A Note on Output Effects of Price Discrimination

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Introduction

M. L. Greenhut and H. Ohta (1972) demonstrated a greater output for a spatial monopolist pricing discriminatorily than would be the case under f.o.b. pricing. They used as their referential departure point the linear demand curve. Furthermore, Greenhut with Norman and Hung (1986) demonstrated that the linear demand curve is a member of the less convex demand set and that what applies to linear demand therefore holds for a vast body of demand curve types, such as those given by \( p = a - bq^x/x, \ x > 0 \). They further note that even when \(-1 < x < 0\) (the so-called more convex demand set), the firm’s delivered price schedule is linearly positive with greater freight cost, albeit the discrimination then proceeds against distant buyers. However, they did not evaluate the output effects under these more convex demands when the subject firm prices discriminatorily vis à vis a f.o.b. mill. It is the purpose of this paper to ascertain whether the output effects claimed in the Greenhut–Ohta (1972) paper (henceforth GO) also apply to the more convex demand curve set.

To provide our argument as inclusively as possible, we shall first directly compare the outputs for the two (alternative) price policies under the condition where \( x = -1/2 \) (i.e., a particular member of the more convex demand curve set): subsequently, we shall compare the marginal revenue curve under discrimination (call it DMR) with the aggregate simple f.o.b. mill marginal revenue curve (refer to it as SMR). For all comparisons, we shall initially conceive of a given limit distance \( R \) under the condition \(-1 < x < 0\) which, to repeat, we particularize by assuming \( x = -1/2 \).
II. The Analysis

Consider the demand form:

\[ q = [-(\frac{1}{2b})(a-m-tr)]^{-2}, \]  

where \( m \) = mill price, \( t \) is the freight rate, \( r \) the distance from the seller's place to a particular buyer's site, and \( a, b \) parameters are positive numbers. Total aggregate demand under f.o.b. mill pricing over the market length \( R \) is:

\[ Q_f = \frac{4b^2}{t} \left[ \frac{1}{(a-m-tr)} - \frac{1}{(a-m)} \right] \]  

(2)

Via:

\[ \pi = (m-c)Q_f, \]  

(3)

the optimal f.o.b. mill price \( m \) is then:

\[ m_f = c + [(c-a)tr + (c-a)^2]^{1/2} \]  

(4)

Substituting (4) into (2) establishing the f.o.b. mill firm's profit maximizing output:

\[ Q_f = \frac{4b^2}{t} \left[ \frac{tR}{(a-c-tr-[(c-a)tr+(c-a)^2]^{1/2})} \right] \]  

(5)

Determining the discriminatory mill price requires two basic steps:

\[ \pi_{\text{max}} = (m-c)q = (\frac{1}{-2b})^{-2}(m-c)(a-m-tr)^{-2} \]  

(6)

and \[ \frac{\partial \pi}{\partial m} = (\frac{1}{-2b})^{-2}(a-m-tr)^{-2}[1+2(m-c)(a-m-tr)^{-1}] = 0 \]

hence \[ m_D = 2c + tr - a \]  

(7)

Total discriminatory output is then

\[ Q_D = \int_0^R (\frac{1}{-2b}(a-m-tr))^{-2} dr = -b^2 \left[ \frac{tR}{(a-c-tr)^{-1}} \right] = \frac{b^2}{t} \left[ \frac{1}{a-c-tr} - \frac{1}{a-c} \right] \]  

(8)

Output comparisons of (5) and (8) establish

\[ Q_f = \frac{4b^2}{t} \left[ \frac{tR}{\left[ X - ((-Y)(-X))^\frac{1}{2} \right] \left[ Y - ((-Y)(-X))^\frac{1}{2} \right]} \right] \]  

(5)

and \[ Q_D = \frac{b^2}{t} \left[ \frac{1}{X} - \frac{1}{Y} \right] = \frac{b^2}{t} \left[ \frac{tR}{XY} \right] \]  

(8)'
where \( X = a - c - tR \) and \( Y = a - c \). It follows that

\[
\frac{Q_d}{Q_f} = \frac{1}{4} \left[ 2 - \frac{X + Y}{(XY)^{\frac{1}{2}}} \right]
\]

(9)

Hence \( \frac{Q_d}{Q_f} > 1 \) if \( \left[ \frac{X + Y}{(XY)^{\frac{1}{2}}} \right] < -2 \) or alternatively \( Q_d > Q_f \)

if \( X + Y + 2(XY)^{\frac{1}{2}} < 0 \)

By substitution, we obtain:

\[
X + Y + 2(XY)^{\frac{1}{2}} = -\left\{ (c + tR - a)^{\frac{1}{2}} - (c - a)^{\frac{1}{2}} \right\}^2 < 0
\]

Therefore \( Q_d > Q_f \).

Alternatively, consider the aggregate demand curve used in determining the profit maximizing f. o. b. mill price. From equation (2) we have

\[
m = \frac{(2a - tR)}{2} + \frac{\left[ (tR)^2 + \frac{(4b)^2}{Q} R \right]}{2}^{\frac{1}{2}}
\]

(11)

converting to revenues and differentiating yields

\[
\frac{\partial R}{\partial Q} = MR = \frac{2a - tR}{2} + \frac{1}{2} \left( (tR)^2 + \frac{(4b)^2}{R} R \right)^{\frac{1}{2}} - \frac{1}{4} \left( (tR)^2 + \frac{(4b)^2}{R} R \right)^{-\frac{1}{2}} \left( \frac{4b^2}{Q} \right) R
\]

(12)

From equation (1), the inverse demand is

\[
m = a - tr + 2bq^{-\frac{1}{2}}
\]

(13)

and since

\[
\frac{\partial R}{\partial q} = MR_d = a - tr + bq^{-\frac{1}{2}}
\]

(14)

we obtain:

\[
q = \left[ \frac{-1}{b} (MR + tr - a) \right]^{-2}
\]

(15)

\[
Q = \int_0^k qdr = \frac{b^2}{t} \left( -\frac{1}{MR - a} - \frac{1}{MR + tR - a} \right)
\]

(16)

Rewriting (16) provides MR in terms of Q (i.e., the sum of the individual MR's), namely
\[ MR_D = \frac{(2a - tR)}{2} + \frac{1}{2} \left[ (tR)^2 + \frac{4t^2 R}{Q} \right]^{\frac{1}{2}} \]  \hspace{1cm} (17)

It is manifest via (12) and (17) that \( MR_D > MR_I \) for all \( Q \). The GO result thus applies under the more convex demand curve set as well as in their paper based on the less convex demand set. But note the following differences:

III. Conclusion

In contrast with GO (1972), where the greater output involved a larger market area under spatial price discrimination than under f.o.b. pricing, the result for extremely convex demands applies even more forcefully. In fact, our present finding was based on the same market size \( R \) under either pricing system. Hence, our finding contrasts sharply from that of Beckmann (1976), who worked with linear demand and a fixed (same size) market, and for that type of demand correctly derived identical \( Q_0, Q_I \), not our \( Q_0 > Q_I \). Moreover, since profits are greater under discriminatory pricing, we propose—as did GO (1975) for the linear demand case—that under competition if \( N = 0 \) when \( R_I = R_D \) more firms will enter the market under discriminatory pricing. This entry will produce a still lower set of prices and smaller distances for the discriminatory sellers, hence a greatly expanded total output. We obtain the same basic long-run result that was established by GO (1975), albeit obtaining it now even more emphatically since demands of the more convex order provide greater output per firm for identical market length besides having the greater number of firms in the competitive zero profit equilibrium.

References


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1) As noted in the paper’s introduction, the linear demand is a member of the less convex demand curve set.