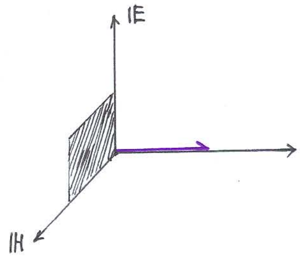
2.2 Poynting Vector와 전력



- 평면파는 $E_x\bar{x}\times H_y\bar{y}=P\bar{z}$ 형태, $(=E_x H_y \sin 90^0\ \bar{z})$

즉, P는 평면을 뚫고 나가는 형태이므로

$\nabla\cdot(\mathbb{E}\times\mathbb{H})$를 계산해 보자

$\nabla\cdot(\mathbb{E}\times\mathbb{H})=$ by vector 등식

$$=\mathbb{H}\cdot(\nabla\times\mathbb{E})-\mathbb{E}\cdot(\nabla\times\mathbb{H})$$

where,

$$\nabla\times E=-\frac{\partial B}{\partial t}=-\mu\frac{\partial H}{\partial t}$$

$$\nabla\times H=\mathbb{J}+\frac{\partial\mathbb{D}}{\partial t}=\mathbb{J}+\epsilon\frac{\partial\mathbb{E}}{\partial t}\quad : \textit{source 포함}\qquad \nabla\times H=J$$

$$=\mathbb{H}\cdot\left(-\mu\frac{\partial H}{\partial t}\right)-E\cdot\left(J+\epsilon\frac{\partial E}{\partial t}\right)$$

$$=-\mu H\cdot\frac{\partial H}{\partial t}-E\cdot J-\epsilon E\cdot\frac{\partial E}{\partial t}$$

where,

$$\frac{\partial}{\partial t}(\mu H\cdot H)=\mu\frac{\partial}{\partial t}(H\cdot H)$$
$$=\mu\left[\frac{\partial H}{\partial t}\cdot H+H\cdot\frac{\partial H}{\partial t}\right]$$
$$=2\mu\frac{\partial H}{\partial t}\cdot H$$

그러므로 우변 첫 번째 항

$$\therefore\quad \mu\frac{\partial H}{\partial t}\cdot H=\frac{1}{2}\frac{\partial}{\partial t}(\mu H\cdot H)=\frac{1}{2}\frac{\partial}{\partial t}(\mu H^2)$$

두 번째 항

$$E\cdot J=E\cdot(J_s+J_c)\,;\textit{source current + conduction current}$$
$$=E\cdot(J_s+\sigma E)$$
$$=E\cdot J_s+\sigma E^2$$

세 번째 항

$$=\varepsilon\mathbb{E}\cdot\frac{\partial\mathbb{E}}{\partial t}=\frac{1}{2}\frac{\partial}{\partial t}(\varepsilon\mathbb{E}\cdot\mathbb{E})$$

$$\therefore\quad \epsilon\frac{\partial E}{\partial t}\cdot E=\frac{1}{2}\frac{\partial}{\partial t}(\epsilon E^2)$$

$$\therefore\quad \nabla\cdot(E\times H)=-\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2+\frac{1}{2}\mu H^2\right)-E\cdot J_s-\sigma E^2 \qquad (2\text{-}48)$$

식 (2-48) 양변을 체적 적분하면

$$\int \nabla \cdot (\mathrm{E}\times\mathrm{H})\, dv = -\frac{\partial}{\partial t}\int\left(\frac{1}{2}\epsilon\,\mathrm{E}^2+\frac{1}{2}\mu\,\mathrm{H}^2\right)dv-\int \mathrm{E}\cdot\mathrm{J}_s\, dv-\int \sigma\,\mathrm{E}^2 dv$$

by Divergence

$$\underbrace{\int (\mathrm{E}\times\mathrm{H})\cdot d\mathrm{S}}_{\text{V 체적 밖으로 송출되는 전력}} = \underbrace{-\frac{\partial}{\partial t}\int\left(\frac{1}{2}\epsilon\,E^2+\frac{1}{2}\mu\,H^2\right)dv}_{\text{V 체적내에 축적된 E, H 에너지의 시간 변화율}}\underbrace{-\int \mathrm{E}\cdot\mathrm{J}_s\, dv}_{\text{전원전류 }\mathrm{J}_s\text{에 의한 공급전력}}\underbrace{-\int \sigma\,\mathrm{E}^2 dv}_{\text{Joule 열 손실}}$$

$$\therefore\ -\int \mathrm{E}\cdot\mathrm{J}_s\, dv = \frac{\partial}{\partial t}\int\left(\frac{1}{2}\epsilon\,E^2+\frac{1}{2}\mu\,H^2\right)dv+\int \sigma\,\mathrm{E}^2 dv+\int(\mathrm{E}\times\mathrm{H})\cdot d\mathrm{S} \qquad (2\text{-}49)$$

즉, 식 (2-49)는 Closed Surface 내에 source가 있는 경우임.

• Source가 Closed surface 밖에 있으면

$$\int (\mathrm{E}\times\mathrm{H})\cdot d\mathrm{S} = -\frac{\partial}{\partial t}\int\left(\frac{1}{2}\epsilon\,E^2+\frac{1}{2}\mu\,H^2\right)dv-\int \sigma\,\mathrm{E}^2 dv$$

where,

$$\begin{aligned}\mathrm{E}\times\mathrm{H} = \mathrm{P}\ &[\mathrm{W/m^2}],\ \mathcal{S}\\ &= \textit{J.H. Poynting}\\ &= \textit{Poynting Vector}\\ &= \textit{Poynting Theorem}\end{aligned}$$

$$\int \mathrm{P}\cdot d\mathrm{S} = -\frac{\partial}{\partial t}\int\left(\frac{1}{2}\epsilon\,E^2+\frac{1}{2}\mu\,H^2\right)dv-\int \sigma\,\mathrm{E}^2 dv$$

식 (2-49)에서

$\mathrm{E},\ \mathrm{H},\ \mathrm{J}_s = \textit{Sinusoidal signals} \Rightarrow \textit{Phasor}$(1-4절 참조)

식 (2-49)는

$$-\int \mathrm{E}\cdot\mathrm{J}_s^{\,*}\, dv = \frac{1}{2}j\omega\int(\epsilon\mathrm{E}\cdot\mathrm{E}^*+\mu\,\mathrm{H}\cdot\mathrm{H}^*)dv+\int\sigma\,\mathrm{E}\cdot\mathrm{E}^* dv+\int(\mathrm{E}\times\mathrm{H}^*)\cdot d\mathrm{S}$$

또한 시간 평균값이 필요하므로

$$\begin{aligned}-\frac{1}{2}\int \mathrm{E}\cdot\mathrm{J}_s^{\,*}\, dv &= \frac{1}{2}j\omega\int\left(\frac{1}{2}\epsilon\mathrm{E}\cdot\mathrm{E}^*+\frac{1}{2}\mu\,\mathrm{H}\cdot\mathrm{H}^*\right)dv \\ &\quad+\frac{1}{2}\int\sigma\,\mathrm{E}\cdot\mathrm{E}^* dv+\frac{1}{2}\int(\mathrm{E}\times\mathrm{H}^*)\cdot d\mathrm{S}\end{aligned} \qquad (2\text{-}51)$$

where,

$$-\frac{1}{2}\int \mathrm{E}\cdot \mathrm{J_s}^{*}dv = P_s$$: 전원 전력

$$\frac{\epsilon}{4}\int \mathrm{E}\cdot \mathrm{E}^{*}\, dv = W_e$$: 전계 energy

$$\frac{\mu}{4}\int \mathrm{H}\cdot \mathrm{H}^{*}\, dv = W_m$$: 자계 energy

$$\frac{\sigma}{2}\int \mathrm{E}\cdot \mathrm{E}^{*}\, dv = P_{loss}$$: 저항성 전력

$$\frac{1}{2}\int \mathrm{E}\times \mathrm{H}^{*}\cdot d\mathrm{S} = P_0$$: 이동 전력(by wave)

∴ 식 (2-51)

$$P_s = 2j\omega(W_m + W_e) + P_{loss} + P_0 \qquad (2-53)$$

예제 2-5 풀어보세요.

식(2-14)의 Poynting Vector를 구하여라.

sol> 식2-14 > $E(z) = E_{x0}\, e^{-jk_z z} \cdot \bar{x}$

$$S = E \times H^{*}$$

$$= \left(E_{x0}\, e^{\cancel{-jk\cdot r}}\, \bar{x}\right) \times \frac{1}{\eta} \underbrace{\left(\bar{z} \times E_{x0}\, e^{-jk\cdot r} \cdot \bar{x}\right)^{*}}_{\frac{1}{\eta}\left(\bar{y} \cdot E_{x0}\, e^{\cancel{jk\cdot r}}\right)}$$

$$= \frac{1}{\eta} |E_{x0}|^{2} \cdot \bar{z}$$

$|E_0|^2/\eta$ (W/m^2)의 전력밀도가 $\bar{z}$ 방향으로 흐르고 있다.