THE MATHEMATICAL ASSOCIATION OF AMERICA
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AMERICAN INVITATIONAL MATHEMATICS EXAMINATION (AIME)

SOLUTIONS PAMPHLET

Tuesday, March 13, 2007

This Solutions Pamphlet gives at least one solution for each problem on this year's AIME and shows that all the problems can be solved using precalculus mathematics. When more than one solution for a problem is provided, this is done to illustrate a significant contrast in methods, e.g., algebraic vs geometric, computational vs. conceptual, elementary vs. advanced. The solutions are by no means the only ones possible, nor are they necessarily superior to others the reader may devise.

We hope that teachers inform their students about these solutions, both as illustrations of the kinds of ingenuity needed to solve nonroutine problems and as examples of good mathematical exposition. Routine calculations and obvious reasons for proceeding in a certain way are often omitted. This gives greater emphasis to the essential ideas behind each solution.

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1. (Answer: 083)

   Because $24 = 3 \cdot 2^3$, a square is divisible by 24 if and only if it is divisible by $3^2 \cdot 2^4 = 144$. Furthermore, a perfect square N^2 less than 10^6 is a multiple of 144 if and only if N is a multiple of 12 less than 10^3. Because 996 is the largest multiple of 12 less than 10^3, there are $\frac{996}{12} = 83$ such positive integers less than 10^3 and 83 positive perfect squares which are multiples of 24.

2. (Answer: 052)

   Let t be Al's travel time. Then $t-2$ is Bob's time, and $t-4$ is Cy's time, and $t \geq 4$. If Cy is in the middle, then $10(t-2) - 8(t-4) = 8(t-4) - 6t$, which has no solution. If Bob is in the middle, then $10(t-2) - 8(t-4) = 6t - 10(t-2)$, which has solution $t = 4/3$. But $t \geq 4$, so this is impossible. If Al is in the middle, then $6t - 8(t-4) = 10(t-2) - 6t$, which has solution $t = 26/3$. In this case, Al is 52 feet from the start and is $44/3$ feet from both Bob and Cy. Thus the required distance is 52.

3. (Answer: 015)

   The complex number $z = 9 + bi$, so $z^2 = (81 - b^2) + 18bi$ and $z^3 = (729 - 27b^2) + (243b - b^3)i$. These two numbers have the same imaginary part, so $243b - b^3 = 18b$. Because b is not zero, $243 - b^2 = 18$, and $b = 15$.

4. (Answer: 105)

   All four positions will be collinear if and only if the difference in the number of revolutions made by each pair of planets is an integer multiple of $\dfrac{1}{2}$. When the outermost planet has made r revolutions, the middle and innermost planets will have made $\dfrac{140r}{84} = \dfrac{5}{3}r$ and $\dfrac{140r}{60} = \dfrac{7}{3}r$ revolutions, respectively. Thus it is necessary and sufficient that $\dfrac{2}{3}r = \dfrac{1}{2}k$ for some integer k, so the smallest positive solution for r is $\dfrac{3}{4}$. Hence $n = \dfrac{3}{4} \cdot 140 = 105$.

5. (Answer: 539)

   Note that a temperature T converts back to T if and only if $T+9$ converts back to $T+9$. Thus it is only necessary to examine nine consecutive temperatures. It is easy to show that 32 converts back to 32, 33 and 34 both convert back to 34, 35 and 36 both convert back to 36, 37 and 38 both convert back to 37, and 39 and 40 both convert back to 39. Hence out of every nine consecutive temperatures starting with 32, five are converted correctly and four are not. For $32 \leq T < 32 + 9 \cdot 107 = 995$. There are $107 \cdot 5 = 535$ temperatures that are converted correctly. The remaining six temperatures $995, 996, \ldots, 1000$ behave like $32, 33, \ldots, 37$, so four of the remaining six temperatures are converted correctly. Thus there is a total of $535 + 4 = 539$ temperatures.

OR

Because one Fahrenheit degree is 5/9 of a Celsius degree, every integer Celsius temperature is the conversion of either one or two Fahrenheit temperatures (nine Fahrenheit temperatures are being converted to only five Celsius temperatures) and converts back to one of those temperatures. The Fahrenheit temperatures 32 and 1000 convert to 0 and 538, respectively, which convert back to 32 and 1000. Therefore there are 539 Fahrenheit temperatures with the required property, corresponding to the integer Celsius temperatures from 0 to 538.

6. (Answer: 169)

The set of move sequences can be partitioned into two subsets, namely the set of move sequences that contain 26, and the set of move sequences that do not contain 26. If a move sequence does not contain 26, then it must contain the subsequence 24, 27. To count the number of such sequences, note that there are six ways for the frog to get to 24 (five ways that go through 13, and one way that does not go through 13), and there are four ways for the frog to get from 27 to 39, since any of 27, 30, 33, 36 can be the second to last element of a move sequence. Thus there are 24 move sequences that do not contain 26. If a move sequence contains 26, then it either contains 13 or it does not. If such a move sequence does not contain 13, then it must contain the subsequence 12, 15, and in this case, there is one way for the frog to get from 0 to 12, and there are 4 ways for it to get from 15 to 26. If a move sequence contains 13 and 26, then there are five ways for the frog to get from 0 to 13, and there are are five ways for it to get from 13 to 26. Thus there are $1 \cdot 4 + 5 \cdot 5 = 29$ ways for the frog to get from 0 to 26, and there are five ways for the frog to get from 26 to 39, for a total of 145 move sequences that contain 26. Thus there are a total of $24 + 145 = 169$ move sequences for the frog.

OR

There are five sequences from 0 to 13, five from 13 to 26, and five from 26 to 39. There are four sequences from 0 to 26 not containing 13 and four sequences from 13 to 39 not containing 26. Finally, there are four sequences from 0 to 39 containing neither 13 nor 26. Thus there are $5 \cdot 5 \cdot 5 + 2 \cdot 4 \cdot 5 + 4 = 169$ sequences in all.

7. (Answer: 477)

First note that

$$\lceil x \rceil - \lfloor x \rfloor = \begin{cases} 1, & \text{if } x \text{ is not an integer} \\ 0, & \text{if } x \text{ is an integer} \end{cases}.$$

Thus for any positive integer k,

$$\lceil \log_{\sqrt{2}} k \rceil - \lfloor \log_{\sqrt{2}} k \rfloor = \begin{cases} 1, & \text{if } k \text{ not an integer power of } \sqrt{2} \\ 0, & \text{if } k \text{ an integer power of } \sqrt{2} \end{cases}.$$

The integers k, $1 \le k \le 1000$, that are integer powers of $\sqrt{2}$ are described by $k = 2^j$, $0 \le j \le 9$. Thus

$$\sum_{k=1}^{1000} k(\lceil \log_{\sqrt{2}} k \rceil - \lfloor \log_{\sqrt{2}} k \rfloor) = \sum_{k=1}^{1000} k - \sum_{j=0}^{9} 2^j = \frac{1000 \cdot 1001}{2} - 1023 = 499477.$$

The requested remainder is 477.

8. (Answer: 030)

Because $P(x)$ has three roots, if $Q_1(x) = x^2 + (k-29)x - k$ and $Q_2(x) = 2x^2 + (2k-43)x + k$ are both factors of $P(x)$, then they must have a common root r. Then $Q_1(r) = Q_2(r) = 0$, and $mQ_1(r) + nQ_2(r) = 0$, for any two constants m and n. Taking $m = 2$ and $n = -1$ yields the equation $15r + 3k = 0$, so $r = \frac{-k}{5}$. Thus $Q_1(r) = \frac{k^2}{25} - (k-29)\left(\frac{k}{5}\right) - k = 0$, which is equivalent to $4k^2 - 120k = 0$, whose roots are $k = 30$ and 0. When $k = 30$, $Q_1(x) = x^2 + x - 30$ and $Q_2(x) = 2x^2 + 17x + 30$, and both polynomials are factors of $P(x) = (x+6)(x-5)(2x+5)$. Thus the requested value of k is 30.

9. (Answer: 737)

Let T_1 and T_2 be the points of tangency on AB so that $\overline{O_1T_1}$ and $\overline{O_2T_2}$ are radii of length r. Let circles O_1 and O_2 be tangent to each other at point T_3, let circle O_1 be tangent to the extension of $\overline{AC}$ at E_1, and let circle O_2 be tangent to the extension of $\overline{BC}$ at E_2. Let D_1 be the foot of the perpendicular from O_1 to $\overline{BC}$, let D_2 be the foot of the perpendicular from O_2 to $\overline{AC}$, and let $\overline{O_1D_1}$ and $\overline{O_2D_2}$ intersect at P. Let r be the radius of O_1 and O_2. Note that $O_1T_3 = O_2T_3 = r$, $CD_1 = O_1E_1 = r$, and $CD_2 = O_2E_2 = r$. Because triangles ABC and O_1O_2P are similar, $\dfrac{O_1O_2}{AB} = \dfrac{O_1P}{AC} = \dfrac{O_2P}{BC}$. Because $O_1O_2 = 2r$ and $AB = \sqrt{30^2 + 16^2} = 34$, $O_1P = \dfrac{30r}{17}$ and $O_2P = \dfrac{16r}{17}$. By equal tangents, $AE_1 = AT_1 = x$ and $BE_2 = BT_2 = y$. Thus

$$AB = 34 = AT_1 + T_1T_2 + BT_2$$

$$= AT_1 + O_1O_2 + BT_2$$
$$= x + 2r + y.$$

Also, $O_1P = E_1D_2$, so $\frac{30r}{17} = AD_2 + AE_1 = 30 - r + x$, and $O_2P = E_2D_1$, so $\frac{16r}{17} = BD_1 + BE_2 = 16 - r + y$. Adding these two equations produces $\frac{46r}{17} = 46 - 2r + x + y$. Substituting $x + y = 34 - 2r$ yields $\frac{46r}{17} = 80 - 4r$. Thus $r = \frac{680}{57}$, and $p + q = 737$.

OR

Draw $\overline{O_1A}$ and $\overline{O_2B}$ and note that these segments bisect the external angles of the triangle at A and B, respectively. Thus $\angle T_1O_1A = \dfrac{1}{2}(\angle A)$ and $T_1A = r\tan\angle T_1O_1A = r\tan\dfrac{1}{2}(\angle A)$. Similarly $T_2B = r\tan\dfrac{1}{2}(\angle B)$. By the half-angle identity for tangent,

$$\tan\left(\frac{1}{2}\angle A\right) = \frac{\sin\angle A}{\cos\angle A + 1} = \frac{16/34}{(30/34) + 1} = \frac{1}{4},$$

and similarly $\tan\left(\frac{1}{2}(\angle B)\right) = \frac{3}{5}$. Then $34 = AB = AT_1 + T_1T_2 + T_2B = r/4 + 2r + 3r/5 = 57r/20$ and $r = 680/57$. Thus $p + q = 737$.

10. (Answer: 860)

There are $\binom{6}{3}$ ways to shade three squares in the first column. Given a shading scheme for the first column, consider the ways the second, third, and fourth columns can then be shaded. If the shaded squares in the second column are in the same rows as those of the first column, then the shading pattern for the last two columns is uniquely determined. Thus there are $\binom{6}{3} = 20$ shadings in which the first two columns have squares in the same rows shaded. Next shade the second column so that two of the shaded squares are in the same rows as shaded squares in the first column. Given a shading of the first column, there are $\binom{3}{2}$ ways to choose the two shaded rows in common, then $\binom{3}{1}$ ways to choose the third shaded square in this column. This leaves two rows with no shaded squares, so the squares in these rows must be shaded in the third and fourth columns. There are also two rows with one shaded square each, one of these in the first column and one in the second. For the first of these rows the square can be shaded in the same row in the third or fourth column, for two choices, and this uniquely determines the shading in the second of these rows. Thus there are $\binom{6}{3} \cdot \binom{3}{2} \cdot \binom{3}{1} \cdot 2 = 360$ shadings in which the first two columns have two shaded rows in common. Next shade the second column so that only one of the shaded squares is in the same row as a shaded square in the first column. The row containing the shaded square in both columns can be selected in $\binom{3}{1}$ ways and the other two shaded squares in the second column in $\binom{3}{2}$ ways. This leaves one row with no shaded squares, so the squares in this row in the third and fourth columns must be shaded. There are four rows each with one square shaded in the first two columns. Two of these rows must be shaded in the third column, and two in the fourth. This can be done in $\binom{4}{2}$ ways. Thus there are $\binom{6}{3}\cdot\binom{3}{1}\cdot\binom{3}{2}\cdot\binom{4}{2} = 1080$ shadings in which the first two columns have one shaded row in common. Finally, if the first two columns have no shaded row in common, then the shading of the second column is uniquely determined. The three squares to be shaded in the third column can be selected in $\binom{6}{3}$ ways, and the shading for the fourth column is then uniquely determined. There are $\binom{6}{3}\cdot\binom{6}{3} = 400$ shading patterns in which the first two columns have no shaded row in common. Thus the total number of shadings is $20 + 360 + 1080 + 400 = 1860$, and the requested remainder is 860.

OR

The three shaded squares in the first column can be chosen in $\binom{6}{3} = 20$ ways. For $0 \le k \le 3$, there are $\binom{3}{k}\binom{3}{3-k}$ ways to choose the shaded squares in the second column so that k rows have two shaded squares. No shaded square in the third column can be in one of these rows. In each case there are also k rows with no shaded squares, and each of these rows must contain a shaded square in the third column. The remaining $3-k$ shaded squares in the third column must be chosen from the remaining $6-2k$ rows. Thus there are $\binom{6-2k}{3-k}$ ways to choose the shaded squares in the third column. In each case, the shaded squares in the fourth column are uniquely determined. Therefore $N = 20\sum_{k=0}^{3}\binom{3}{k}\binom{3}{3-k}\binom{6-2k}{3-k} = 20(20+54+18+1) = 1860$, and the requested remainder is 860.

11. (Answer: 955)

First, if k is a nonnegative integer, and n is a positive integer then $\sqrt{n^2+k} < n + \frac{1}{2} \iff n^2 + k < n^2 + n + \frac{1}{4} \iff k < n + \frac{1}{4} \iff k = 0, 1, 2, \ldots, n$. Similarly, $n - \frac{1}{2} < \sqrt{n^2 - k} \iff n^2 - n + \frac{1}{4} < n^2 - k \iff k < n - \frac{1}{4}$. So if k is a positive integer, the second inequality is satisfied when $k = 1, 2, 3, \ldots, n-1$. Thus a positive integer n is the value of $b(p)$ precisely when p is one of the $(n+1)+(n-1) = 2n$ integers $n^2-(n-1)$, $n^2-(n-2)$, …, n^2-1, n^2, n^2+1, …, n^2+n.

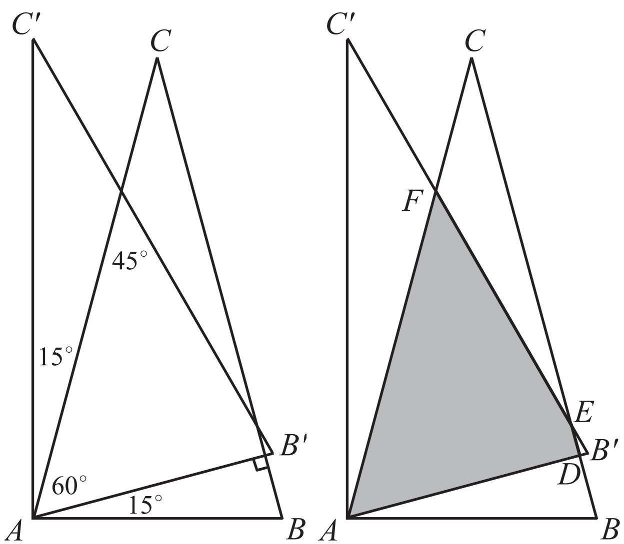
Next, observe that $44^2 = 1936 < 1936 + 44 = 1980 < 2007 < 45^2 = 2025$. Therefore each integer $n = 1, 2, 3, \ldots, 44$ contributes $n \cdot 2n = 2n^2$ to the sum. Consequently,

$$S = \sum_{p=1}^{2007} b(p) = \sum_{p=1}^{1980} b(p) + \sum_{p=1981}^{2007} b(p) = \left(\sum_{n=1}^{44} 2n^2\right) + 27 \cdot 45$$
$$= 2\left(\frac{44 \cdot 45 \cdot 89}{6}\right) + 1215 = 59955.$$

Thus the requested remainder is 955.

12. (Answer: 875)

Let $[XYZ]$ represent the area of triangle XYZ.



For future use, $\sin 75° = \cos 15° = (\sqrt{6}+\sqrt{2})/4$. Let B' and C' be the images of B and C respectively under the given rotation. Let D denote the point at which $\overline{BC}$ intersects $\overline{AB'}$, let E denote the point at which $\overline{BC}$ intersects $\overline{B'C'}$, and let F denote the point at which $\overline{AC}$ intersects $\overline{B'C'}$. Then the region common to the two triangles (shaded in the figure on the right) is $ADEF$, and its area is $[ADEF] = [AB'F] - [EB'D]$. Note that $\angle B + \angle B'AB = 75° + 15° = 90°$ implies $\overline{AB'} \perp \overline{BC}$. Because $AB' = AB = 20$, the Law of Sines applied to $\triangle B'FA$ gives $B'F = 20 \sin 60°/\sin 45° = 20\sqrt{3/2} = 10\sqrt{6}$, and thus
$[AB'F] = \frac{1}{2} \cdot 20 \cdot 10\sqrt{6} \sin 75° = 100\sqrt{6}\left(\frac{\sqrt{6}+\sqrt{2}}{4}\right) = 50(3+\sqrt{3})$.

Note that $B'D = 20(1 - \cos 15°)$, $BD = 20 \sin 15°$, and $\triangle EB'D \sim \triangle ABD$. Because $[ABD] = \frac{1}{2} \cdot 20 \cos 15° \cdot 20 \sin 15° = 100 \sin 30° = 50$, it follows that $[EB'D] = 50(\frac{1-\cos 15^\circ}{\sin 15^\circ})^2$.

Using $\cos^2 15^\circ = \frac{1+\cos 30^\circ}{2} = \frac{2+\sqrt{3}}{4}$ and $\sin^2 15^\circ = \frac{1-\cos 30^\circ}{2} = \frac{2-\sqrt{3}}{4}$ yields

$$\begin{aligned}\left(\frac{1-\cos 15^\circ}{\sin 15^\circ}\right)^2 &= \frac{1-2\cos 15^\circ + \cos^2 15^\circ}{\sin^2 15^\circ}\\ &= \frac{1-(\sqrt{6}+\sqrt{2})/2 + (2+\sqrt{3})/4}{(2-\sqrt{3})/4}\\ &= (6+\sqrt{3}-2\sqrt{6}-2\sqrt{2})(2+\sqrt{3})\\ &= 15+8\sqrt{3}-6\sqrt{6}-10\sqrt{2}.\end{aligned}$$

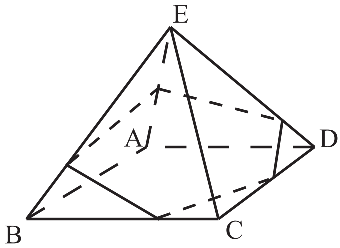
Finally,

$$\begin{aligned}[ADEF] &= [AB'F] - [EB'D]\\ &= 50(3+\sqrt{3}) - 50(15+8\sqrt{3}-6\sqrt{6}-10\sqrt{2})\\ &= 50(10\sqrt{2}-7\sqrt{3}+6\sqrt{6}-12),\end{aligned}$$

so $(p-q+r-s)/2 = 25(10+7+6+12) = 875$.

13. (Answer: 80)

Place the pyramid on a coordinate system with A at $(0,0,0)$, B at $(4,0,0)$, C at $(4,4,0)$, D at $(0,4,0)$ and with E at $(2,2,2\sqrt{2})$. Let R, S, and T be the midpoints of $\overline{AE}$, $\overline{BC}$, and $\overline{CD}$ respectively. The coordinates of R, S, and T are respectively $(1,1,\sqrt{2})$, $(4,2,0)$ and $(2,4,0)$. The equation of the plane containing R, S, and T is $x+y+2\sqrt{2}z = 6$. Points on $\overline{BE}$ have coordinates of the form $(4-t,t,t\sqrt{2})$, and points on $\overline{DE}$ have coordinates of the form $(t,4-t,t\sqrt{2})$. Let U and V be the points of intersection of the plane with $\overline{BE}$ and $\overline{DE}$ respectively. Substituting into the equation of the plane yields $t = \dfrac{1}{2}$ and $\left(\dfrac{7}{2},\dfrac{1}{2},\dfrac{\sqrt{2}}{2}\right)$ for U, and $t = \dfrac{1}{2}$ and $\left(\dfrac{1}{2},\dfrac{7}{2},\dfrac{\sqrt{2}}{2}\right)$ for V. Then $RU = RV = \sqrt{7}$, $US = VT = \sqrt{3}$ and $ST = 2\sqrt{2}$. Note also that $UV = 3\sqrt{2}$. Thus the pentagon formed by the intersection of the plane and the pyramid can be partitioned into isosceles triangle RUV and isosceles trapezoid $USTV$ with areas of $3\sqrt{5}/2$ and $5\sqrt{5}/2$ respectively. Therefore the total area is $4\sqrt{5}$ or $\sqrt{80}$, and $p = 80$.



14. (Answer: 224)

The fact that the equation $a_{n+1}a_{n-1} = a_n^2 + 2007$ holds for $n \geq 2$ implies that $a_n a_{n-2} = a_{n-1}^2 + 2007$ for $n \geq 3$. Subtracting the second equation from the first one yields $a_{n+1}a_{n-1} - a_n a_{n-2} = a_n^2 - a_{n-1}^2$, or $a_{n+1}a_{n-1} + a_{n-1}^2 = a_n a_{n-2} + a_n^2$. Dividing the last equation by $a_{n-1}a_n$ and simplifying produces $\frac{a_{n+1}+a_{n-1}}{a_n} = \frac{a_n+a_{n-2}}{a_{n-1}}$. This equation shows that $\frac{a_{n+1}+a_{n-1}}{a_n}$ is constant for $n \geq 2$. Because $a_3 a_1 = a_2^2 + 2007$, $a_3 = 2016/3 = 672$. Thus $\frac{a_{n+1}+a_{n-1}}{a_n} = \frac{672+3}{3} = 225$, and $a_{n+1} = 225a_n - a_{n-1}$ for $n \geq 2$. Note that $a_3 = 672 > 3 = a_2$. Furthermore, if $a_n > a_{n-1}$, then $a_{n+1}a_{n-1} = a_n^2 + 2007$ implies that

$$a_{n+1} = \frac{a_n^2}{a_{n-1}} + \frac{2007}{a_{n-1}} = a_n\left(\frac{a_n}{a_{n-1}}\right) + \frac{2007}{a_{n-1}} > a_n + \frac{2007}{a_{n-1}} > a_n.$$

Thus by mathematical induction, $a_n > a_{n-1}$ for all $n \geq 3$. Therefore the recurrence $a_{n+1} = 225a_n - a_{n-1}$ implies that $a_{n+1} > 225a_n - a_n = 224a_n$ and therefore $a_n \geq 2007$ for $n \geq 4$. Finding a_{n+1} from $a_{n+1}a_{n-1} = a_n^2 + 2007$ and substituting into $225 = \frac{a_{n+1}+a_{n-1}}{a_n}$ shows that $\frac{a_n^2+a_{n-1}^2}{a_n a_{n-1}} = 225 - \frac{2007}{a_n a_{n-1}}$. Thus the largest integer less than or equal to the original fraction is 224.

15. (Answer: 989)

In the following, let $[XYZ]$ = the area of triangle XYZ.
In general, let $AB = s$, $FA = a$, $DC = c$, $EF = x$, $FD = y$, and $AE = t$. Then $EC = s - t$. In triangle AEF, angle A is $60°$, and so $[AEF] = \frac{1}{2} \cdot \sin 60° \cdot AE \cdot AF = \frac{\sqrt{3}}{4}at$. Similarly, $[BDF] = \frac{\sqrt{3}}{4}(s-a)(s-c)$, $[CDE] = \frac{\sqrt{3}}{4}c(s-t)$, and $[ABC] = \frac{\sqrt{3}}{4}s^2$. It follows that

$$\begin{aligned}[DEF] &= [ABC] - [AEF] - [BDF] - [CDE] \\ &= \frac{\sqrt{3}}{4}[s^2 - at - (s-a)(s-c) - c(s-t)] \\ &= \frac{\sqrt{3}}{4}[a(s-t) + ct - ac].\end{aligned}$$

The given conditions then imply that $56 = 5(s-t) + 2t - 10$, or $5(s-t) + 2t = 66$. Because $\angle A = \angle DEF = 60°$, it follows that $\angle AEF + \angle AFE = 120° = \angle AEF +$

$\angle CED$, implying that $\angle AFE = \angle CED$. Note also that $\angle C = \angle A$. Thus $\triangle AEF \sim \triangle CDE$. Consequently, $\frac{AE}{AF} = \frac{CD}{CE}$, or $\frac{t}{5} = \frac{2}{s-t}$. Thus $t(s-t) = 10$ or $s - t = \frac{10}{t}$. Substituting this into the equation $5(s-t) + 2t = 66$ gives $5 \cdot 10 + 2t^2 = 66t$. Solving this quadratic equation gives $t = \frac{33 \pm \sqrt{989}}{2}$, and hence $s = t + \frac{10}{t} = \frac{231}{10} \pm \frac{3}{10}\sqrt{989}$. Repeated applications of the Law of Cosines show that both values of s produce valid triangles. Thus $r = 989$.

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