# 5 장 도파관과 동축선로

▶ 초기 마이크로파는 마이크로파 전력을 저손실로 전송할 수 있는 도파관과 전송선로의 개발에 관심

- 초기 마이크로파 시스템은 전송 매체로서 도파관과 동축선로에 의존

• 도파관

- 부피가 크고 고비용인 단점이 있으나 고전력 및 저손실의 장점을 가짐

• 동축선로

- 다양한 마이크로파 소자를 만들기 위한 전송선로로서는 부적합하지만 높은 대역폭과 이용에 편리한 장점

• 평판형 전송전로는 스트립 선로, 마이크로 스트립 선로, 슬랏(slot) 선로, 코플래너 선로 등 다양한 구조가 존재하며 마이크로파집적회로(MIC : microwae Integrated Circuits) 구현이 용이

## 5.1 기초방정식(basic Equation)

## (전자파 모드; electormagnetic wave mode)

: Maxwell's eq의 정석적 해석임

※ wave= sinusoidal wave

= propagation direction $=\bar{z}$

= propagation constant$=\beta$

매질 loss =0, $\sigma=0$ (무전원)

도파관 loss =0라 가정

지금까지는

$\mathrm{E}=E_x\bar{\mathrm{x}}$, $\mathrm{H}=H_y\bar{\mathrm{y}}$ 인 경우를 고려하였음.

지금부터는

$\mathrm{E}=E_x\bar{\mathrm{x}}+E_y\bar{\mathrm{y}}+E_z\bar{\mathrm{z}}$ 인 경우를 고려

$\mathrm{H}=H_x\bar{\mathrm{x}}+H_y\bar{\mathrm{y}}+H_z\bar{\mathrm{z}}$

즉, $\mathrm{E}=E_0 e^{j\mathrm{k}\cdot\mathrm{r}}$ 인 경우임

• Maxwell's Eq(1-14, 15)

$$\nabla\times\mathrm{E}=-j\omega\mathrm{B}=-j\omega\mu\mathrm{H}$$

$$=\begin{vmatrix}\bar{\mathrm{x}} & \bar{\mathrm{y}} & \bar{\mathrm{z}}\\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z}\\ E_x & E_y & E_z\end{vmatrix}=j\omega\mu(H_x\bar{\mathrm{x}}+H_y\bar{\mathrm{y}}+H_z\bar{\mathrm{z}})$$

$$\therefore \quad \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \qquad (5\text{-}1)$$
$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y$$
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z$$

$$\nabla \times \mathrm{H} = j\omega \mathrm{D} = j\omega\epsilon \mathrm{E}$$
$$= \begin{vmatrix} \overline{\mathrm{x}} & \overline{\mathrm{y}} & \overline{\mathrm{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = j\omega\epsilon(E_x\,\overline{\mathrm{x}} + E_y\,\overline{\mathrm{y}} + E_z\,\overline{\mathrm{z}})$$

$$\therefore \quad \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = -j\omega\epsilon E_x \qquad (5\text{-}1)$$
$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = -j\omega\epsilon E_y$$
$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -j\omega\epsilon E_z$$

where, E, H는 $+z$ 방향으로 진행

$$\mathrm{E}(x, y, z) = [\mathrm{e}_t(x, y) + e_z(x, y)\overline{\mathrm{z}}]\, e^{-j\beta z} \qquad (5\text{-}2a)$$

where, $\beta$ ; 무손실이므로

$\mathrm{e}_t(x, y)$ : $+z$와 직각 평면상의 횡방향의 전계

(transverse electric filed) → $e_x(x, y)\overline{\mathrm{x}} + e_y(x, y)\overline{\mathrm{y}}$

$e_z(x, y)\overline{\mathrm{z}}$ : $+z$로 진행하는 전계

$\Rightarrow e_x(x, y)\overline{\mathrm{x}} + e_y(x, y)\overline{\mathrm{y}} + e_z(x, y)\overline{\mathrm{z}}$

$$\mathrm{H}(x, y, z) = [\mathrm{h}_t(x, y) + h_z(x, y)\overline{\mathrm{z}}]\, e^{-j\beta z} \qquad (5\text{-}2b)$$

where, $\mathrm{h}_t(x, y)$ : $+z$와 직각 평면상의 횡방향의 자계

(transverse magnetic filed)

그러므로 전계와 자계의 각 방향별 성분은

$$\therefore \quad E_x(x, y, z) = e_x(x, y)\, e^{-j\beta z} \qquad (5\text{-}3)$$
$$E_y(x, y, z) = e_y(x, y)\, e^{-j\beta z}$$
$$E_z(x, y, z) = e_z(x, y)\, e^{-j\beta z}$$
$$H_x(x, y, z) = h_x(x, y)\, e^{-j\beta z}$$
$$H_y(x, y, z) = h_y(x, y)\, e^{-j\beta z}$$
$$H_z(x, y, z) = h_z(x, y)\, e^{-j\beta z}$$

식 (5-3)을 식 (5-1)에 대입

$$\frac{\partial\, e_z\, e^{-j\beta z}}{\partial y} - \frac{\partial\, e_y\, e^{-j\beta z}}{\partial z} = -j\omega\,\mu\, h_x\, e^{-j\beta z} \qquad \text{(5-4a)}$$

$$\frac{\partial\, e_z\, e^{-j\beta z}}{\partial y} - (-j\beta)\, e_y\, e^{-j\beta z} = -j\omega\,\mu\, h_x\, e^{-j\beta z}$$

$$\frac{\partial\, e_z}{\partial y} + j\beta\, e_y = -j\omega\,\mu\, h_x$$

where, 무손실인 경우 : $\frac{\partial}{\partial z} = -j\beta$

손실인 경우 $e^{-\gamma z}$이므로 $\frac{\partial}{\partial z} = -j\gamma$

$$-j\beta\, e_x - \frac{\partial\, e_z}{\partial x} = -j\omega\,\mu\, h_y \qquad \text{(5-4b)}$$

$$\frac{\partial e_y}{\partial x} - \frac{\partial\, e_x}{\partial y} = -j\omega\,\mu\, h_z \qquad \text{(5-4c)}$$

$$\frac{\partial\, h_z}{\partial y} + j\beta\, h_y = j\omega\,\epsilon\, e_x \qquad \text{(5-4d)}$$

$$-j\beta\, h_x - \frac{\partial\, h_z}{\partial x} = j\omega\,\epsilon\, e_y \qquad \text{(5-4b)}$$

$$\frac{\partial h_y}{\partial x} - \frac{\partial\, h_x}{\partial y} = j\omega\,\epsilon\, e_z \qquad \text{(5-4c)}$$

$e_x$, $e_y$, $h_x$, $h_y$ 로 정리하면

$$(c) \quad e_x = \frac{-j}{k_c^2}\left(\beta\frac{\partial\, e_z}{\partial\, x} + \omega\mu\frac{\partial\, h_z}{\partial y}\right) \qquad \text{(5-5)}$$

$$(d) \quad e_y = \frac{j}{k_c^2}\left(-\beta\frac{\partial\, e_z}{\partial\, y} + \omega\mu\frac{\partial\, h_z}{\partial x}\right)$$

$$(a) \quad h_x = \frac{j}{k_c^2}\left(\omega\,\epsilon\,\frac{\partial\, e_z}{\partial\, y} - \beta\frac{\partial\, h_z}{\partial x}\right)$$

$$(b) \quad h_y = \frac{-j}{k_c^2}\left(\omega\,\epsilon\,\frac{\partial\, e_z}{\partial\, x} + \beta\frac{\partial\, h_z}{\partial y}\right)$$

where, $k_c^2 = k^2 - \beta^2$

$k^2 = \omega^2 \epsilon\, \mu$

식 (5-5)에서 $e_z$, $h_z$만 알면 나머지 성분($e_x$, $e_y$, $h_x$, $h_y$ ) 파악 가능

그러므로 $e_z$, $h_z$를 구하기 위해 helmholtz's Eq를 이용

$$\nabla^2 e_z + k^2 e_z = 0$$

$$\frac{\partial^2 e_z}{\partial x^2} + \frac{\partial^2 e_z}{\partial y^2} + \frac{\partial^2 e_z}{\partial z^2} = -k^2 e_z$$

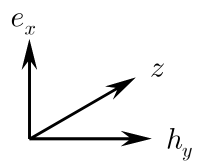
$$\frac{\partial}{\partial z} = -j\beta \quad \rightarrow \quad \frac{\partial^2}{\partial z^2} = \beta^2$$

$$\therefore \quad \frac{\partial^2 e_z}{\partial x^2} + \frac{\partial^2 e_z}{\partial y^2} = -\frac{\partial^2 e_z}{\partial z^2} - k^2 e_z$$
$$= \beta^2 e_z - k^2 e_z$$
$$= (\beta^2 - k^2) e_z$$
$$= -k_c^2 e_z$$

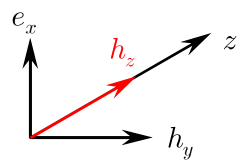
$$\therefore \quad \frac{\partial^2 h_z}{\partial x^2} + \frac{\partial^2 h_z}{\partial y^2} = -k_c^2 h_z$$

위식을 이용하여 $e_z$, $h_z$를 구한다.

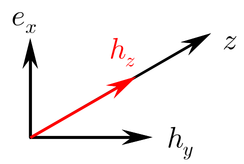
where, $e_z = h_z = 0$ : TEM




$e_z = 0$, $h_z \neq 0$ : TE, Hwave



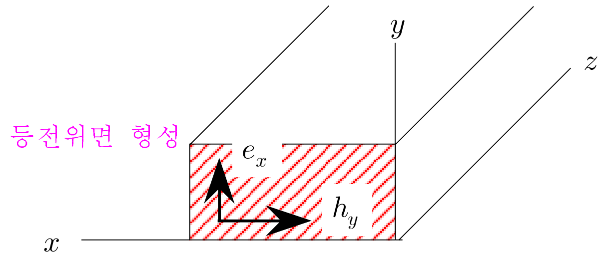

$e_z \neq 0$, $h_z = 0$ : TM, Ewave



(1) TEM 모드(Transverse Electric and Magnetic mode)

$e_z = h_z = 0$ → (5-5c)

$$e_x = \frac{-j}{k_c^2}\left(\beta \frac{\partial 0}{\partial x} + \omega\mu \frac{\partial 0}{\partial y}\right)$$




$\therefore \; k_c^2 = 0$ 이여야

$= k^2 - \beta^2$

$\therefore \; k^2 = \beta^2 = \omega^2 \epsilon \mu$

$\therefore \; k = \omega\sqrt{\epsilon\mu} = \beta = \frac{2\pi}{\lambda}$ (무손실의 경우)

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{\epsilon\mu}} = c$$

한편, $\nabla \cdot \mathrm{E} = \frac{\rho}{\epsilon_0}$ → $\nabla \cdot \mathrm{E} = 0$

$$= \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y}$$

$$\mathrm{E} = -\nabla V = -\left(\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}\right)$$

즉, TEM의 전계분포 = 정전계 전계분포

그러므로 TEM → waveguide로 전송

→ 도파관(도체) 벽 → 등전위면형성 ⇒ energy 전송불가

(2) TE 모드(Transverse Electric mode) ; H wave

$e_z = 0, \quad h_z \neq 0$ → (5-5)

$$e_x = \frac{-j}{k_c^2}\ \omega\mu \frac{\partial h_z}{\partial y} \quad (5\text{-}5)'$$

$$e_y = \frac{j}{k_c^2}\ \omega\mu \frac{\partial h_z}{\partial x}$$

$$h_x = \frac{-j}{k_c^2}\ \beta \frac{\partial h_z}{\partial x}$$

$$h_y = \frac{-j}{k_c^2} \beta \frac{\partial h_z}{\partial y}$$

where, $\frac{e_x}{h_y} = \frac{\omega\mu}{\beta} = Z_{TE} = \frac{-e_y}{h_x} = \frac{j\omega\mu}{\gamma}$ (5-25)

= wave impedance

(3) TM 모드(Transverse Magnetic mode) ; E wave

$e_z \neq 0, \quad h_z = 0$ → (5-5)

$$e_x = \frac{-j}{k_c^2}\ \beta \frac{\partial e_z}{\partial x} \quad (5\text{-}5)''$$

$$e_y = \frac{-j}{k_c^2}\ \beta \frac{\partial e_z}{\partial y}$$

$$h_x = \frac{j}{k_c^2}\ \omega\epsilon \frac{\partial e_z}{\partial y}$$

$$h_y = \frac{-j}{k_c^2}\ \omega\epsilon \frac{\partial e_z}{\partial x}$$

where, $\frac{e_x}{h_y} = \frac{\beta}{\omega\mu} = Z_{TM} = \frac{-e_y}{h_x} = \frac{\gamma}{j\omega\mu}$ (5-43)

= wave impedance

# 5-2. 구형도파관(Rectangular Waveguide)

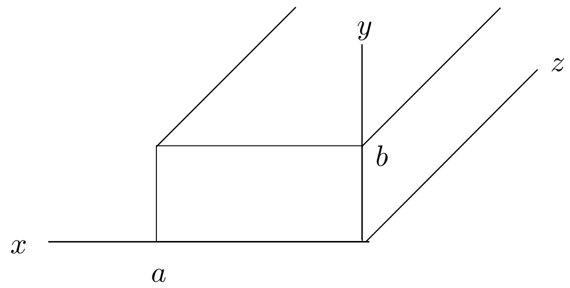
• 마이크로파 신호를 전송하는 전송시스템 중 가장 일찍 사용된 형태

- 결합기, 검파기, 분리기(isolator), 감쇠기, slot line(슬롯선로)에 사용

• 1GHz ~ 220GHz 범위에 사용

## [1] TE mode(H wave) ; $e_z = 0$

식 (5-5)를 조정




$$e_x = \frac{-j}{k_c^2}\ \omega\mu \frac{\partial h_z}{\partial y}$$

$$e_y = \frac{j}{k_c^2}\ \omega\mu \frac{\partial h_z}{\partial x}$$

$$h_x = \frac{-j}{k_c^2}\ \beta \frac{\partial h_z}{\partial x}$$

$$h_y = \frac{-j}{k_c^2} \beta \frac{\partial h_z}{\partial y}$$

where, $k_c^2 = k^2 - \beta^2$

$h_z(x, y)$는 helmholtz's Eq (2-8)를 만족해야 하므로

$$\frac{\partial^2 h_z}{\partial x^2} + \frac{\partial^2 h_z}{\partial y^2} = -k_c^2 h_z \qquad (5\text{-}12)$$

where, $h_z = X(x)\, Y(y)\, e^{-j\beta z}$

$$X''\, Y + X\, Y'' = -k_c^2\, X\, Y$$

양변을 $X\,Y$로 나누면

$$X'' \frac{1}{X} + \frac{1}{Y}\ Y'' = -k_c^2 \qquad (5\text{-}14)$$

↓ ↓ ↓

$X$만의 함수 $Y$만의 함수 상수

$$\therefore \quad \frac{1}{X}\frac{d^2 X}{dx^2} = -k_x^2 \quad ;\ x\text{방향 파수} \qquad (5\text{-}15a)$$

$$\frac{1}{Y}\frac{d^2 X}{dy^2} = -k_y^2 \quad ;\ y\text{방향 파수} \qquad (5\text{-}15b)$$

$$\therefore \quad k_x^2 + k_y^2 = k_c^2 \qquad (5\text{-}16)$$

$\therefore \quad \frac{1}{X}\frac{d^2X}{dx^2} = -k_x^2$ 의 해(solution)

$$X = Ae^{jk_x x} + Be^{-jk_x x}$$
$$= A\cos k_x x + B\sin k_x x$$

$\frac{d^2X}{dx^2} + k_x^2 X = 0$ 의 form

$\frac{\partial^2 E_x}{\partial z^2} + k_z^2 E_x = 0$ 일 때

$E_x = E_{x0}\, e^{-jk_z z}$

마찬가지로

$$Y = C\cos k_y y + D\sin k_y y$$

한편 (5-5)'에서 보면

$e_x \propto \frac{\partial h_z}{\partial y}$ 이므로

$$\frac{\partial h_z}{\partial y} = \frac{\partial}{\partial y}(XYe^{-j\beta z}) \qquad (5\text{-}18a)$$
$$= \frac{\partial}{\partial y}[(A\cos k_x x + B\sin k_x x)(C\cos k_y y + D\sin k_y y)\, e^{-j\beta z}]$$
$$= (A\cos k_x x + B\sin k_x x)(-k_y C\sin k_y y + k_y D\cos k_y y)\, e^{-j\beta z}$$

where, 경계조건

$e_x(x, y) = 0 \quad ; \ y = 0,\ b$에서 이므로

① $y = 0$에서 $e_x = 0$일려면

$-k_y C\sin k_y 0 = 0$

$k_y D\cos k_y 0 = k_y D \quad \Rightarrow \quad \therefore\ D = 0$

② $y = b$에서 $e_x = 0$일려면

$-k_y C\sin k_y b = 0$ 이여야 하므로

$$\therefore \quad k_y b = n\pi\ (n = 0, 1, 2, \cdots) \quad \Rightarrow \quad \therefore \quad k_y = \frac{n\pi}{b}$$

또한 $e_y \propto \frac{\partial h_z}{\partial x}$ 이므로

$$\frac{\partial h_z}{\partial x} = \frac{\partial}{\partial x}(XYe^{-j\beta z}) \qquad (5\text{-}18b)$$
$$= \frac{\partial}{\partial x}[(A\cos k_x x + B\sin k_x x)(C\cos k_y y + D\sin k_y y)\, e^{-j\beta z}]$$
$$= (-k_x A\cos k_x x + k_y B\sin k_x x)(C\sin k_y y + D\cos k_y y)\, e^{-j\beta z}$$

where, 경계조건

$e_y(x, y) = 0 \quad ; \ x = 0,\ a$에서 이므로

① $x = 0$에서 $e_y = 0$일려면

$$-k_x\ A\sin k_x\,0 = 0$$
$$k_x\,B\cos k_x\,0 = k_x\,B \quad \Rightarrow \quad \therefore\ B = 0$$

② $x = a$에서 $e_y = 0$일려면

$-k_x\ A\sin k_y\ a = 0$ 이여야 하므로

$$\therefore\quad k_x a = m\pi\,(m = 0,\,1,\,2,\cdots) \qquad \Rightarrow \quad \therefore\quad k_x = \frac{m\pi}{s}$$

그러므로 경계조건에 맞는 solution은

$$X = A\cos k_x\,x$$
$$Y = C\cos k_y\,y$$

where, $k_x = \dfrac{m\pi}{a}$, $k_y = \dfrac{n\pi}{b}$

$$\begin{aligned}\therefore\quad h_z &= X(x)\,Y(y)\,e^{-j\beta z} \\ &= A\,C\cos k_x\,x\cos k_y\,y\,e^{-j\beta z} \\ &= H_{mn}\cos\frac{m\pi x}{a}\ \cos\frac{n\pi y}{b}\ e^{-j\beta_{mn}z} \\ &= H_z\,(x,\,y,\,z)\end{aligned} \qquad (5\text{-}20)$$

if, $m = n = 0$

$h_z = H_{mn} = constant$

$\therefore\quad e_x = e_y = h_x = h_y = 0$ : 무의미

$\therefore\quad TE_{00}$ mode는 없다.

$\therefore\quad TE$ mode는 $TE_{01}$, $TE_{10}$ mode부터 시작(최조모드)

그러므로 식 (5-5)와 식 (5-20)을 이용하면 $TE_{mn}$ 모드의 횡전자계 성분을 구할 수 있음.

즉,

$$E_x = \frac{j\,\omega\,\mu\,n\pi}{k_c^2\,b}\,H_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\ e^{-j\beta_{mn}z} \qquad (5\text{-}21)$$
$$E_y = \frac{-j\,\omega\,\mu\,m\pi}{k_c^2\,a}\,H_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\ e^{-j\beta_{mn}z}$$
$$H_x = \frac{j\,\beta_{mn}\,m\pi}{k_c^2\,a}\,H_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\ e^{-j\beta_{mn}z}$$
$$H_y = \frac{j\,\beta_{mn}\,n\pi}{k_c^2\,b}\,H_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\ e^{-j\beta_{mn}z}$$

where, $k_c^2 = k_2 - \beta^2$

$$\begin{aligned}\therefore\quad \beta_{mn} &= \sqrt{k^2 - k_c^2} = \sqrt{k^2 - (k_x^2 + k_y^2)} \\ &= \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}\end{aligned} \qquad (5\text{-}22)$$

if, 매질 loss $\neq 0$

$$e^{-j\beta_{mn}z} \rightarrow \quad e^{-\gamma_{mn}z}$$

[2] TM mode(E wave) ; $h_z = 0$

식 (5-40)유도 Repot