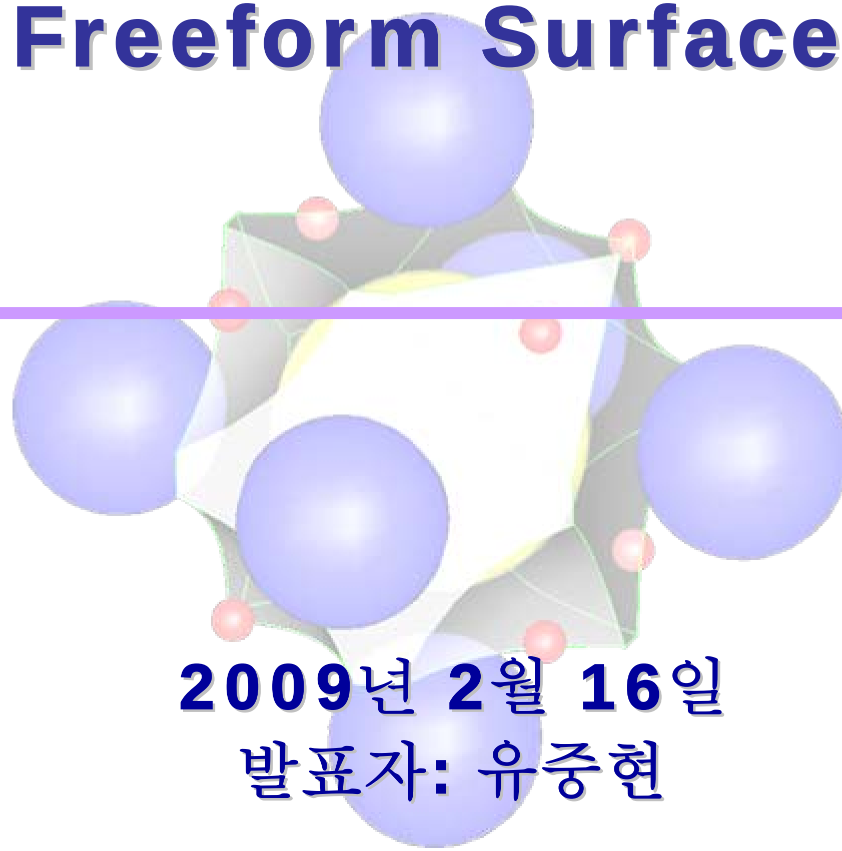
Architectural geometry: Freeform Surfaces

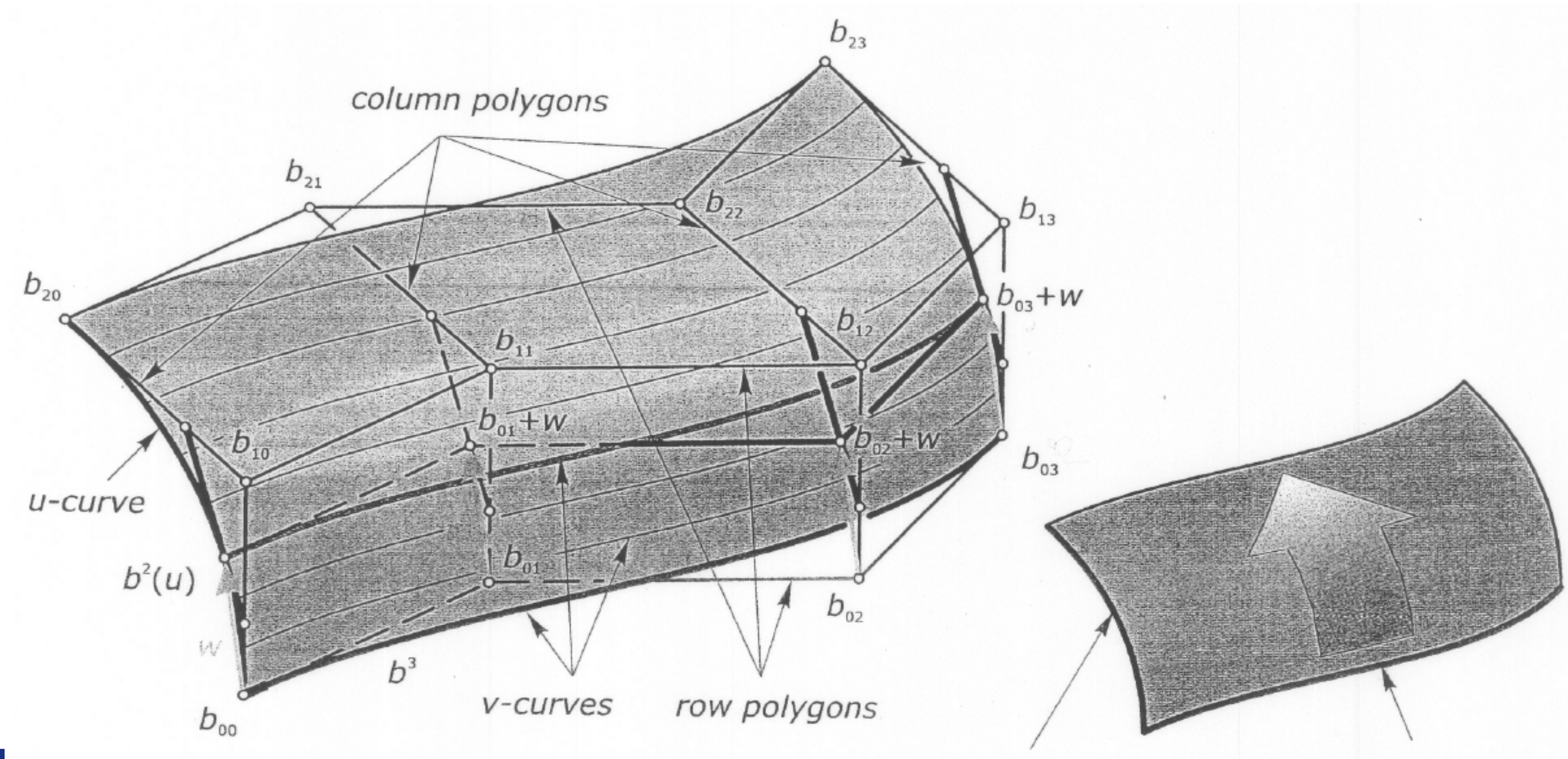


2009년 2월 16일

발표자: 유중현

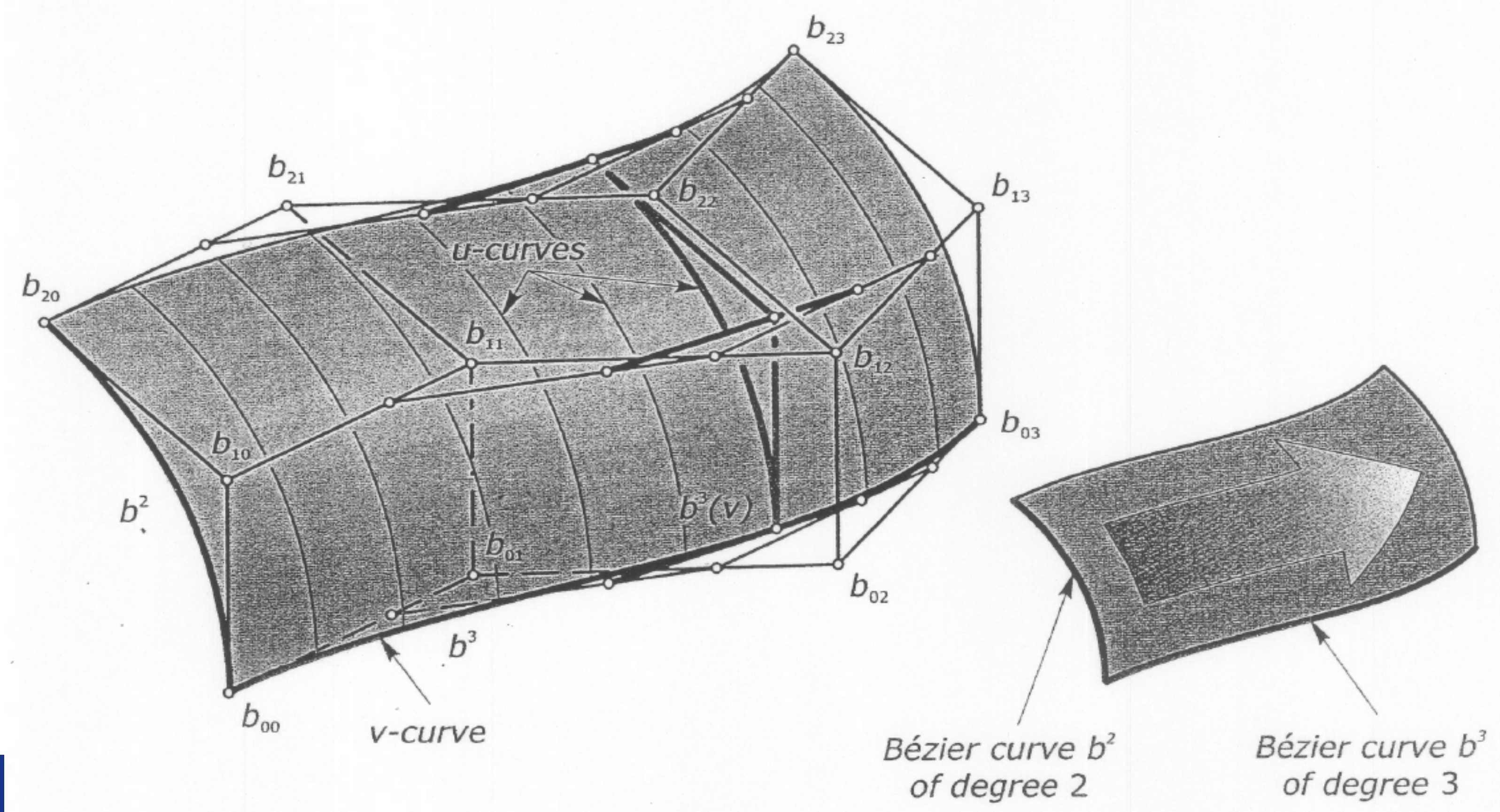
Translational Bézier surface

- Input: two Bézier curves (control polygons)
- 4 column polygons
- Quadrilaterals are parallelogram.



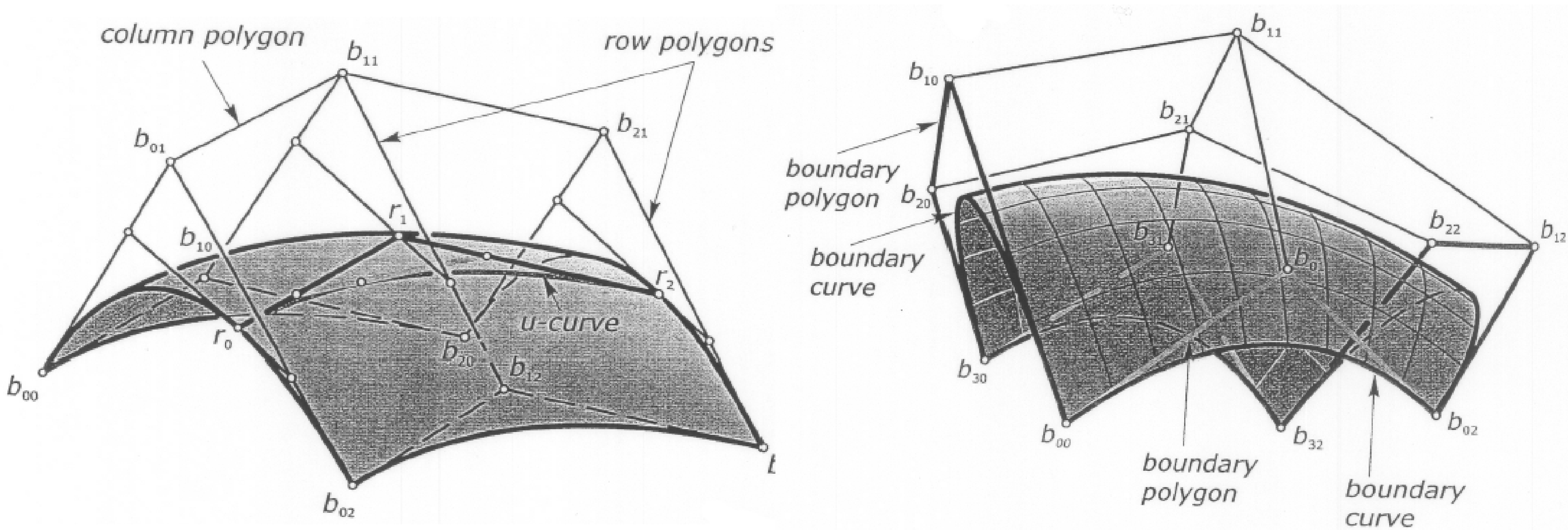
Translational Bézier surface

- Input: two Bézier curves (control polygons)
- 3 row polygons
- Quadrilaterals are parallelogram.



Tensor product Bézier surface

- Input: a set of control points (control mesh)
- Generalization of translational Bézier surface



Tensor product surface

- **The extension of parametric curve**

$$
\begin{aligned}
S(u,v) &= \sum_i \sum_j BF_{i,j,m,n}(u,v)P_{ij} \\
&= \sum_i \sum_j BF_{i,m}(u)BF_{j,n}(v)P_{ij}
\end{aligned}
$$

BF: basis function

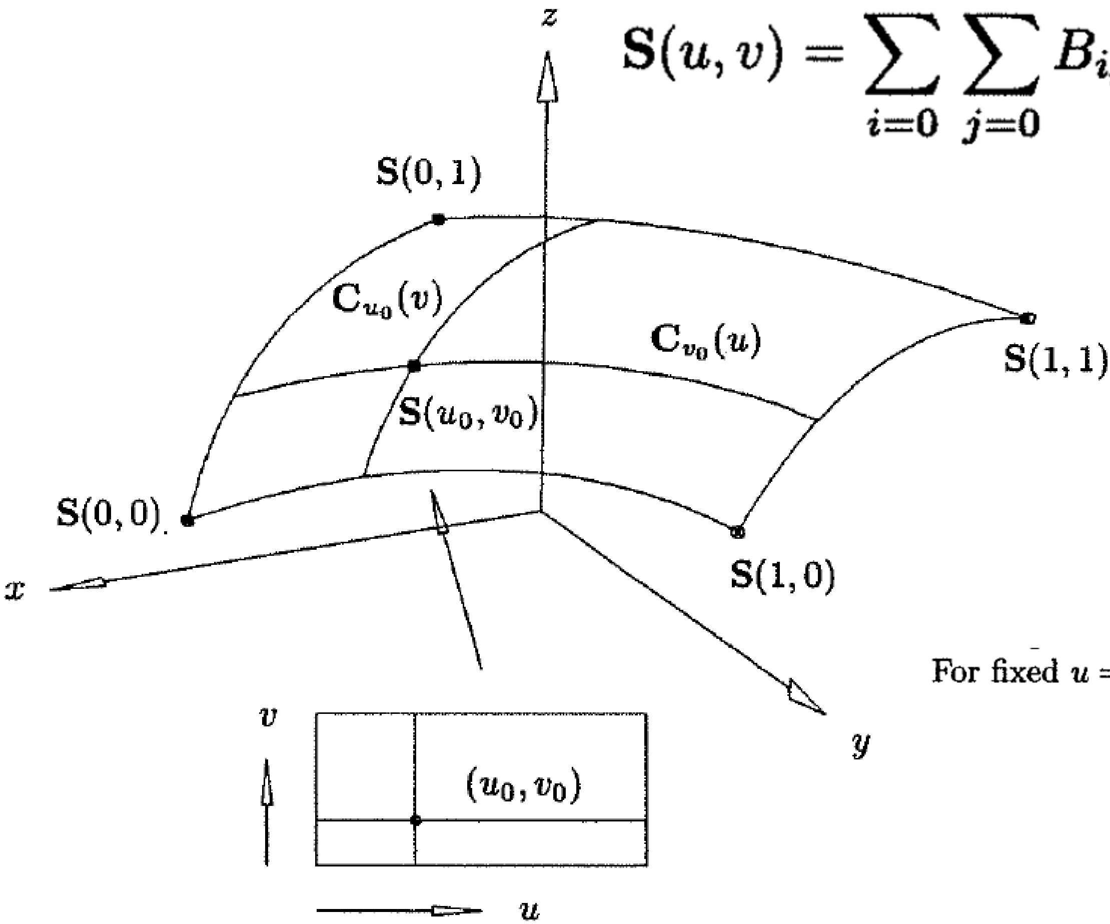
P : control point

m : the degree for u-direction

n : the degree for v-direction

Bézier surface

$$\mathbf{S}(u,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}B_{i,n}(u)B_{j,m}(v)\mathbf{P}_{i,j} \qquad 0\le u,v\le 1$$



For fixed $u=u_0$

$$\mathbf{C}_{u_0}(v)=\mathbf{S}(u_0,v)=\sum_{i=0}^{n}\sum_{j=0}^{m}B_{i,n}(u_0)B_{j,m}(v)\mathbf{P}_{i,j}$$

$$=\sum_{j=0}^{m}B_{j,m}(v)\left(\sum_{i=0}^{n}B_{i,n}(u_0)\mathbf{P}_{i,j}\right)$$

$$=\sum_{j=0}^{m}B_{j,m}(v)\mathbf{Q}_j(u_0)$$

where $\quad \mathbf{Q}_j(u_0)=\sum_{i=0}^{n}B_{i,n}(u_0)\mathbf{P}_{i,j} \qquad j=0,\ldots,m$

Basis functions

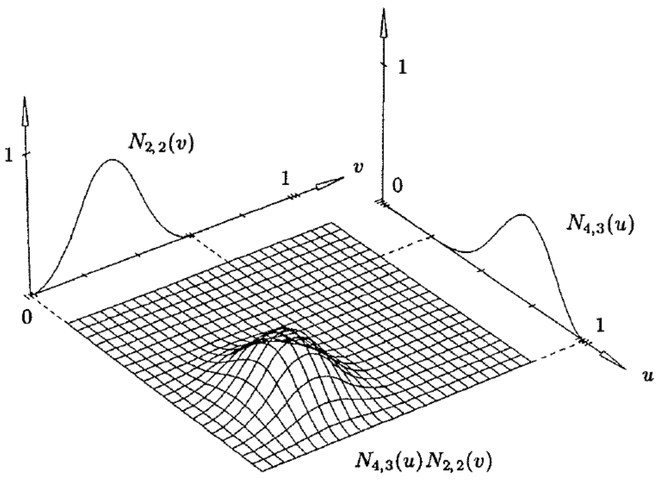


Figure 3.19. Cubic × quadratic basis functions. (a) $N_{4,3}(u)N_{4,2}(v)$; (b) $N_{4,3}(u)N_{2,2}(v)$; $U = \{0,0,0,0,\ {}^1\!/_4,\ {}^1\!/_2,\ {}^3\!/_4,\ 1,1,1,1\}$ and $V = \{0,0,0,\ {}^1\!/_5,\ {}^2\!/_5,\ {}^3\!/_5,\ {}^3\!/_5,\ {}^4\!/_5,\ 1,1,1\}$.

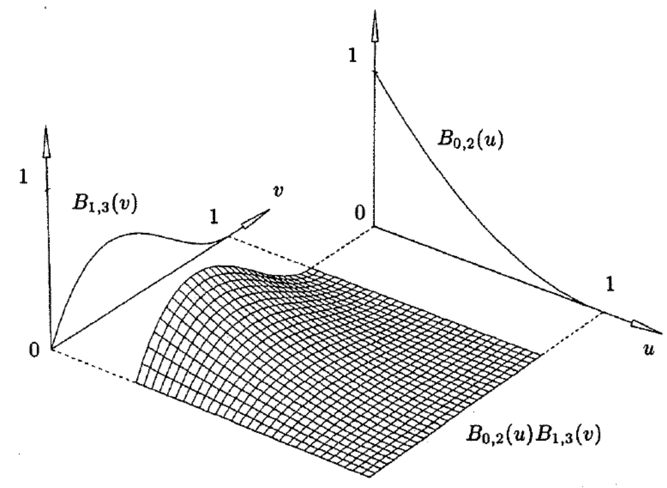


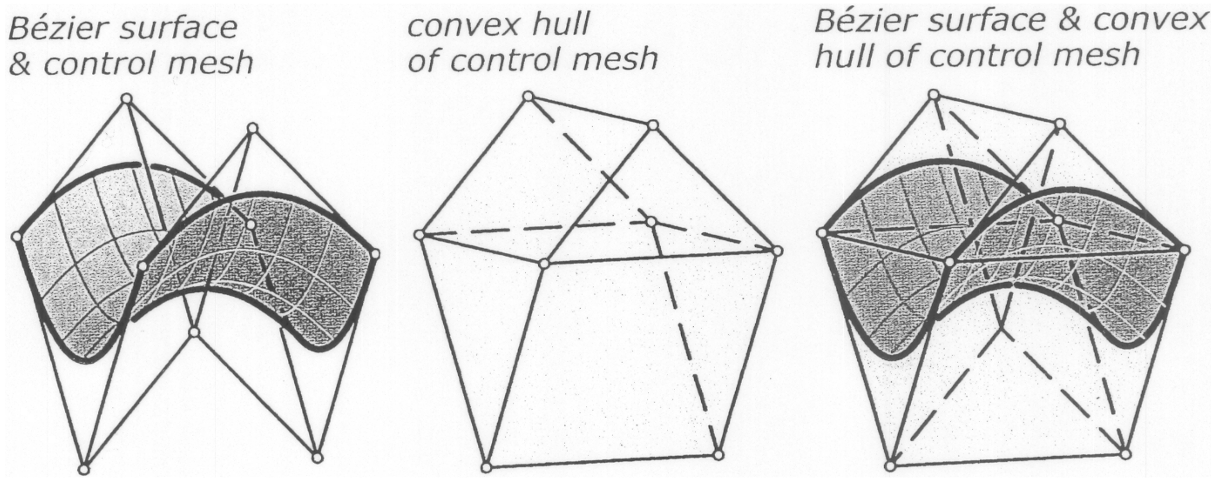
Figure 1.24. (a) The Bézier tensor product basis function, $B_{0,2}(u)B_{1,3}(v)$; (b) a quadratic × cubic Bézier surface.

B-Spline basis function for 3 X 2

Bernstein basis function for 2 X 3

Properties of Bézier surface

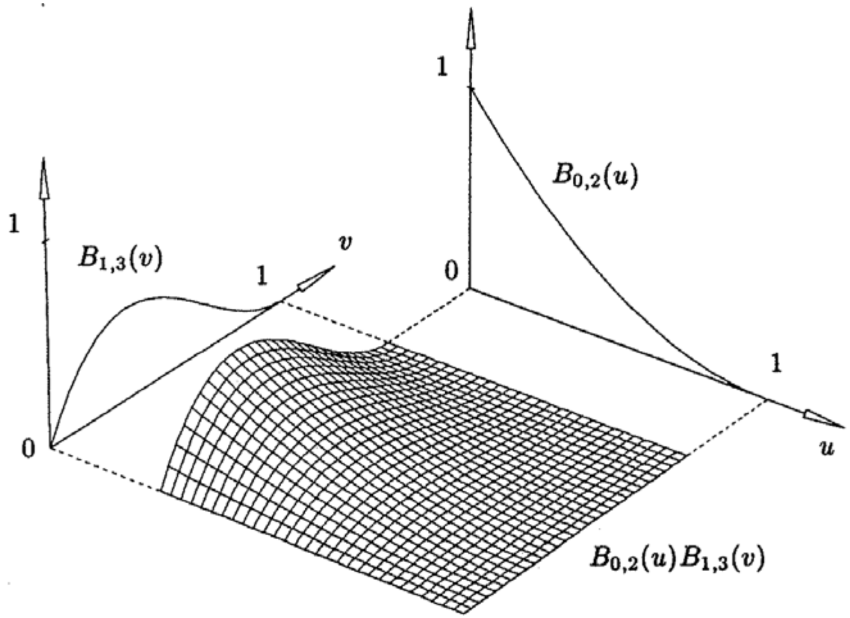
- Each boundary polygon defines a boundary Bézier curve on the Bézier surface
- Corner point interpolation
- Convex hull property

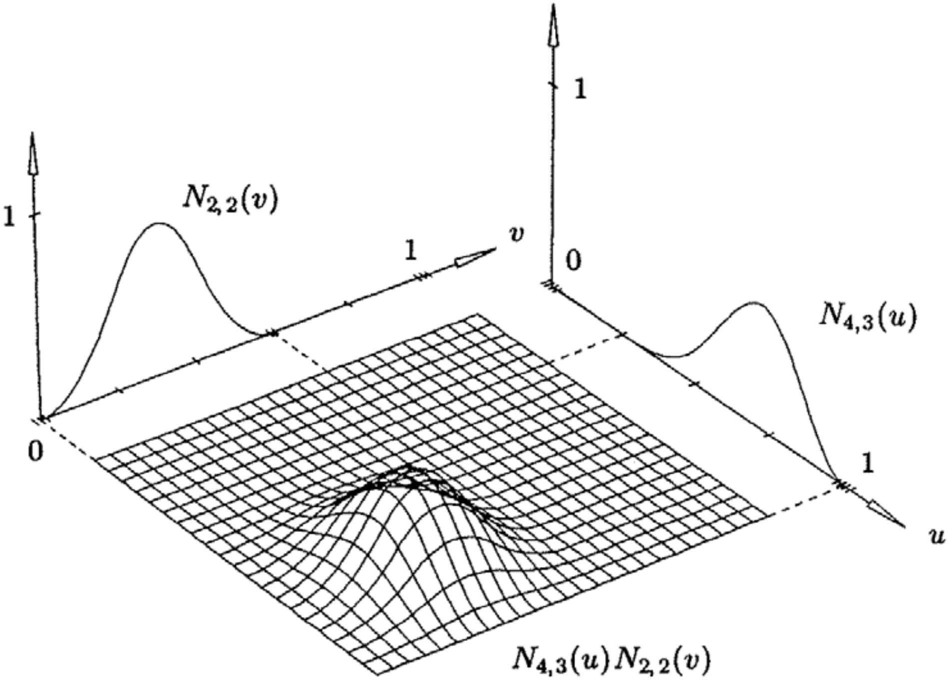


- Affine invariance

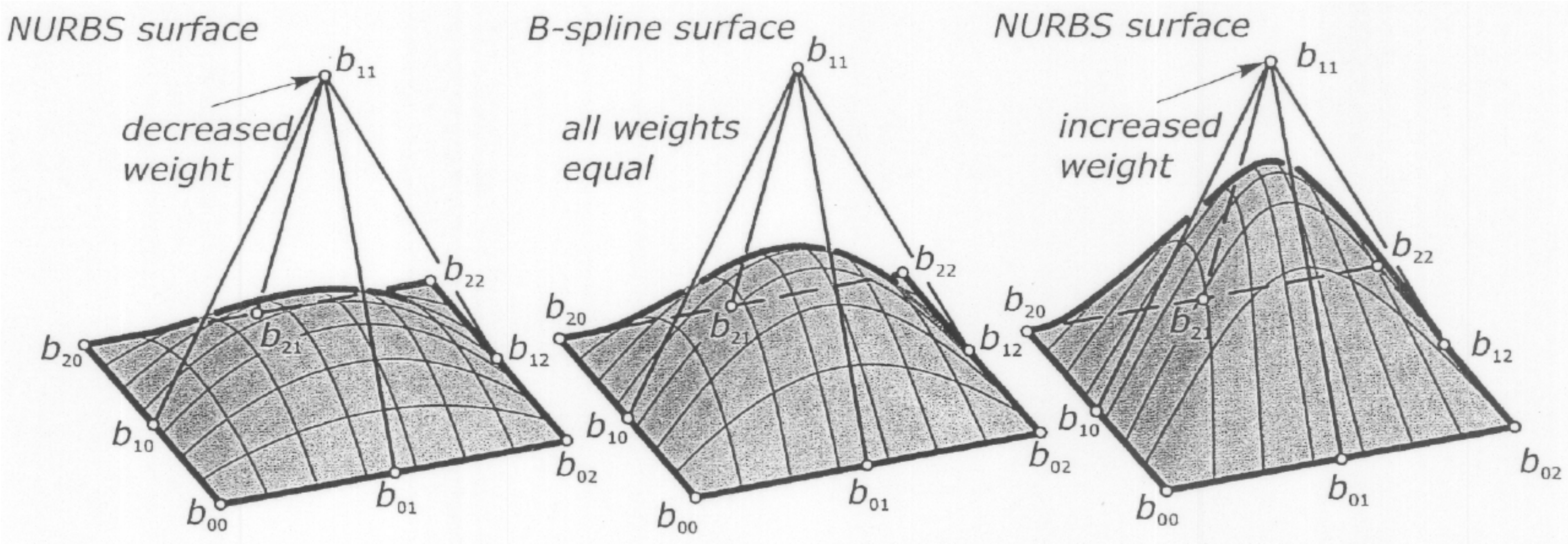
B-Spline / NURBS surfaces

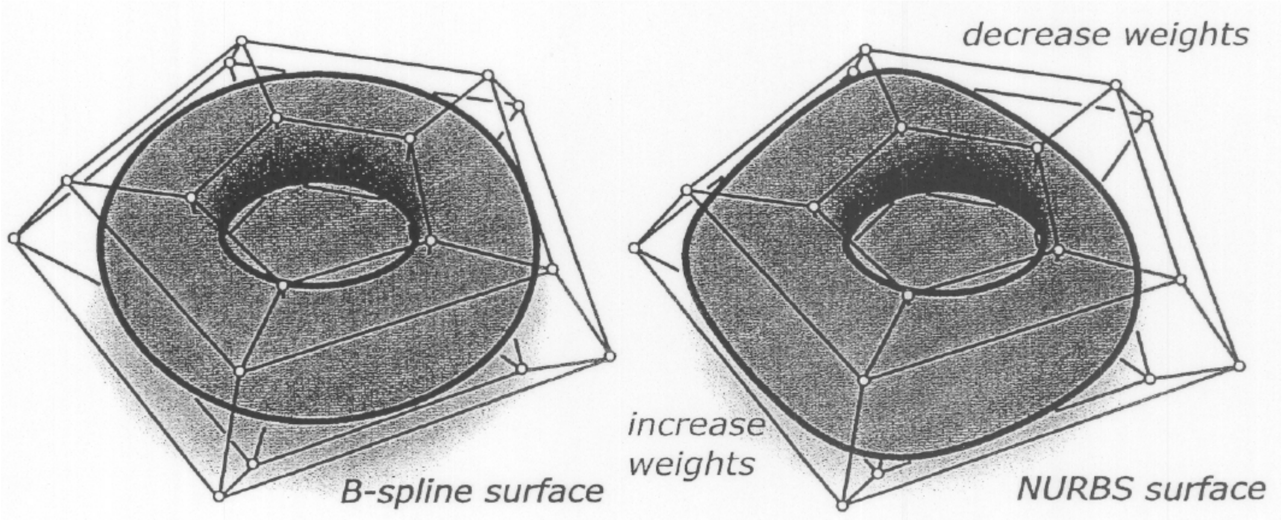
- **Bézier surface**
  - Local control is not available
  - High degree is required as the size of control mesh increases.





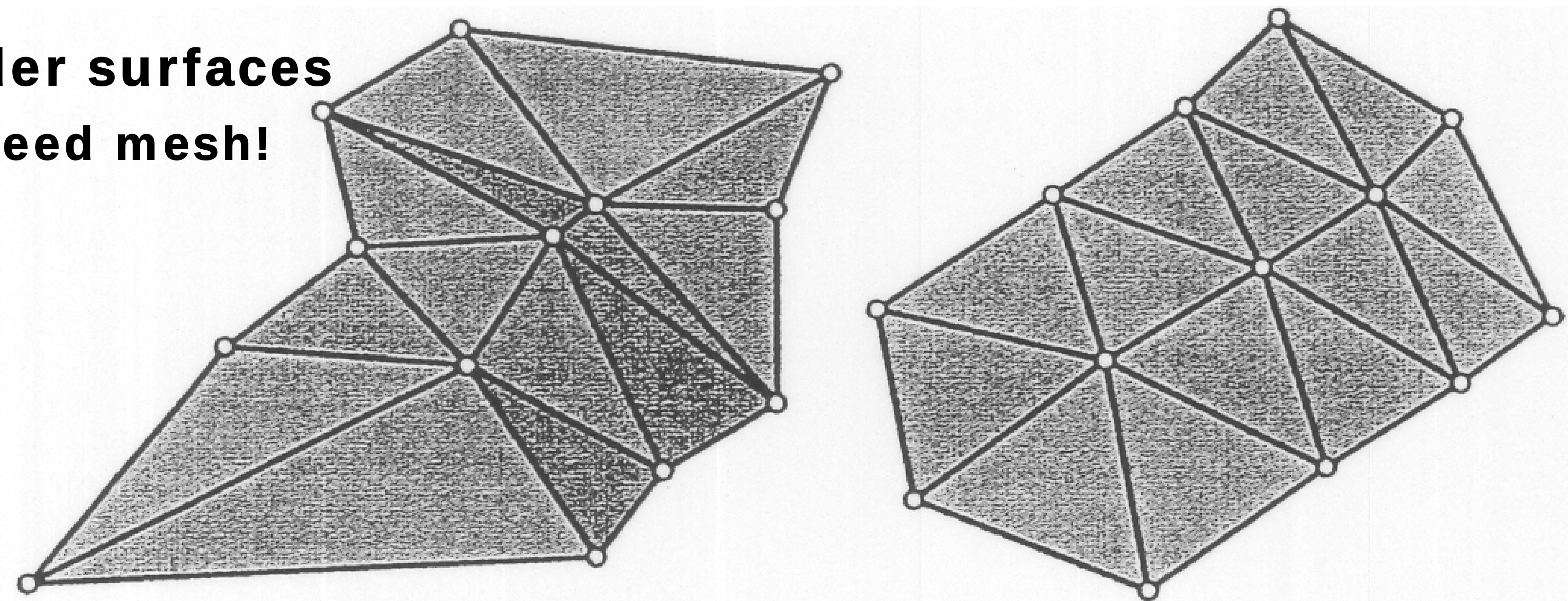
B-Spline / NURBS surfaces





Mesh

- A set of polygons
  - Mesh topology (connectivity)
  - Mesh geometry (vertex coordinates)
- To render curves
  - To make piecewise line segments from sample points on curves
- To render surfaces
  - We need mesh!



Mesh refinement

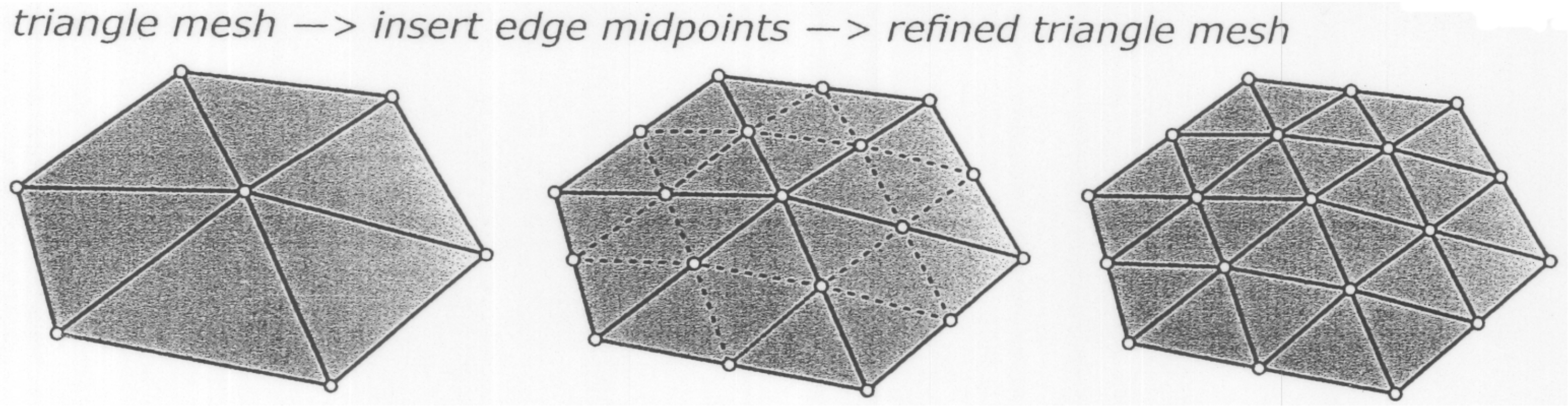
- Two-step procedure
  - Change the connectivity: the number of vertices and the way they are connected
  - Change the geometry: position of vertices

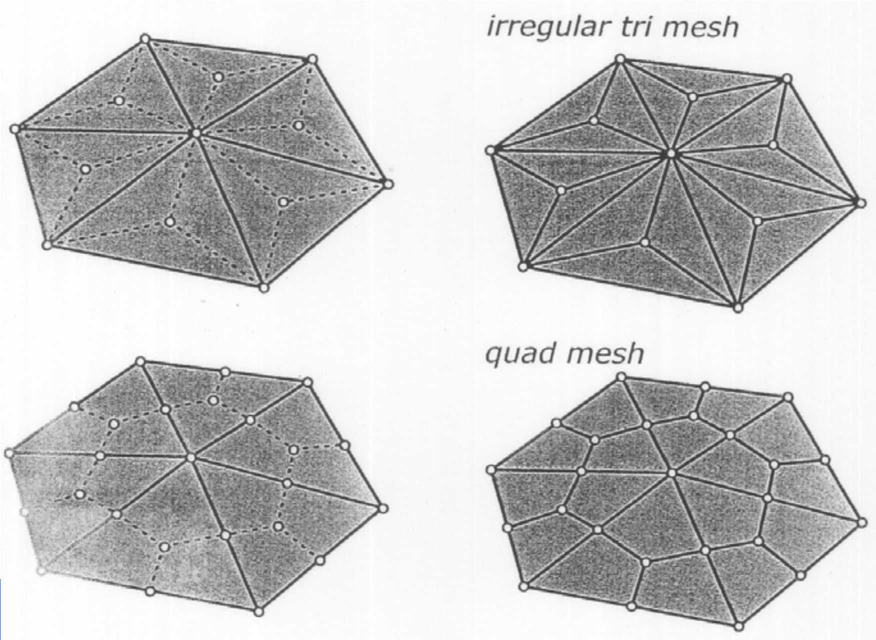
- Triangle / quad mesh
  - Edge midpoint insertion
  - Barycenter insertion

- Planar / Nonplanar mesh

Mesh refinement

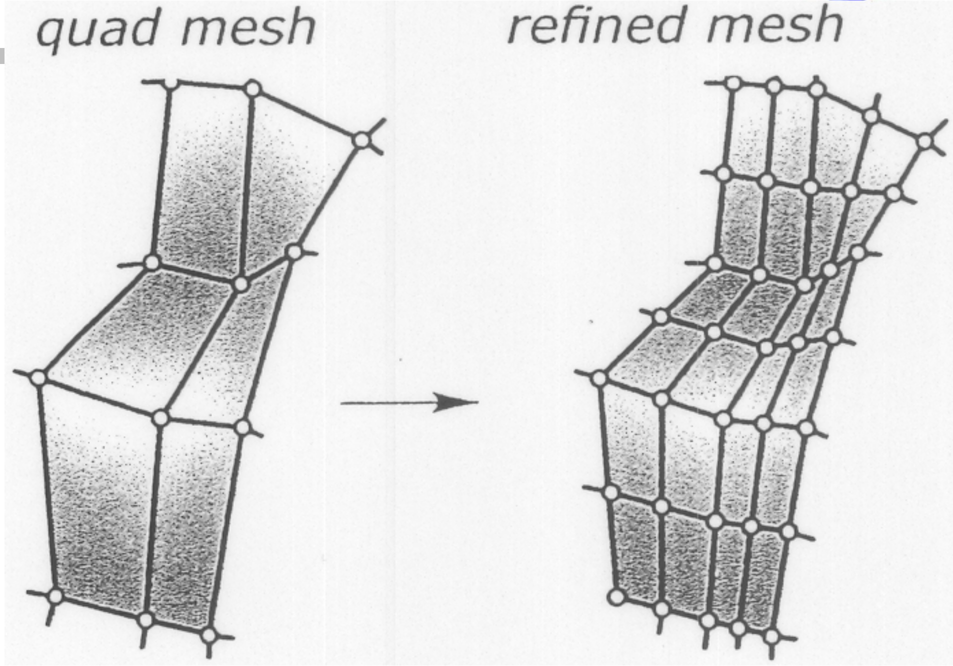
- **Triangle mesh**

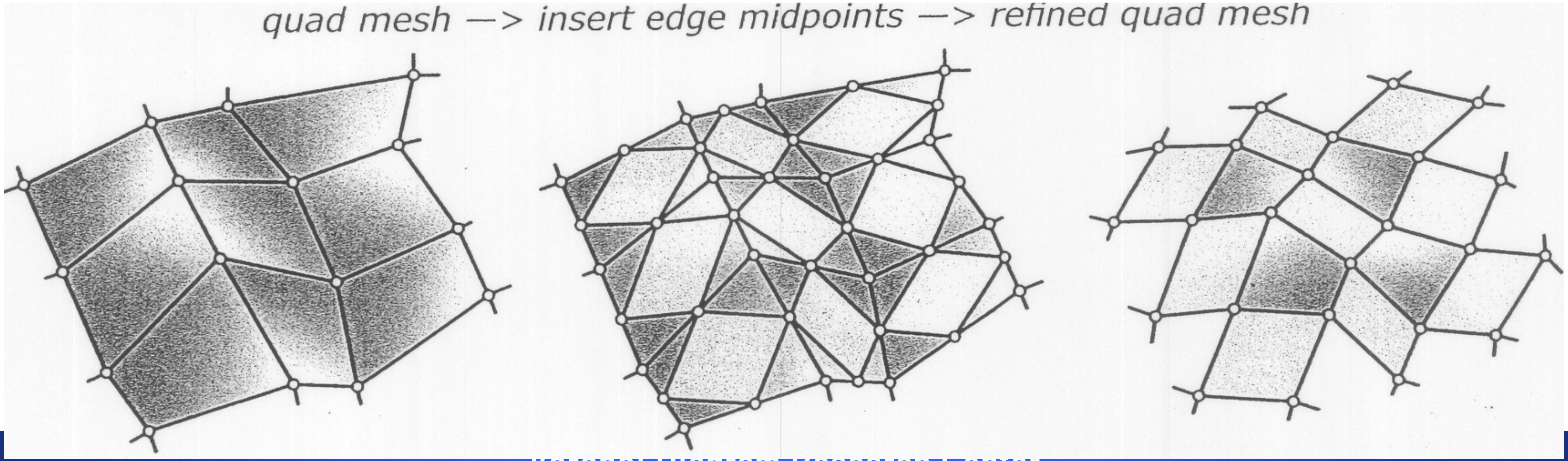




Mesh refinement

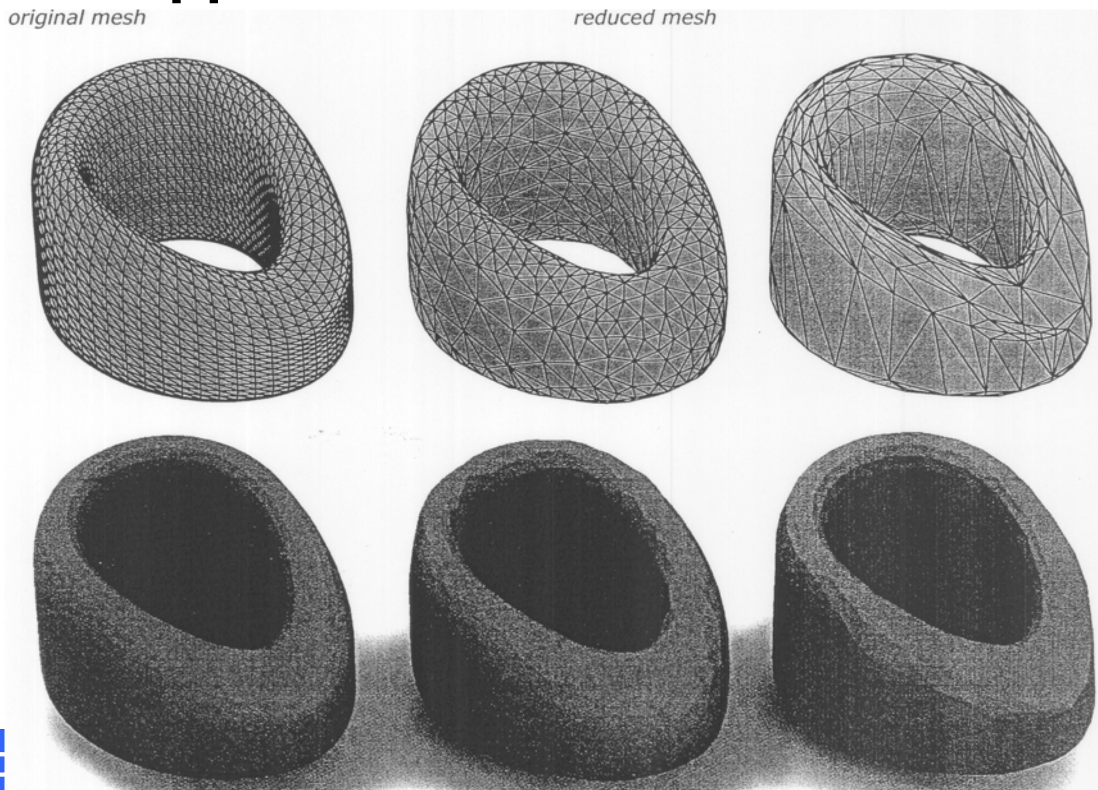
- **Quad mesh**





Mesh decimation

- The reverse process of mesh refinement
- Data reduction process
- Acceleration of downstream applications:
  - Simulations
  - Fast rendering



Mesh quality

- Visual appearance
  - Too many irregular vertices might not look good
- Simulation based FEM (Finite Element Method)
  - Thin triangles should be avoided
  - Holes in meshes should be removed

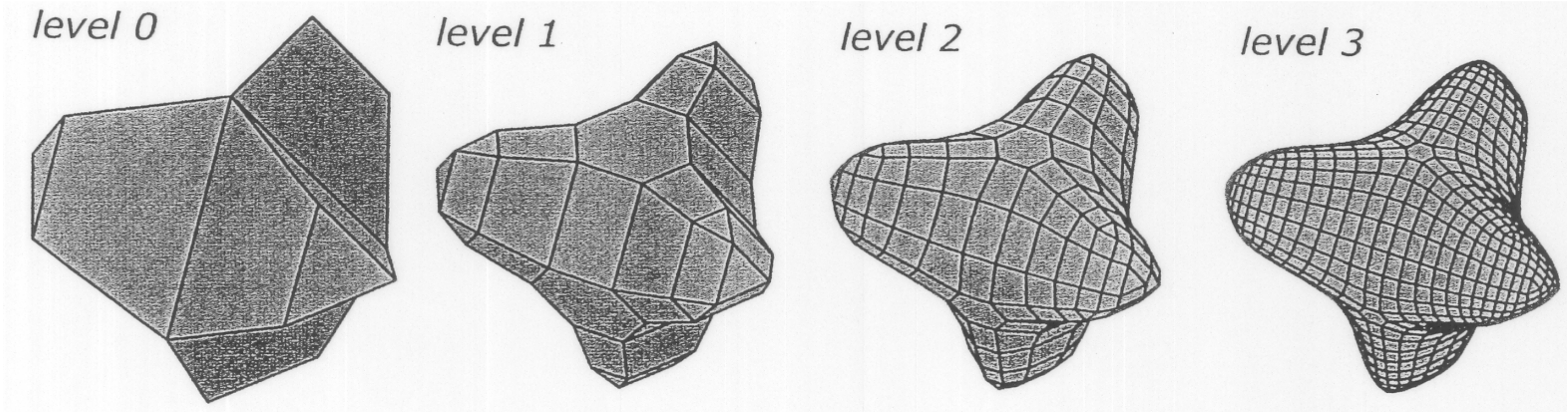
Subdivision surface

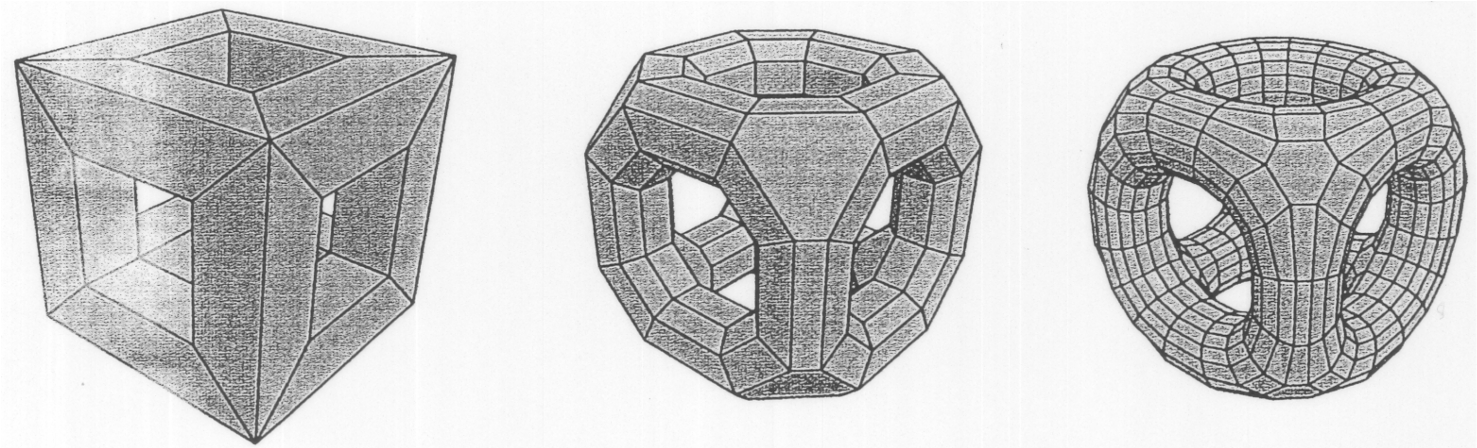
- For modeling surfaces with a more general topology
  - Mesh + Subdivision rule

- Quad mesh
  - Doo-Sabin subdivision
  - Catmull-Clark subdivision

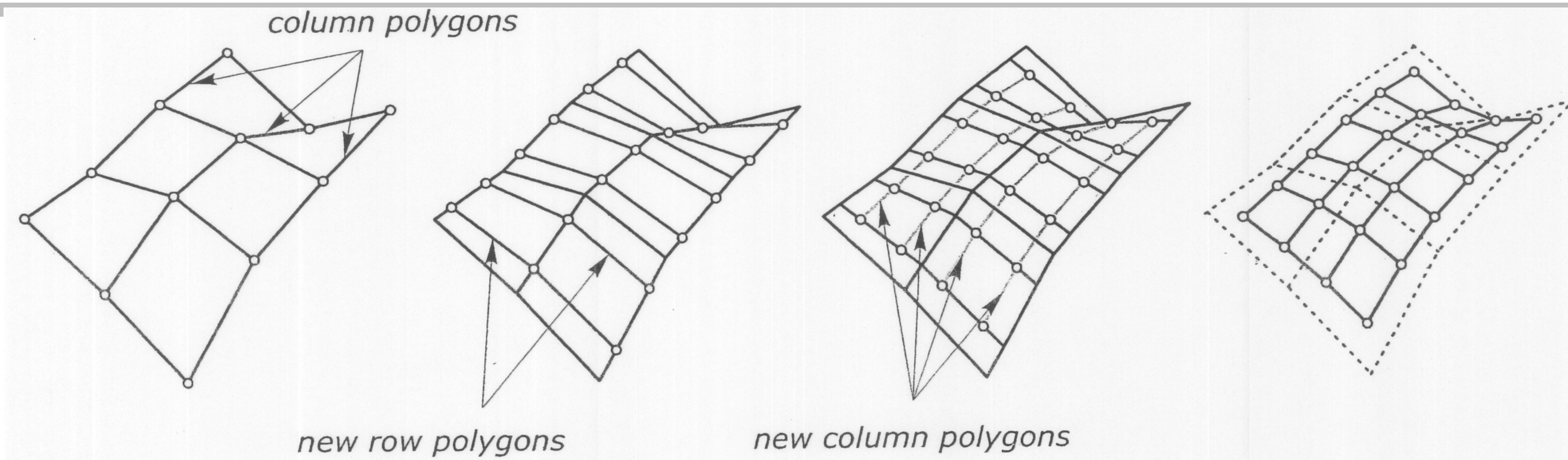
- Triangle mesh
  - Loop subdivision

Doo-Sabin subdivision





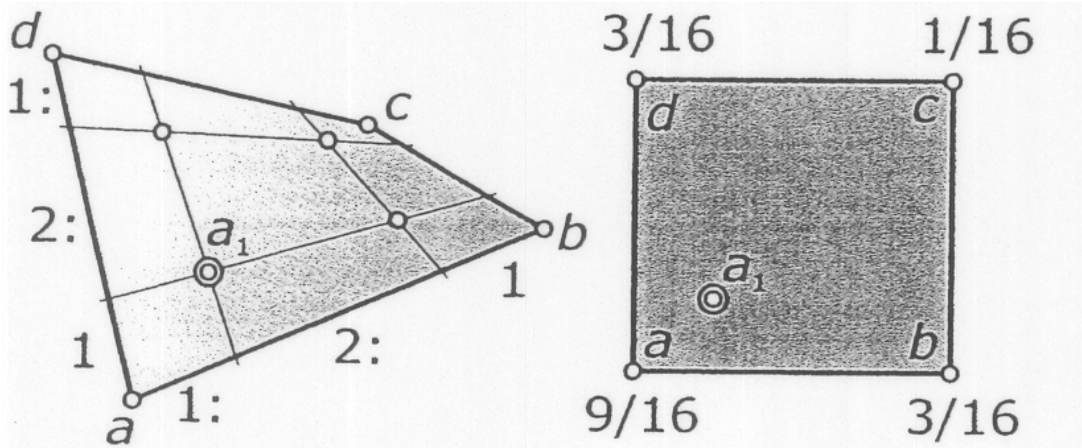
Doo-Sabin subdivision



$$a' = \frac{3}{4}a + \frac{1}{4}b$$

$$d' = \frac{3}{4}d + \frac{1}{4}c$$

$$a_1 = \frac{3}{4}a' + \frac{1}{4}d'$$



$a_1 = (9/16)a + (3/16)b + (3/16)d + (1/16)c$

Multi-resolution modeling

- Editing operations can be performed to any level of meshes

  - Initial mesh
  - Editing
  - Subivision
  - Editing
  - Subdivision
  - ...

- A change made at a coarse level has a broader influence than a chagne at a fine level