to be monopolized in this analysis and total revenue (\( TR \)) of \( x \) is \( p \cdot x = p(az_i)az_i \), the marginal revenue of \( x \) for \( DI_m \) is

\[ (1) \quad \frac{dTR}{dx} = p'(x) \cdot x + p(x) = g_m(x) = g_m(az_m) \]

And the marginal revenue of \( x \) in terms of \( z_m \) \( (=dTR/dz_m) \) is

\[ (2) \quad \frac{dTR}{dz_m} = a(p'(az_m)az_m + p(az_m)) = ag_m(az_m) = ag_m(x) \]

Note that the marginal revenue of \( x \) in terms of \( z_m \) is obtained by multiplying \( a \) to the marginal revenue of \( x \). Hence, the inverse derived demand for \( z_m \) for \( DI_m \) is given by the marginal revenue of \( x \).

\[ (3) \quad w_m(z_m) = a(p'(az_m)az_m + p(az_m)) = g_m(az_m) = ag_m(x) \]

where \( w_m \) is the price of \( z_m \) for \( DI_m \).

On the other hand, since the other \( DI \) is the oligopoly which is consisted of \( n \) identical firms behaving as Cournot-type in retailing good \( x \), the inverse derived demand for \( z_o \) by \( DI_o \) is also given by marginal revenue of \( x \) as shown in the case of monopoly above. To obtain the marginal revenue of \( z_o \) at \( DI_o \), at first, it is required to derive the

\textsuperscript{5)} See Vernon Graham (1971), Greenhut and Ohta (1976, 1979), and Westfield (1981).
\textsuperscript{6)} See Gould (1979).
individual marginal revenue of $z_{ok}$ which is obtained by

\[(4) \ w_o(z_o) = a\{p'(az_o)az_{ok} + p(az_o)\} \text{ where } k=1,2,...,n\]

where $w_o$ is the price of $z_o$ for DI$_o$ and $z_{ok}$ is the amount of input $z$ demanded by individual firm $k$ in DI$_o$, and $z_o=\sum_{k=1}^{n}z_{ok}$ since market demand $z_o$ must equal with the market supply. And $w_o$ is the price of $z_o$ to DI$_o$, in other words, there exists perfect competition in purchasing input $z$ in DI$_o$.

Then, the total market demand for input $z_o$ in DI$_o$ involves summing

\[(4) \text{ for all } i=1,2,...,n.\]

This aggregation establishes

\[(5) \ w_o(z_o) = a\{p(az_o) + (1/n)\sum_{k=1}^{n}p'(az_o)az_{ok}\} \]

\[= a\{p(az_o) + (1/n)p'(az_o)az_o\} \]

\[= g_o(az_o) = ag_o(x)\]

where $g_o(x) (=g_o(az_o))$ and $ag_o(x)$ implies the marginal revenue of $x$ and $z$ respectively at DI$_o$. Equation (5) brings about the inverse derived demand $z_o$ for DI$_o$. Note that from equation (3) and (5), if $n$ is equal to one, $g_o$ becomes to $g_m$, that is, the equation (5) is identical to equation (3).

It is assumed that $w'_i < 0$ ($i=m,o$) implying $z_i(w_i) = w_i^{-1}(w_i)$ is defined. In addition, it is assumed that the profit to the UM from the DI$_i$, $\Pi_i(w_i) = z_i(w_i)(w_i - c)$ is concave in $w_i$ for $w_i > c$.

First, we can derive some preliminary results. A UM who can practise third-degree price discrimination solves

\[\text{Max } \Pi_i(w_i), \quad i = m, o \]

\[w_i\]

\[\text{7) The price of } x \text{ is expected to vary as each firm alters his supply while conjecturing that other sellers' supplies are fixed. The term } ap'(az_o)az_{ok} \text{ in equation (4) stems from this Cournot condition.}\]
The solution yields the discriminatory prices denoted by \( w_{id} \), satisfies

\[
\Pi_i'(w_{id}) = w_{id} \left[ 1 - \left\{ 1/\varepsilon_i(z_i(w_{id})) \right\} \right] -c = 0, \quad i = m, o
\]

where \( \varepsilon_i = -z_i' \cdot (w_i/z_i) \) is the elasticity of derived demand \( z \) for DI\( i \). Consider the more interesting case in which either \( w_{md} > w_{od} \) or \( w_{md} < w_{od} \). Then, using (6), we have the following relationship among the elasticities:

\[
(7-1) \quad \varepsilon_m(z_m(w_{md})) < \varepsilon_o(z_o(w_{od})) \quad \text{or,}
\]

\[
(7-2) \quad \varepsilon_m(z_m(w_{md})) > \varepsilon_o(z_o(w_{od})).^8
\]

A UM who cannot practise price discrimination solves

\[
\text{Max } [\Pi_m(w) + \Pi_o(w)]
\]

\[
w
\]

Denoting the solution value by \( w^* \), it satisfies

\[
(8) \quad \Pi_m'(w^*) + \Pi_o'(w^*) = w^*(1 - 1/\eta(w^*)) -c = 0
\]

where

\[
\eta(w^*) = \varepsilon_m(z_m(w^*))((z_m(w^*)/(z_m(w^*) + z_o(w^*))) + \varepsilon_o(z_o(w^*))
\]

\[
\{(z_o(w^*)/(z_m(w^*) + z_o(w^*))\},
\]

elasticity of aggregate derived demand, we assume that \( z_i(w^*) > 0 \) for \( i = m, o \), or that both DIIs employ some \( z \) at a price of \( w^* \).

Since \( \Pi_i'(w) > (\leq)0 \) for \( w < (>)w_{id} \) (from the concavity of \( \Pi_i \)), it follows from (6), (7), and (8) that

\[
(9-1) \quad w_{md} > w^* > w_{od}, \quad \text{or}
\]

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8) \( \varepsilon_m \) is greater or smaller than \( \varepsilon_o \), which depends on the type of demand curve \( p(x) \). This will be specified in next section.
(9-2) \( w_{md} < w* < w_{od} \) \(^9\)

Adopting Robinson's terminology of 'strong' and "weak" markets, I refer to the relatively inelastic demander of z as the strong DI and the relatively elastic demander of z as the weak DI. Hence, equation 9 suggests that either DI\(_m\) or DI\(_o\) could be a strong or an weak DI depending on the market conditions, which will be specified at next section.

Since the degree of \( \varepsilon_i \) decides the type of market (strong or weak), it is important for the evaluation of integration policy to define \( \varepsilon_i \) in analytical form. In other words, we have to define the conditions in which \( \varepsilon_m \) is more elastic or inelastic than \( \varepsilon_o \). These conditions will decide which market is strong (or weak). Since \( \varepsilon_i \) is the elasticity of derived demand for \( z_i \) by DI\(_i\), it is to be defined from equation (3) for \( \varepsilon_m \) and from the equation (5) for \( \varepsilon_o \). However, it is impossible to define \( \varepsilon_i \) in analytical form since equation (3) and (5) cannot be specifically defined in terms of \( w_i \), in other words, \( z_i(w_i) = w_i^{-1}(w_i) \) cannot be specified in analytical form. Hence, the elasticities in the function of \( x \), i.e. \( z_i \) (not \( w_i \)) is derived from equation (3) and (5), which will be used later to compare the size of input price elasticities indirectly.

From equation (3), the elasticity is obtained as follows,

\[
(10-1) \quad \varepsilon_m = -(dx/dw_m)(w_m/x) = -(dz_m/dw_m)(w_m/z_m) \quad \text{(since } x = az_m) \\
= - \left( \frac{p'(az_m)az_m + p}{(p''(az_m)az_m + 2p'(az_m))az_m} \right) \\
= - \frac{g_m(az_m)/g_m'(az_m)az_m}{(x)/g_m'(x)x}.
\]

From equation (5), the elasticity is obtained as follows

\(^9\) First note that (9-1) corresponds to (7-1). Then, since \( \Pi_i(w), i=1,2 \), is concave so too is \( \Pi_{m+o} \). For \( w > w_{md} \), \( \Pi',(w) < 0, i=m, o \), so that \( \Pi_{m+o} \) is declining at this point. Thus using (8), it follows that \( w^* \) cannot be greater than \( w_{md} \). Analogously, \( w^* \) cannot be less than \( w_{od} \). By using same way, we can explain the case of (9-2) through (7-1). We will show at next section that either (9-1) or (9-2) is possible depending on the type of basic demand curve of output x.
(10-2) $\varepsilon_o = -(dx/dw_o)(w_o/x) = -(dz/dw_o)(w_o/z_o)$ (since $x=az_o$)

$$= -((1/n)p'(az_o)az_o + p)/ [p'(az_o) + (1/n)(p''(az_o)az_o + p'(az_o))]]az_o$$

$$= - g_o(az_o)/g_o'(az_o)az_o = - g_o(x)/g_o'(x)x.$$  

Equation (10) is the elasticity $\varepsilon_i$ in terms of $x$. From the equation (10), we know that at given $x$, $\varepsilon_o$ equals to $\varepsilon_m$ when the number of firm $n$ is equal to one, which implies the special case of $\varepsilon_o$. Hence, to compare the size of $\varepsilon_i$, we simply decide the sign of $\partial \varepsilon_o/\partial n$, and apply $n=1$ for the $\varepsilon_m$.

At first, let us figure out the sign of $(\partial \varepsilon_o/\partial n)$ at given $x(=az)$, which is, $\partial \varepsilon_o/\partial n = (\partial/\partial n)(g_o(x)/g_o'(x)x) = (x/n)(g_o'(g_o-p) - g_o(g_o-p'))/(g_o'x)^2$, then,

(11) $\partial \varepsilon_o/\partial n \cong 0$ as $g_o'/g_o \cong (g_o'-p')/(g_o-p)$.

Since $g_o(x) = p + (p'/n)x$, the relationship (11) can be also written in the functional form of the final output demand $p(x)$. The relationship (11) shows that $\varepsilon_m$ is greater or smaller than $\varepsilon_o$ at given $x$ (or $z_o$), which depends on the shape of basic demand curve $p(x)$ and its marginal revenue curve. It is useful to explain graphically the results of elasticity with respect to the number of firms.

Figure F-1
Note that on figure (F-1), \( g_o(x) \) exists above \( g_m(x) \) since \( n \) is greater than one. Based on (11), figure (F-1) shows that if \( n \) is greater than one \( \varepsilon^a \equiv \varepsilon^b \) at given \( z \) as \( g_o/g_o \equiv (g_o'p)/(g_o-p) \). However, these results is derived at given output \( z (=x/a) \), not at given input price \( w_i \). Since the analysis of discriminatory input price policy requires to compare \( \varepsilon_o \) with \( \varepsilon_m \) at given input price \( w_i \), we have to decide the sign of \( \partial \varepsilon_i/\partial w \). As we did in equation (10), since \( \varepsilon_i \) is defined in the function of \( z \), it is impossible to decide the sign of \( \partial \varepsilon_i/\partial w \) directly. However, \( w \) and \( z \) move in opposite direction along the derived input demand curve since we assume that \( w_i' < 0 \), which indicates that the sign of \( \partial \varepsilon_i/\partial w \) is opposite to the sign of \( \partial \varepsilon_i/\partial z \). Now let us defined the value of \( \partial \varepsilon_i/\partial z \) from equation (10-2)

\[
\partial \varepsilon_i/\partial z = -a((g_o')^2a_o - g_o(g_o''a_o + g_o')) / [g_o'z_o]^2
\]

Thus,

\[
(12) \quad \partial \varepsilon_i/\partial z \equiv 0 \text{ as } g_o'/g_o \equiv [(g_o''a_o + g_o')/g_o'a_o] = (g_o''x + g_o')/g_o'x
\]

Note that equation (12) can also be specified in the functional form of \( p(x) \) since \( g(x) = p + (p'/x)x \). As we mentioned, since the sign of \( \partial \varepsilon_i/\partial w_i \) is opposite to the sign of \( \partial \varepsilon_i/\partial z \), we can specify the sign of \( \partial \varepsilon_i/\partial w_i \) from equation (12) as follows

\[
(13) \quad \partial \varepsilon_i/\partial w_i \equiv 0 \text{ as } g_o'/g_o \equiv [(g_o''a_o + g_o')/g_o'a_o] = (g_o''x + g_o')/g_o'x
\]

Now utilizing the equations (11), (12), and (13), we may classify four different cases in which the sizes of \( \varepsilon_m \) and \( \varepsilon_o \) can be compared.
In the case of case 1 and case 4, it is not decided which market has a more or less elastic derived input demand curve, in other words, which market is weak (or strong). This condition says that $\varepsilon^b$ is greater or smaller than $\varepsilon^c$ on Figure-1. However, in case of 3, $\varepsilon^c$ is greater than $\varepsilon^b$ on Figure-1. This implies that the elasticity of derived input demand curve at given input price for oligopoly downstream industry(DL_o) is more elastic than for monopolized downstream industry(DL_m). This situation indicates that DL_o is an weak market and DL_m is a strong market. On the other hands, in case of 2, $\varepsilon^b$ is greater than $\varepsilon^c$ on Figure-1, which is completely opposite to the case 3. Thus, DL_o is a strong market and DL_m is an weak market. This result is a rather extra-ordinary thing in the sense of traditional theory since the input demand curve under the oligopoly seems to be more elastic than that under the monopoly.\(^{10}\)

III. The Analysis

Assuming that the UM cannot distinguish the DIs or prevent arbitrage, he must supply both DIs at same price. By integrating forward with either DI, the UM acquires a monopoly in that DI, so that he can supply the non-integrated DI at any price without fear of entry into the newly integrated industry. The optimal price to the non-integrated DI is the

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\(^{10}\) Perry(1978) argues this type of result by comparing monopoly and competitive market. However, he did not specify the condition in detail.
appropriate discriminatory price. In contrast to the case already discussed in the literature with competitive DIs, the set of industries into which the UM can profitably integrate is unconstrained and the supply price of $z$ following integration can fall at the UM’s optimum.

After integration takes place, a strategy of charging the appropriate discriminatory price to the non-integrated DI and internally transferring $z$ in the integrated industry at the pre-integration price ($w$) would make integration profitable. The UM can further increase profits, however, by internally transferring $z$ in the integrated industry at the marginal cost. Hence, there is a twofold incentive to integrate, where the second part of the gain from integration comes from eliminating the successive monopoly (or market imperfection) effect in the integrated industry and is not present in the alternative case on which the downstream industries are competitive.

Integration with either DI exhausts all the gains from price discrimination since only one independent DI remains. There is an incentive to integrate with the last DI to resolve the successive-monopoly (or imperfect market) distortion. Then if the act of integration is costless and not prohibited, the UM optimally integrates with both DIs. However, integration may not be costless. Moreover, there may be differential cost associated with integration into each industry. A list of sources of integration includes anything that reduces the profits obtainable in a frictionless world from vertical integration. Some examples of these costs are any legal costs necessitated by vertical integration, any cost of bargaining for the eventual split of increased profits, and anticipated costs from possible anti-trust prosecution following integration. On the other hand, even if the integration cost is negligible, the authority may allow the UM to integrate with only one of the DIs because of a desire to maintain certain facets of an anti-trust policy.

As long as there exist any conditions which prohibit the integration with both DIs, the analysis of integration with only one DI is directly relevant. Then questions arise as to which DI should be selected for
integration by the UM. As far as social welfare is concerned, the result depends largely on the type of consumer demand that holds for the final product and the type of market structure of downstream industries.

IV. The Results

Consider the case of integrating with only one industry among two downstream industries (monopoly downstream industry and oligopoly downstream industry). Here, we only consider the case 3 and the case 4 on Table-1 since the market conditions (strong or weak) cannot be definitely specified at the case 1 and the case 2 on Table-1.

Result 1: If the basic demand curve of output \( x \) and its marginal revenue curve satisfies the condition \( (g_o'' x + g_o')/(g_o' x) > (g_o'/g_o) > (g_o' - p')/(g_o - p) \), which is the case 3 on Table-1, integration by the UM with the monopolized downstream industry (the oligopolized downstream industry) leads to a lower (higher) supply price of \( z \). The internal transfer price of \( z \) in the newly integrated industry falls to \( c \), the marginal cost of input.

Proof of result 1. Under the condition \( (g_o'' x + g_o')/(g_o' x) > (g_o'/g_o) > (g_o' - p')/(g_o - p) \), the oligopoly downstream industry (\( DI_o \)) is an weak market and the monopolized downstream industry (\( DI_m \)) is a strong market as shown in previous section. Following integration with the monopolized downstream industry (the oligopoly downstream industry), the optimal supply price of \( z \) equals the discriminatory price \( w_{\text{ml}} \) (\( w_{\text{ml}} \)). The first statement then follows immediately from (7-1) and (9-1). After integration, it is definitely optimal to transfer \( z \) in the newly integrated firm at the marginal cost \( c \). It follows that the internal price 'falls' after integration since \( w_{\text{ml}} > c \), i.e., \( o \) Q.E.D.

Result 2: If the basic demand curve of output \( x \) and its marginal
revenue curve satisfies the condition \((g_o' - p')/(g_o - p) > (g_o'/g_o) > (g_o'' x + g_o')(g_o'x)\), which is the case 2 on Table-1, integration by the UM with the oligopoly downstream industry (The monopolized downstream industry) leads to a lower (higher) supply price of \(z\). The internal transfer price of \(z\) in the newly integrated industry falls to \(c\), the marginal cost of input.

Proof of result 2. Under the condition \((g_o' - p')/(g_o - p) > (g_o'/g_o) > (g_o'' x + g_o')(g_o'x)\), the monopolized downstream industry (DI\(_m\)) is an weak market and the oligopolized downstream industry (DI\(_o\)) is a strong market as shown in previous section. Following integration with the DI\(_o\) (DI\(_m\)), the optimal supply price of \(z\) equals the discriminatory price \(w_{md}\). The first statement then follows immediately from (7-2) and (9-2). After integration, it is definitely optimal to transfer \(z\) in the newly integrated firm at the marginal cost \(c\). It follows that the internal price 'fall' after integration since \(w_{ad} > c\), i=m, o. Q.E.D.

The intuition of results above is straightforward. The strong(weak) DI puts forward(downward) pressure on the supply price of \(z\), and his elimination as a market-demander through integration reverses this effect. Here it is desirable to note that result 2 is rather contradictory to the traditional micro-economic theory. As well known, the oligopoly market is mostly more competitive that the monopoly market. This implies that the market demand curve of oligopoly is more elastic than the one of monopoly under the identical basic market demand curve. Hence, integration by the UM with the oligopoly downstream industry(the monopolized downstream industry) should lead to higher(lower) supply price of input \(z\), which is identical to Result 1 and opposite to result 2. As shown in results above, to specify the direction of change of input price after integration, it is required to analyse the type of basic demand curve of final output \(x\) and its market structure in detail. The specification as to the type of demand curve can therefore be expected to be relevant to the matter of welfare effects, which can now be discussed with advantage.
Result 3: If the basic demand curve of output $x$ and its marginal revenue curve satisfies the condition, \((g_o''x + g_o')/(g_o'x) > (g_o'/g_o) > (g_o' - p')/(g_o - p)\) integration by the UM with the monopolized downstream industry (DI$_m$) results in a Pareto improvement. Integration by the UM with the oilgopolized downstream industry (DI$_o$) may or may not lead to a social welfare improvement.

Proof of result 3: First, note that the DI$_o$ is an weak market and DI$_m$ is a strong market under the condition \((g_o''x + g_o')/(g_o'x) > (g_o'/g_o) > (g_o' - p')/(g_o - p)\). Let $p^m(w_i)$ be the monopoly price of $x$ given that the price of $z_i$ is $w_i$. Of course, $p^m > 0$ for $x(p^m) > 0$. If integration is with DI$_m$ (the strong DI), then, using result 1, $p^m_0(w_{oi}) < p^m_0(w^*)$; i.e. the final output price in the non-integrated industry declines. On the other hand, $p^m_0(w_{md}) < p^m_m(w^*)$ if the integration is with DI$_o$ (the weak DI). In the either case, $p^m_1(c) < p^m_1(w^*)$, where the former is the price that prevails in the DI$_m$ (the strong DI), consumers in both DI$_s$ are better off since both final prices decline: the DI$_o$ (the weak DI) is better off since the supply price $z$ declines, and the UM and the DI$_m$ (the strong DI) are better off since their joint profits rise.

In the event of integration with the DI$_o$ (the weak DI), since supply price of $z$ rises, the DI$_m$ (the strong DI) is hurt and so too are consumers in this market. However, both UM and the DI$_o$ (the weak DI) are better off as in the case of integration with the strong DI. Neither effect dominates in general, as can be shown upon request of the author. Q.E.D.

Result 4: If the basic demand curve of output $x$ and its marginal revenue curve satisfies the condition, \((g_o''x + g_o')/(g_o'x) < (g_o'/g_o) < (g_o' - p')/(g_o - p)\), integration by the UM with the oligopoly downstream industry (DI$_o$) results in a Pareto improvement. Integration by the UM with the monopolized downstream industry (DI$_m$) may or may not lead to a social welfare improvement.

Proof of result 4. As shown in previous section, the DI$_m$ is an weak
market and DI₀ is a strong market under the condition, \( \frac{(g₀''x + g₀')}{(g₀'x)} < \frac{(g₀' - p')}{(g₀ - p)} \). This is exactly opposite to the case for proving the result 3. Hence, the same method used in proving the result 3 can be applied to prove result 4. The only difference is that the strong market (the weak market) becomes the weak market (the strong market). If integration is with DI₀ (the strong DI) which is weak DI under the case of result 3, then, using result 1, \( pₘ⁽ⁿ⁽ₘ⁾(wₘ) < pₘ⁽ⁿ⁽ₘ⁾(w⁎) \); i.e. the final output price in the non-integrated industry declines. On the other hand, \( pₘ⁽ⁿ⁽ₘ⁾(wₘ) > pₘ⁽ⁿ⁽ₘ⁾(w⁎) \) if the integration is with DIₘ (the weak DI) which is strong DI under the case of result 3. In either case, \( pₘ⁽ⁿ⁽ₘ⁾(c) < pₘ⁽ⁿ⁽ₘ⁾(w⁎) \), where the former is the price that prevails in the integrated industry following integration. Then, based on this findings, the same explanation used in proving the result 3 could be applied for the proof of result 4. Q.E.D.

As in the case of result 3 and 4, among two downstream industries (monopoly and oligopoly), simple selection of the monopolized DI which is less competitive market structure in the traditional sense does not always increase social welfare nor does it always result in Pareto improvement. The selection of the downstream industry and the related impact on welfare requires detailed analysis of demand convexity of the basic demand curve of final output \( x \) (and also the derived input demand curve) and market structure.

V. Conclusion

The result of this paper has important implications for anti-trust proceedings. If the DIs are competitive and the UM integrates into one DI with price discrimination resulting, there will be a social welfare loss in most cases. Furthermore, consumers of the non-integrated DIs will definitely be worse off since output prices will increase. If the nation’s antitrust laws take the narrow view that vertical integration should be illegal if it adversely affects any consumer group, preventing vertical
integration might be a costly mistake when DIs are not competitive. Under the condition that DIs are monopolized or oligopoly, integration that entails price discrimination can lead to a Pareto improvement. Also it has been shown that vertical integration among downstream industries with heterogeneous market structure requires more detailed analysis of the shape of basic demand curve of final output (and also the derived input demand curve) and market structure. The naive view that for the social welfare improvement the upstream monopolist should integrate with the less competitive firm in the traditional sense (here, monopolized DI) is valid to the limited extent of applying only when the demand curve belongs to a specific demand curve type.

References

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