081110 mon

< 11주차 >

* 손실 ($\sigma \neq 0$)

$$\begin{cases} \nabla^2 \mathbb{E} + k_c^2 \mathbb{E} = 0 \\ \nabla^2 \mathbb{H} + k_c^2 \mathbb{E} = 0 \end{cases}$$

$k_c^2 = \omega^2 \mu \varepsilon_c$, $\varepsilon_c = \varepsilon \left(1 + \sigma / j\omega\varepsilon\right)$

k_c = Wave number.

* 무손실 ($\sigma = 0$)

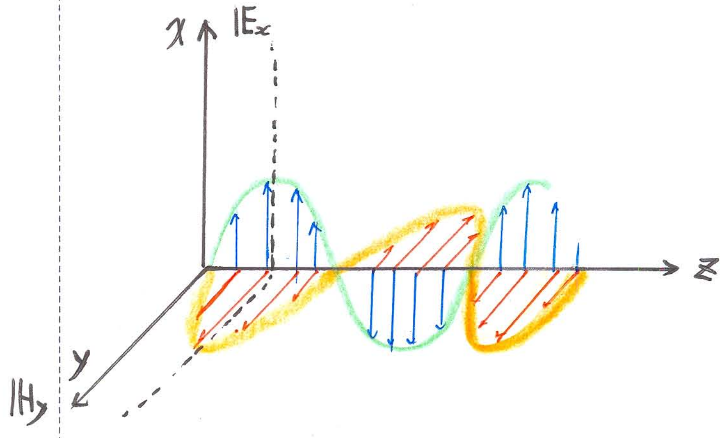
$$\begin{cases} \nabla^2 \mathbb{E} + k^2 \mathbb{E} = 0 \\ \nabla^2 \mathbb{H} + k^2 \mathbb{H} = 0 \end{cases}$$

$k^2 = \omega^2 \varepsilon \mu$

$k = \omega \sqrt{\varepsilon \mu}$

$= \omega / v = \dfrac{2\pi}{\lambda}$

< Wave number.
propagation constant >



$$\frac{\partial^2 E_x}{\partial z^2} = -\mu\varepsilon \frac{\partial^2 E_x}{\partial t^2}$$

순시식

$$* \begin{cases} \dfrac{\partial^2 E_x}{\partial z^2} + k_z^2 E_x = 0 \\ \\ \dfrac{\partial^2 H_y}{\partial z^2} + k_z^2 H_y = 0 \end{cases}$$

$$\begin{cases} \mathbb{E}_x(z) = E_{x0}\, e^{-jk_z \cdot z} \cdot \bar{x} \\ \\ \mathbb{H}_y(z) = H_{y0}\, e^{-jk_z \cdot z} \cdot \bar{y} \end{cases}$$

$$= \frac{1}{\eta} E_{x0}\, e^{-jk_z \cdot z} \cdot \bar{y}$$

where, $\eta = \sqrt{\mu / \varepsilon}$

고유임피던스.

$$\therefore \quad \mathbb{E}_{(z,t)} = R_e\left[\ \mathbb{E}_{(z)} \cdot e^{j\omega t}\ \right]$$

$$= E_{x0} \cdot \cos(\omega t - k_z \cdot z)\ \bar{x}$$

$$\mathbb{H}_{(z,t)} = R_e\left[\ \mathbb{H}_{(z)} \cdot e^{j\omega t}\ \right]$$

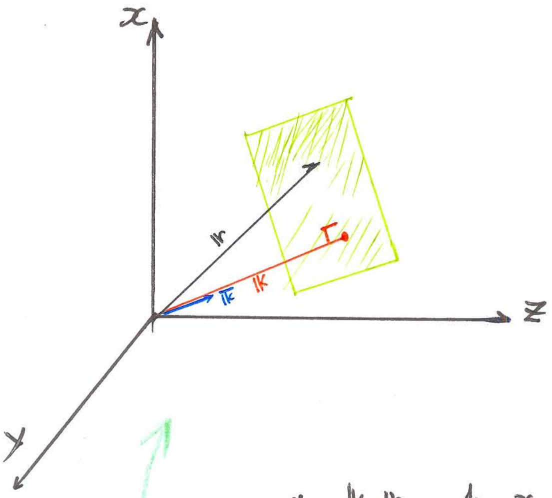
$$= H_{y0} \cdot \cos(\omega t - k_z \cdot z)\ \bar{y}$$

* 이제 $\mathbb{E}$가 $\bar{y}$ 방향 성분을 갖고, $\bar{x}$. $\bar{y}$ 방향으로

propagation 하면, (by 2-14)

$$\mathbb{E}(x,z) = E_0\, e^{-jk_x \cdot x \,-\, jk_z \cdot z} \cdot \bar{y}$$

(교재 그림 2-3)



$$\begin{cases} \mathbb{r} = x \cdot \bar{x} + y \cdot \bar{y} + z \cdot \bar{z} \\ \mathbb{k} = k_x \cdot \bar{x} + k_y \cdot \bar{y} + k_z \cdot \bar{z} \end{cases}$$

* $\mathbb{k} \cdot \mathbb{r} = k_x \cdot x + k_y \cdot y + k_z \cdot z$

$= k_x \cdot x + k_z \cdot z =$ Constant.

(y성분 없다!)

$$\therefore\ \mathbb{E}(x,z) = E_0 \cdot e^{-j\,\mathbb{k} \cdot \mathbb{r}}\ \bar{y}$$

$$= \mathbb{E}_0\, e^{-j\,\mathbb{k} \cdot \mathbb{r}}$$

$$= E_0\, e^{-jk \cdot \bar{\mathbb{k}} \cdot \mathbb{r}} \cdot \bar{y}$$

(식 2-23)

$$\mathbb{H}(x, z) = \frac{1}{-j\omega\mu} \cdot \nabla \times \mathbb{E}$$ 로 부터.

$$= \frac{1}{-j\omega\mu} \nabla \times \left(E_0 \, e^{-jk \cdot \mathbb{k} \cdot \mathbb{r}} \right) \bar{\mathbb{y}}$$

Cross product.

행렬 연산!

$$= \frac{1}{-j\omega\mu} \begin{vmatrix} \bar{\mathbb{x}} & \bar{\mathbb{y}} & \bar{\mathbb{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_0 e^{-jk \mathbb{k} \cdot \mathbb{r}} & 0 \end{vmatrix}$$

$$= \frac{1}{-j\omega\mu} \left[-\frac{\partial}{\partial x} E_0 \, e^{-jk_x \cdot x - jk_z \cdot z} \cdot \bar{\mathbb{x}} + \frac{\partial}{\partial x} E_0 \, e^{-jk_x \cdot x - jk_z \cdot z} \cdot \bar{\mathbb{z}} \right]$$

$$= \frac{1}{-j\omega\mu} \left[-(-jk_z) E_0 \, e^{-jk_x \cdot x - jk_z \cdot z} \cdot \bar{\mathbb{x}} + (-jk_x) E_0 \cdot e^{-jk_x \cdot x - jk_z \cdot z} \cdot \bar{\mathbb{z}} \right]$$

$$= \frac{E_0}{\omega\mu} \left(-k_z \bar{\mathbb{x}} + k_z \cdot \bar{\mathbb{z}} \right) e^{-jk_x \cdot x - jk_z \cdot z}$$

$$= \frac{1}{\omega\mu}\left(k_x \mathbb{Z} - k_z \mathbb{X}\right) \cdot \underline{E_0\, e^{-jk_x\cdot \mathbb{X} - jk_z\cdot \mathbb{Z}}}$$

$$= \frac{1}{\omega\mu}\left(k_x \mathbb{Z} - k_z\cdot \mathbb{X}\right) \cdot \underline{E_y}$$

$$= \frac{1}{\omega\mu}\left(k_x \cdot E_y \cdot \mathbb{Z} - k_z \cdot E_y \cdot \mathbb{X}\right)$$

$$= \frac{1}{\omega\mu}\left(k_x\cdot \mathbb{X} + k_y\cdot \mathbb{Y} + k_z\cdot \mathbb{Z}\right) \times \left(E_x\cdot \mathbb{X} + E_y\cdot \mathbb{Y} + E_z\cdot \mathbb{Z}\right)$$

$*$ where, $k_y = 0 = E_x = E_z$

$$= \frac{1}{\omega\mu}\left(\mathbb{K} \times \mathbb{E}\right)$$

$$= \frac{1}{\omega\mu}\, k\cdot \bar{\mathbb{K}} \times \mathbb{E}$$

$$= \frac{1}{\eta}\, \bar{\mathbb{K}} \times \mathbb{E} \qquad (\text{식 } 2\text{-}25)$$

if. 매질 = free Space

$$\mathbb{H} = \frac{k}{\omega\mu} \cdot \mathbb{k} \times \mathbb{E}$$

$$= \frac{\omega\sqrt{\varepsilon_0\mu_0}}{\omega\mu_0} \mathbb{k} \times \mathbb{E}$$

$$= \sqrt{\frac{\varepsilon_0}{\mu_0}} \mathbb{k} \times \mathbb{E}$$

$$= \frac{1}{\eta_0} \mathbb{k} \times \mathbb{E} = \frac{1}{120\pi} \mathbb{k} \times \mathbb{E}$$

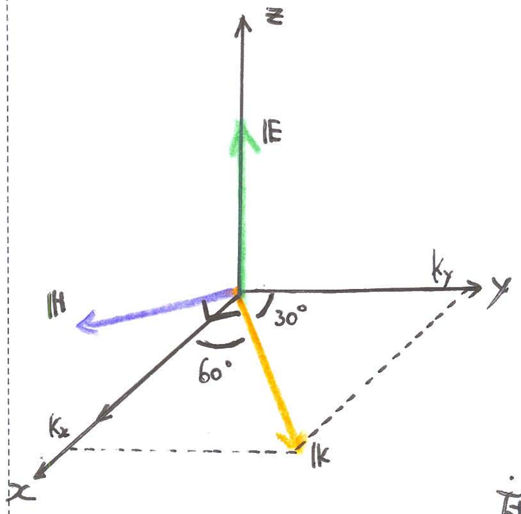
Where, $\eta_0 = 120\pi = \sqrt{\frac{\mu_0}{\varepsilon_0}} \doteqdot 376.7\,\Omega$

Free space Impedence

<예제 2-1>

풀어보세요.

교재 26p. < 예제 2-2 >



전계 E가 z 방향 성분을 가지고,

xy 방향에서 x축으로부터

반시계 방향으로 60° 입사하는 경우.

전계와 자계의 순시값은?

Sol> 먼저 전계부터 보면,

전계강도 E는 z 성분을 갖고 k 방향, 즉 x와 y 방향으로

propagation 하므로,

$$\mathbf{E}(x,y) = E_0 \, e^{-j\mathbf{k}\cdot\mathbf{r}} \, \hat{z}$$

$$= E_0 \, e^{-jk_x \cdot x - jk_y \cdot y} \cdot \hat{z}$$

where, $\mathbf{k} = k_x \cdot \hat{x} + k_y \cdot \hat{y}$

여기서 k_x 와 k_y 를 구해야 함.

$$\begin{cases} k_x = k\cos 60° = k/2 \\ k_y = k\cos 30° = \frac{\sqrt{3}}{2}k \end{cases}$$

$$\therefore \mathbb{E}(x,y) = E_0 \, e^{-j\frac{k}{2}\cdot x - \frac{\sqrt{3}}{2}k\cdot y} \cdot \bar{z}$$

phasor 값.

전체의 순시값은.

$$\mathbb{E}(x.y.t) = R_e\left[E_0 \, e^{-j\frac{k}{2}\cdot x - \frac{\sqrt{3}}{2}k\cdot y} \cdot e^{j\omega t} \cdot \bar{z} \right]$$

$$= E_0 \cos\left(\omega t - \frac{1}{2}k\cdot x - \frac{\sqrt{3}}{2}k\cdot y \right) \bar{z}$$

한편 $\mathbb{H}$ 를 구하면 (by 2-2k)

" $\mathbb{H} = \frac{k}{\omega\mu} \bar{\mathbb{K}} \times \mathbb{E}$ "

$$\mathbb{H}(x.y) = \frac{1}{\omega\mu}\left(k_x \bar{x} + k_y \bar{y} \right) \times \left(E_0 e^{-j\frac{k}{2}x - j\frac{\sqrt{3}}{2}ky} \cdot \bar{z} \right)$$

$$= \frac{1}{\omega\mu}\left(k\cos 60^\circ \bar{x} + k\cos 30^\circ \bar{y} \right) \times \left(E_0 \, e^{-j\frac{k}{2}x - j\frac{\sqrt{3}}{2}ky} \cdot \bar{z} \right)$$

$$= \frac{1}{\omega\mu}\left(\frac{1}{2}k\cdot\bar{x} + \frac{\sqrt{3}}{2}k\bar{y} \right) \times \left(E_0 \, e^{-j\frac{k}{2}x - j\frac{\sqrt{3}}{2}ky} \cdot \bar{z} \right)$$

$$= \frac{k}{\omega\mu}\left[\frac{1}{2} E_0 e^{-j\frac{k}{2}x - j\frac{\sqrt{3}}{2}ky} \cdot (-\hat{y}) + \frac{\sqrt{3}}{2} E_0 e^{-j\frac{k}{2}x - j\frac{\sqrt{3}}{2}k\cdot y} \cdot (\hat{x}) \right]$$

$$\therefore \mathbb{H}(x.y) = \frac{\sqrt{3}k}{2\omega\mu} E_0 e^{-j\frac{k}{2}x - j\frac{\sqrt{3}}{2}k\cdot y} \cdot \hat{x}$$

$$- \frac{k}{2\omega\mu} \cdot E_0 \cdot e^{-j\frac{k}{2}x - j\frac{\sqrt{3}}{2}k\cdot y} \cdot \hat{y}$$

phasor 식.

∴ 자계의 순시식은

$$\mathbb{H}(x.y.t) = \mathrm{Re}\left[\mathbb{H}(x,y) \cdot e^{j\omega t} \right]$$

$$= \frac{\sqrt{3}k}{2\omega\mu} E_0 \cos(\omega t - \tfrac{1}{2}kx - \tfrac{\sqrt{3}}{2}ky) \cdot \hat{x} - \frac{k}{2\omega\mu} E_0 \cos(\omega t - \tfrac{1}{2}kx - \tfrac{\sqrt{3}}{2}ky) \cdot \hat{y}$$

$$= \frac{\sqrt{3}}{2\eta} E_0 \cos(\omega t - \tfrac{1}{2}kx - \tfrac{\sqrt{3}}{2}ky) \cdot \hat{x} - \frac{1}{2\eta} E_0 \cos(\omega t - \tfrac{1}{2}kx - \tfrac{\sqrt{3}}{2}ky) \cdot \hat{y}$$

where, $\frac{1}{\eta} = \frac{k}{\omega\mu} = \sqrt{\frac{\varepsilon}{\mu}}$

$\mathbb{E}(x.y.t)$ 와 동위상.

[2] 손실매질에서 평면파.

$\sigma \neq 0$, $\mathbb{J} = \sigma \mathbb{E}$.

* Maxwell's Eq

$$\begin{cases} \nabla \times \mathbb{E} = -j\omega\mu \mathbb{H} \\ \nabla \times \mathbb{H} = \sigma \mathbb{E} + j\omega\varepsilon \mathbb{E} \end{cases} \qquad (식 2-26)$$

위식에 대한 helmholtz's Eq.

$$\nabla^2 \mathbb{E} + k_c^2 \mathbb{E} = 0$$

where, $k_c^2 = \omega^2 \mu \varepsilon_c$

$$\varepsilon_c = \varepsilon\left(1 + \sigma/_{j\omega\varepsilon}\right)$$

* 무손실 매질 ($\sigma = 0$)

Helmholtz's Eq.

$$\nabla^2 \mathbb{E} + k^2 \mathbb{E} = 0$$

where, $k^2 = \omega^2 \mu \varepsilon$ (식 2-7)

$\therefore k^2 = k_c^2$

= Complex Propagation Constant.

$= -\gamma^2$ (전송선로의 (식 3-21) 과 같다)

$$\therefore \gamma = \alpha + j\beta = j\omega\sqrt{\varepsilon\mu} \cdot \sqrt{1 - j\frac{\sigma}{\omega\varepsilon}} \quad \text{(식 2-28)}$$

여기서 매질 = 손실 매질 이므로,

$\varepsilon = \varepsilon' - j\varepsilon''$ 를 대입.

$$\gamma = j\omega\sqrt{\varepsilon' - j\varepsilon''}\sqrt{1 - j\frac{\sigma}{\omega(\varepsilon' - j\varepsilon'')}}$$

$$= j\omega\sqrt{(\varepsilon' - j\varepsilon'')\mu - j\left(\frac{\cancel{(\varepsilon' - j\varepsilon'')}\mu\sigma}{\omega\cancel{(\varepsilon' - j\varepsilon'')}}\right)}$$

$$= j\omega\sqrt{\mu\varepsilon' - j\mu\varepsilon'' - j\frac{\mu\sigma}{\omega}}$$

$$= j\omega\sqrt{\mu\varepsilon'\left(1 - j\frac{\varepsilon''}{\varepsilon'} - j\frac{\mu\sigma}{\omega\varepsilon'}\right)}$$

$$= j\omega\sqrt{\mu\varepsilon'\left[1 - j\left(\frac{\varepsilon''}{\varepsilon'} + \sigma/\omega\varepsilon'\right)\right]}$$

$$\therefore \gamma = j\omega\sqrt{\mu\varepsilon'\left(1 - j\left(\frac{\omega\varepsilon'' + \sigma}{\omega\varepsilon'}\right)\right)}$$

where, $\dfrac{\omega\varepsilon'' + \sigma}{\omega\varepsilon'}$ = (식 1-56)

= Loss tangent "δ"

δ = delta.

$$\therefore \gamma = j\omega\sqrt{\mu\varepsilon'(1 + \tan\delta)}$$

if. frequency → 높아지면,

μ-wave , $\omega\varepsilon'' \gg \sigma$

$\tan\delta = \omega\varepsilon'' / \omega\varepsilon' = \varepsilon'' / \varepsilon' =$ 허수/실수.

$=$ 식(2-28) 에서 $\sigma / \omega\varepsilon$

where, $\varepsilon' \gg \varepsilon'' \rightarrow \tan\delta = \varepsilon'' / \varepsilon' = 0$

〈 good dielectric material 〉

또는 $\omega\varepsilon \gg \sigma$

$\tan\delta = \sigma / \omega\varepsilon$ 손실은 분극 전류에 의한 것.

"Good dielectric material"

$\omega\varepsilon \ll \sigma$

$\tan\delta = \sigma / \omega\varepsilon$ 손실은 도전전류에 의한 것.

" Good Conductor "

ex. 습기 땅의 경우.

$$\varepsilon_r = 10$$

$$\sigma = 10^{-2} \ (S/m)$$

$$\text{loss tangent} = \sigma/\omega\varepsilon \quad \begin{cases} 1\text{KHz} : 1.8 \times 10^{4} \\ \quad \text{ㄴ 양도체.} \\ 10\text{GHz} : 1.8 \times 10^{-3} \\ \quad \text{ㄴ 유전체.} \end{cases}$$

그러므로 손실이 있는 매질에서

Helmholtz's Eq

$$\begin{cases} \nabla^2 \mathbb{E} - \gamma^2 \mathbb{E} = 0 \\ \nabla^2 \mathbb{H} - \gamma^2 \mathbb{H} = 0 \end{cases} \qquad (\text{식 } 2\text{-}29)$$

if. $\mathbb{E} = E_x \hat{x}$ 이고 진행방향 $\hat{z}$ (by 2-29)

$$\underbrace{\frac{d^2 E_x}{dz^2} - \gamma^2 E_x = 0}$$

$$\therefore E_x(z) = E_0^+ e^{-\gamma z} + E_0^- e^{+\gamma z} \quad (식\ 2\text{-}31)$$

$$= E_0^+ e^{-\alpha z} e^{-j\beta z} + E_0^- e^{+\alpha z} e^{+j\beta z}$$

시간영역 (순시식) 에서,

$$E_x(z, t) = R_e \left[E_0^+ e^{-\alpha z} e^{-j\beta z} e^{j\omega t} + E_0^- e^{+\alpha z} e^{+j\beta z} \cdot e^{j\omega t} \right]$$

$$= \underbrace{E_0^+ e^{-\alpha z}}_{손실} \cos(\omega t - \beta z) + \underbrace{E_0^- e^{+\alpha z}}_{손실} \cos(\omega t + \beta z)$$

자계 H는 (by 2-12a)

$$H_y(z) = \frac{j}{\omega\mu} \cdot \frac{dE_x}{dz}$$

$$= \frac{-j\gamma}{\omega\mu} \left(E_0^+ e^{-\gamma z} - E_0^- e^{+\gamma z} \right)$$

(식 2-33)

where, $\frac{E_x}{H_y} = \frac{j\omega\mu}{\gamma} = \eta$ (식 2-34)

$\therefore \underbrace{\gamma = jk}_{\sigma = 0,\ \alpha = 0}$

$$\therefore \gamma = j\beta = j \cdot \frac{2\pi}{\lambda}$$

$$= j \cdot k$$

즉, $\beta = k = \gamma$

phasor constant

wave number

propagation.

* 정리.

$$
\begin{cases}
\text{손실} \rightarrow \text{Helmholtz's Eq} \begin{cases} \mathbf{E} \\ \mathbf{H} \end{cases} \\[4em]
\text{무손실} \rightarrow \text{Helmholtz's Eq} \begin{cases} \mathbf{E} \\ \mathbf{H} \end{cases}
\end{cases}
$$