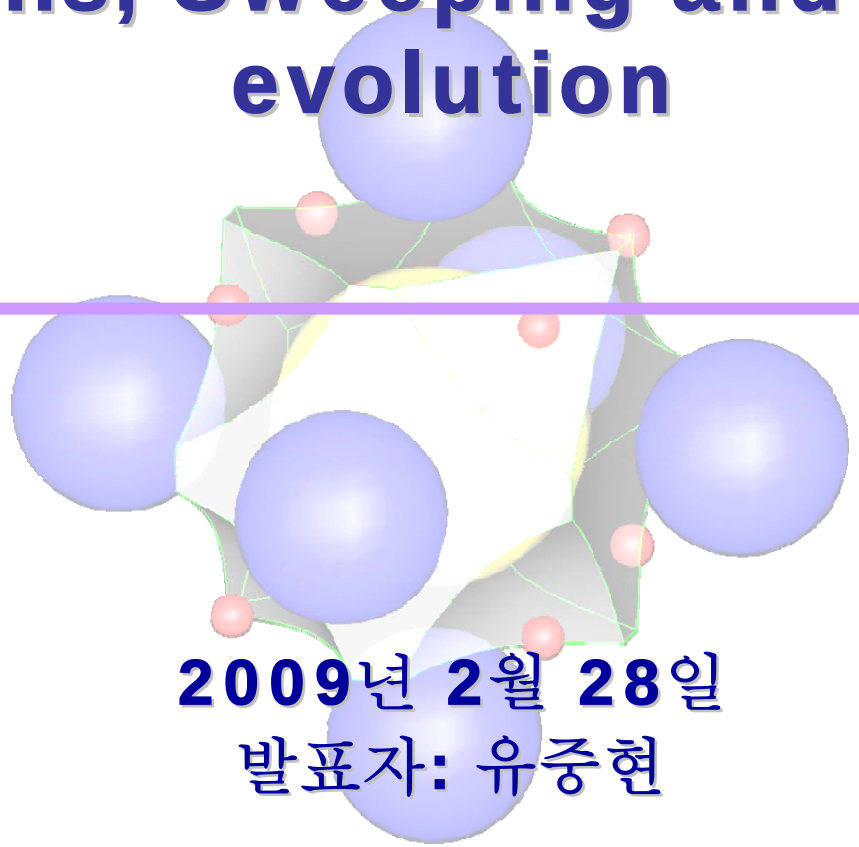
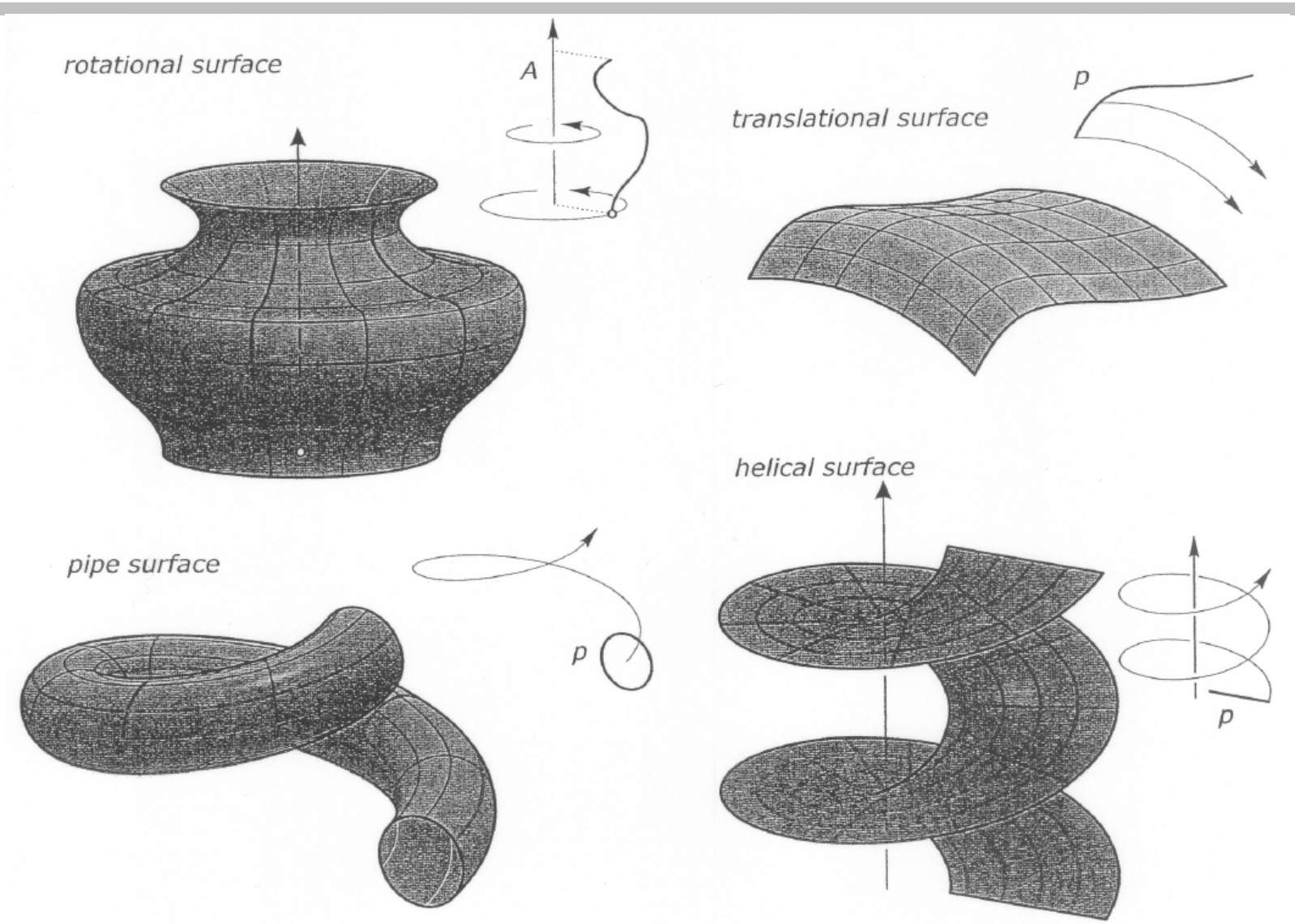
Architectural geometry: Motions, Sweeping and Shape evolution



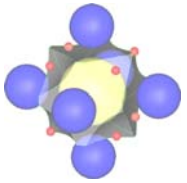
2009년 2월 28일

발표자: 유중현

Surfaces via motions



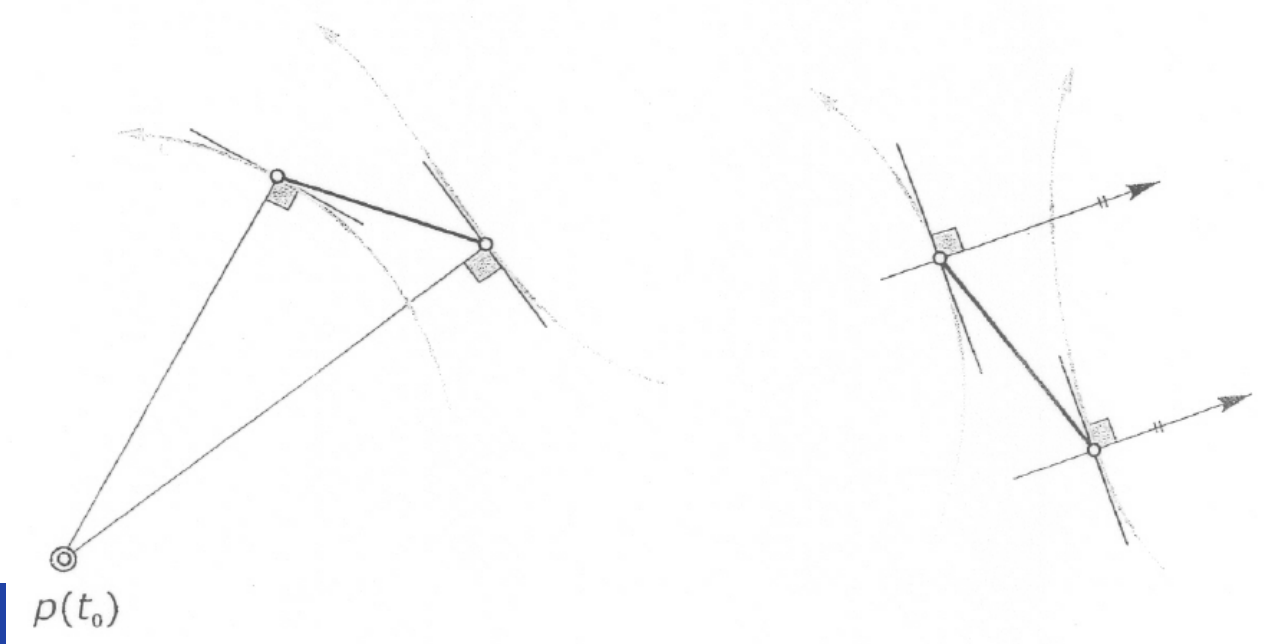
Kinematic geometry



- (Geometry of motions)
- Basics for understanding of surface generation via motions
- Design and functionality can not be fully separated
  - Mechanical design
  - Architectural design
- Various surface classes are based on kinematic geometry
  - Surfaces via motion of profile curves
  - Surfaces via motion of profile curves which changes their shapes

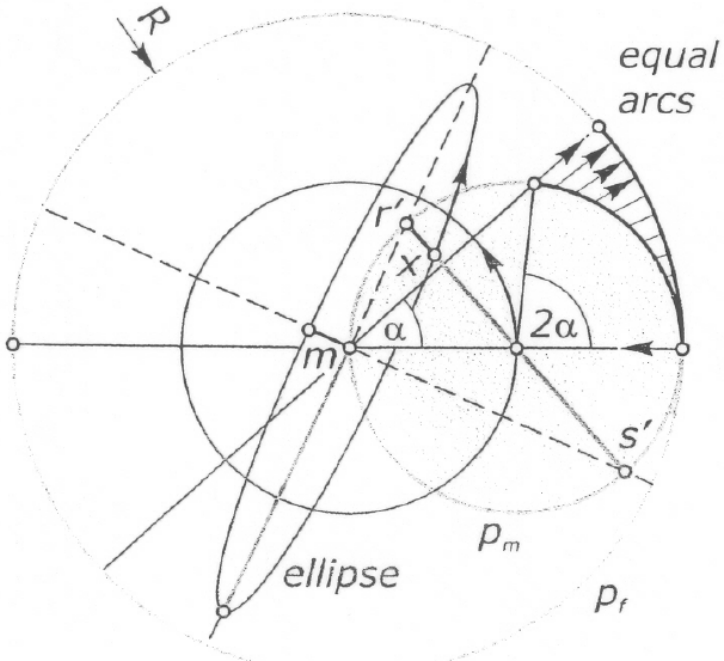
Motions in a plane

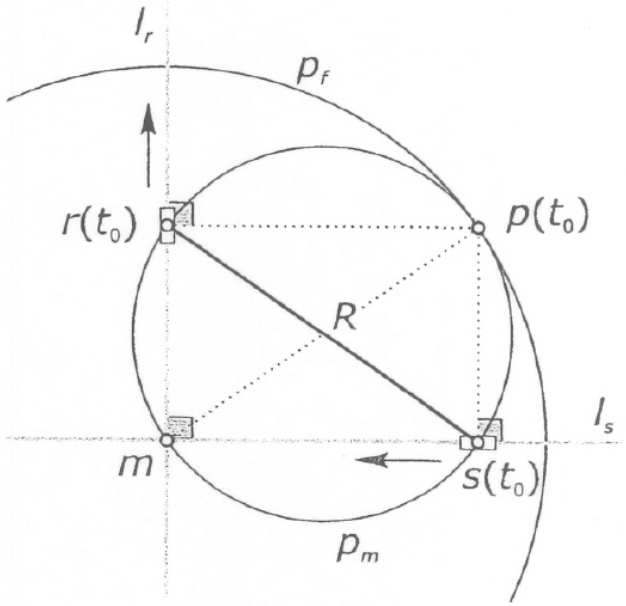
- **Moving system *M* and fixed system *F* under a continuous motion**
  - **M(t): position of M in F at time t**
  - **Path (or trajectory)**
  - **Path tangent / normal**
  - **Pole p(t) at time t**
  - **Polehode: a set of poles in a system M or F**

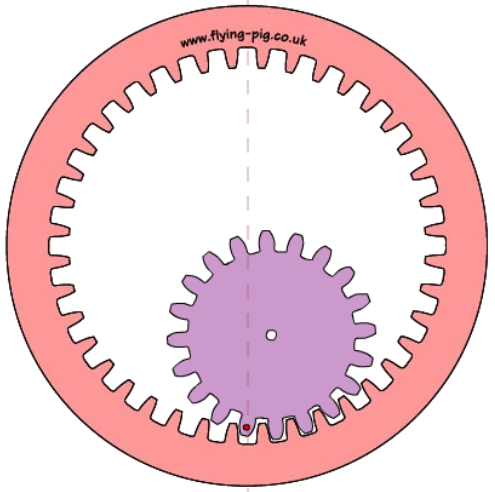


Motions in a plane

- **Cardan motion**
  - **Moving line segment with two points being in a constant speed**
  - **The rolling motion of a circle p_m in a circle p_f**

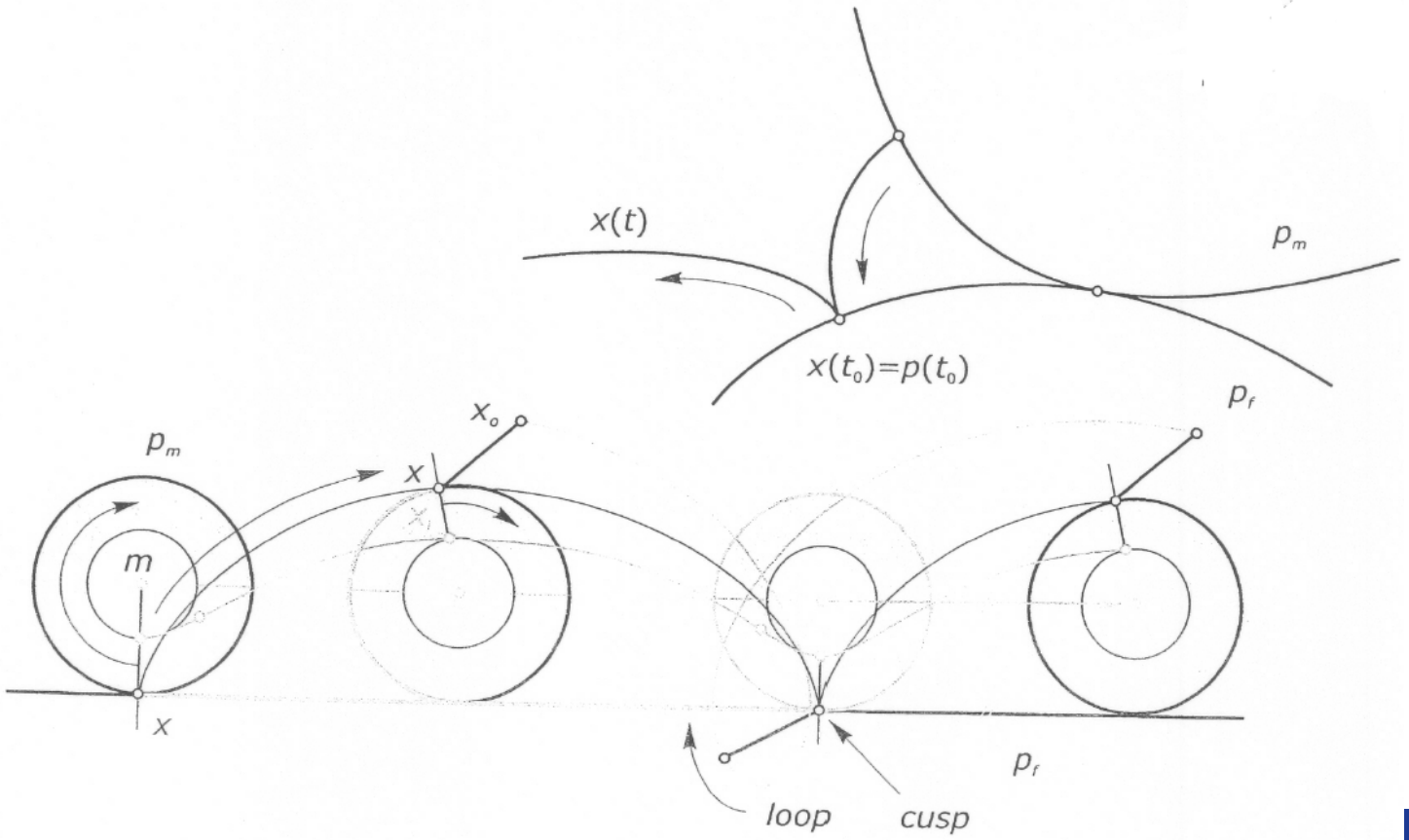






Motions in a plane

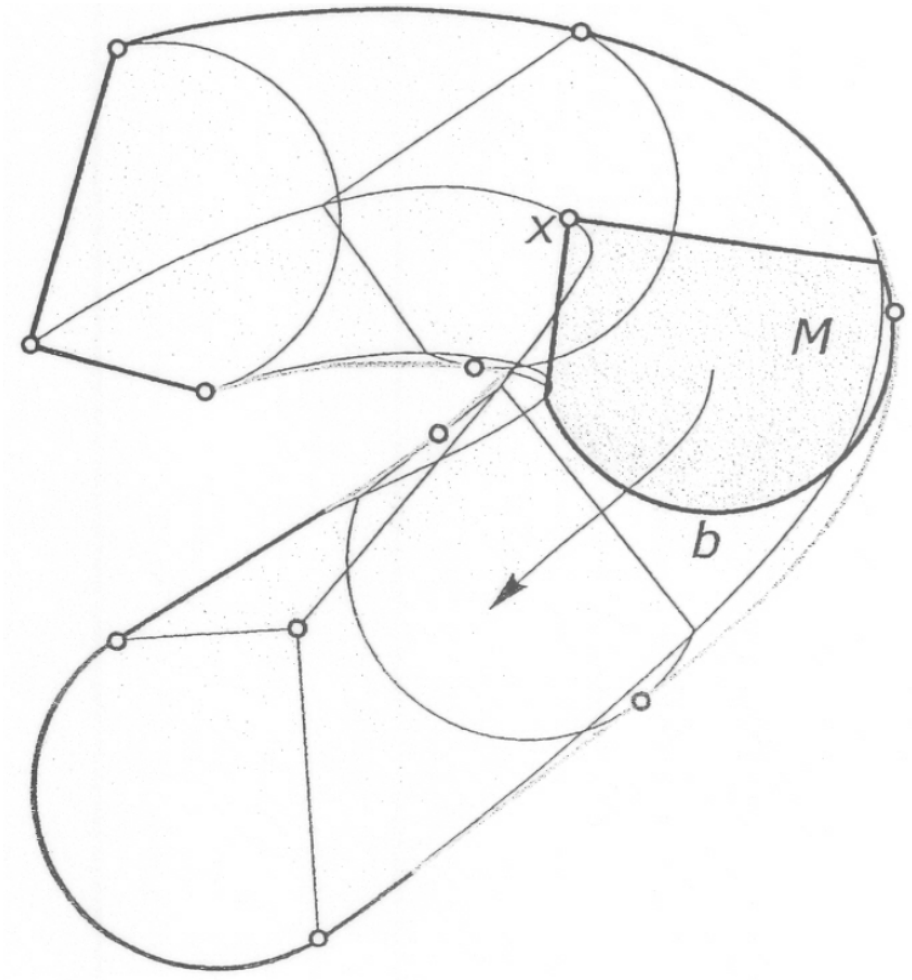
- **The rolling motion of a circle p_m on a straight line p_f**
  - **Cycloids**
  - **Cusps and loops**



Motions in a plane

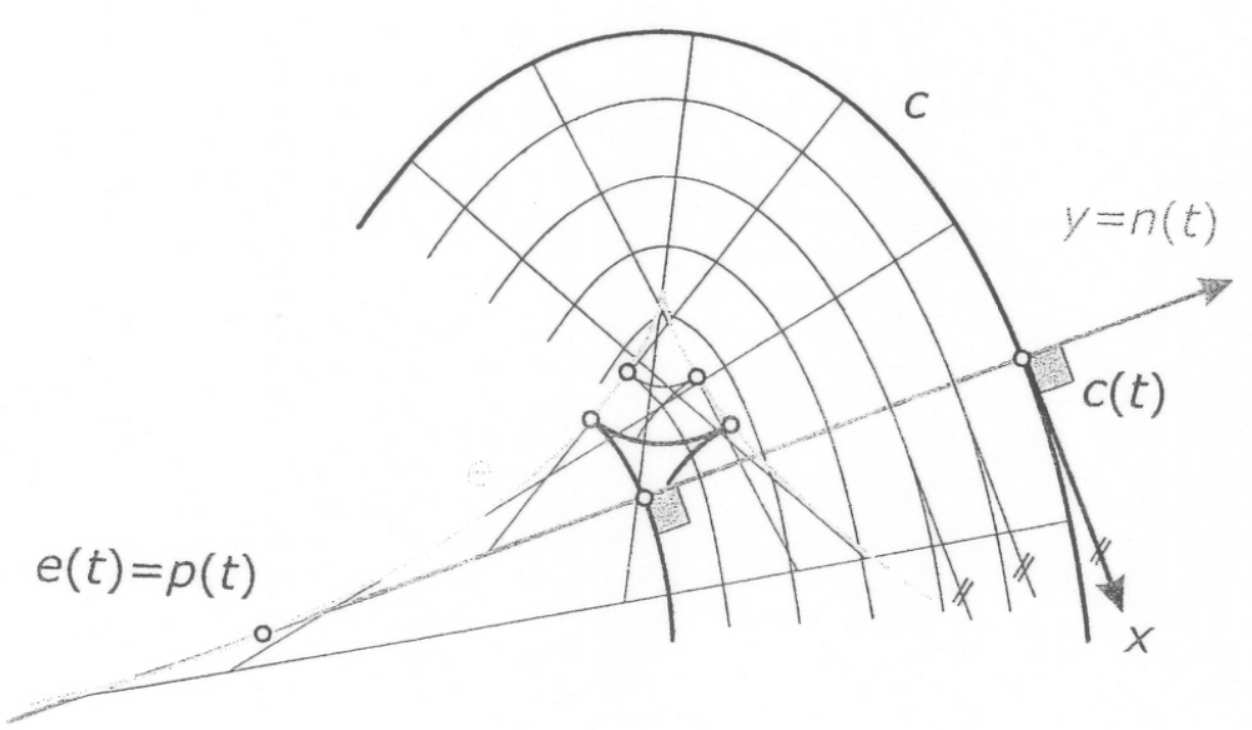
- **Swept area** consists of two parts:
  - **Trajectory of a corner** of the object M
  - **Envelopes of boundary curves** of M

- **Application**
  - An interference free motion
  - Collision free path
    - Robotics

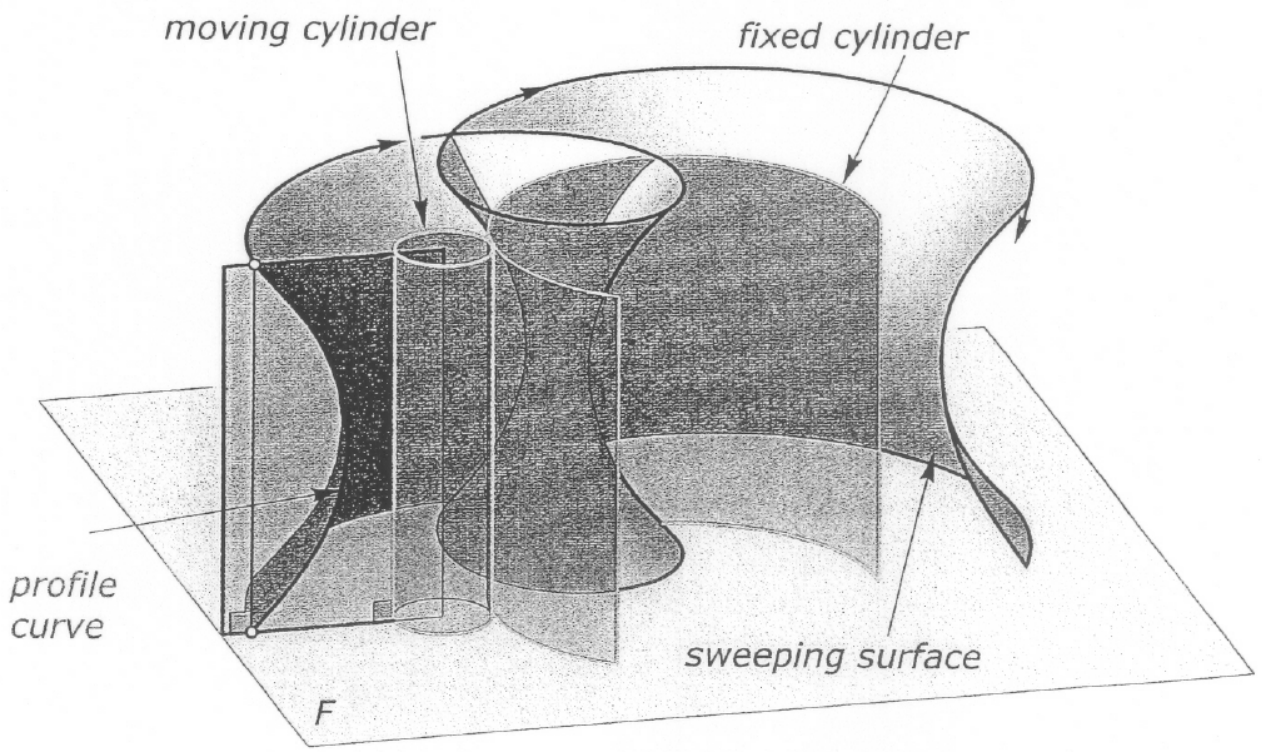


Motions in a plane

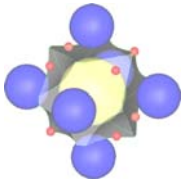
- **Motion of Frenet frame along a planar curve c**
  - **Fixed polhode: evolue of c**
  - **Moving polhode: y(t) (curve normal)**



Spatial motions

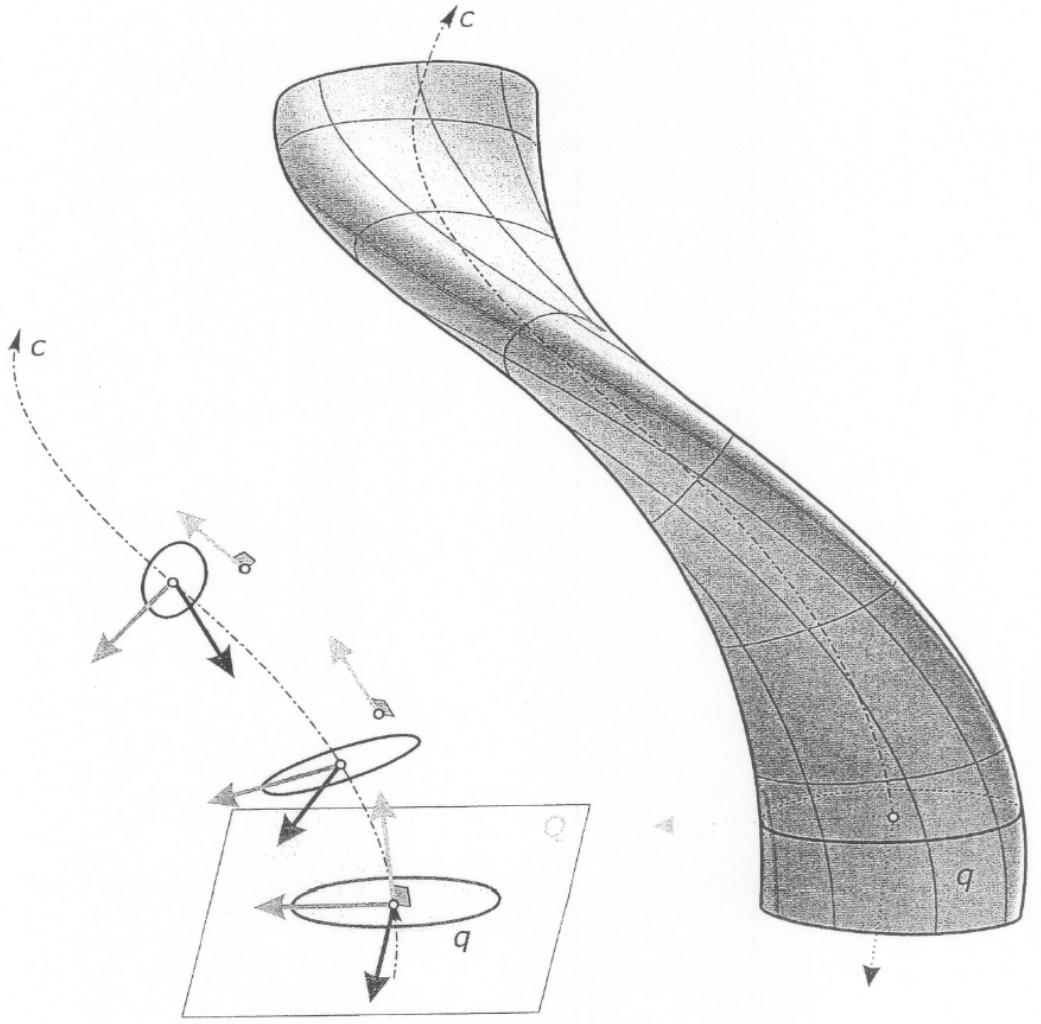


Spatial motions



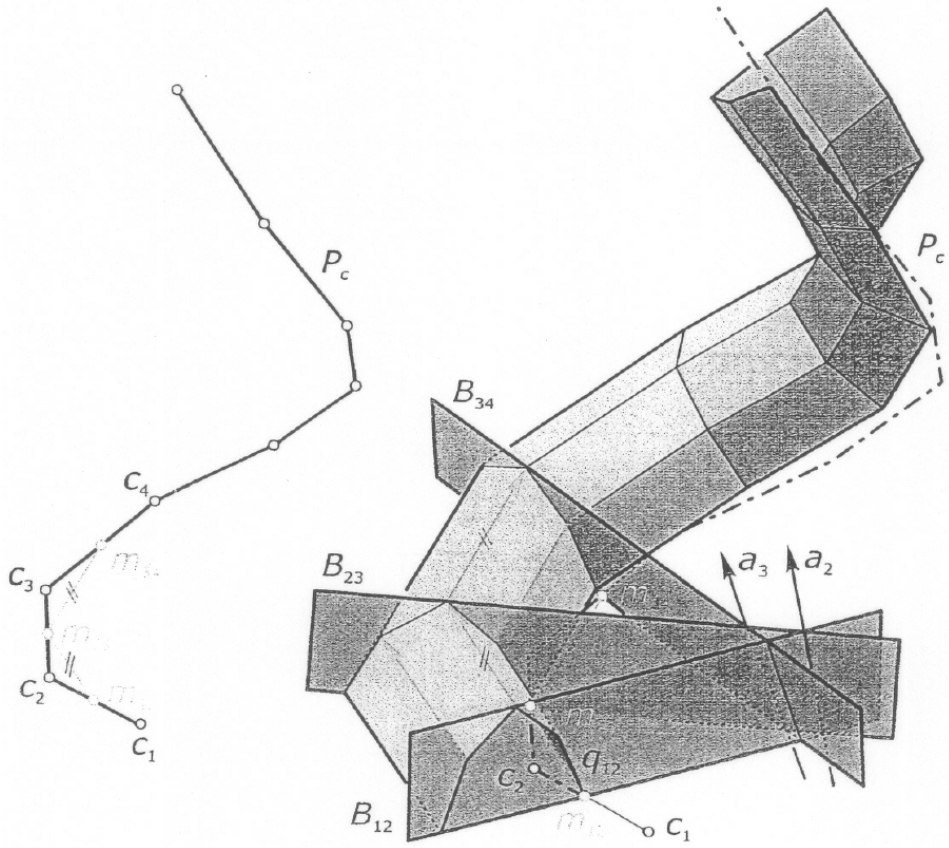
Sweeping

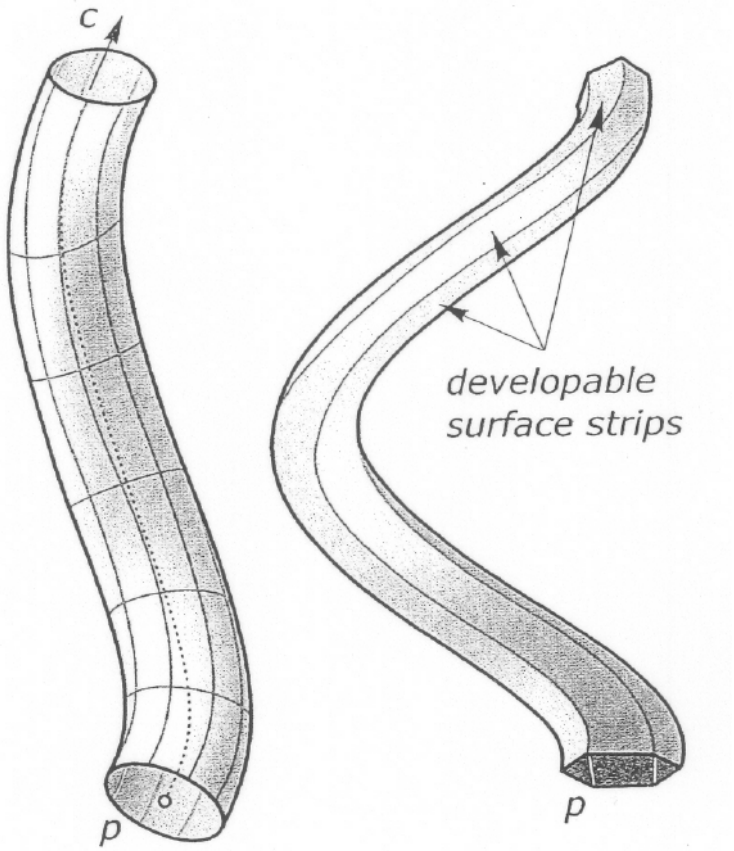
- Path c, source q
- Degree of freedom: rotation about the tangent of c



Sweeping

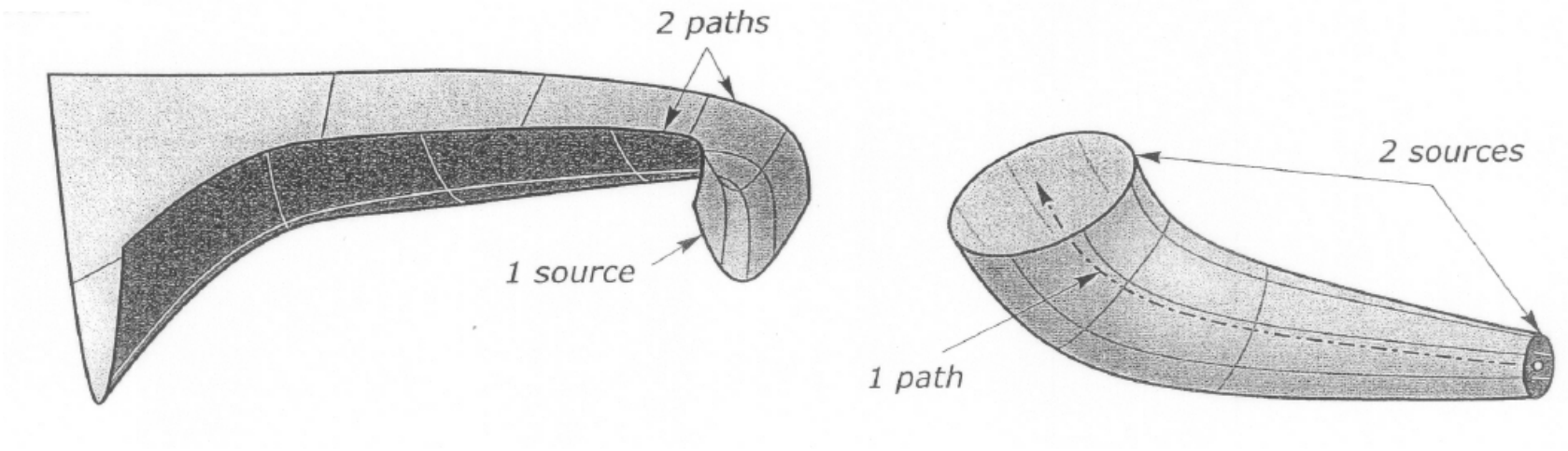
- **Rotation minimizing frame**



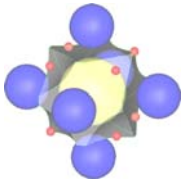


Sweeping

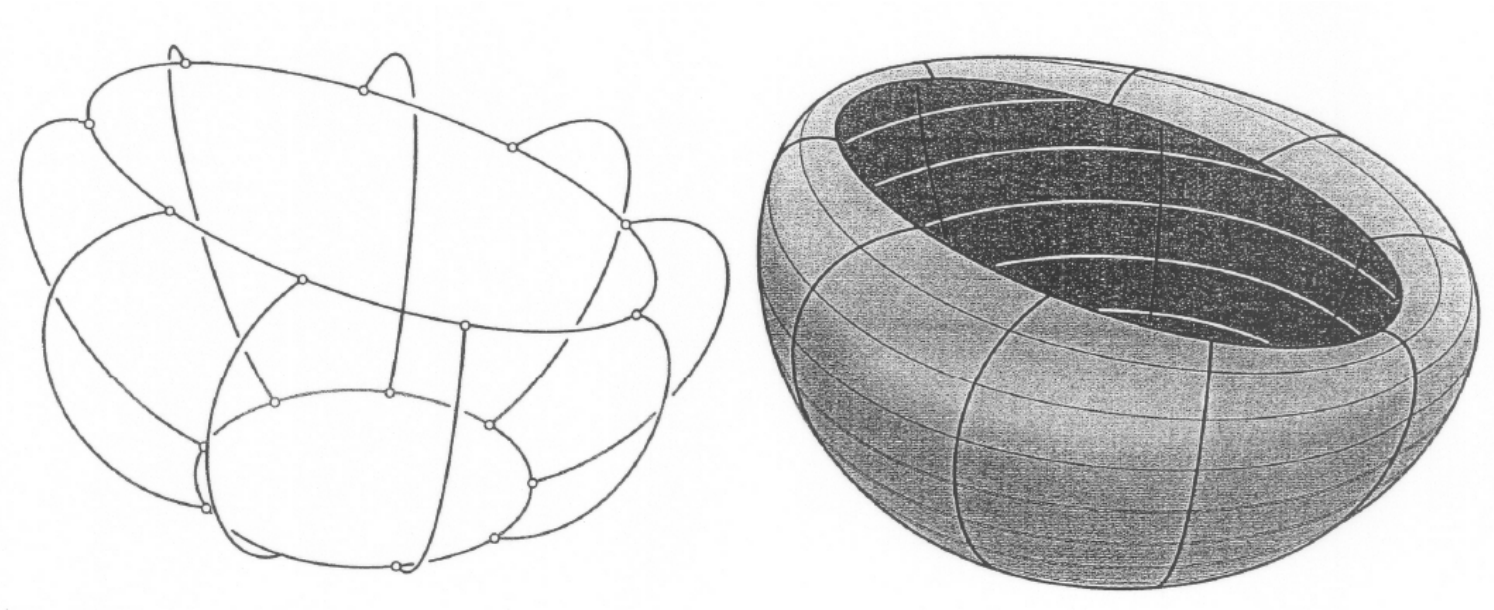
- **Sweeping with several sources and paths**



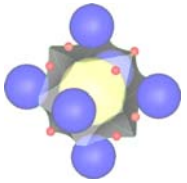
Skinning



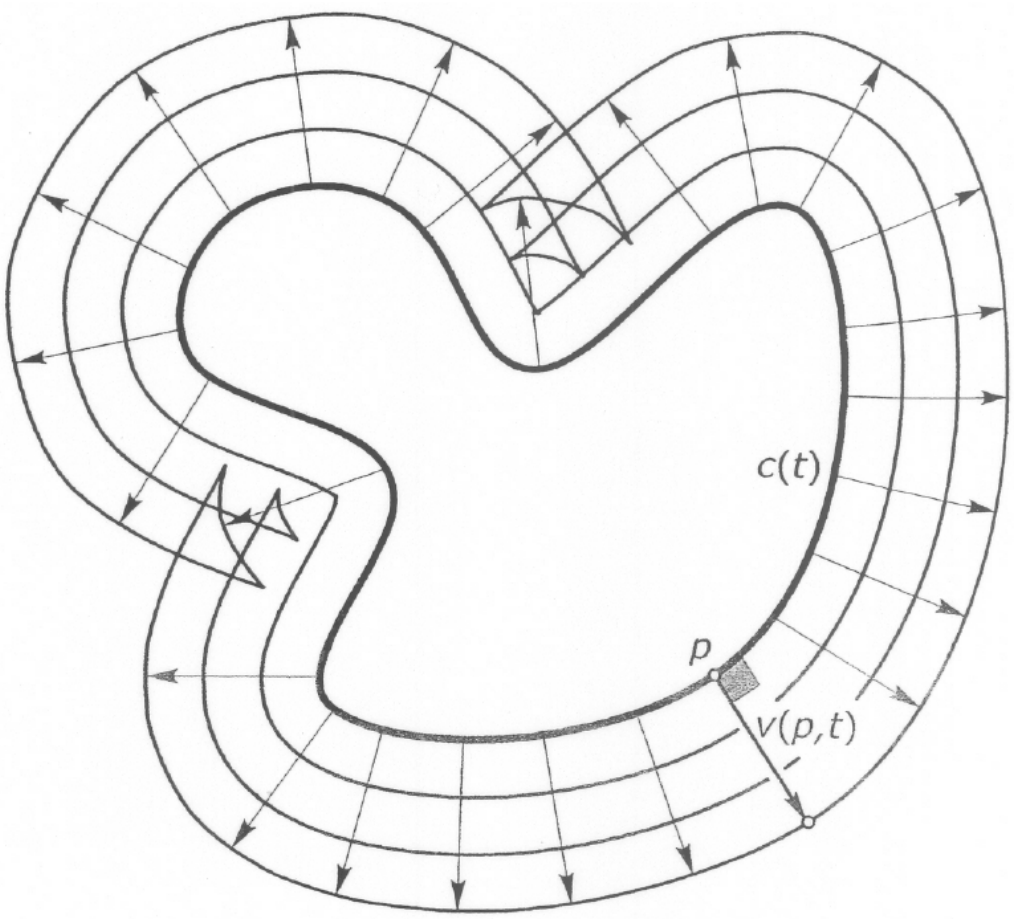
- Wrap a surface (skin) over a given network of curves
- A lot of degree of freedom
  - Depends on a specific modeling system and options (algorithm)



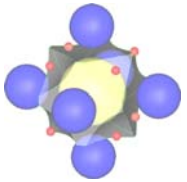
Curve evolution



- The design of changing curves
  - Useful for modeling surfaces with a profile curve
- Evolutions of curves (curve flows)
- v(p, t): normal component of c(t)

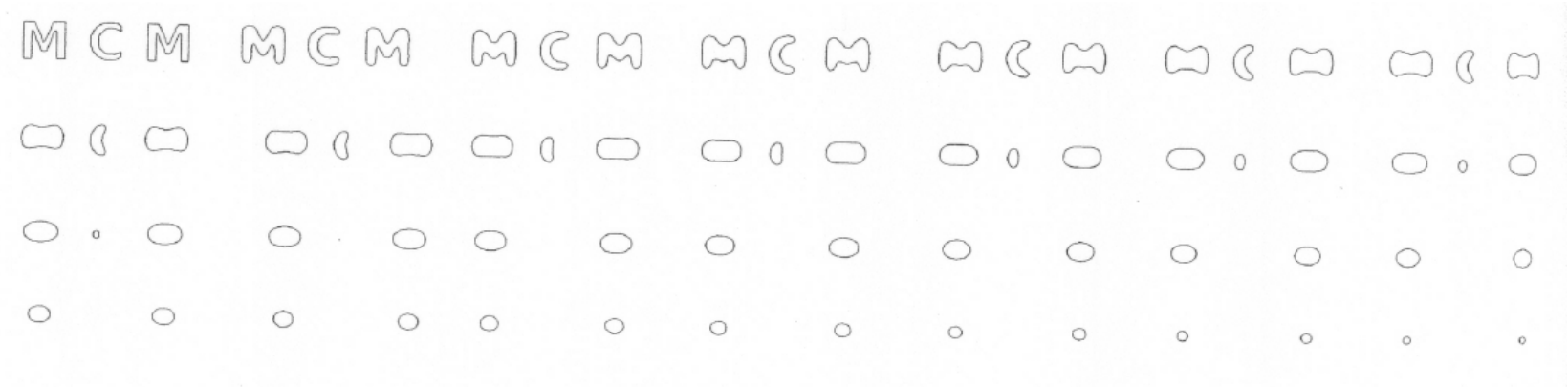


Curve evolution



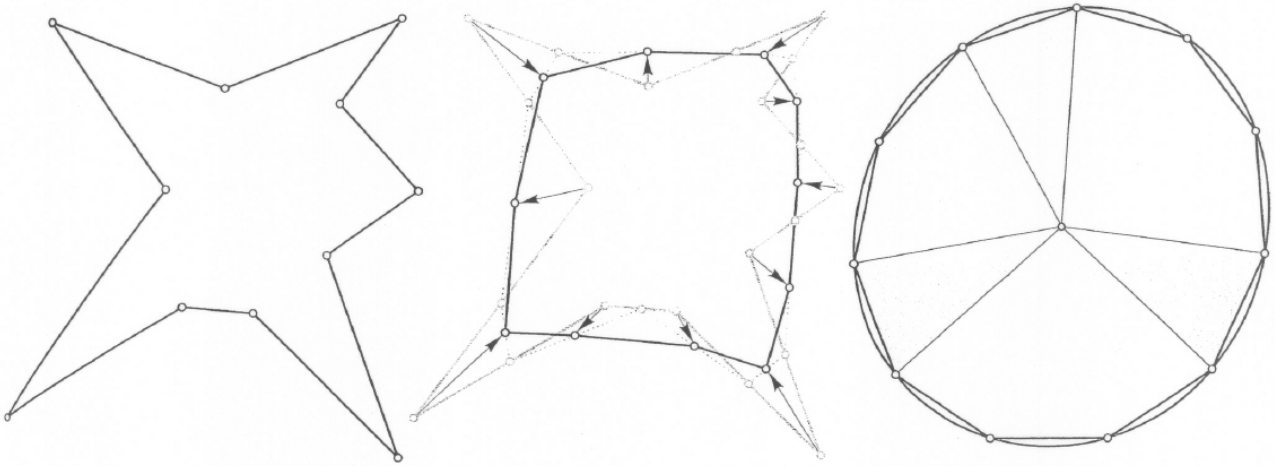
- **Curvature flow**
  - **v(p, t): curvature of c(t)**

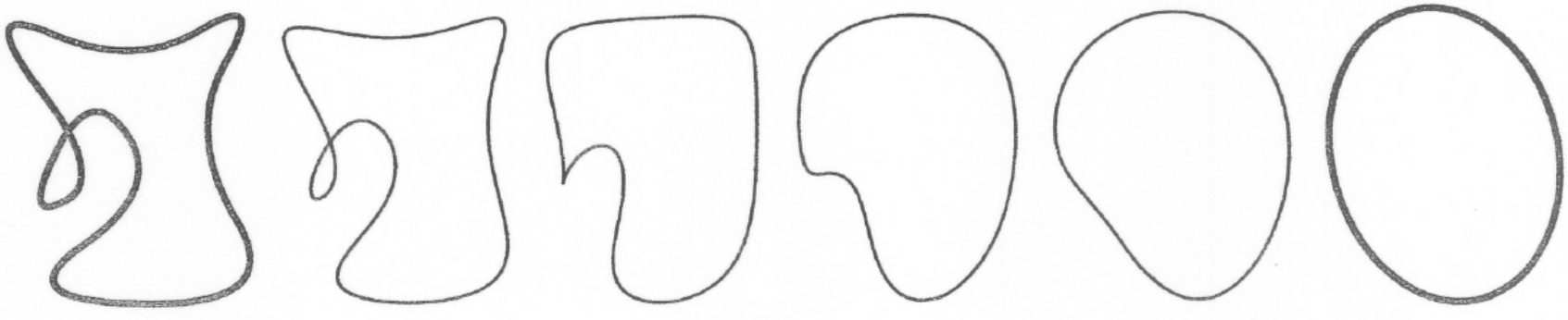
- **Evolution of c(t) will shrink c(t) to a point**



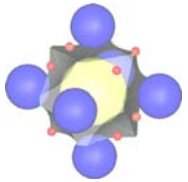
Curve evolution

- **Simple polygon evolution**
  - **Affinely regular polygon**
  - **Ellipse**

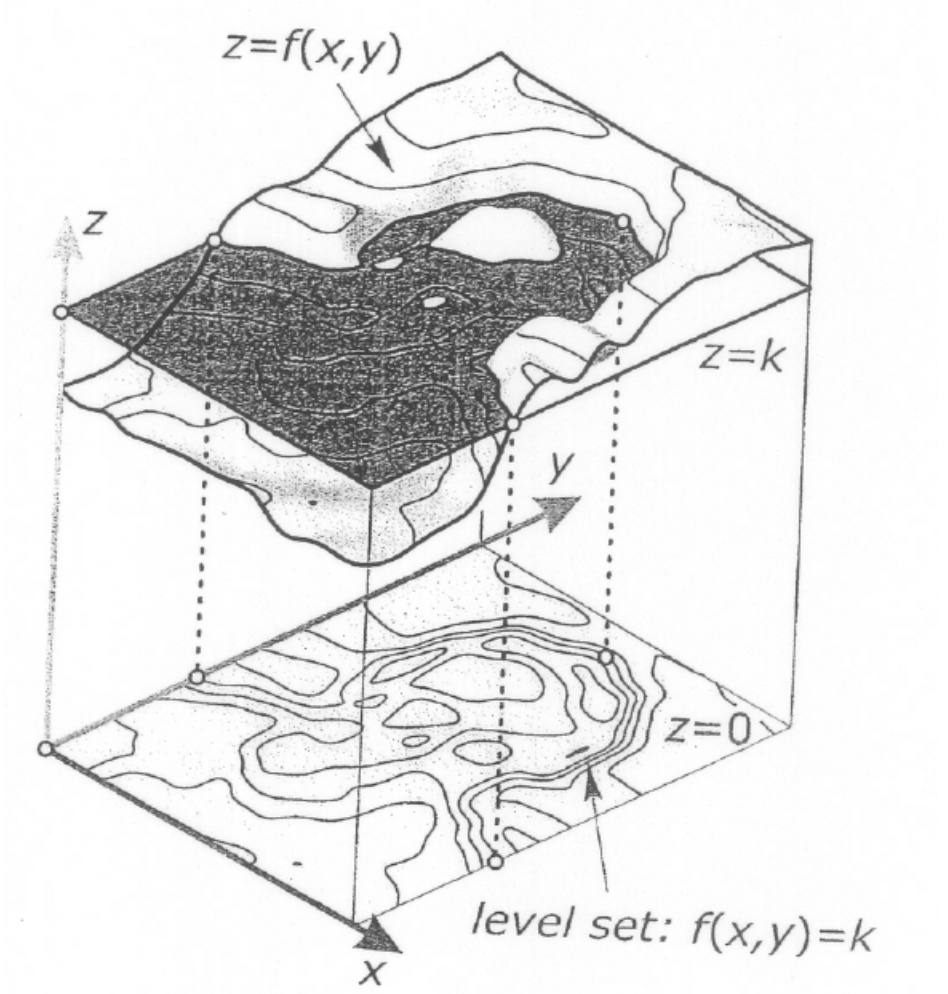




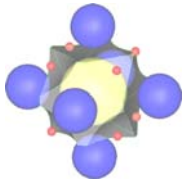
Metaballs and modeling with implicit surfaces



- **Implicit representation**
  - Curve : f(x, y) = 0
  - Surface : f(x, y, z) = 0
- **Level set**
  - f(x, y) = k
  - f(x, y, z) = k

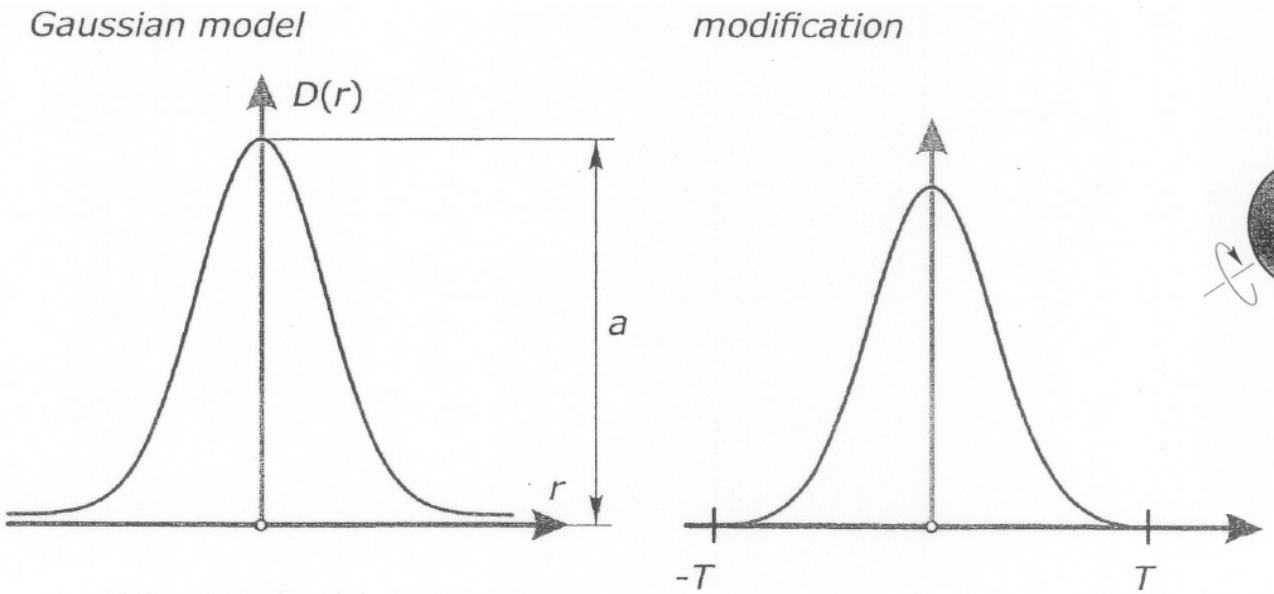


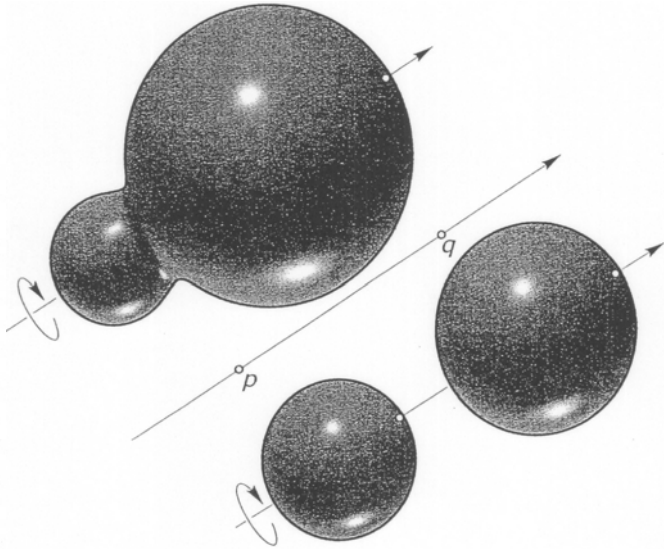
Metaballs and modeling with implicit surfaces



- **Distance-based functions (Df)**
- **Meta-balls**
  - **Blobs or soft objects**

$$f(x,y,z) = \sum_i Df_i(x,y,z), \quad f(x,y,z) = k$$





$$P(x,y,z) = a \cdot \exp(-b \cdot [(x-p_1)^2 + (y-p_2)^2 + (z-p_3)^2])$$