THE MATHEMATICAL ASSOCIATION OF AMERICA
AMERICAN MATHEMATICS COMPETITIONS

**24th Annual**

# AMERICAN INVITATIONAL MATHEMATICS EXAMINATION (AIME)

# SOLUTIONS PAMPHLET

**Tuesday, March 7, 2006**

This Solutions Pamphlet gives at least one solution for each problem on this year's AIME and shows that all the problems can be solved using precalculus mathematics. When more than one solution for a problem is provided, this is done to illustrate a significant contrast in methods, e.g., algebraic vs geometric, computational vs. conceptual, elementary vs. advanced. The solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers inform their students about these solutions, both as illustrations of the kinds of ingenuity needed to solve nonroutine problems and as examples of good mathematical exposition. Routine calculations and obvious reasons for proceeding in a certain way are often omitted. This gives greater emphasis to the essential ideas behind each solution. *Remember that reproduction of these solutions is prohibited by copyright.*

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1. (Answer: 084)
Because $AB^2 + BC^2 = AC^2$ and $AC^2 + CD^2 = DA^2$, it follows that $DA^2 = AB^2 + BC^2 + CD^2 = 18^2 + 21^2 + 14^2 = 961$, so $DA = 31$. Then the perimeter of $ABCD$ is $18 + 21 + 14 + 31 = 84$.

2. (Answer: 901)
The least possible value of $S$ is $1 + 2 + 3 + \cdots + 90 = 4095$, and the greatest possible value is $11 + 12 + 13 + \cdots + 100 = 4995$. Furthermore, every integer between 4096 and 4994, inclusive, is a possible value of $S$. To see this, let $\mathcal{A}$ be a 90-element subset the sum of whose elements is $S$, and let $k$ be the smallest element of $\mathcal{A}$ such that $k+1$ is not an element of $\mathcal{A}$. Because $S \le 4994$, conclude that $k \ne 100$. Therefore, for every 90-element subset with sum $S$, where $S \le 4994$, a 90-element subset with sum $S+1$ can be obtained by replacing $k$ by $k+1$. Thus there are $4995 - 4095 + 1 = 901$ possible values of $S$.

3. (Answer: 725)
The desired integer has at least two digits. Let $d$ be its leftmost digit, and let $n$ be the integer that results when $d$ is deleted. Then for some positive integer $p$, $10^p \cdot d + n = 29n$, and so $10^p \cdot d = 28n$. Therefore 7 is a divisor of $d$, and because $1 \le d \le 9$, it follows that $d = 7$. Hence $10^p = 4n$, so $n = \dfrac{10^p}{4} = \dfrac{100 \cdot 10^{p-2}}{4} = 25 \cdot 10^{p-2}$. Thus every positive integer with the desired property must be of the form $7 \cdot 10^p + 25 \cdot 10^{p-2} = 10^{p-2}(7 \cdot 10^2 + 25) = 725 \cdot 10^{p-2}$ for some $p \ge 2$. The smallest such integer is 725.

**OR**

The directions for the AIME imply that the desired integer has at most three digits. Because it also has at least two digits, it is of the form $abd$ or $cd$, where $a$, $b$, $c$, and $d$ are digits, and $a$ and $c$ are positive. Thus $bd \cdot 29 = abd$ or $d \cdot 29 = cd$. Note that no values of $c$ and $d$ satisfy $d \cdot 29 = cd$, and that $d$ must be 0 or 5. Thus $b0 \cdot 29 = ab0$ or $b5 \cdot 29 = ab5$. But $b0 \cdot 29 = ab0$ implies $b \cdot 29 = ab$, which is not satisfied by any values of $a$ and $b$. Now $b5 \cdot 29 = ab5$ implies that $b5 < 1000/29 < 35$, and so $b = 1$ or $b = 2$. Because $15 \cdot 29 = 435$ and $25 \cdot 29 = 725$, conclude that the desired integer is 725.

4. (Answer: 124)
Let $P = 1!\,2!\,3!\,4! \cdots 99!\,100!$. Then $N$ is equal to the number of factors of 5 in $P$. For any positive integer $k$, the number of factors of 5 is the same for $(5k)!$, $(5k+1)!$, $(5k+2)!$, $(5k+3)!$, and $(5k+4)!$. The number of factors of 5 in $(5k)!$ is 1 more than the number of factors of 5 in $(5k-1)!$ if $5k$ is not a multiple of 25; and the number of factors of 5 in $(5k)!$ is 2 more than the number of factors of 5 in $(5k-1)!$ if $5k$ is a multiple of 25 but not 125.

Thus

$N = 4 \cdot 0 + 5(1+2+3+4+6+7+8+9+10+12+13+14+15+16+18+19+20+21+22) + 24.$

This sum is equal to

$$5 \cdot \left( \frac{22 \cdot 23}{2} - (5 + 11 + 17) \right) + 24 = 1124,$$

so the required remainder is 124.

**OR**

Let $P = 1!\,2!\,3!\,4! \cdots 99!\,100!$. Then $N$ is equal to the number of factors of 5 in $P$. When the factorials in $P$ are expanded, 5 appears 96 times (in $5!, 6!, \ldots, 100!$), 10 appears 91 times, and, in general, $n$ appears $101 - n$ times. Every appearance of a multiple of 5 yields a factor of 5, and every appearance of a multiple of 25 yields an additional factor of 5. The number of multiples of 5 is $96 + 91 + 86 + \cdots + 1 = 970$, and the number of multiples of 25 is $76 + 51 + 26 + 1 = 154$, so $P$ ends in $970 + 154 = 1124$ zeros. The required remainder is 124.

5. (Answer: 936)
Expand $(a\sqrt{2} + b\sqrt{3} + c\sqrt{5})^2$ to obtain $2a^2 + 3b^2 + 5c^2 + 2ab\sqrt{6} + 2ac\sqrt{10} + 2bc\sqrt{15}$, and conclude that $2a^2 + 3b^2 + 5c^2 = 2006, 2ab = 104$, $2ac = 468$, and $2bc = 144$. Therefore $ab = 52 = 2^2 \cdot 13$, $ac = 234 = 2 \cdot 3^2 \cdot 13$, and $bc = 72 = 2^3 \cdot 3^2$. Then $a^2b^2c^2 = ab \cdot ac \cdot bc = 2^6 \cdot 3^4 \cdot 13^2$, and $abc = 2^3 \cdot 3^2 \cdot 13 = 936$.

Note that $a = \dfrac{abc}{bc} = \dfrac{2^3 \cdot 3^2 \cdot 13}{2^3 \cdot 3^2} = 13$ and similarly that $b = 4$ and $c = 18$. These values yield $2a^2 + 3b^2 + 5c^2 = 2006$, as required.

6. (Answer: 360)
For any digit $x$, let $x'$ denote the digit $9 - x$. If $0.\overline{abc}$ is an element of $\mathcal{S}$, then $0.\overline{a'b'c'}$ is also in $\mathcal{S}$, is not equal to $0.\overline{abc}$, and

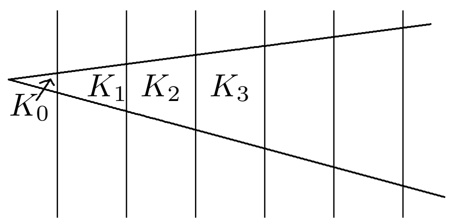
$$0.\overline{abc} + 0.\overline{a'b'c'} = 0.\overline{999} = 1.$$

It follows that $\mathcal{S}$ can be partitioned into pairs so that the elements of each pair add to 1. Because $\mathcal{S}$ has $10 \cdot 9 \cdot 8 = 720$ elements, the sum of the elements of $\mathcal{S}$ is $\frac{1}{2} \cdot 720 = 360$.

**OR**

Recall that $0.\overline{abc} = abc/999$, so the requested sum is 1/999 times the sum of all numbers of the form $abc$. To find the sum of their units digits, notice that each of the digits 0 through 9 appears $720/10 = 72$ times, and conclude that their sum is $72(0+1+2+\cdots+9) = 72\cdot 45$. Similarly, the sum of the tens digits and the sum of the hundreds digits are both equal to $72\cdot 45$. The requested sum is therefore $72\cdot 45(100+10+1)/999 = 360$.

7. (Answer: 408)

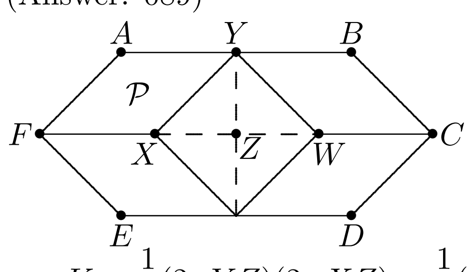



Without loss of generality, choose a unit of length equal to the distance between two adjacent parallel lines. Let $x$ be the distance from the vertex of the angle to the closest of the parallel lines that intersect the angle. Denote the areas of the regions bounded by the parallel lines and the angle by $K_0, K_1, K_2, \ldots$, as shown. Then for $m > 0$ and $n > 0$,

$$\begin{aligned}\frac{K_m}{K_0} &= \frac{K_0+K_1+K_2+\cdots+K_m}{K_0} - \frac{K_0+K_1+K_2+\cdots+K_{m-1}}{K_0}\\ &= \left(\frac{x+m}{x}\right)^2 - \left(\frac{x+m-1}{x}\right)^2 = \frac{2x+2m-1}{x^2}, \quad \text{so}\\ \frac{K_m}{K_n} &= \frac{K_m/K_0}{K_n/K_0} = \frac{2x+2m-1}{2x+2n-1}.\end{aligned}$$

Thus the ratio of the area of region $\mathcal{C}$ to the area of region $\mathcal{B}$ is $\dfrac{K_4}{K_2} = \dfrac{2x+7}{2x+3}$, and so $\dfrac{2x+7}{2x+3} = \dfrac{11}{5}$. Solve this equation to obtain $x = 1/6$. Also, the ratio of the area of region $\mathcal{D}$ to the area of region $\mathcal{A}$ is $\dfrac{K_6}{K_0} = \dfrac{2x+11}{x^2}$. Substitute 1/6 for $x$ to find that the requested ratio is 408.

8. (Answer: 089)




Draw the diagonals of rhombus $\mathcal{T}$. Let $Z$ be their point of intersection, and let $X$ and $Y$ be the shared vertices of rhombuses $\mathcal{P}$ and $\mathcal{T}$, with $Y$ on $\overline{AB}$. Let $YZ = x$ and $XY = z$. Consequently $\sqrt{2006} = [\mathcal{P}] = FX\cdot YZ = zx$, so $z = \sqrt{2006}/x$. Thus

$$K = \frac{1}{2}(2\cdot YZ)(2\cdot XZ) = \frac{1}{2}(2x)\left(2\sqrt{z^2-x^2}\right) = 2x\sqrt{\frac{2006}{x^2}-x^2} = \sqrt{8024-4x^4}.$$

There are $\lfloor\sqrt{8023}\rfloor = 89$ positive values of $x$ that yield a positive square for the radicand, so there are 89 possible values for $K$.

**OR**

Define $X$, $Y$, and $Z$ as in the first solution, and let $W$ be the shared vertex of rhombuses $\mathcal{Q}$ and $\mathcal{T}$, $W \neq Y$, let $\alpha = m\angle AYX$, let $\beta = m\angle XYW$, and let $z$ be the length of the sides of the rhombuses. Then $\beta + 2\alpha = 180°$, and the area of each of the four rhombuses is $z^2 \sin\alpha = \sqrt{2006}$. Therefore

$$K = z^2 \sin\beta = z^2 \sin 2\alpha = 2z^2 \sin\alpha\cos\alpha = 2\sqrt{2006}\,\cos\alpha.$$

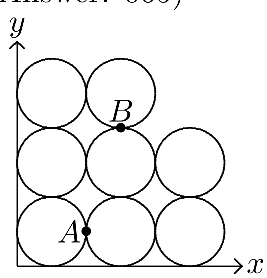
Thus $1 \le K < 2\sqrt{2006} = \sqrt{8024}$, so $1 \le K \le 89$, and there are 89 possible values for $K$.

9. (Answer: 046)
Note that

$$\log a_1 + \log a_2 + \cdots + \log a_{12} = \log(a_1 a_2 \cdots a_{12}) = \log\left(a \cdot ar \cdot \cdots \cdot ar^{11}\right) = \log\left(a^{12} r^{66}\right),$$

where the base of the logarithms is 8. Therefore $a^{12}r^{66} = 8^{2006} = 2^{3\cdot 2006}$, so $a^2 r^{11} = 2^{1003}$. Because $a$ and $r$ are positive integers, each must be a factor of $2^{1003}$. Thus $a = 2^x$ and $r = 2^y$ for nonnegative integers $x$ and $y$. Hence $2x + 11y = 1003$, and each ordered pair $(a, r)$ corresponds to exactly one ordered pair $(x, y)$ that satisfies this equation. Because $2x$ is even and 1003 is odd, $y$ must be odd, so $y$ has the form $2k - 1$, where $k$ is a positive integer. Then $1003 = 2x + 11y = 2x + 22k - 11$, so $x = 507 - 11k$. Therefore $507 - 11k \ge 0$, and so $1 \le k \le \lfloor 507/11 \rfloor = 46$. Thus there are 46 possible ordered pairs $(a, r)$.

10. (Answer: 065)




Such a line is unique. This is because when a line with equation $y = 3x + d$ intersects $\mathcal{R}$, then as $d$ decreases, the area of the part of $\mathcal{R}$ above the line is strictly increasing and the area of the part of $\mathcal{R}$ below the line is strictly decreasing. The symmetry point of the two circles that touch at $A(1, 1/2)$ is $A$, and so any line passing through $A$ divides their region into two regions of equal area. Similarly, any line passing through $B(3/2, 2)$ divides the region consisting of the two circular regions that touch at $B$ into two regions of equal area. Of the remaining four circles, two of them are on either side of line $AB$. Thus line $AB$ divides $\mathcal{R}$ into two regions of equal area. Because the slope of line $AB$ is 3, line $AB$ is line $\ell$, and it has equation $y - (1/2) = 3(x - 1)$ or $6x = 2y + 5$. Then $a^2 + b^2 + c^2 = 36 + 4 + 25 = 65$.

11. (Answer: 458)
Let $S(n)$ be the number of permissible towers that can be constructed from $n$ cubes, one each with edge-length $k$ for $1 \le k \le n$. Observe that $S(1) = 1$ and $S(2) = 2$. For $n \ge 2$, a tower of $n+1$ cubes can be constructed from any tower of $n$ cubes by inserting the cube with edge-length $n+1$ in one of three positions: on the bottom, on top of the cube with edge-length $n$, or on top of the cube with edge-length $n-1$. Thus, from each tower of $n$ cubes, three different towers of $n+1$ cubes can be constructed. Also, different towers of $n$ cubes lead to different towers of $n+1$ cubes, and each tower of $n+1$ cubes becomes a permissible tower of $n$ cubes when the cube with edge-length $n+1$ is removed. Hence, for $n \ge 2$, $S(n+1) = 3S(n)$. Because $S(2) = 2$, it follows that $S(n) = 2 \cdot 3^{n-2}$ for $n \ge 2$. Hence $T = S(8) = 2 \cdot 3^6 = 1458$, and the requested remainder is 458.

12. (Answer: 906)
The given equation implies that

$$\begin{aligned}\cos^3 3x + \cos^3 5x &= (2\cos 4x \cos x)^3 \\ &= (\cos(4x+x) + \cos(4x-x))^3 \\ &= (\cos 5x + \cos 3x)^3.\end{aligned}$$

Let $y = \cos 3x$ and $z = \cos 5x$. Then $y^3 + z^3 = (y+z)^3$. Expand and simplify to obtain $0 = 3yz(y+z)$. Thus $y = 0$ or $z = 0$ or $y + z = 0$, that is, $\cos 3x = 0$ or $\cos 5x = 0$ or $\cos 5x + \cos 3x = 0$. The solutions to the first equation are of the form $x = 30 + 60j$, where $j$ is an integer; the second equation has solutions of the form $x = 18 + 36k$, where $k$ is an integer. The third equation is equivalent to $\cos 4x \cos x = 0$, so its solutions are of the form $x = 22\frac{1}{2} + 45m$ and $x = 90 + 180n$, where $m$ and $n$ are integers. The solutions in the interval $100 < x < 200$ are 150, 126, 162, 198, $112\frac{1}{2}$, and $157\frac{1}{2}$, and their sum is 906.

13. (Answer: 899)
Note that $S_n$ is defined as the sum of the greatest powers of 2 that divide the $2^{n-1}$ consecutive even numbers $2, 4, 6, \ldots, 2^n$. Of these, $2^{n-2}$ are divisible by 2 but not 4, $2^{n-3}$ are divisible by 4 but not $8, \ldots, 2^0$ are divisible by $2^{n-1}$ but not $2^n$, and the only number not accounted for is $2^n$. Thus

$$S_n = 2 \cdot 2^{n-2} + 2^2 \cdot 2^{n-3} + \cdots + 2^{n-1} \cdot 2^0 + 2^n = (n+1)2^{n-1}.$$

In order for $S_n = 2^{n-1}(n+1)$ to be a perfect square, $n$ must be odd, because if $n$ were even, then the prime factorization of $S_n$ would have an odd number of factors of 2. Because $n$ is odd, $n+1$ must be a square, and because $n+1$ is even, $n+1$ must be the square of an even integer. The greatest $n < 1000$ that is 1 less than the square of an even integer is $30^2 - 1 = 899$.

14. (Answer: 183)
The feet of the unbroken tripod are the vertices of an equilateral triangle $ABC$, and the foot of the perpendicular from the top to the plane of this triangle is at the center of the triangle. By the Pythagorean Theorem, the distance from the center to each vertex of the triangle is 3. Place a coordinate system so that the coordinates of the top $T$ are $(0,\ 0,\ 4)$ and the coordinates of $A$, $B$, and $C$ are $(3,\ 0,\ 0)$, $(-3/2,\ 3\sqrt{3}/2,\ 0)$, and $(-3/2,\ -3\sqrt{3}/2,\ 0)$, respectively. Let the break point $A'$ be on $\overline{TA}$. Then $TA' : A'A = 4 : 1$. Thus the coordinates of $A'$ are

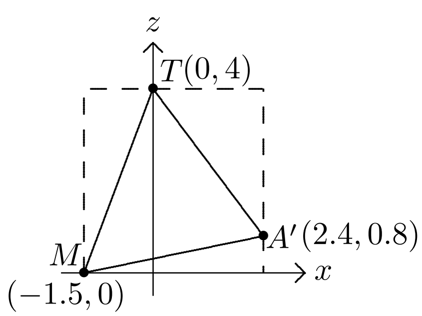
$$\frac{4}{5}(3,\ 0,\ 0) + \frac{1}{5}(0,\ 0,\ 4) = (12/5,\ 0,\ 4/5).$$

Note that the coordinates of $M$, the midpoint of $\overline{BC}$, are $(-3/2,\ 0,\ 0)$. The perpendicular from $T$ to the plane of $\triangle A'BC$ will intersect this plane at a point on $\overline{MA'}$. This segment lies in the $xz$-plane and has equation $8x - 39z + 12 = 0$ in this plane. Then $h$ is the distance from $T$ to line $MA'$, and is equal to

$$\frac{|\,8 \cdot 0 - 39 \cdot 4 + 12|}{\sqrt{8^2 + (-39)^2}} = \frac{144}{\sqrt{1585}},$$

so $\lfloor m + \sqrt{n} \rfloor = 144 + 39 = 183$.

**OR**




Place a coordinate system as in the first solution. Note that $\triangle A'MT$ is in the $xz$-plane. In this plane, circumscribe a rectangle around $\triangle A'MT$ with its sides parallel to the axes. Then

$$[A'MT] = 4(3.9) - \tfrac{1}{2}(4)(1.5) - \tfrac{1}{2}(3.2)(2.4) - \tfrac{1}{2}(3.9)(0.8) = 7.2.$$

Thus $h = \dfrac{2[A'MT]}{A'M} = \dfrac{14.4}{\sqrt{15.85}} = \dfrac{144}{\sqrt{1585}}$, so $\lfloor m + \sqrt{n} \rfloor = 144 + 39 = 183$.

**OR**

The feet of the unbroken tripod are the vertices of an equilateral triangle $ABC$, and the foot of the perpendicular from the vertex to the plane of this triangle is at the center of the triangle. Let $T$ be the top of the tripod, let $O$ be the center of $\triangle ABC$, let $A'$ be the break point on $\overline{TA}$, and let $M$ be the midpoint of $\overline{BC}$. Apply the Pythagorean Theorem to $\triangle TOA$ to conclude that $OA = 3$. Therefore $\triangle ABC$ has sides of length $3\sqrt{3}$. Notice that $A'$, $M$, and $T$ are all equidistant from $B$ and $C$, so the plane determined by $\triangle TA'M$ is perpendicular to $\overline{BC}$, and so $h$ is the length of the altitude from $T$ in $\triangle TA'M$. Because

$$\frac{1}{2}A'M \cdot h = [TA'M] = \frac{1}{2}A'T \cdot TM \sin\angle A'TM,$$

it follows that

$$h = \frac{A'T \cdot TM \sin\angle A'TM}{A'M}.$$

The length of $\overline{A'T}$ is 4, and $TM = \sqrt{TB^2 - BM^2} = \sqrt{25 - (27/4)} = \sqrt{73}/2$. To find $A'M$, note that $A'M^2 = A'C^2 - CM^2$, and that $A'C^2 = A'T^2 + TC^2 - 2A'T \cdot TC \cos\angle A'TC$. But $\cos\angle A'TC = \cos\angle ATC = \dfrac{5^2 + 5^2 - (3\sqrt{3})^2}{2 \cdot 5 \cdot 5} = \dfrac{23}{50}$,
so $A'C^2 = 4^2 + 5^2 - 2 \cdot 4 \cdot 5 \cdot (23/50) = 113/5$, and $A'M = \sqrt{\dfrac{113}{5} - \dfrac{27}{4}} = \sqrt{\dfrac{317}{20}}$.
Now $\cos\angle A'TM = \dfrac{16 + 73/4 - 317/20}{2 \cdot 4 \cdot \sqrt{73}/2} = \dfrac{23}{5\sqrt{73}}$,
so $\sin^2\angle A'TM = 1 - \dfrac{23^2}{25 \cdot 73}$, and $\sin\angle A'TM = \dfrac{36}{5\sqrt{73}}$. Thus

$$h = \frac{4 \cdot \frac{\sqrt{73}}{2} \cdot \frac{36}{5\sqrt{73}}}{\sqrt{\frac{317}{20}}} = \frac{144}{\sqrt{1585}},$$

so $\lfloor m + \sqrt{n} \rfloor = 144 + 39 = 183$.

15. (Answer: 027)
The condition $|x_k| = |x_{k-1}+3|$ is equivalent to $x_k^2 = (x_{k-1}+3)^2$. Thus

$$\sum_{k=1}^{n+1} x_k^2 = \sum_{k=1}^{n+1}(x_{k-1}+3)^2 = \sum_{k=0}^{n}(x_k+3)^2 = \left(\sum_{k=0}^{n} x_k^2\right) + \left(6\sum_{k=0}^{n} x_k\right) + 9(n+1), \quad \text{so}$$

$$x_{n+1}^2 = \sum_{k=1}^{n+1} x_k^2 - \sum_{k=0}^{n} x_k^2 = \left(6\sum_{k=0}^{n} x_k\right) + 9(n+1), \quad \text{and}$$

$$\sum_{k=0}^{n} x_k = \frac{1}{6}\left[x_{n+1}^2 - 9(n+1)\right].$$

Therefore $\left|\sum_{k=1}^{2006} x_k\right| = \frac{1}{6}\left|x_{2007}^2 - 18063\right|$. Notice that $x_k$ is a multiple of 3 for all $k$, and that $x_k$ and $k$ have the same parity. The requested sum will be a minimum when $|x_{2007}^2 - 18063|$ is a minimum, that is, when $x_{2007}$ is the multiple of 3 whose square is as close as possible to 18063. Check odd multiples of 3, and find that $129^2 < 16900$, $141^2 > 19600$, and $135^2 = 18225$. The requested minimum is therefore $\frac{1}{6}|135^2 - 18063| = 27$, provided there exists a sequence that satisfies the given conditions and for which $x_{2007} = 135$. An example of such a sequence is

$$x_k = \begin{cases} 3k & \text{for } k \le 45, \\ -138 & \text{for } k > 45 \text{ and } k \text{ even}, \\ 135 & \text{for } k > 45 \text{ and } k \text{ odd}. \end{cases}$$

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