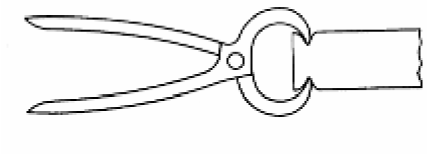
FUNdaMENTALS of Design

Topic 3

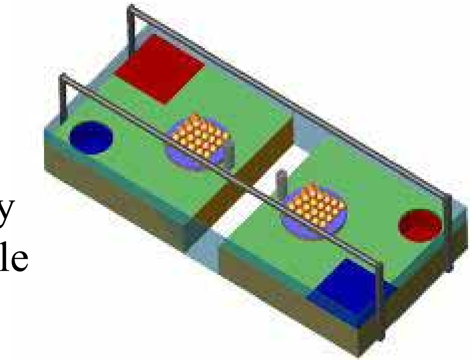
FUNda***MENTAL*** **Principles**

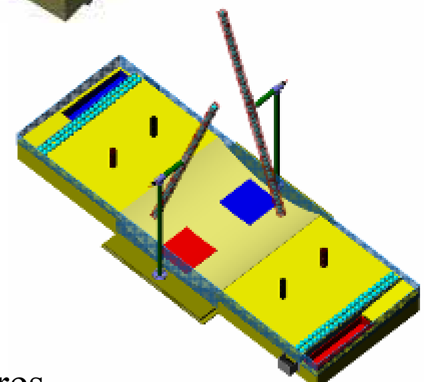


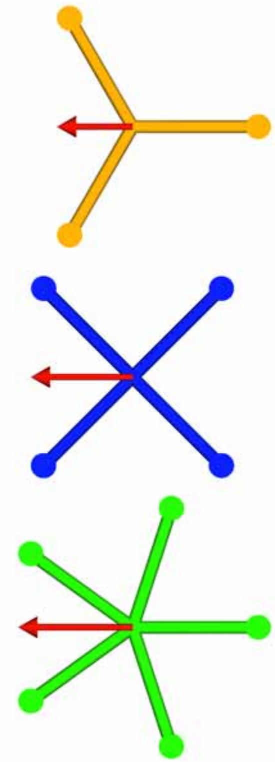
Topic 3
*FUN*da*MENTAL* Principles

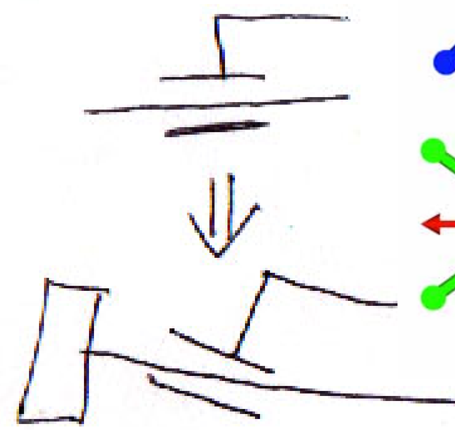
Topics

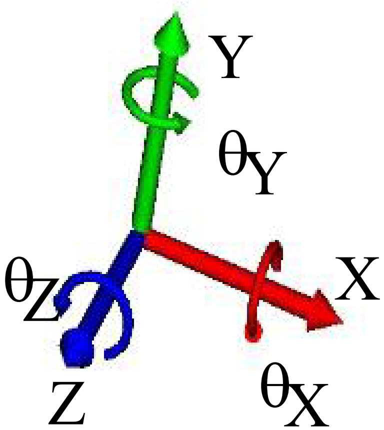
- Occam's Razor
- Newton's Laws
- Conservation of Energy
- Saint-Venant's Principle
- Golden Rectangle
- Abbe's Principle
- Maxwell & Reciprocity
- Self-Principles
- Stability
- Symmetry
- Parallel Axis Theorem
- Accuracy, Repeatability, Resolution
- Sensitive Directions & Reference Features
- Structural Loops
- Preload
- Centers of Action
- Exact Constraint Design
- Elastically Averaged Design
- Stick Figures





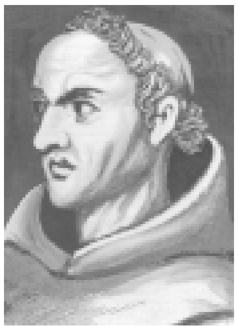




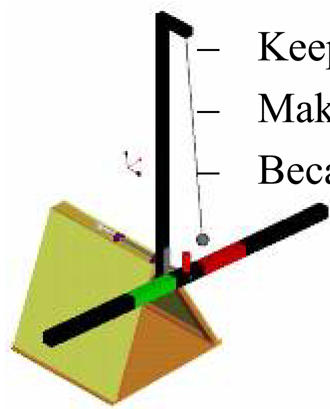


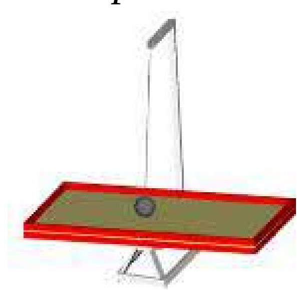
Occam's Razor

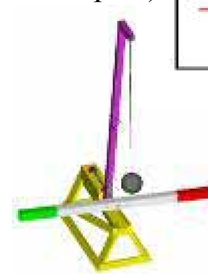
- William of Occam (or Ockham) (1284-1347) was an English philosopher and theologian
  – Ockham stressed the Aristotelian principle that *entities must not be multiplied beyond what is necessary (see Maudslay's maxims on page 1-4)*
  – "Ockham wrote fervently against the Papacy in a series of treatises on Papal power and civil sovereignty. The medieval rule of parsimony, or principle of economy, frequently used by Ockham came to be known as **Ockham's razor**. The rule, which said that *plurality should not be assumed without necessity* (or, in modern English, *keep it simple, stupid*), was used to eliminate many pseudo-explanatory entities" (http://wotug.ukc.ac.uk/parallel/www/occam/occam-bio.html)

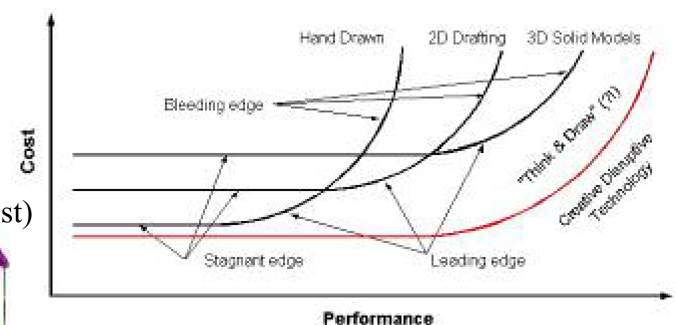


  - ***A problem should be stated in its most basic and simplest terms***
  - ***The simplest theory that fits the facts of a problem is the one that should be selected***
  - ***Limit Analysis is an invaluable way to identify and check simplicity***
- Use fundamental principles as catalysts to help you
  – Keep It Super Simple (KISS)
  – Make It Super Simple (MISS)
  – Because *"Silicon is cheaper than cast iron"* (Don Blomquist)



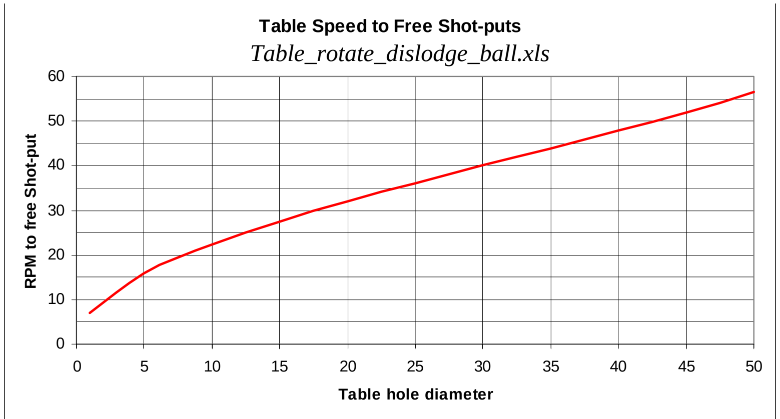


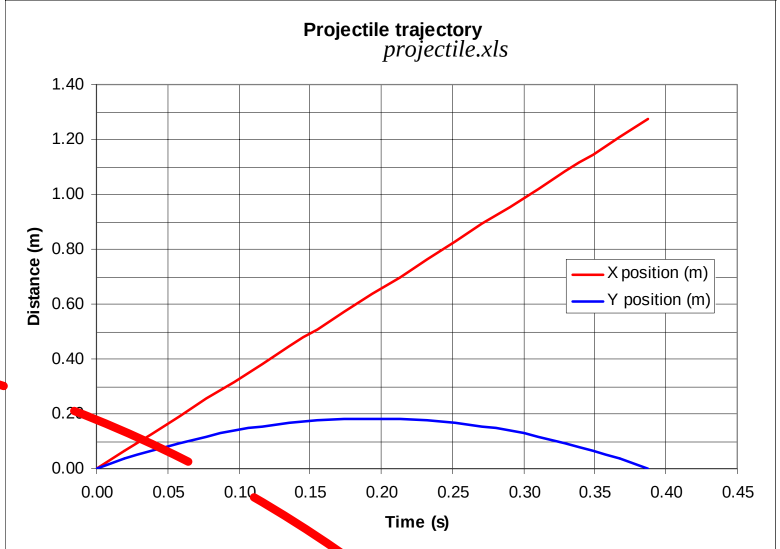




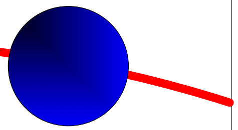
Newton's Laws

- 1st, 2nd, & 3rd "Laws" are invaluable design catalysts that can help launch many an idea!
  - (The only real "law", perhaps, is 300,000 km/second!)
- Conservation of linear momentum
  - If no force is applied, then momentum is constant
- Conservation of angular momentum
  - If no torque is applied to a body about an axis, angular momentum is constant about that axis
    - A force coincident with an axis does not apply torque about that axis





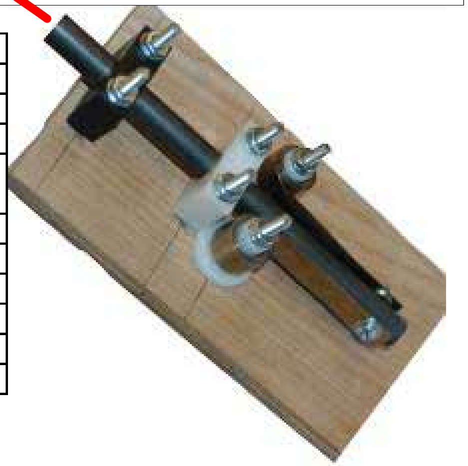
See Projectile_motion.xls



Sir Isaac Newton (1642 - 1727)

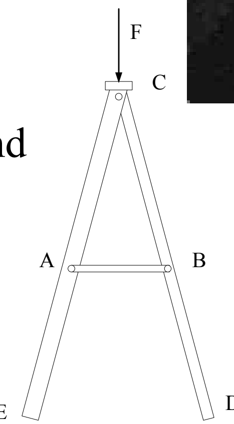
Isaac probably would have LOVED snowboarding!

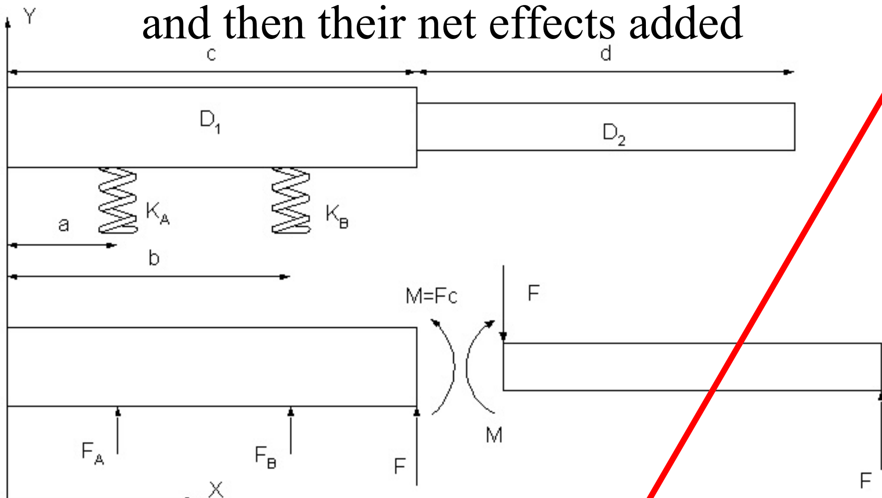
Power_budget_estimate.xls										
Power & energy budget for individual moves, total (S) for simultaneous moves, and cumulative										
Last modified 9/01/03 by Alex Slocum										
Enters numbers in **BOLD,** Results in RED						Power (Watts)			Energy (N-m)	
Axis	Move #	Force (N)	Velocity (m/s)	Distance (m)	Efficiency, net system	Move	Battery dissipation	Σ power for move #	Energy for move	Σ Energy
Drive to pucks	**1**	**3**	**0.2**	**1**	**29%**	2.10	8.30		52.0	52.0
Lower arm	**1**	**0.5**	**0.5**	**0.04**	**29%**	0.88	8.30	11.28	0.7	52.8
Scoop	**2**	**3**	**0.2**	**0.02**	**29%**	2.10	3.00	5.10	0.5	53.3
Raise arm	**3**	**3**	**0.2**	**0.05**	**29%**	2.10	3.00	5.10	1.3	54.5
Drive to goal	**4**	**2**	**0.2**	**0.5**	**29%**	1.40	3.00	4.40	11.0	65.6
Dump pucks	**5**	**0.1**	**0.5**	**0.05**	**29%**	0.18	3.00	3.18	0.3	65.9



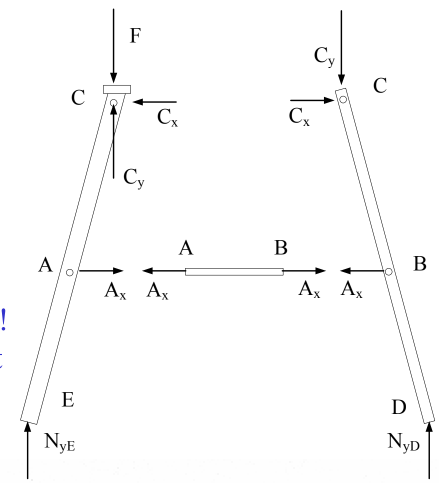
Newton: *Free Body Diagrams & Superposition*

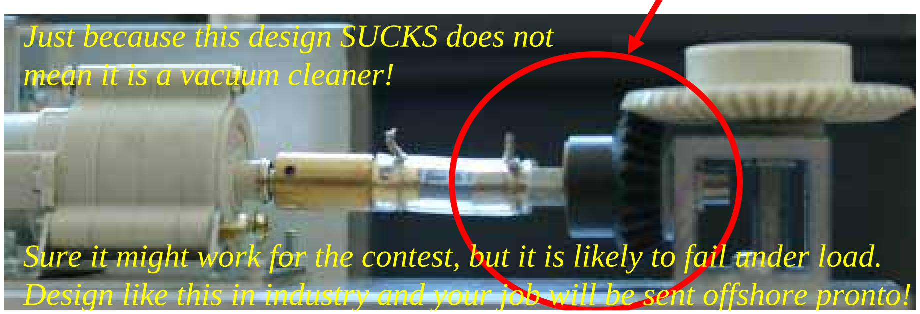
- *Free body diagrams* are a grpahical representation of Newton's third law
  - They allow a designer to show ***components*** and their relationship to each other with respect to forces transmitted between them
    - Invaluable for properly visualizing loads on ***components***
    - In order to properly constrain a ***component***, one has to understand how it is loaded and constrained
- *Superposition* allows a complex load to be broken up into ***components*** each of which can be applied one at a time, and then their net effects added

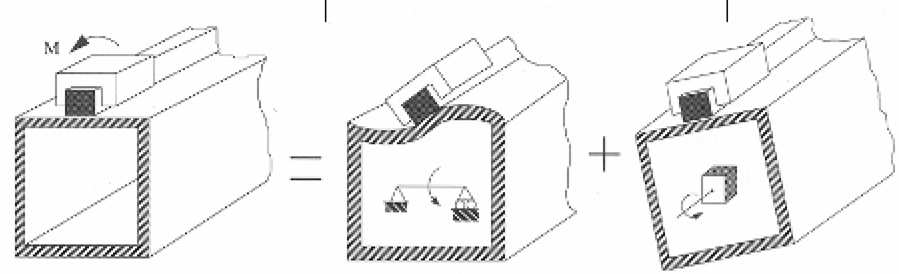




What supports the other end of the shaft to which the gear is attached? How will the gear-tooth radial forces be resisted? A simple FBD of every component can be a critical design synthesis catalyst. FDBs are critical to helping identify how to properly support components! (in a few pages, Saint-Venant will…)







Conservation of Energy…

- What goes in must come out:

$$E_{\text{energy in}} = n_{\text{efficiency}} E_{\text{energy out}}$$

$$F_{\text{force in}} \times d_{\text{distance in}} = n_{\text{efficiency}} F_{\text{force out}} \times d_{\text{distance out}} \Rightarrow 9N \times 1m = 9Nm$$

$$\Gamma_{\text{torque (or moment) in}} \times \alpha_{\text{distance in}} = n_{\text{efficiency}} \Gamma_{\text{torque (or moment) out}} \times \alpha_{\text{distance out}} \Rightarrow 9Nm \times 2\pi = 18\pi Nm$$

Conservation of Energy:

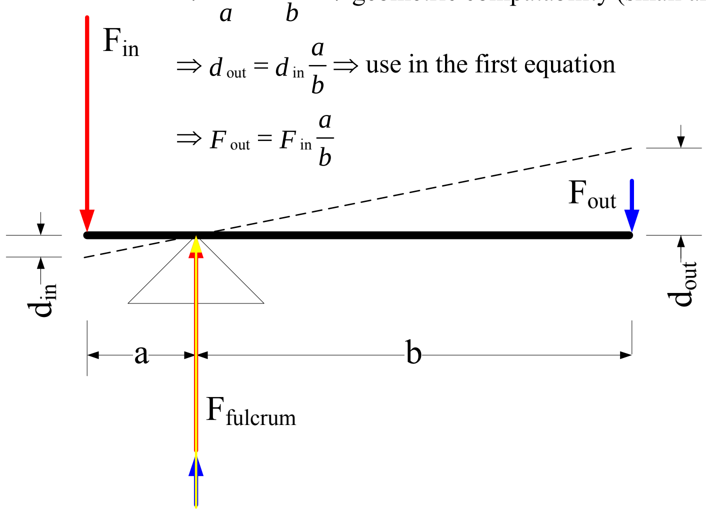
Assume F_{in} and d_{in} are known inputs; find F_{out} and d_{out}:

$F_{in} \times d_{in} = F_{out} \times d_{out} \Rightarrow$ 1 equation and 2 unknowns

$\Rightarrow \frac{d_{in}}{a} = \frac{d_{out}}{b} \Rightarrow$ geometric compatability (small angles)

$\Rightarrow d_{out} = d_{in}\frac{a}{b} \Rightarrow$ use in the first equation

$\Rightarrow F_{out} = F_{in}\frac{a}{b}$



Force and Moment Equilibrium (Newton's First Law):

Assume F_{in} is a known input; find F_{out} and $F_{fulcrum}$

$\sum F_y = 0 \Rightarrow -F_{in} - F_{out} + F_{fulcrum} = 0 \Rightarrow$ 1 equation and 2 unknowns

$\sum M_A = 0 \Rightarrow F_{in} \times a = F_{out} \times b \Rightarrow$ 1 equation and 1 unknown

$\Rightarrow F_{out} = F_{in}\frac{a}{b} \Rightarrow$ use in the first equation

$\Rightarrow F_{fulcrum} = F_{in} + F_{out} = F_{in}\pi\left(1+\frac{a}{b}\right)$

Conservation of Energy:

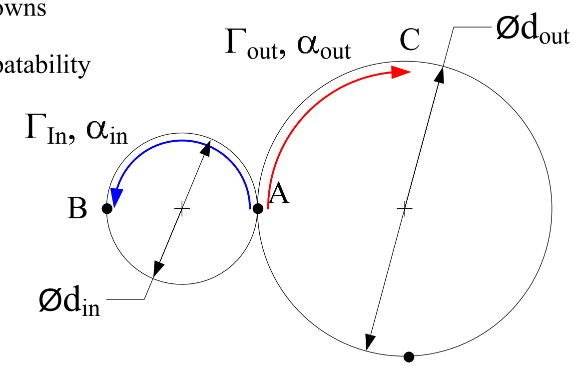
Assume Γ_{in} and α_{in} are known inputs ($\alpha_{in}=\pi$); find Γ_{out} and α_{out}:

$\Gamma_{in} \times \alpha_{in} = \Gamma_{out} \times \alpha_{out} \Rightarrow$ 1 equation, 2 unknowns

$\Rightarrow \pi\frac{d_{in}}{2} = \alpha_{out}\frac{d_{out}}{2} \Rightarrow$ geometric compatability

$\Rightarrow \Gamma_{in} \times \pi = \Gamma_{out} \times \pi\frac{d_{in}}{d_{out}}$

$\Rightarrow \Gamma_{out} = \Gamma_{in}\frac{d_{out}}{d_{in}}$



Conservation of Energy:

Assume torque applied to screw is Γ_{in} over one revolution ($\alpha_{in}=2\pi$)

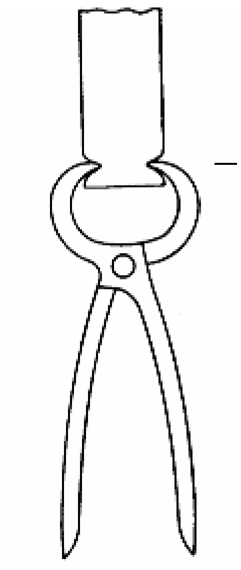
Lead ℓ is defined as distance nut travels in one screw revolution

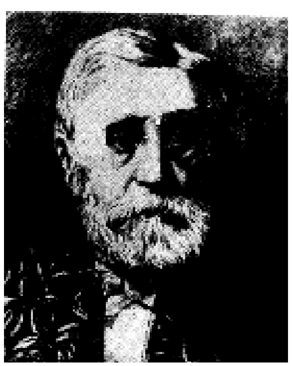
$\Gamma_{in} \times 2\pi \times \eta_{efficiency} = F_{out} \times \ell \Rightarrow$ 1 equation, 1 unknown

$\Rightarrow F_{out} = \frac{2\pi\eta\Gamma_{in}}{\ell}$

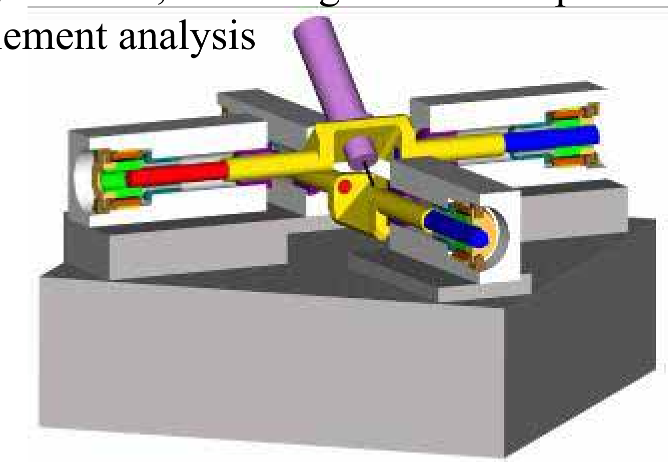
Saint-Venant's Principle

- Saint-Venant's Principle
  – Saint-Venant did extensive research in the theory of elasticity, and many times he relied on the assumption that local effects of loading do not affect global strains
    - e.g., bending strains at the root of a cantilever are not influenced by the local deformations of a point load applied to the end of a cantilever
  – The engineering application of his general observations are profound for the development of conceptual ideas and initial layouts of designs:
    - To NOT be affected by local deformations of a force, be several characteristic dimensions away
      – How many seats away from the sweaty dude do you want to be?
      – Several can be interpreted as 3-5
    - To have control of an object, apply constraints over several characteristic dimensions
      – These are just initial layout guidelines, and designs must be optimized using closed-form or finite element analysis



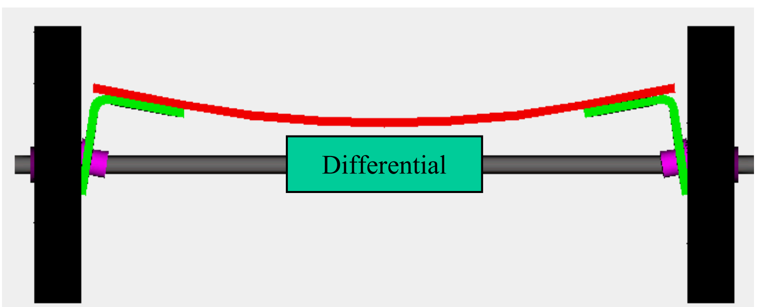


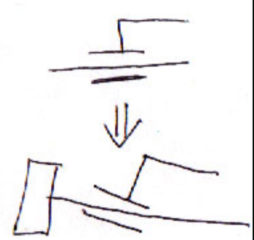
Barré de Saint-Venant 1797-1886



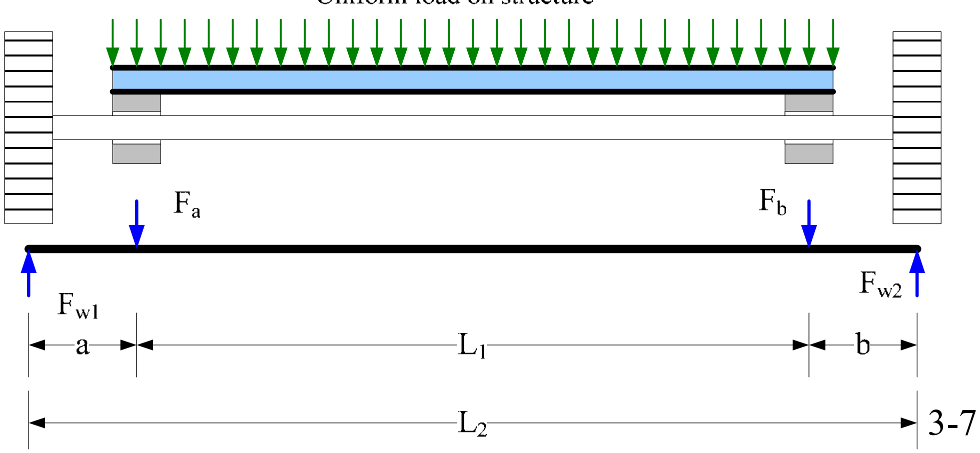
Saint-Venant's Principle: *Structures*

- To NOT feel something's effects, be several characteristic dimensions away!
  - If a plate is 5 mm thick and a bolt passes through it, you should be 3 plate thicknesses away from the bolt force to not cause any warping of the plate!
    - Many bearing systems fail because bolts are too close to the bearings
- To DOMINATE and CONTROL something, control several characteristic dimensions
  - If a column is to be cantilevered, the anchor region should be 3 times the column base area
  - Most machines that suffer from "lawn furniture syndrome" have inadequate proportions
    - Diagonal braces or gussets, that are 3-5 x the column base width, can make a column appear to be cantilevered





Uniform load on structure

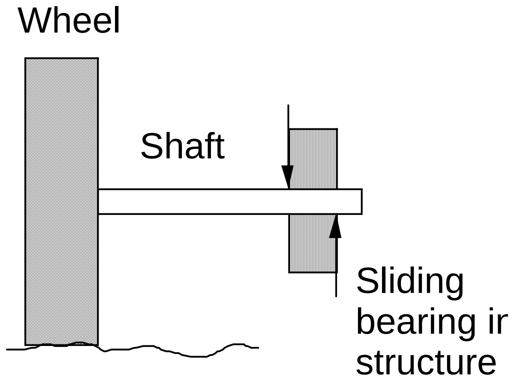


Bearings_rotary_spacing_axle_support.xls

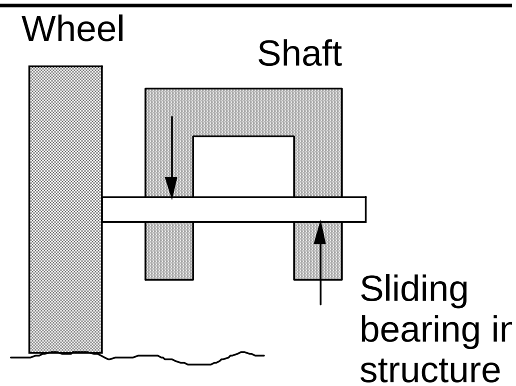
Design Parameters	Thin plate	Laminate
Number of support axles, N	2	2
Bearing length, Lb (m)	0.01	0.01
Bottom beam length, L_2 (m)	0.25	0.25
Axle diameter, d (m)	0.006	0.006
Axle modulus, Eaxle (Pa)	2.00E+11	2E+11
Total load on top beam, F (N)	50	50
Top beam length, L_1 (m)	0.2	0.2
Distance wheels to 1st bearing, a (m)	0.025	0.025
Distance wheels to 2nd bearing, b (m)	0.225	0.225
Top beam top layer thickness, tu (m)	0.0015	0.0015
Top beam bottom layer thickness, tb (m)		0.0015
Top beam layer (top and bottom) modulus, E (Pa)	7.00E+10	7.00E+10
Top beam laminate spacer thickness, tlam (m)		0.01
Top beam laminate spacer modulus, Elam(Pa)		1.00E+10
Top beam front-to-back width, width (m)	0.3	0.3
Top beam centroid position from top, y(m)		0.0065
Top beam EI per axle, EI, EIlam (N-m^2)	3	1922
Axle EI, EIaxle (N-m^2)	13	13
Load per unit width, w (N/m)	250	250
Upper beam slope at bearings, alpha1 (rad)	-2.82E-02	-4.33E-05
Axle slope at bearings, alpha2 (rad)	-0.0049	-0.0049
Net slope at bearings, alphabear (rad)	0.0233	-0.0049
Change in bearing diametral clearance, delta (mm)	0.2331	-0.0487

Saint-Venant's Principle: *Bearings*

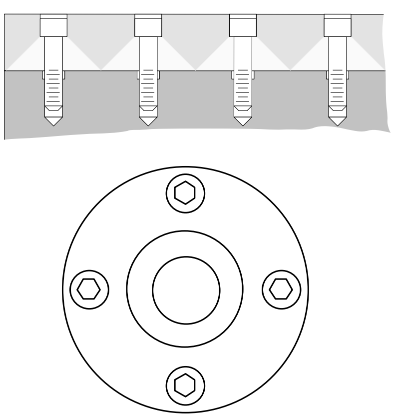
- Saint-Venant: *Linear Bearings:*
  - Make friction (μ) low and L/D>1, 1.6:1 very good, 3:1 awesome
  - Every year some students try L/D<1 and their machines jam!
    - Wide drawers guided at the outside edges can jam
    - Wide drawers guided by a central runner do not!
    - If L/D<1, actuate both sides of the slide!
- Saint-Venant: *Rotary Bearings* (see page 7-27 for a fun case-study!) :
  - L/D>3 if you are to have the bearings "build the shaft into a wall"
  - IF L/D<3, BE careful that slope from shaft bending does not KILL the bearing!
- Bolting bearings in pace: beware the zone under a bolt that deforms due to bolt pressure!

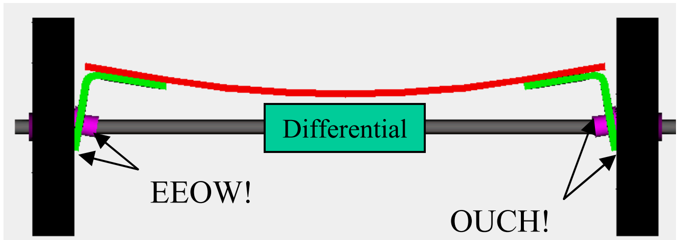


!!Non Optimal!!



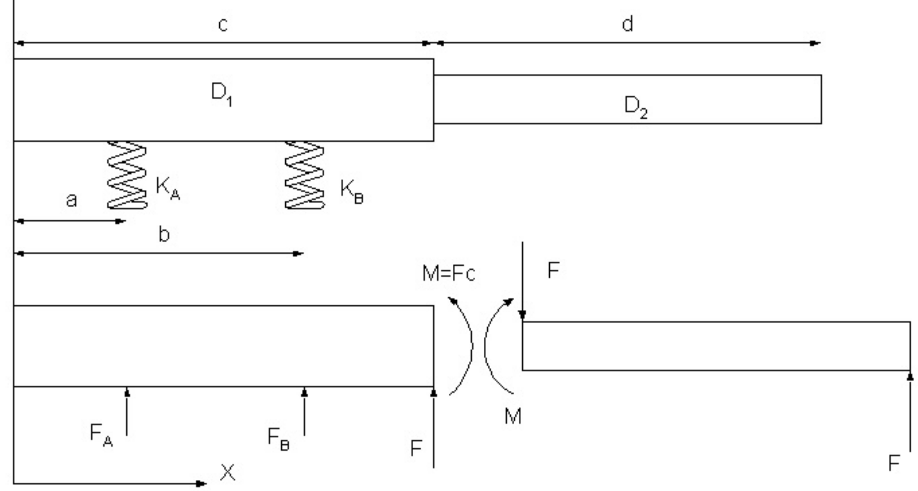
!!Optimal!!





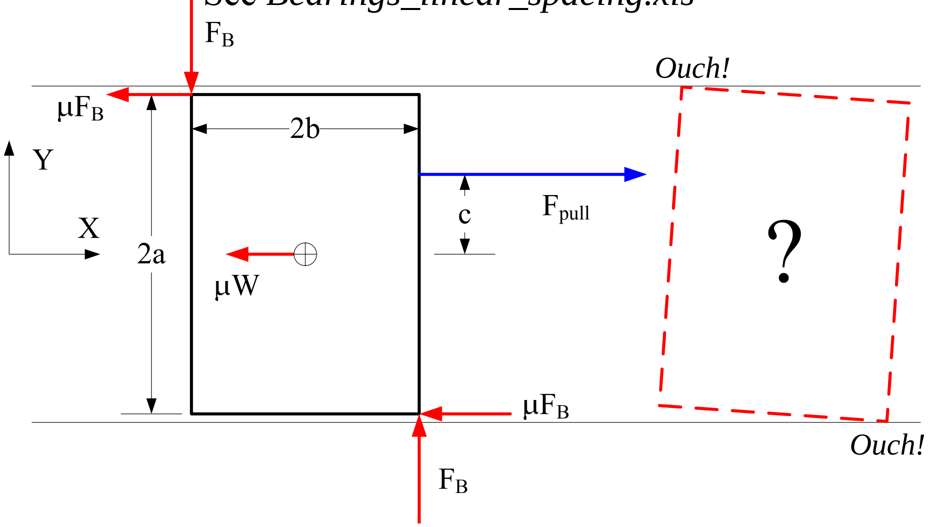
Saint-Venant's Principle: *Bearings*

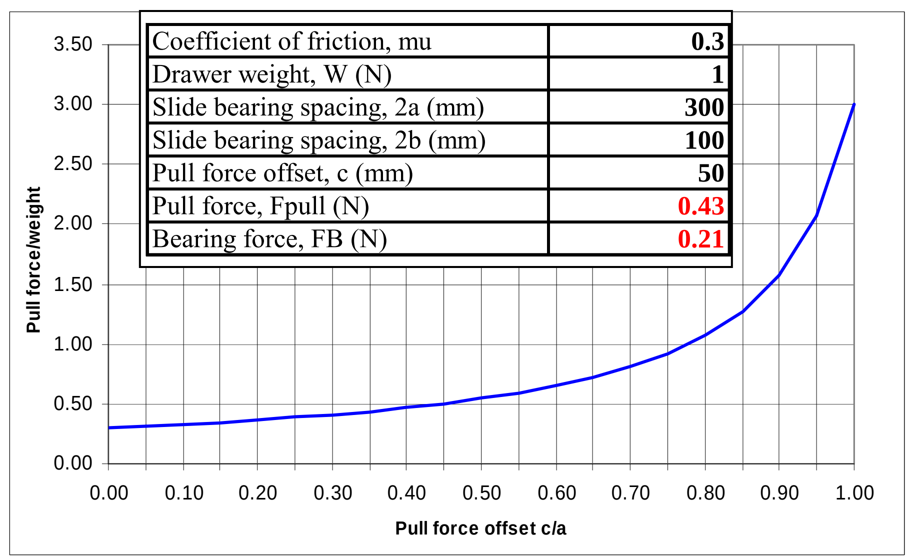
- Model of a shaft supported by bearings: Minimize the deflection of the ends of the beam
  - See *Bearings_rotary_spacing.xls*)



Design Parameters	Values	
a (mm)	50	
b (mm)	20	
c (mm)	250	
d (mm)	100	
Diameter, D_1 (mm)	15	
Diameter, D_2 (mm)	10	
Bearing radial spring constant, KA (N/mm)	2.00E+02	
Bearing radial spring constant, KB (N/mm)	2.00E+02	
Modulus, E (N/mm^2)	6.70E+04	
Tip force, F (N)	10.00	
Moment of inertia, I_1 (mm^4)	2.49E+03	
Moment of inertia, I_2 (mm^4)	4.91E+02	
Spring force, FA (N)	-110.00	
Spring force, FB (N)	100.00	
End deflection of just D_2 segment (mm)	1.01E-01	
End slope of just D_2 segment (rad)	1.52E-03	
Ratio (deflection left end)/(deflection right end)	-0.103	
Position along beam: 0, a, (a+b)/2, b, c, (c+d)	deflection (mm)	slope (rad)
0	-1.20E+00	3.51E-02
50	5.53E-01	3.54E-02
35	2.39E-02	3.52E-02
20	-5.03E-01	3.51E-02
250	7.91E+00	3.78E-02
350	1.17E+01	3.93E-02
Bearing gap closure (for sliding contact bearing supports)		
Bearing width, wb (mm)	5.00	
Diametral clearance loss at first bearing (a) (mm)	0.177	
Diametral clearance loss at first bearing (b) (mm)	0.176	

- Model of the effect of bearing width, friction, and length spacing on the actuation force (drawer jamming)
  - See *Bearings_linear_spacing.xls*

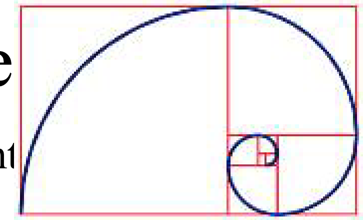


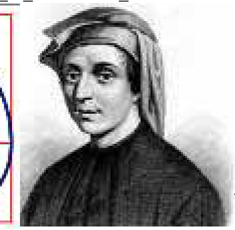


Coefficient of friction, mu	0.3
Drawer weight, W (N)	1
Slide bearing spacing, 2a (mm)	300
Slide bearing spacing, 2b (mm)	100
Pull force offset, c (mm)	50
Pull force, Fpull (N)	0.43
Bearing force, FB (N)	0.21

http://encarta.msn.com/media_461550503_761579463_-1_1/Leonardo_Fibonacci.html

The Golden Rectangle





Leonardo Fibonacci (1170?-1240?)

See http://lady-macbeth.mit.edu/2.007/geekdate/

- The proportions of the *Golden Rectangle* are a natural starting point for preliminary sizing of structures and elements
  - *Golden Rectangle:* A rectangle where when a square is cut from the rectangle, the remaining rectangle has the same proportions as the original rectangle: a/1 = 1/(a-1)
    - See and study *Donald in Mathmagic Land!*
  - Try a *Golden Solid:* 1: 1.618: 2.618, & the diagonal has length 2a = 3.236
  - Example: Bearings:
  - The greater the ratio of the longitudinal to latitudinal (length to width) spacing:
    - The smoother the motion will be and the less the chance of walking (yaw error)
    - First try to design the system so the ratio of the longitudinal to latitudinal spacing of bearing elements is about 2:1
    - For the space conscious, the bearing elements can lie on the perimeter of a golden rectangle (ratio about 1.618:1)
    - The minimum length to width ratio should be 1:1
    - To minimize yaw error
    - Depends on friction too

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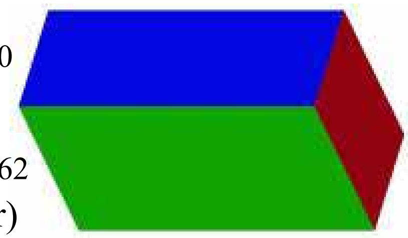
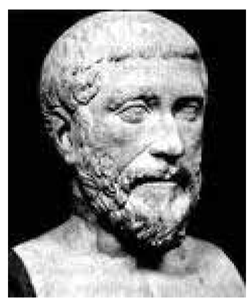
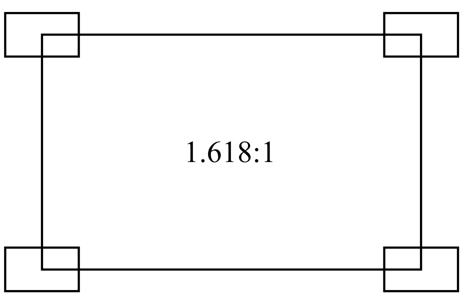


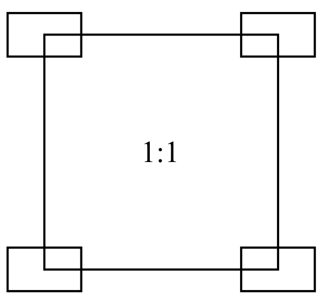
262

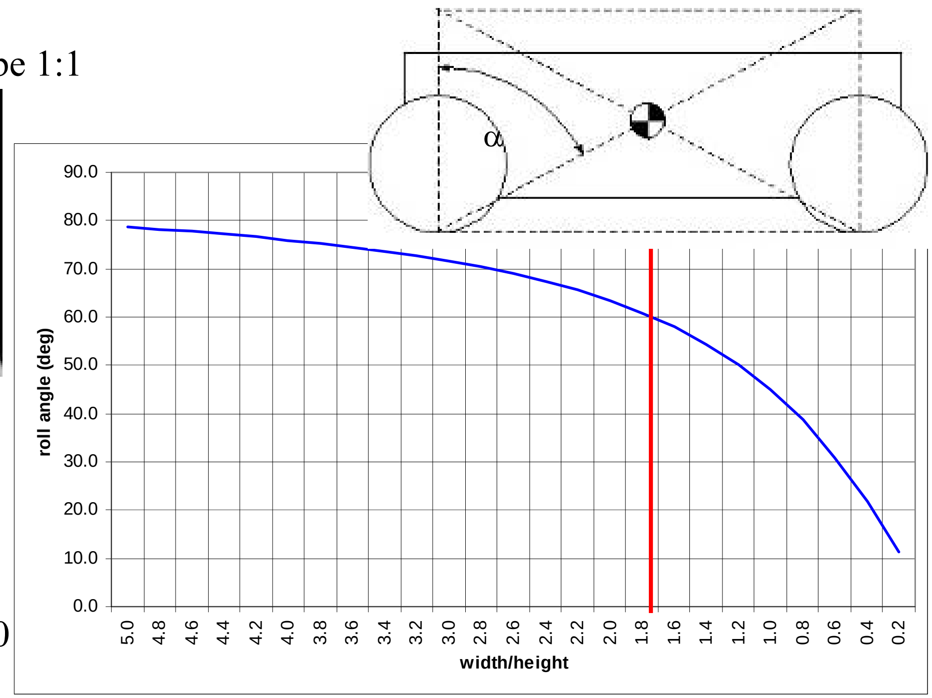
Pythagoras of Samos
569 BC-475 BC



http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Pythagoras.html



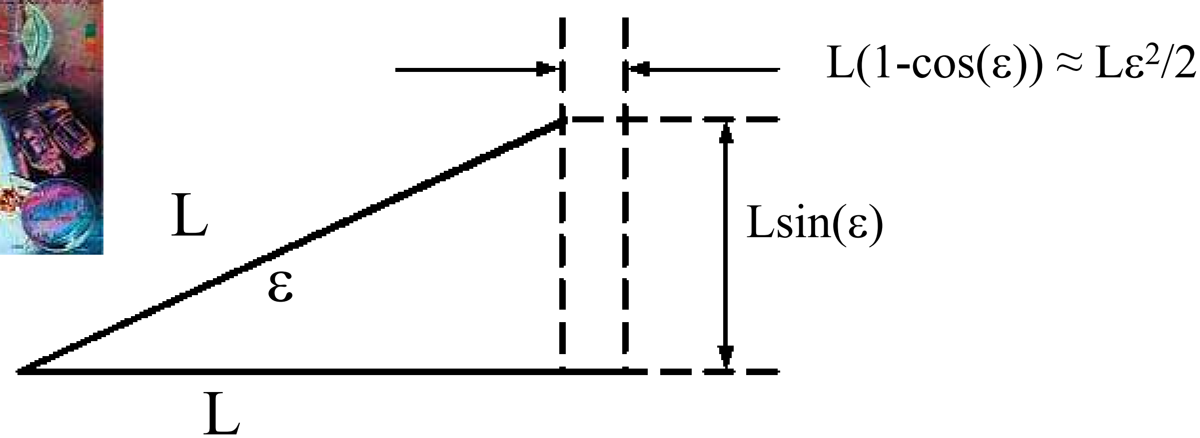




Abbe's Principle

- In the late 1800s, Dr. Ernst Abbe (1840-1905) and Dr. Carl Zeiss (1816-1888) worked together to create one of the world's foremost precision optics companies: Carl Zeiss, GmbH (http://www.zeiss.com/us/about/history.shtml)
- The Abbe Principle (*Abbe errors*) resulted from observations about measurement errors in the manufacture of microscopes:
  - *If errors in parallax are to be avoided, the measuring system must be placed coaxially with the axis along which the displacement is to be measured on the workpiece*
    - Strictly speaking, the term *Abbe error* only applies to measurement errors
- When an angular error is amplified by a distance, e.g., to create an error in a machine's position, the strict definition of the error is a *sine* or *cosine* error

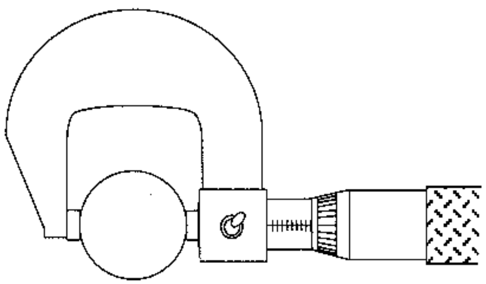
From www.zeiss.com

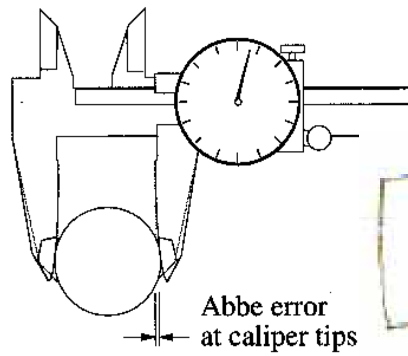


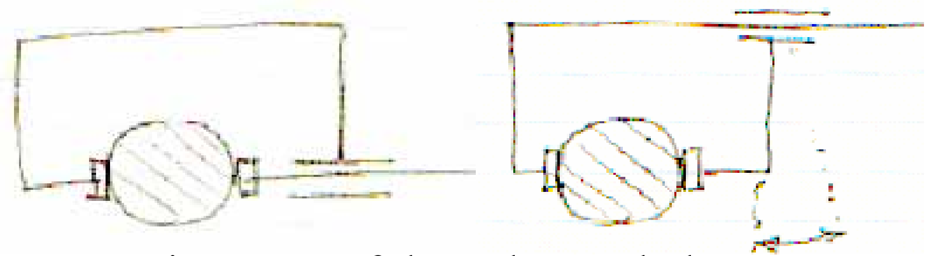
Abbe's Principle: *Locating Components*

- Geometric: Angular errors are amplified by the distance from the source
  - Measure near the source, and move the bearings and actuator near the work!
- Thermal: Temperatures are harder to measure further from the source
  - Measure near the source!

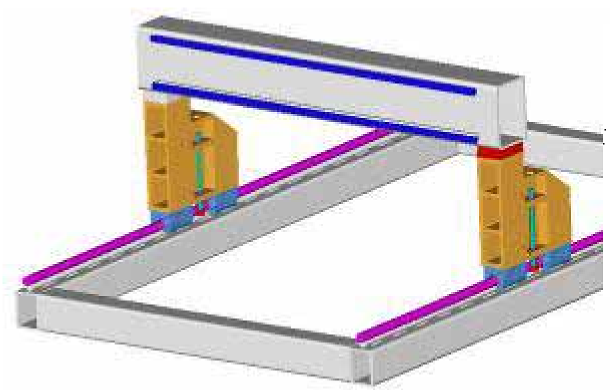
On Brown & Sharpe's vernier caliper: "It was the first practical tool for exact measurements which could be sold in any country at a price within the reach of the ordinary machinist, and its importance in the attainment of accuracy for fine work can hardly be overestimated"







- Thinking of Abbe errors, and the system FRs is a powerful catalyst to help develop DPs, where location of motion axes is depicted schematically
  - Example: Stick figures with arrows indicating motions are a powerful simple means of depicting ***strategy*** or ***concepts***



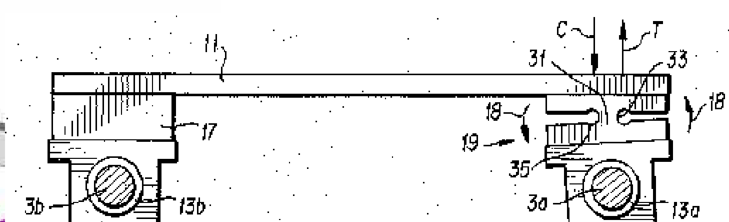
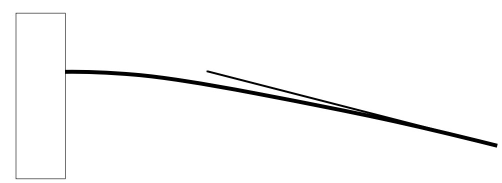
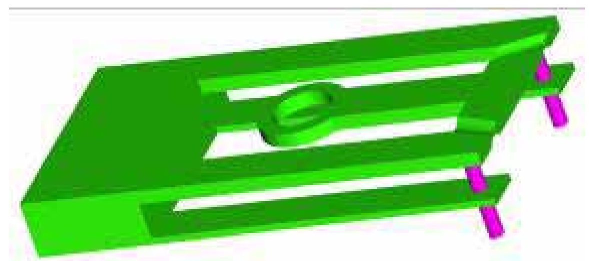


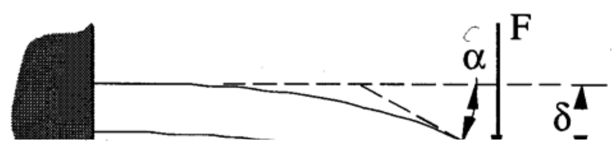
FIG. 3

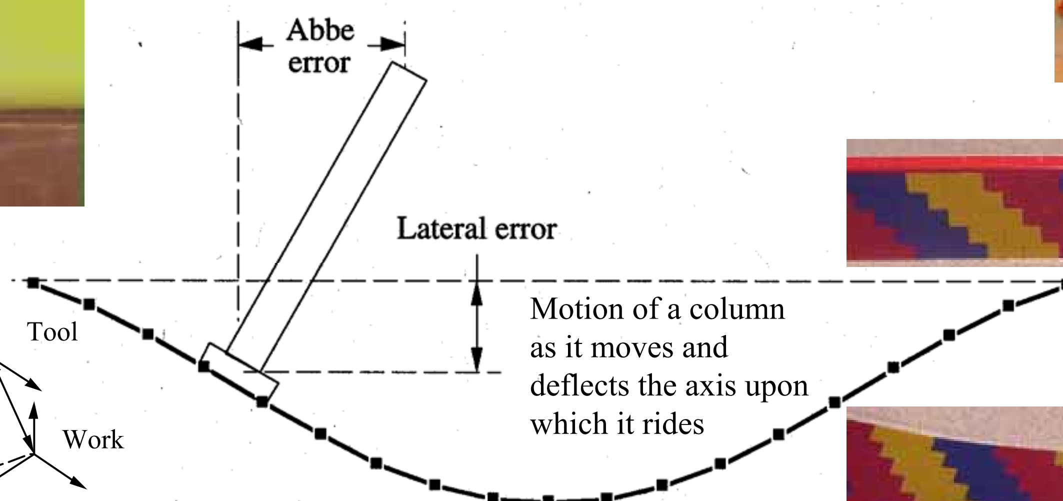


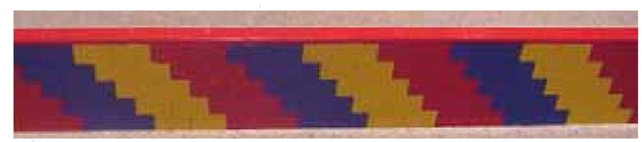


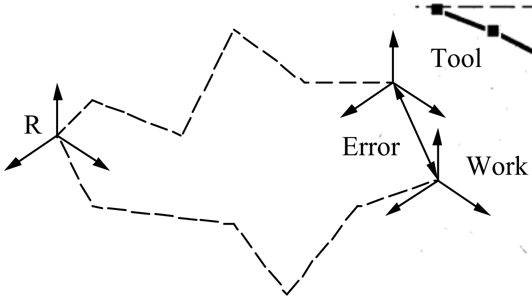
Abbe's Principle: *Cascading Errors*

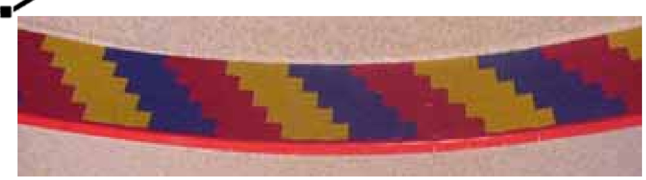
- A small angular deflection in one part of a machine quickly grows as subsequent layers of machine are stacked upon it…
  - A component that tips on top of a component that tips…
  - If *You Give a Mouse a Cookie*… (great kid's book for adults!)
- Error budgeting keeps tracks of errors in cascaded components
  - Designs must consider not only linear deflections, but angular deflections and their resulting *sine errors*…







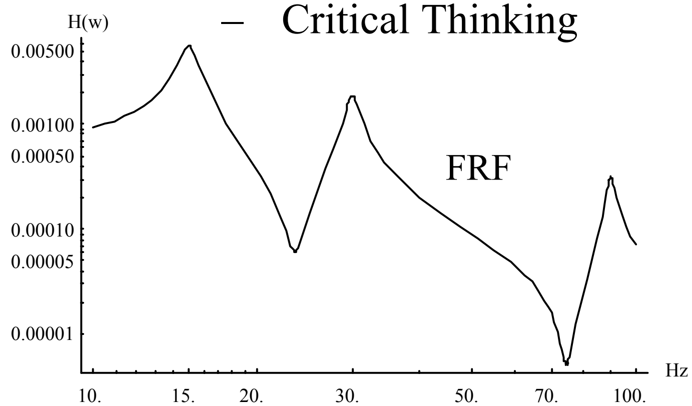




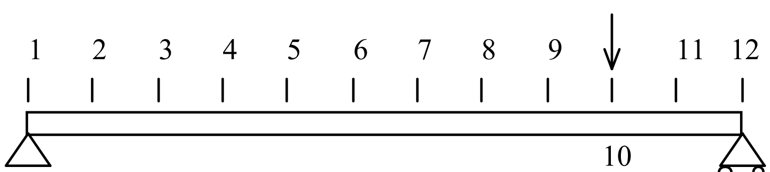
Maxwell & Reciprocity

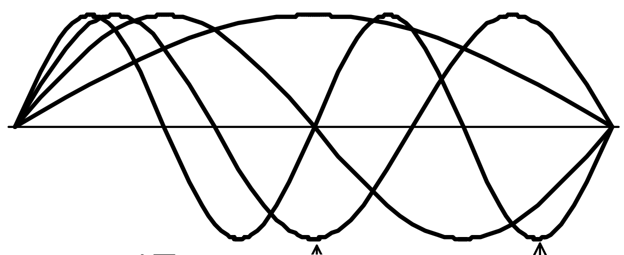
$$\frac{1}{problem} = opportunity! \quad \frac{1}{Ow!} = Ahhhhh!$$

- Maxwell's theory of *Reciprocity*
  - Let *A* and *B* be any two points of an elastic system. Let the displacement of *B* in any direction *U* due to a force *P* acting in any direction *V* at *A* be *u*; and the displacement of *A* in the direction *V* due to a force *Q* acting in the direction *U* at *B* be *v*. Then $Pv = Qu$ (from Roark and Young Formulas for Stress and Strain)
- The principle of *reciprocity* can be extended in philosophical terms to have a profound effect on measurement and development of concepts
  - Reversal
  - Critical Thinking

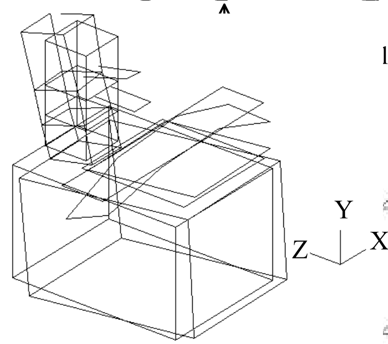


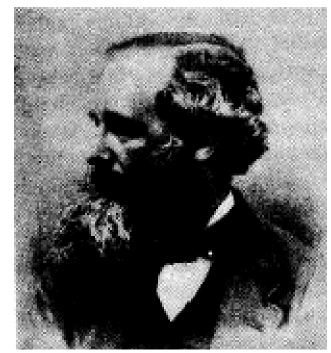
Drive point measurement



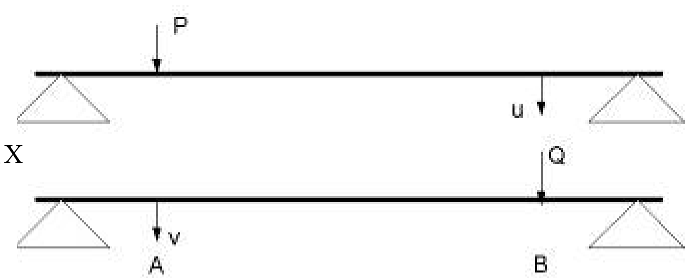


ll modes visible



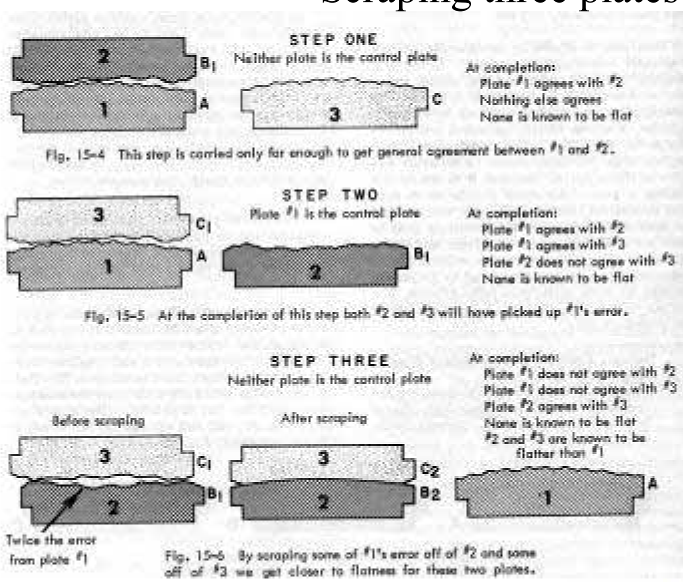


James Clerk Maxwell 1831-1879



Maxwell & Reciprocity: *Reversal*

- *Reversal* is a method used to take out repeatable measuring instrument errors
  - One of the principal methods by which advances in accuracy of mechanical components have been continually made
- There are many application variations for measurement and manufacturing
  - Two bearings rails ground side-by-side can be installed end-to-end
    - A carriage whose bearings are spaced one rail segment apart will not pitch or roll
  - Scraping three plates flat

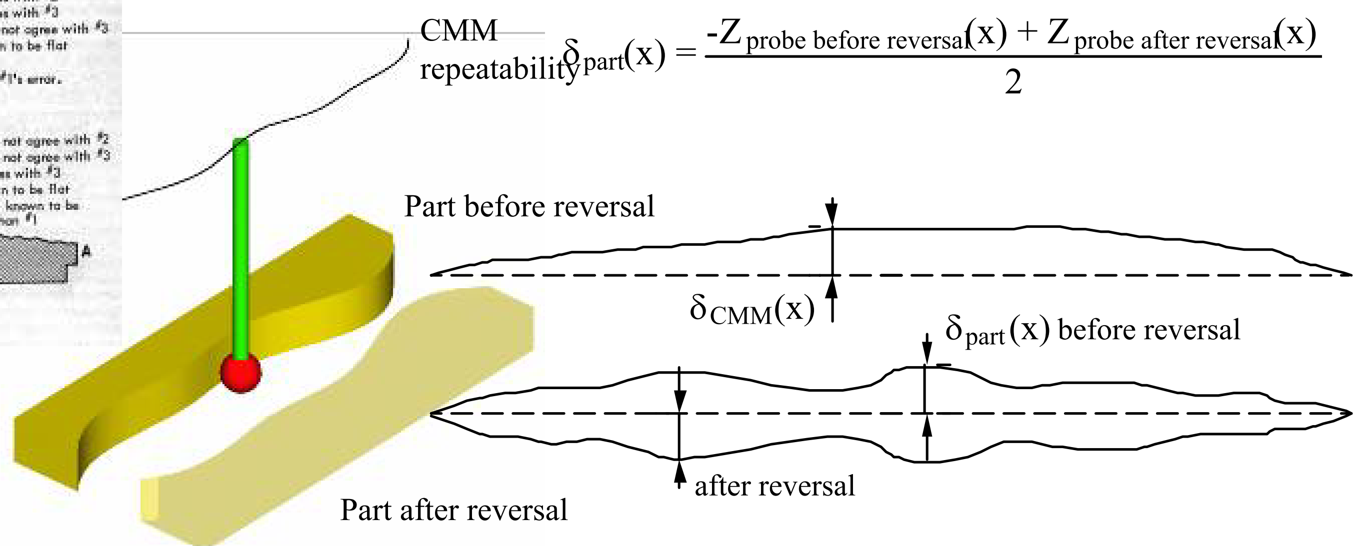


From T. Busch, Fundamentals of Dimensional Metrology, Delmar Publishers, Albany, NY, 1964

$$Z_{\text{probe before reversal}}(x) = \delta_{CMM}(x) - \delta_{part}(x)$$

$$Z_{\text{probe after reversal}}(x) = \delta_{CMM}(x) + \delta_{part}(x)$$

$$\delta_{part}(x) = \frac{-Z_{\text{probe before reversal}}(x) + Z_{\text{probe after reversal}}(x)}{2}$$



Maxwell & Reciprocity: *Critical Thinking*

Rock & Roll Over & Under

- If you are:
  - Happy, turn it around!
  - Unhappy, turn it around!
  - Comfortable on your back, turn over and try lying down on your front……
- You can make a system *insensitive* to its surroundings, or you can *isolate* a system …
- If you cannot solve a problem by starting at the beginning, work backwards!
- Example: Roll-off container passive restraint mechanism
  - In the event of an accident, it keeps an otherwise gravity-held container from flipping off the truck

Reciprocity
It's like velocity
Once up to speed
You have no other need
Late at night
On goes the light
No bounds on curiosity
You create with ferocity

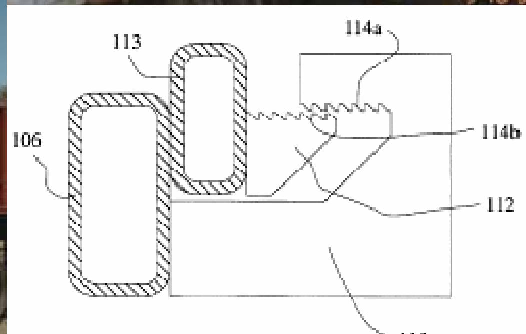


Fig. 7

United States Patent [19] — **Slocum et al.**

[11] **Patent Number:** 5,848,869

[45] **Date of Patent:** Dec. 15, 1998

[54] **CONTAINER RESTRAINING MECHANISM AND METHOD**

[75] Inventors: **Alexander H. Slocum**, Bow; **John William Meskoe**, Concord, both of N.H.

[73] Assignee: **AESOP, Inc.**, Concord, N.H.

[21] Appl. No.: **759,870**

[22] Filed: **Dec. 3, 1996**

[51] **Int. Cl.**[6] **B60P 1/65**

[52] **U.S. Cl.** **414/500**; 414/480; 414/494

[58] **Field of Search** 220/1.5; 414/480, 414/491, 492, 493, 494, 499, 498, 500

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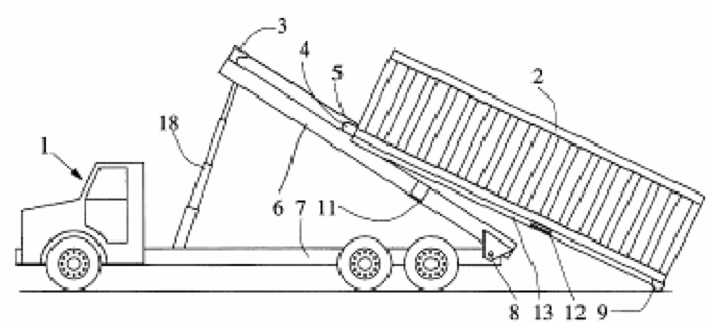
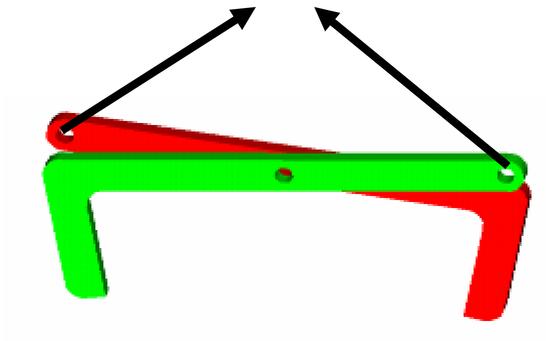
Primary Examiner—Frank E. Werner
Attorney, Agent, or Firm—Rines & Rines

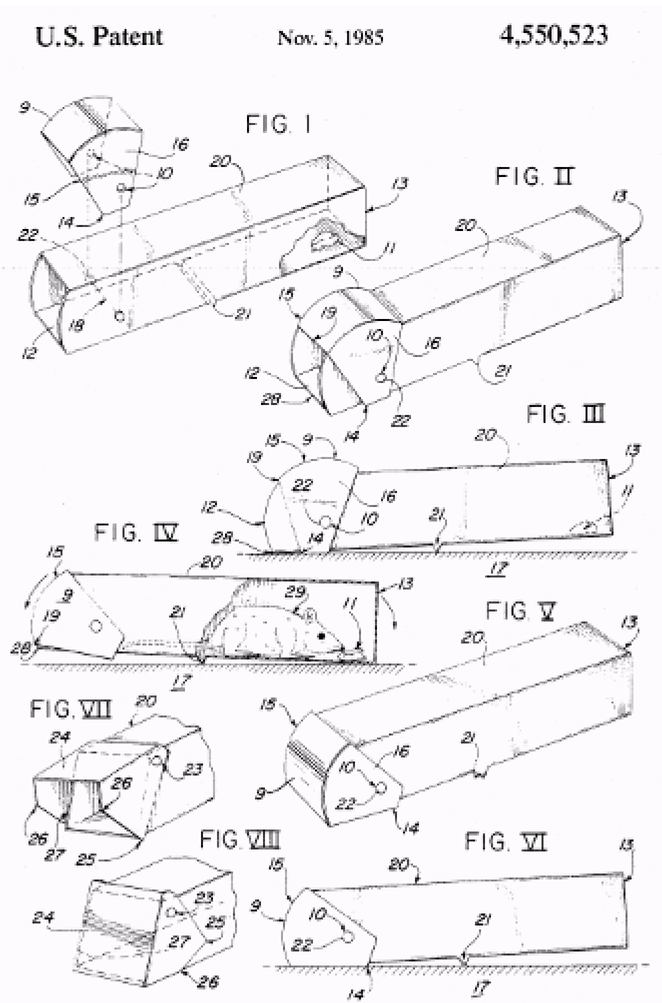
[57] **ABSTRACT**

A system and technique for holding down and restraining a roll-off container in place as it is winched into its final position on the bed of a truck or other transport device, wherein, as the roll-off container is winched into position on the truck, a protuberance on the side of the bottom rail of the roll-off container slides into a mating interlocking cradle attached to the truck. The sliding mate can be of a cantilevered spear into a socket, or more generically, a sliding open-ended male such as a dovetail. In the event of a crash or sudden stop, the roll-off container will thus be retained by the interlocking connection.

5 Claims, 4 Drawing Sheets

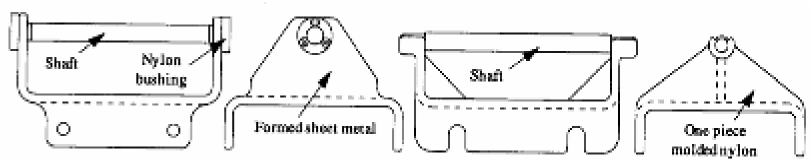




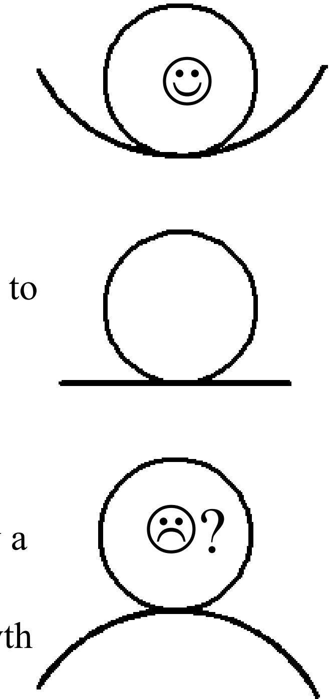


Self-Principles

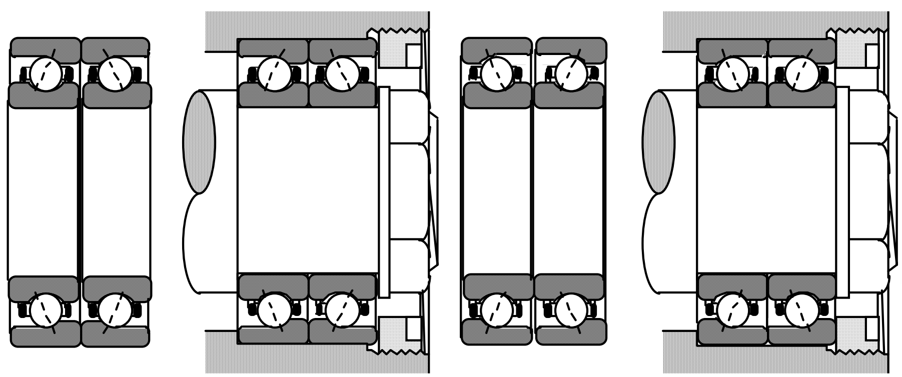
- The manner in which a design reacts to inputs determines its output
  - Reciprocity would philosophically tell us to look for a solution where a potentially detrimental result can be used to cancel the effect
  - Martial artists practice this principle all the time!
- *Self-Help:* A design that uses the inputs to assist in achieving the desired output
  - An initial effect is used to make the device ready for inputs
    - The supplementary effect is that which is induced by the inputs, and it enhances the output
  - *Example:* Airplane doors act like tapered plugs
    - When the door is shut, latches squeeze the seal, making the cabin airtight
    - As the plane ascends and outside air pressure decreases, the higher inner air pressure causes the door to seal even tighter
  - Example: *Back-to-back* angular contact bearings are thermally stable
  - Example: Ice tongs
  - Example: A better mousetrap!
- Other *self-principles* similarly exist:
  - *Self Balancing, Self-Reinforcing, Self-Protecting, Self-Limiting, Self-Damaging, Self-Braking, Self-Starting….*



Stability

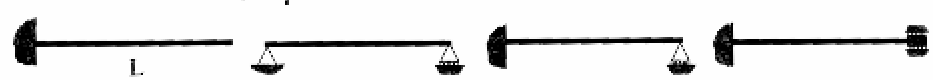


- All systems are either *stable, neutral, or unstable*
  - Saint-Venant's principle was applied to bearing design to reduce the chance of sliding instability (e.g., a drawer jamming)
  - A snap-fit uses an applied force to move from a stable, to a neutrally stable, to an unstable to a final new stable position
  - Wheels allow a system to roll along a flat surface
  - As the load on a tall column increases, infinitesimal lateral deflections are acted on by the axial force to become bending moments, which increase the deflections….
    - Reciprocity says this detrimental effect can be useful: fire sprinklers are activated by a column that buckles when it becomes soft…
  - *Back-to-back* mounted bearings are intolerant of misalignment, but use axial thermal growth to cancel radial thermal growth for constant preload and thermal stability at high speeds
  - *Face-to-face* mounted bearings are tolerant of misalignment, but axial thermal growth adds to radial thermal growth and causes the bearings to become overloaded and seize at high speeds

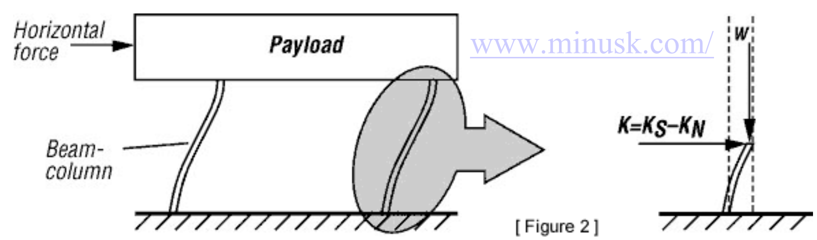


Before mounting | Back-to-back mounting after inner rings are clamped together | Before mounting | Face-to-face mounting after outer rings are clamped together

$$\omega_n = k^2\sqrt{\frac{EI}{A\rho L^4}} \qquad F_{buckle} = \frac{cEI}{L^2}$$



n	Cantilevered k	c	Simply supported k	c	Fixed-simply supported k	c	Fixed-fixed k	c
1	1.875	2.47	3.142	9.87	3.927	20.2	4.730	39.5
2	4.694		6.283		7.069		7.853	
3	7.855		9.425		10.210		10.996	
4	10.996		12.566		13.352		14.137	
n	(2n-1)π/2		nπ		(4n+1)π/4		(2n+1)π/2	



[Figure 2]

Symmetry

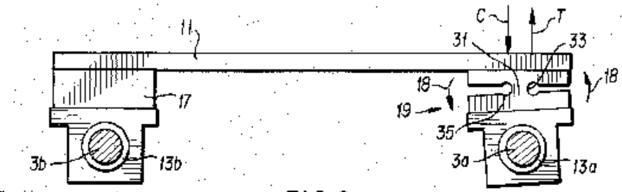
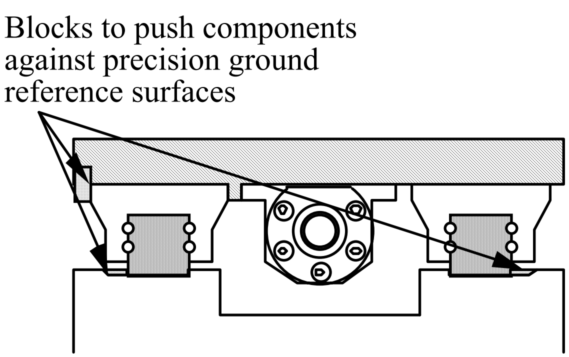
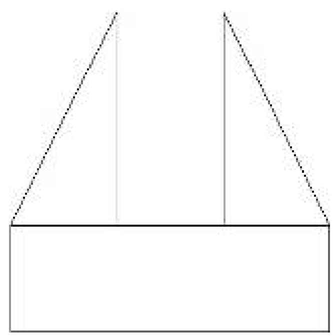
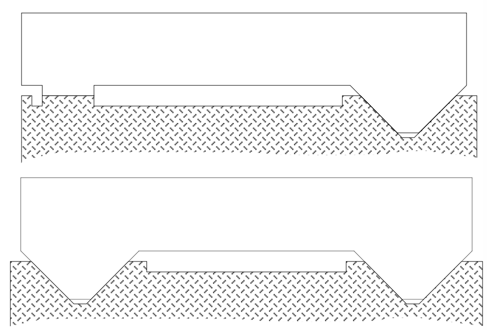


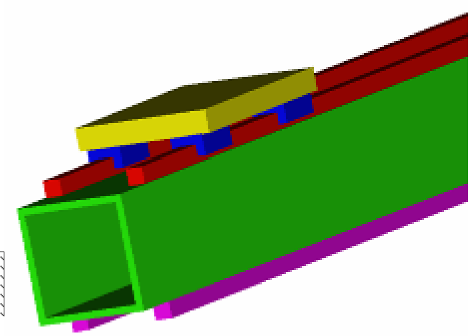
FIG. 3

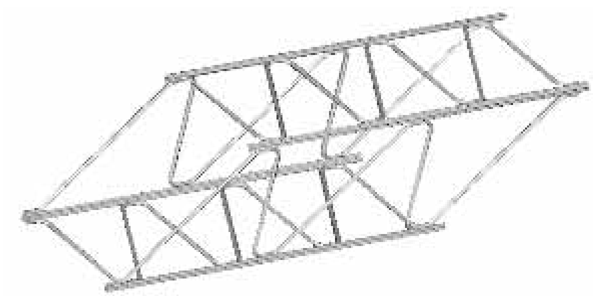
- *Symmetry* can be a powerful design tool to minimize errors
  - Thermal gradient errors caused by bi-material structures can minimize warping errors
    - Steel rails can be attached to an aluminum structure on the plane of the neutral axis
    - Steel rails on an aluminum structure can be balanced by steel bolted to the opposite side
  - Angular error motions can be reduced by symmetric support of elements
- *Symmetry* can be detrimental (Maxwell applied to symmetry)
  - Differential temperature minimized by adding a heat source can cause the entire structure to heat up
    - Only attempt with extreme care
    - Better to isolate the heat source, temperature control it, use thermal breaks, and insulate the structure
  - A long shaft axially restrained by bearings at both ends can buckle
  - Remember-when you generalize, you are often wrong
    - The question to ask, therefore, is “Can symmetry help or hurt this design?”







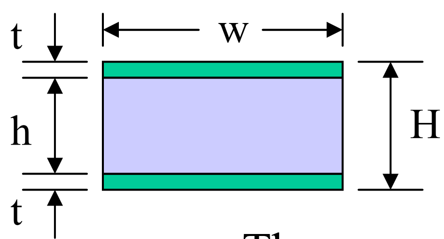




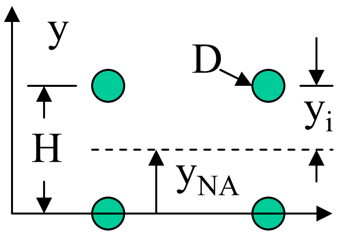
Parallel Axis Theorem

John McBean goes to the extreme!

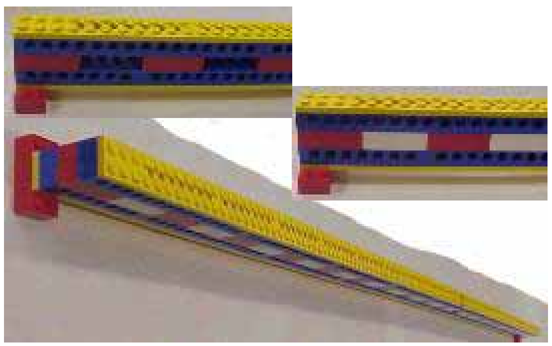
- The *Parallel Axis Theorem* is useful for calculating the moments of inertia for complex objects
  - The stiffness of a design is proportional to the square of the distance y_i of the component structural members' neutral axes from the assembly's neutral axis

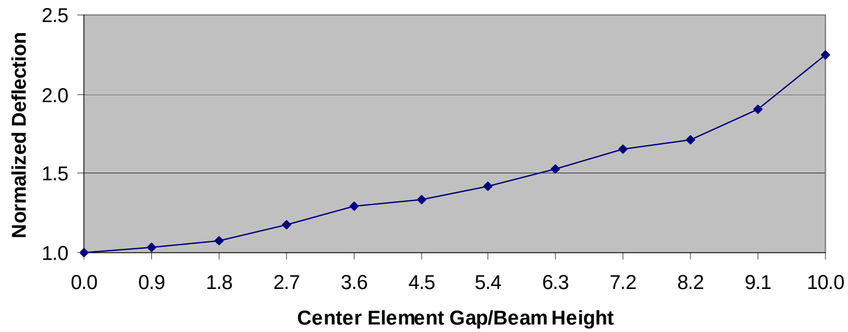


$$I = \sum_{i=1}^{N} I_i + \sum_{i=1}^{N} y_i^2 A_i \qquad y_{NA} = \frac{\sum_{i=1}^{N} y_i A_i}{\sum_{i=1}^{N} A_i}$$

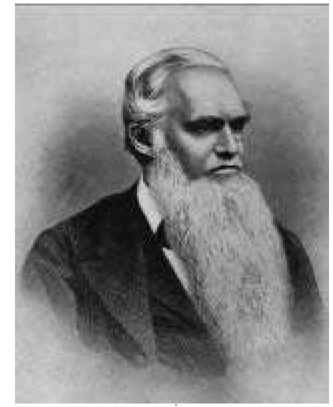


- The assembly's neutral axis is found in the same manner as the center of gravity, and it is located a distance y_{NA} from an arbitrary plane





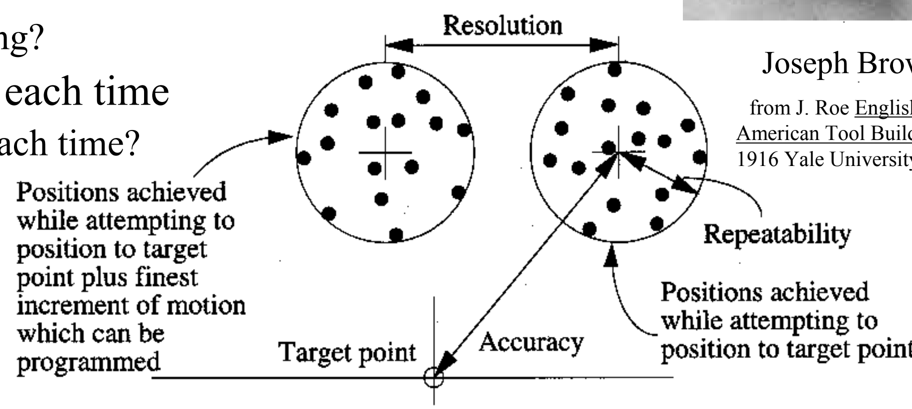
Accuracy, Repeatability, & Resolution



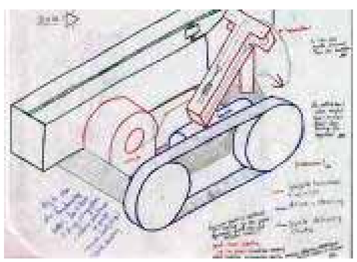
Joseph Brown

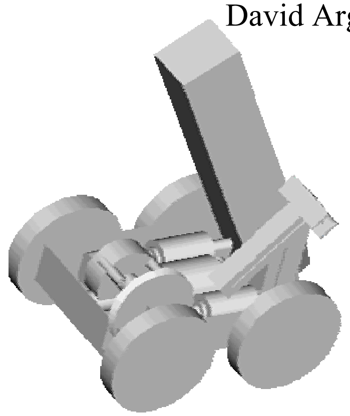
from J. Roe English and American Tool Builders, © 1916 Yale University Press

- Anything you design and manufacture is made from parts
  - Parts must have the desired accuracy, and their manufacture has to be repeatable
- *Accuracy:* the ability to tell the truth
  - Can two machines make exactly the same part?
  - Are the parts the exact size shown on the drawing?
- *Repeatability:* the ability to tell the same story each time
  - Can the machine make the exact same motion each time?
  - Are the parts all the same size?
- *Resolution:* the detail to which you tell a story
  - How fine can you adjust a machine?
  - How small a feature can you make?
- How do these affect the design process?

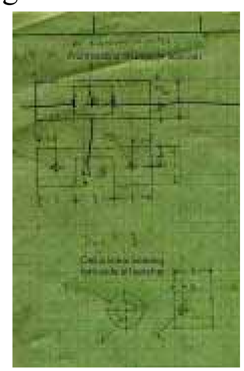


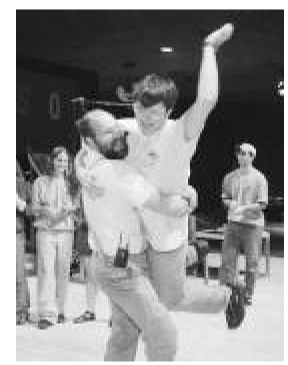
Hook launcher Model	
weight of hook (Kg)	0.05
muzzle velocity	9.4
Number of springs	2
d (draw)	0.095216
Winch model	
radius	0.05
mass	6
w (rpm)	55
torque	2.1
velocity	0.287833

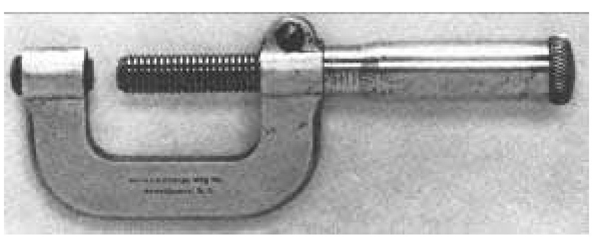




David Arguellis wins "MechEverest" with a machine that repeats every time!







One-inch Micrometer (left) made by Brown & Sharpe, 1868 and Palmer Micrometer (right) brought from Paris by Brown in 1867

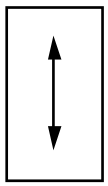
from J. Roe English and American Tool Builders, © 1916 Yale University Press

Accuracy, Repeatability, & Resolution: *Mapping*

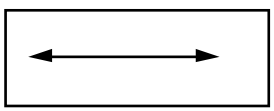
- It is often most important to obtain mechanical *repeatability*, because *accuracy* can often be obtained by the sensor and control system
  - When the error motions of a machine are *mapped*, the controller multiplies the part height by the axis' pitch & roll to yield the sine error for which orthogonal axes must compensate

Eli Whitney

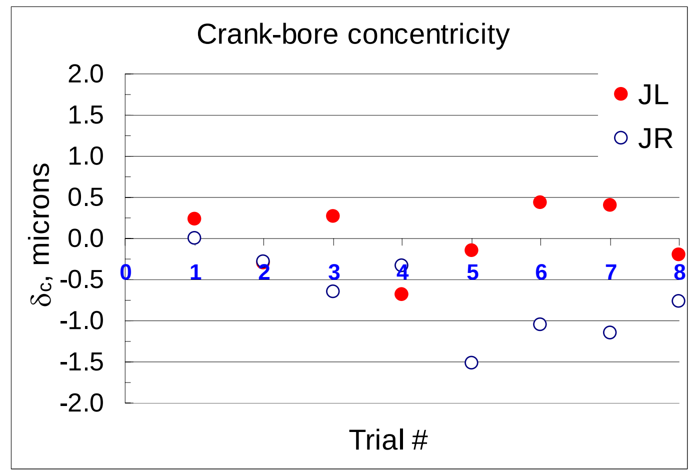
from J. Roe English and American Tool Builders, © 1916 Yale University Press

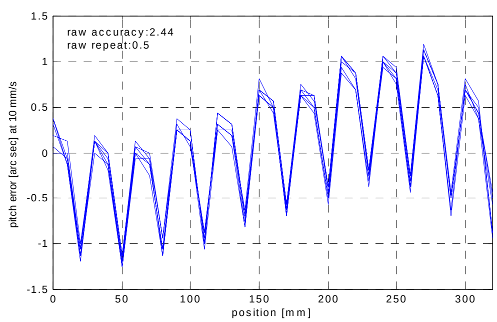


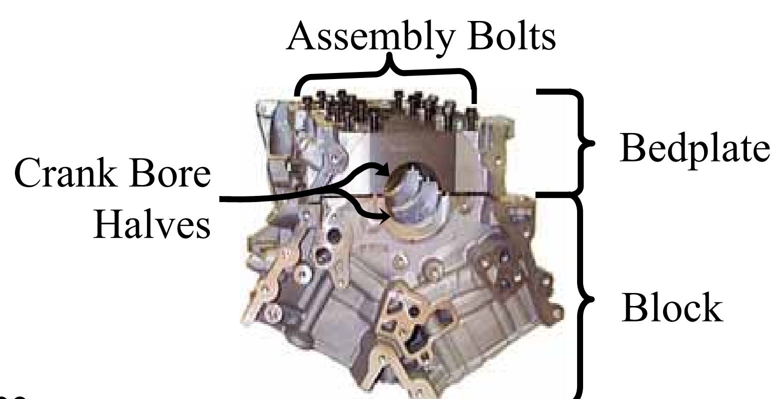
Y axis: Can be used to compensate for straightness errors in the X axis.



X axis: Can be used to compensate for straightness errors in the Y axis.

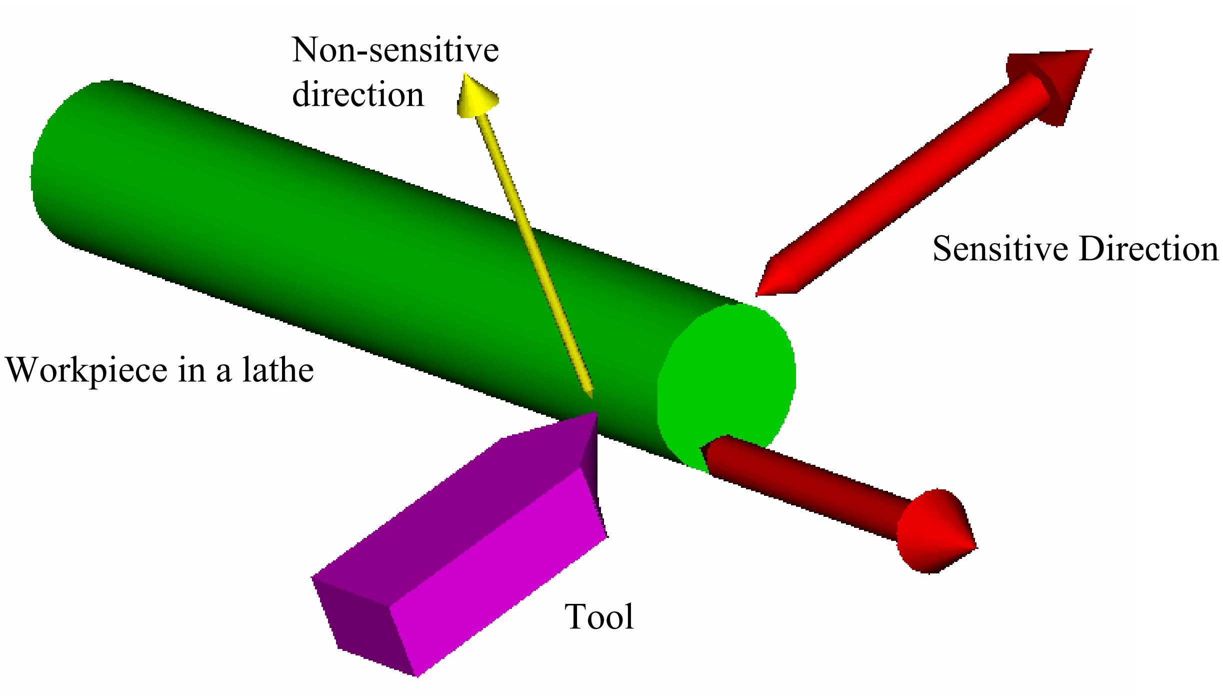


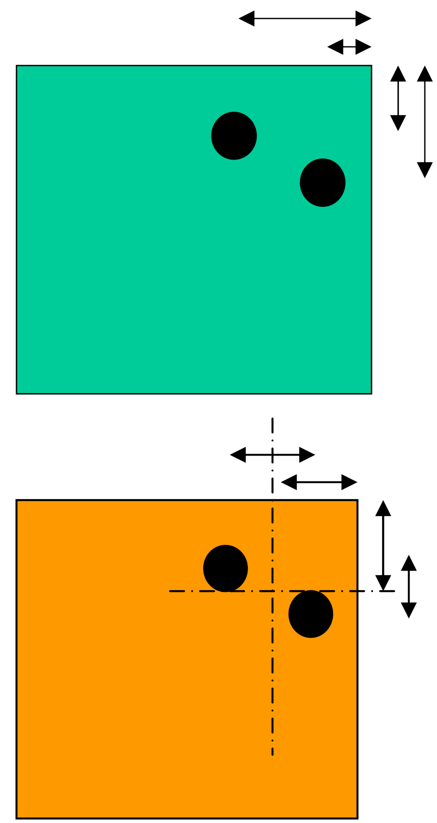




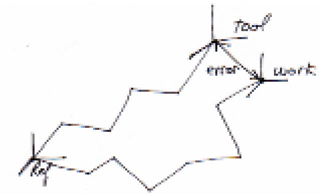
Sensitive Directions & Reference Features

- In addition to *accuracy*, *repeatability*, and *resolution*, we have to ask ourselves, "when is an error really important anyway?"
  - Put a lot of effort into accuracy for the directions in which you need it
    - The *Sensitive Directions*
    - Always be careful to think about where you need precision!

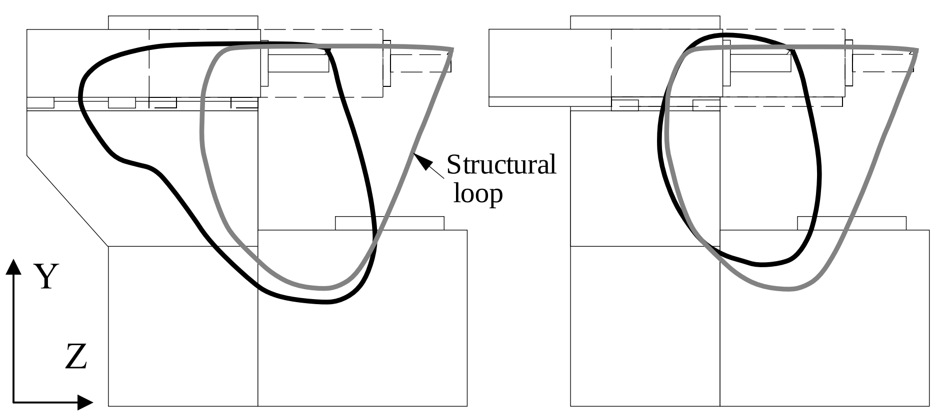




Structural Loops

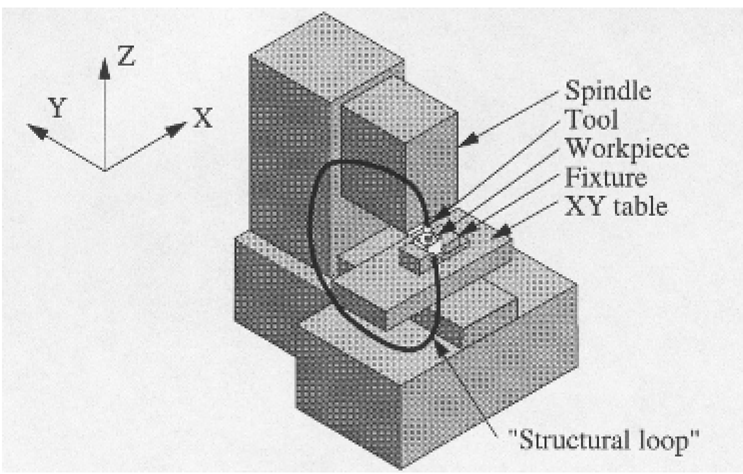


- The *Structural Loop* is the path that a load takes as it travels from the tool through the structure and to the work
  – It contains joints and structural elements that locate the tool with respect to the workpiece
  – It can be represented as a stick-figure to enable a design engineer to create a ***concept***
  – Subtle differences can have a HUGE effect on the performance of a machine
  – The *structural loop* gives an indication of machine stiffness and accuracy
    - *The product of the length of the structural loop and the characteristic manufacturing and component accuracy (e.g., parts per million) is indicative of machine accuracy (ppm)*
    - Long-open *structural loops* have less stiffness and less accuracy

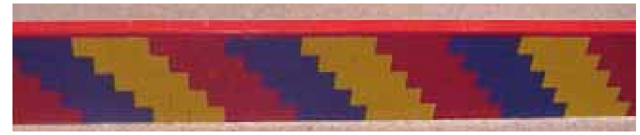


Constant stiffness design

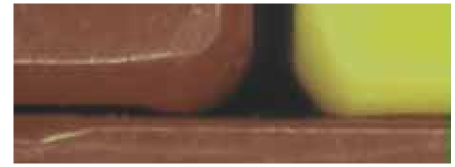
Maximum stiffness design



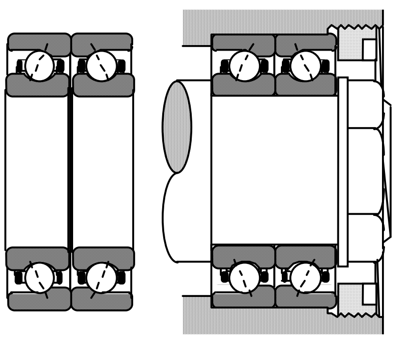
Preload



- Components that move relative to one another generally have tolerances that leave clearances between their mating features
  - These clearances result in *backlash* or wobble which is difficult to control
    - An example is the Lego roller coaster on page 3-10

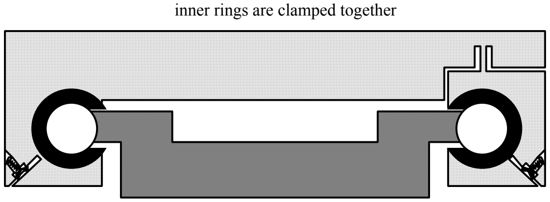


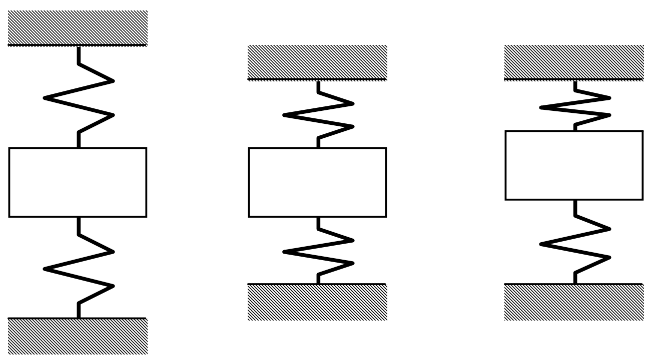
- Because machine elements often have such small compliance, and to account for wear, backlash is often removed with the use of *preload*
  - Preload involves using a spring, or compliance in the mechanism itself, to force components together so there is no clearance between elements
    - However, the compliance in the preload method itself must be chosen such that it locally can deform to accommodate component errors without causing large increases in the forces between components
      - Linear and rotary bearings, gears, leadscrews, and ballscrews are often preloaded
        - » One must be careful when preloading to not too over constrain the system!
      - Structural joints are also often preloaded by bolts



Before mounting

Back-to-back mounting after inner rings are clamped together

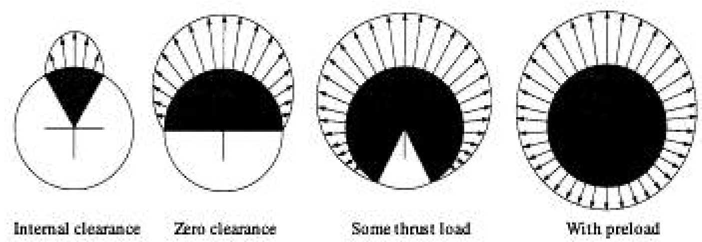


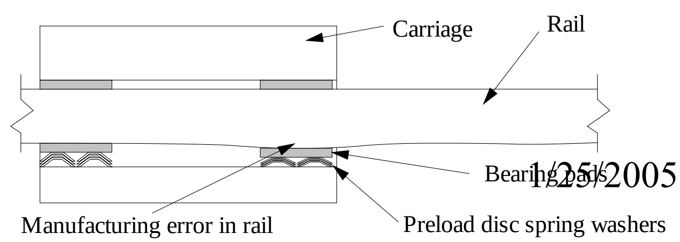


No preload

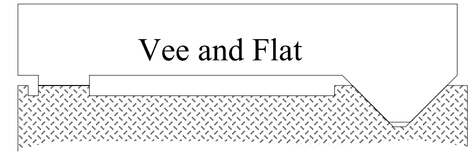
Preloaded

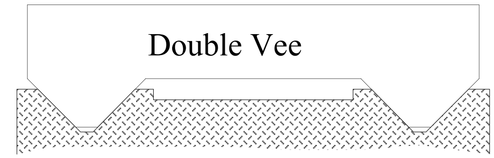
Preloaded with force applied



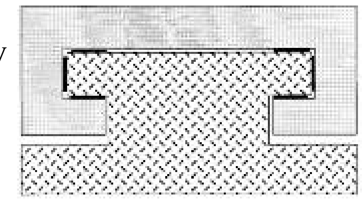


Centers-of-Action

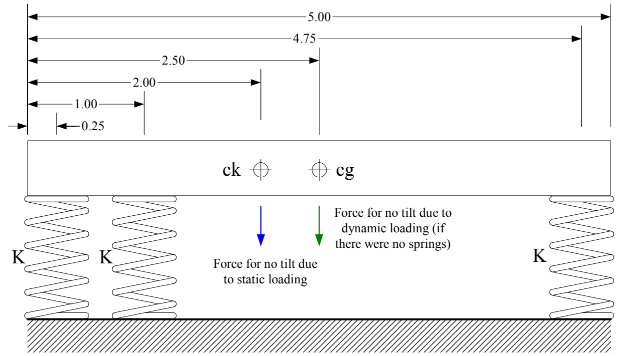


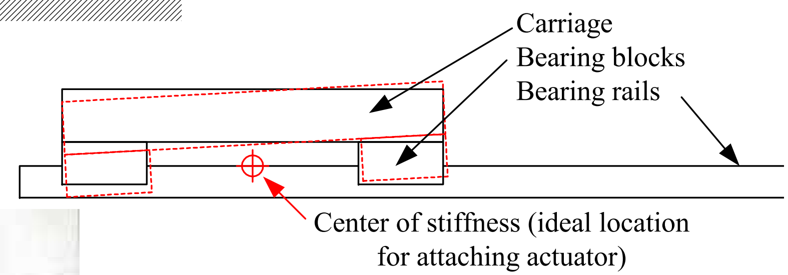


Box way

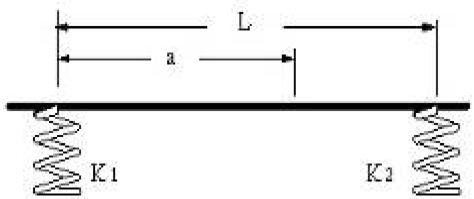


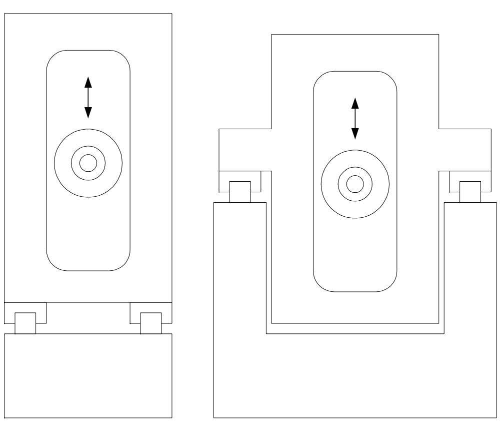
- The *Centers-of-Action* are points at which when a force is applied, no moments are created:
  - *Center-of-Mass*
  - *Center-of-Stiffness*
  - *Center-of-Friction*
  - *Center-of-Thermal Expansion*



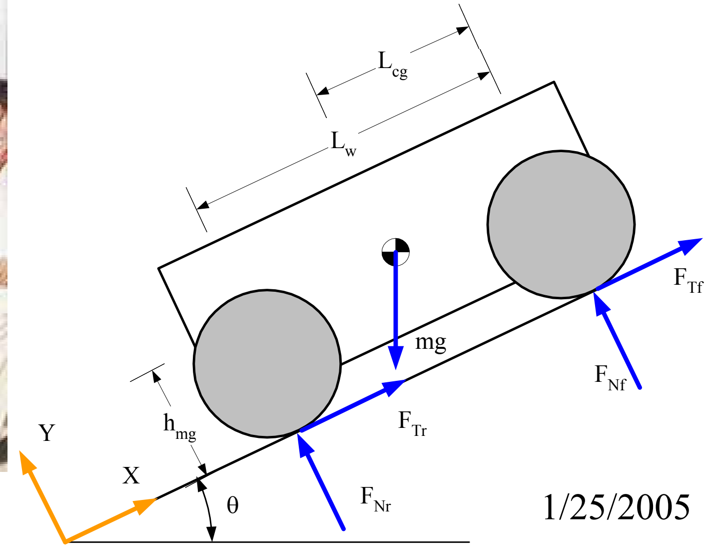


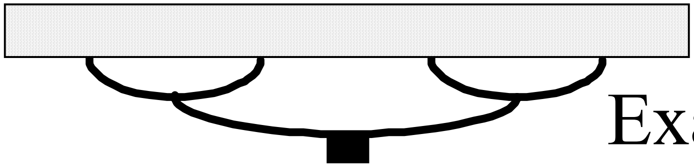
- A system is most robust when forces are applied as near as possible to the *Centers-of-Action*



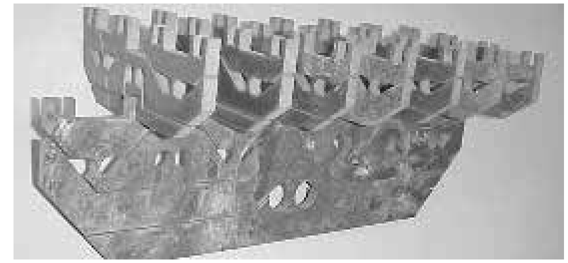


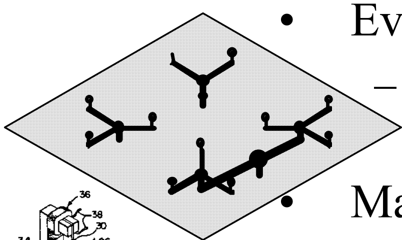
Funny image found on www, photographer not credited, would like to, email slocum@mit.edu





Exact Constraint Design





- Every rigid body has 6 *Degrees of Freedom* (DOF)
  - An exactly constrained design has no chance of deforming or having its function impaired be it by assembly, fastener tightening, thermal expansion, or external loads
- Make sure you have constrained what you want to constrain!
  - For a body to have N degrees of freedom free to move, there must be 6-N bearing reaction points!
  - To resist translation, a force is required.
  - To resist rotation, a moment, or two forces acting as a couple, is required!
    - Saint-Venant rules! Do not constrain a shaft with more than 2 bearings, unless it is very long…

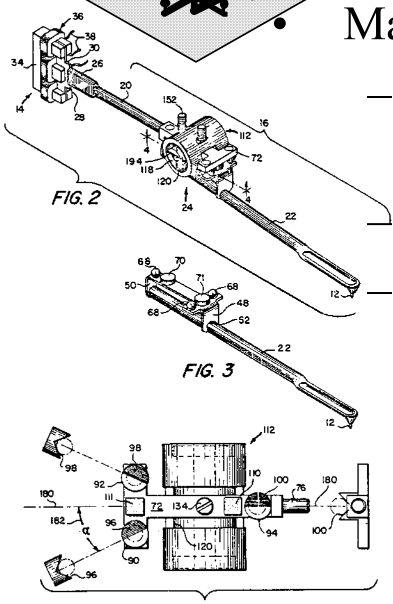
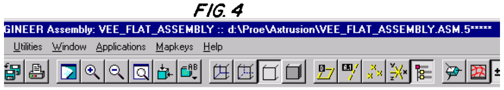
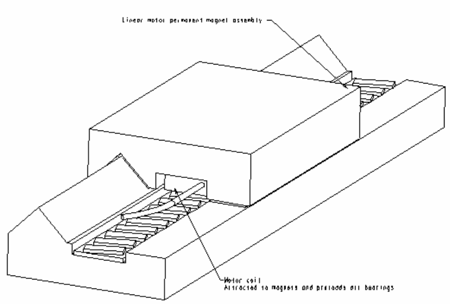


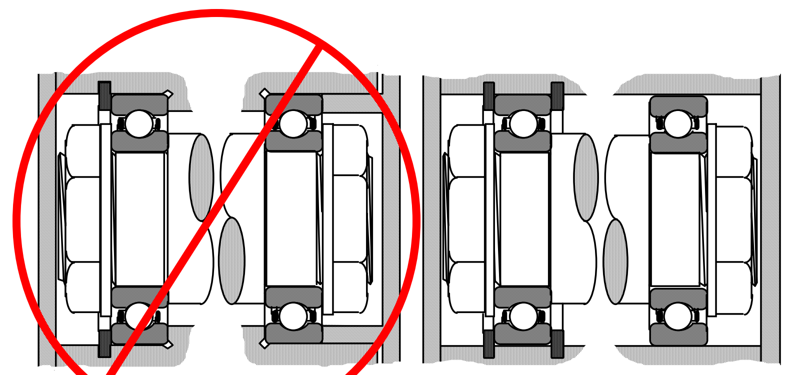
FIG. 2

FIG. 3

FIG. 4

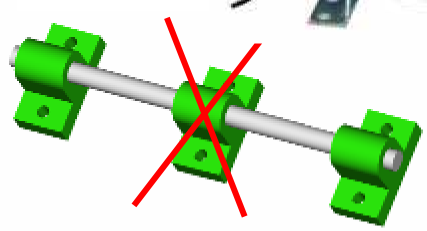


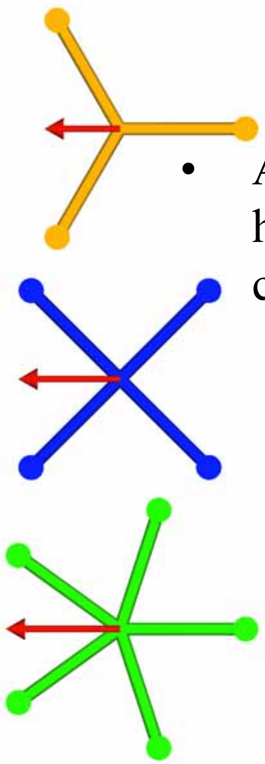


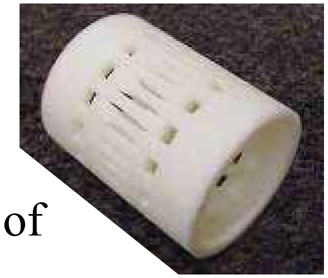


Overconstrained

Properly constrained

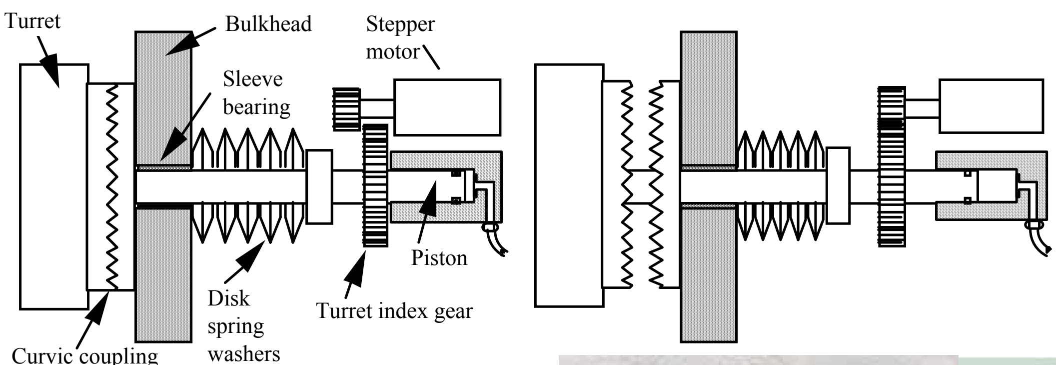




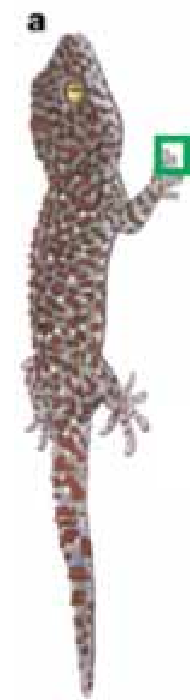


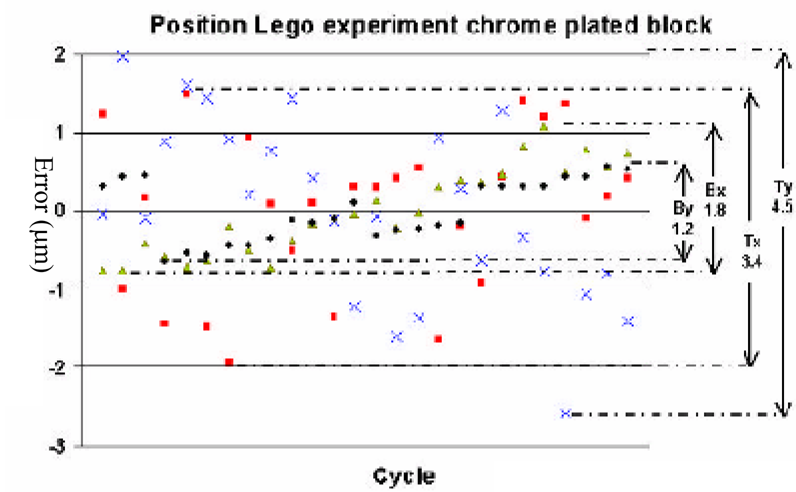
Elastically Averaged Design

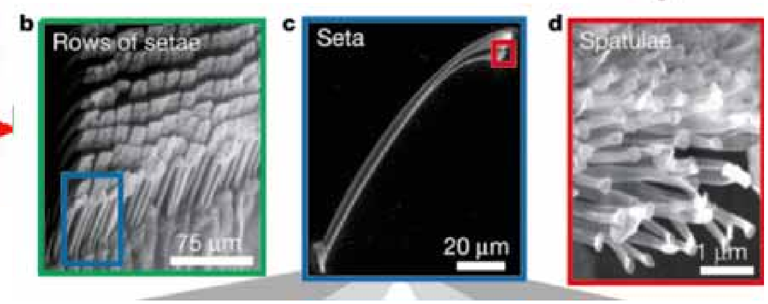
- Applying *Reciprocity* to *Exact Constraint Design* implies that instead of having an exact number of constraints, have an "infinite" number of constraints, so the error in any one will be averaged out!
  - Legos™, five legged chairs, windshield wipers, and Geckos are the most common examples, and many machine components achieve accuracy by elastic averaging



K. Autumn, Y. Liang, W.P. Chan, T. Hsieh, R. Fearing, T.W. Kenny, and R. Full, Dry Adhesive Force of a Single Gecko Foot-Hair, Nature. 405: 681-685 (2000)







Stick Figures

- Use of *fundamental principles* allows a designer to sketch a machine and error motions and coordinate systems just in terms of a *stick figure:*
  - The sticks join at centers of stiffness, mass, friction, and help to:
    - Define the sensitive directions in a machine
    - Locate coordinate systems
    - Set the stage for error budgeting
  - The designer is no longer encumbered by cross section size or bearing size
    - It helps to prevent the designer from locking in too early
- Error budget and preliminary load analysis can then indicate the required stiffness/load capacity required for each "stick" and "joint"
  - Appropriate cross sections and bearings can then be deterministically selected
- It is a "backwards tasking" solution method that is very very powerful!