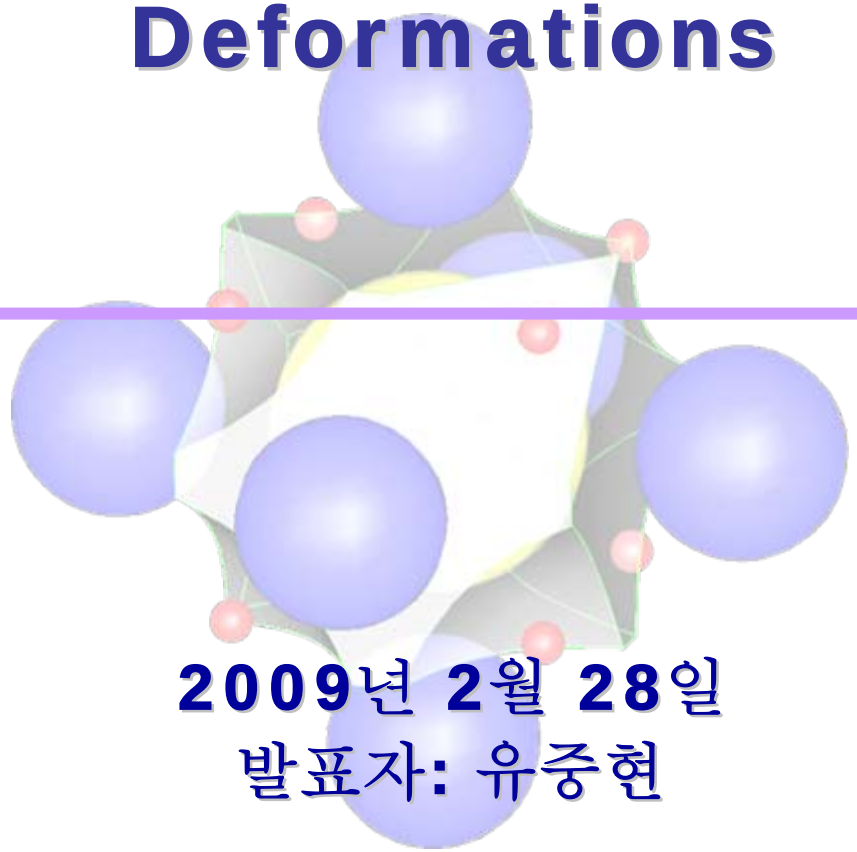
Architectural geometry: Deformations

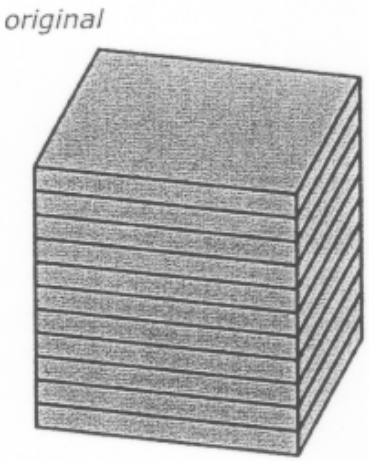


2009년 2월 28일

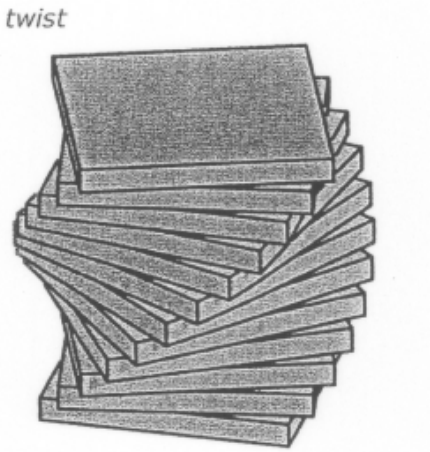
발표자: 유중현

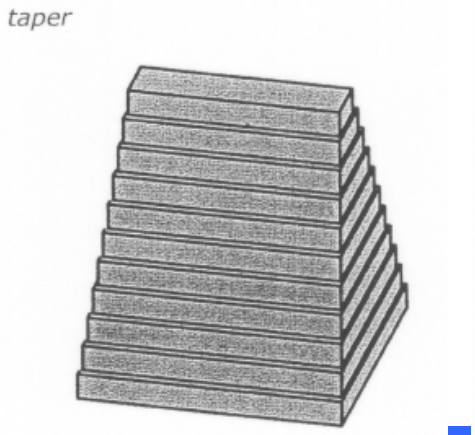
Three-dimensional transformations

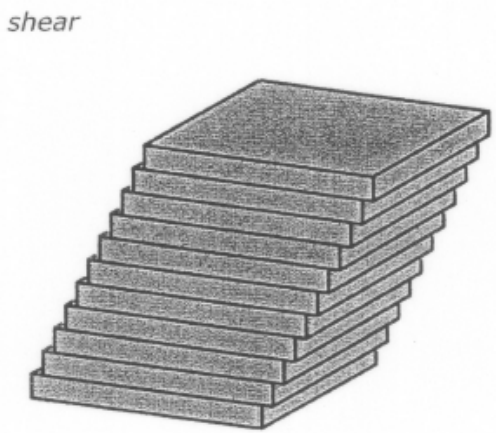
- **Nonlinear transformations for deformation**
  - **cf. Projective and affine map: linear transformation**



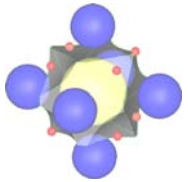
- **Slice-based 3D transformations**
  - **Twist: rotation of slices**
  - **Taper / bulge: scaling of slices**
  - **Shear: translation of slices**



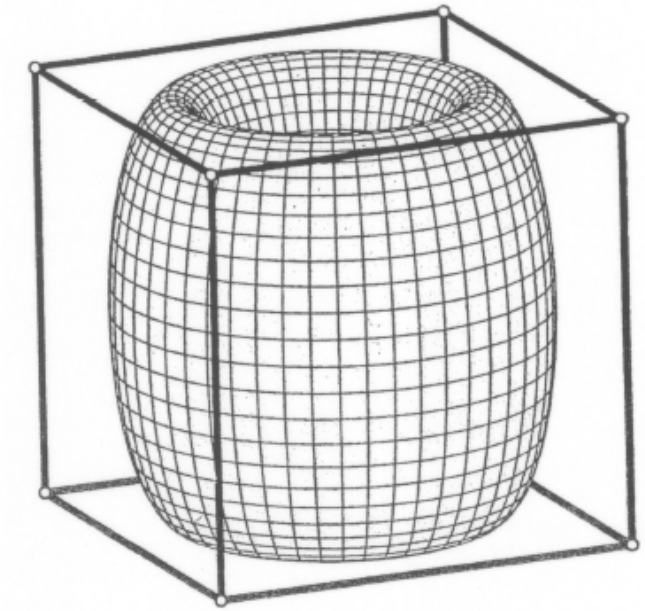


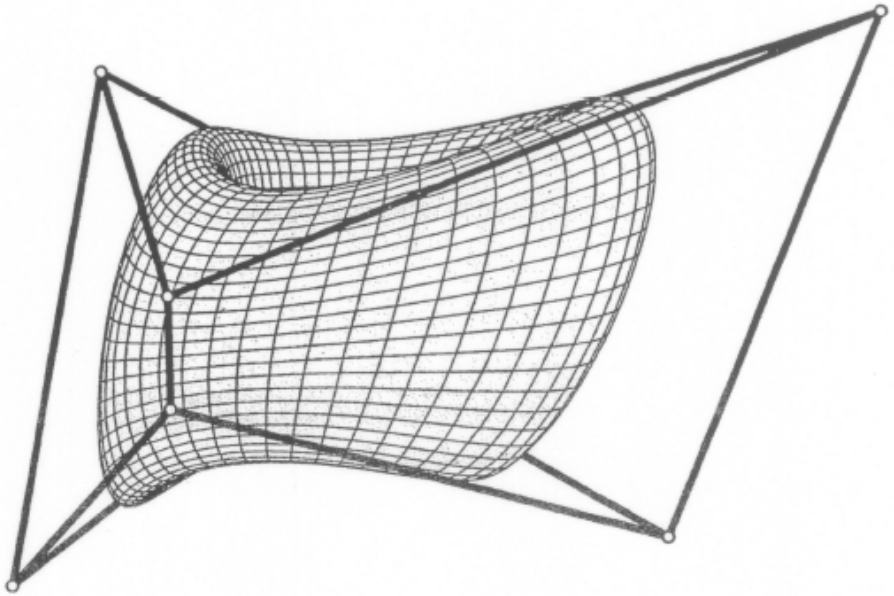


Freeform deformations



- The shape is embedded in a simple solid
- The solid is transformed into a new one using appropriate design handles.

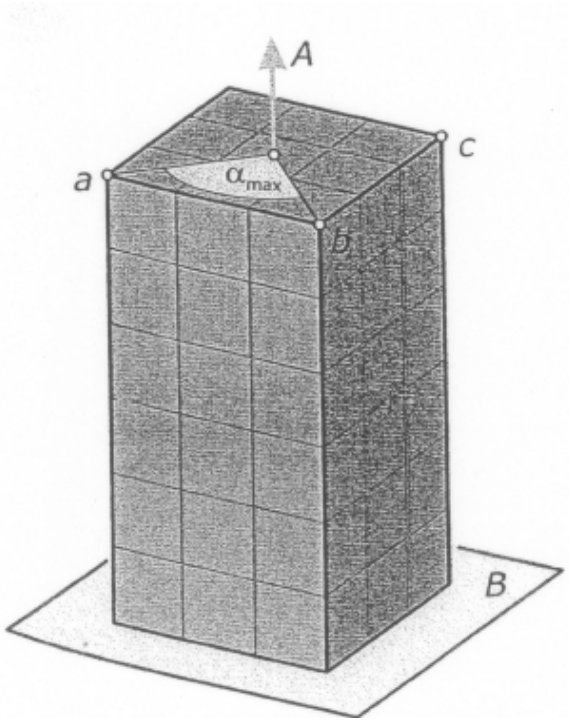


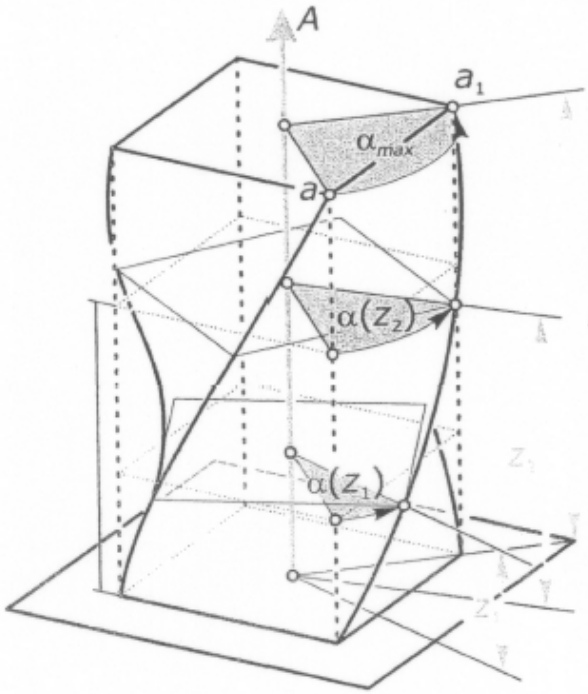


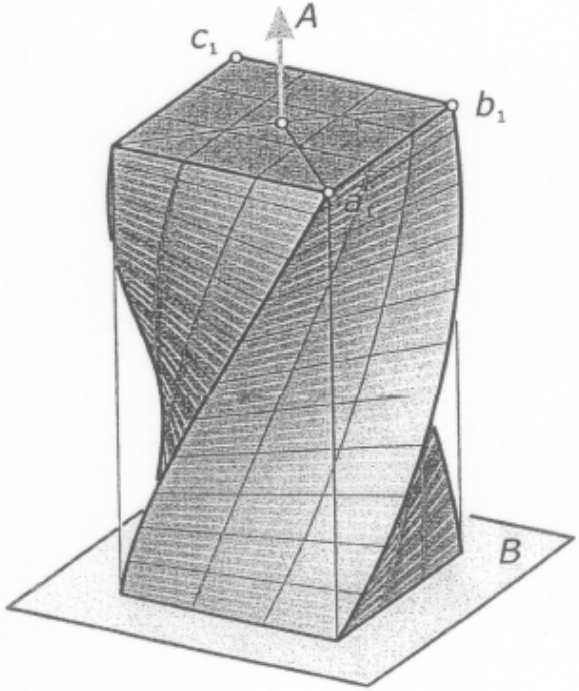
Twisting

- **Input parameters:**
  - **a bottom plane B**
  - **a twist axis A**
  - **maximum rotational angle:** α_{max}

$$\alpha(z) = \frac{z(\alpha_{max})}{h}$$

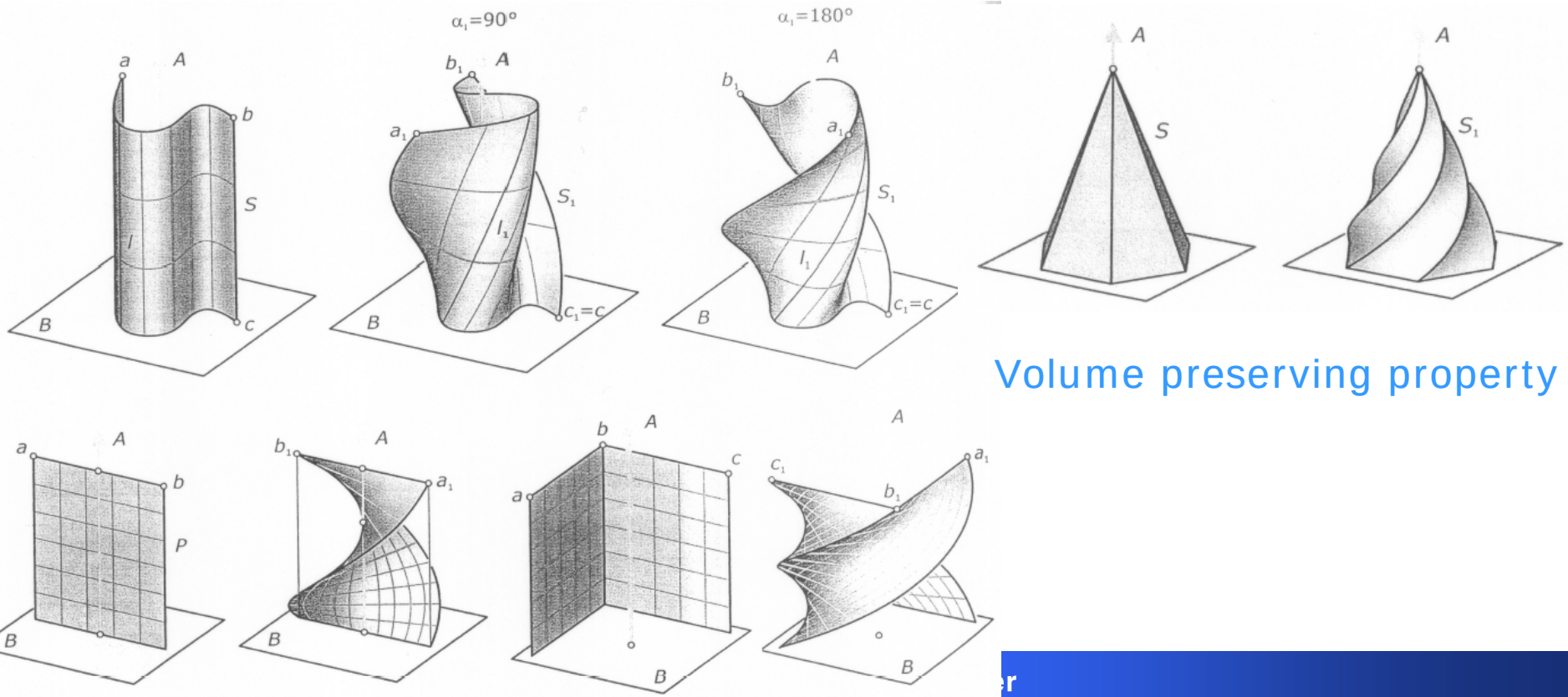






Twisting

- A cylinder surface with rulings parallel to A
  - → A helical surface
- A plane parallel to A
  - → A helical ruled surface



Volume preserving property

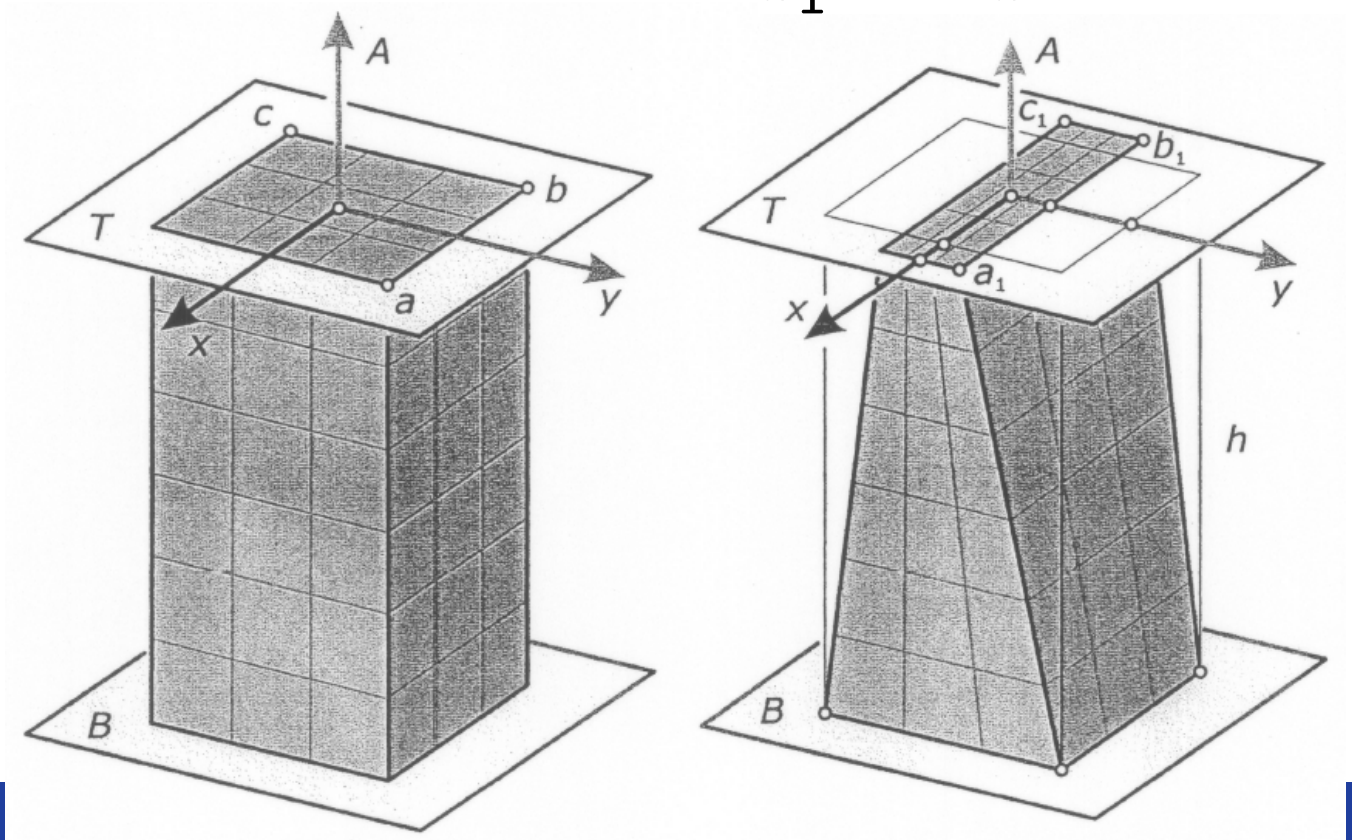
Tapering

- **Input parameters**
  - **B (bottom plane), A (axis)**
  - **Two orthogonal scaling directions with scaling factors**

$$x_1 = vx = x + x\frac{z(v-1)}{h}$$

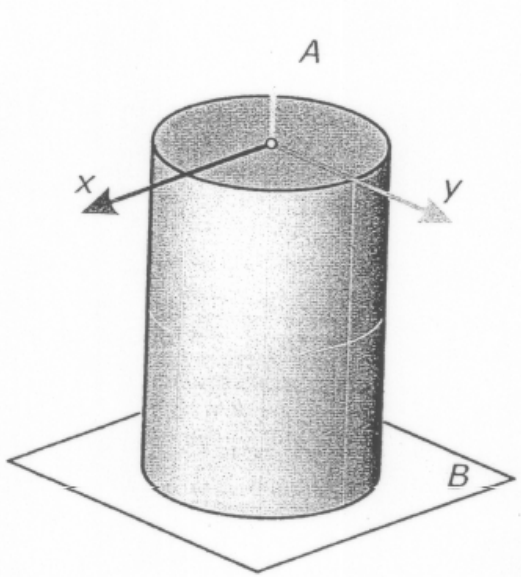
$$y_1 = wy = y + y\frac{z(w-1)}{h}$$

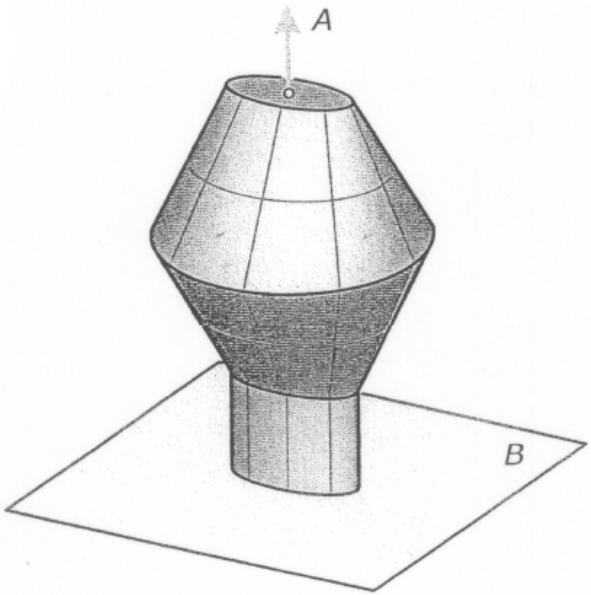
$$z_1 = z$$

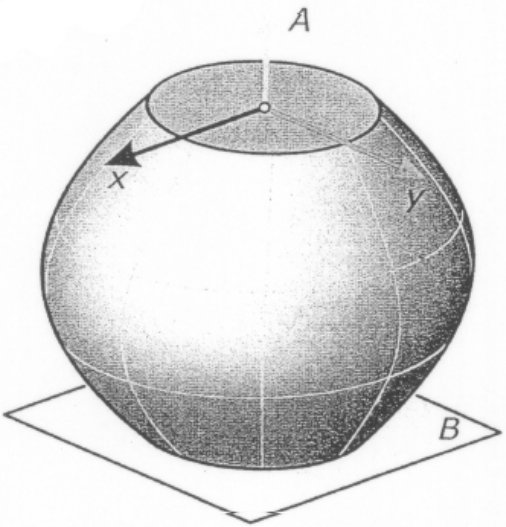


Tapering

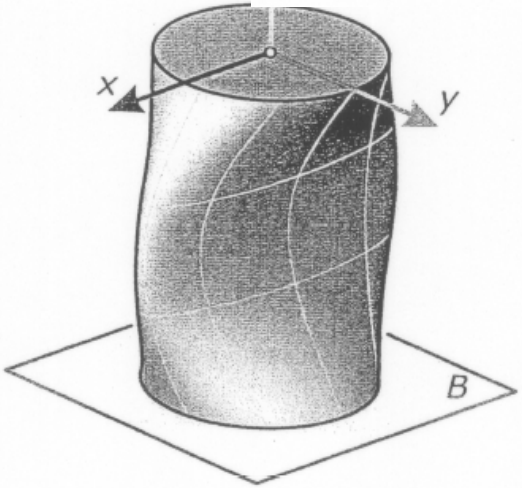
- **Scaling functions**
  - **Linear function**
  - **Piecewise linear function**
  - **Quadratic function: Bulge**



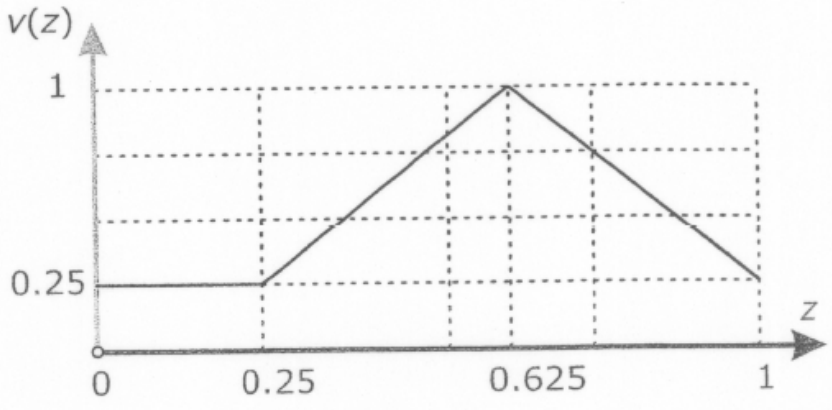




$v(h/2) = 1.33$
$w(h/2) = 1.25$

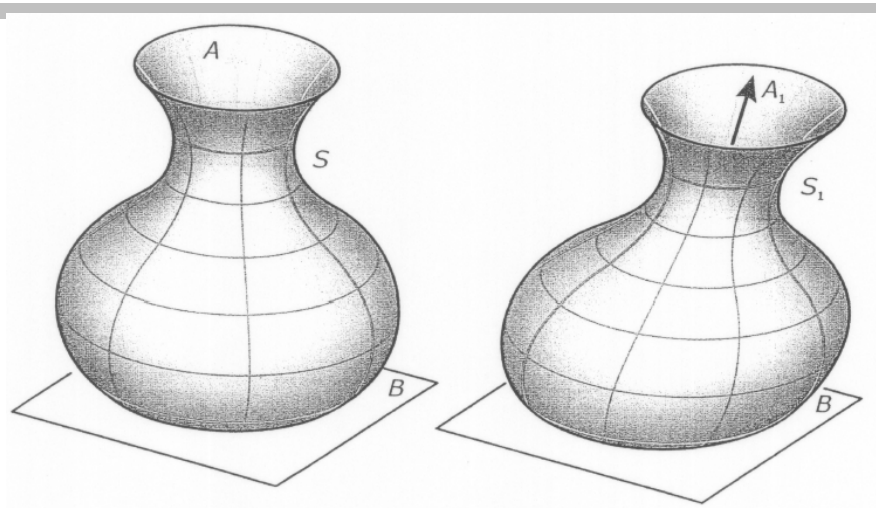


$v(h/2) = 1.17$
$w(h/2) = 0.25$

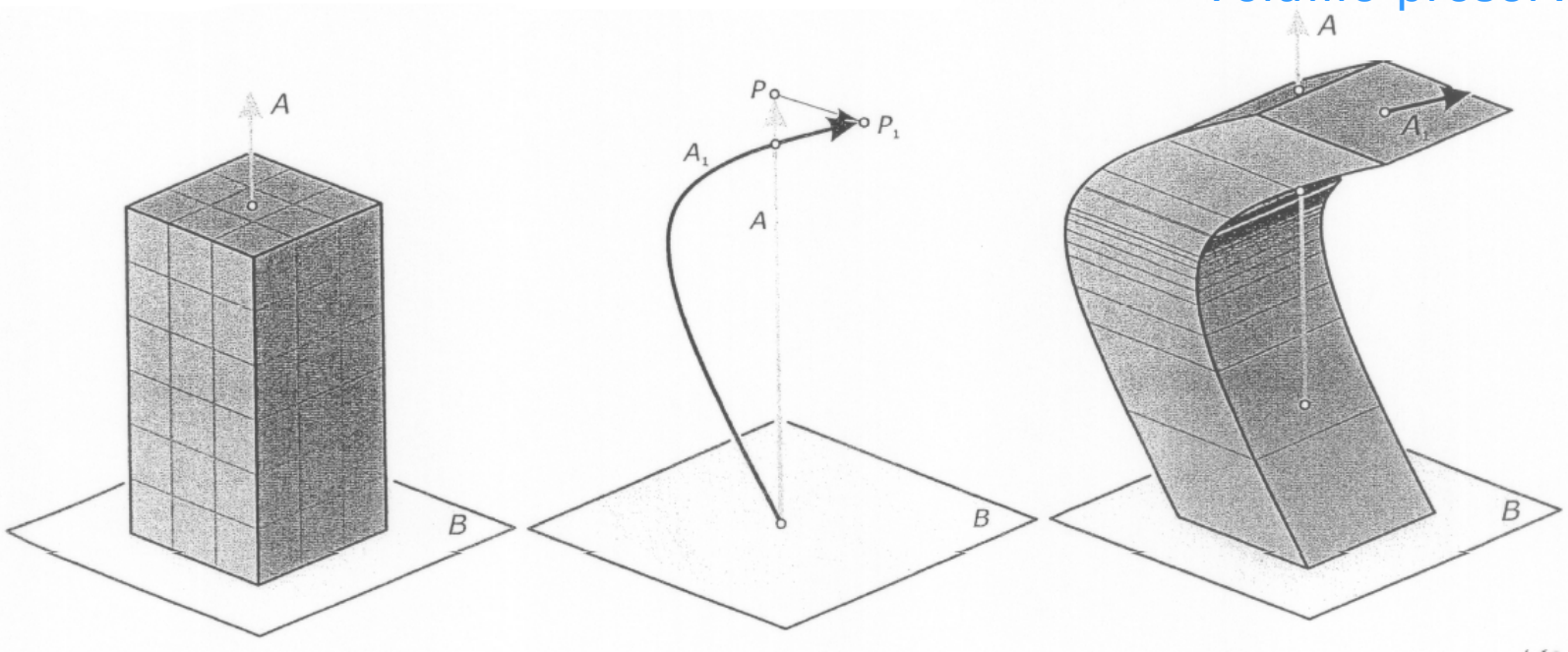


Shearing

- **Input parameters**
  - **Bottom plane (B), axis (A)**
  - **User-defined image curve of A: translation of slices**

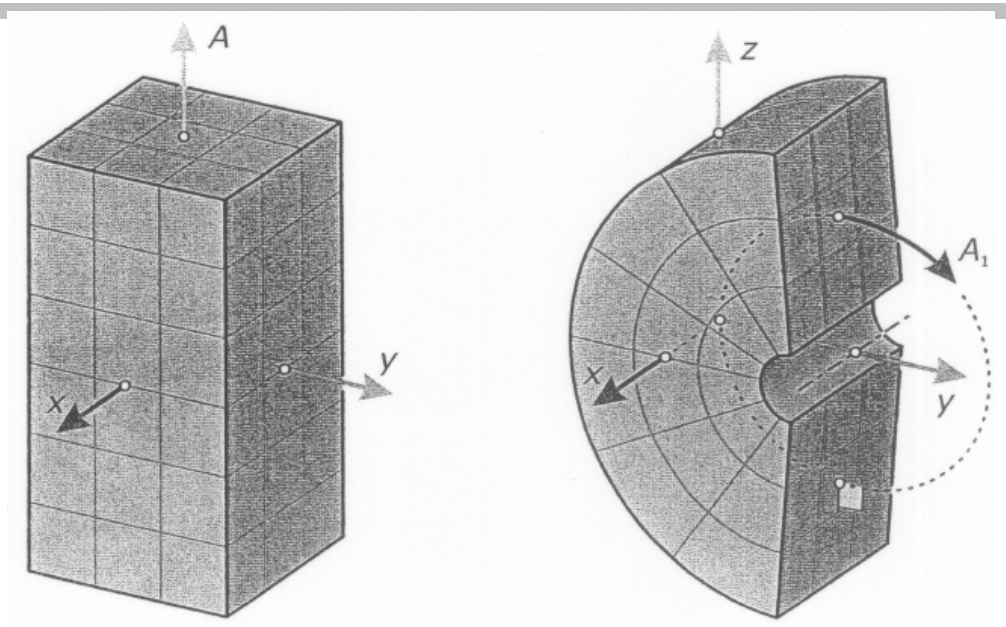


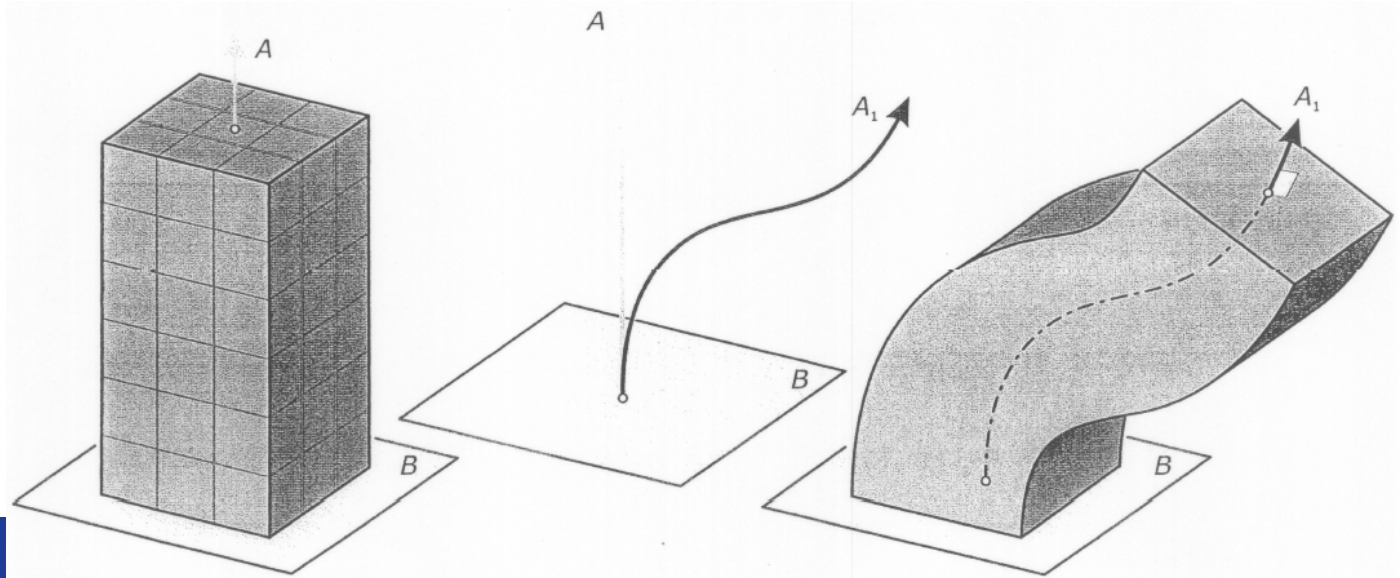
Volume preserving property



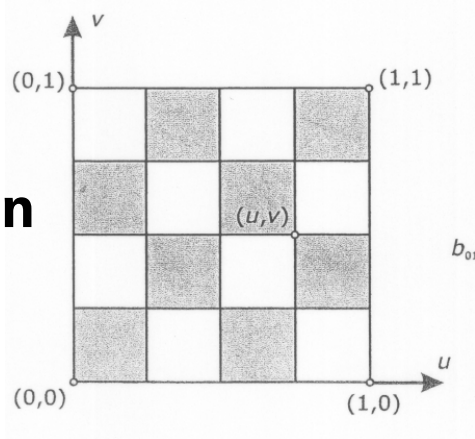
Bending

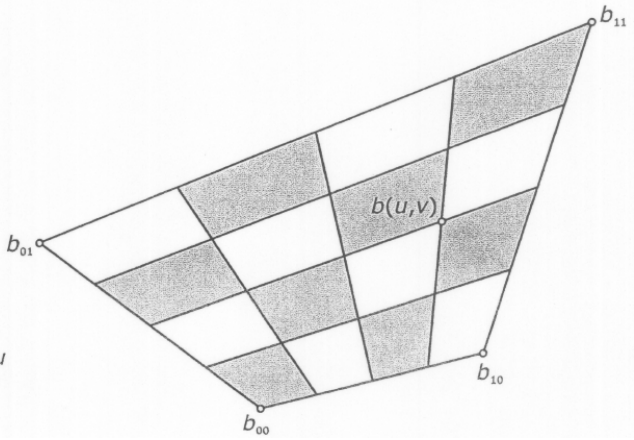
- **Shearing**
  - **Translation and rotation of slices**
  - **Change of a axis A into a general curve**





Freeform deformations

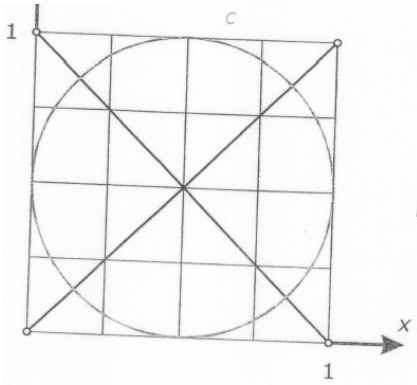


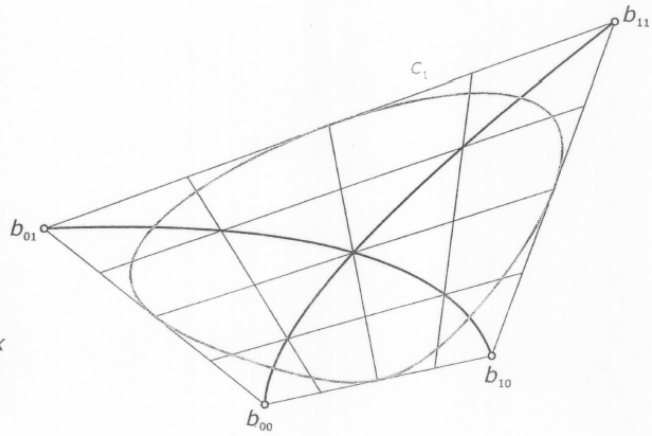


- **Planar Bézier deformation**
  - **Planar deformation of square region**
  - **Bilinear**
  - **Straight line → parabola**

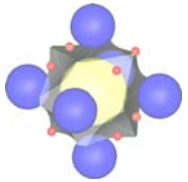
$$b(u,v) = (1-u)(1-v)b_{00} + (1-u)vb_{01} + u(1-v)b_{10} + uvb_{11}$$

$$u, v \in [0, 1]$$



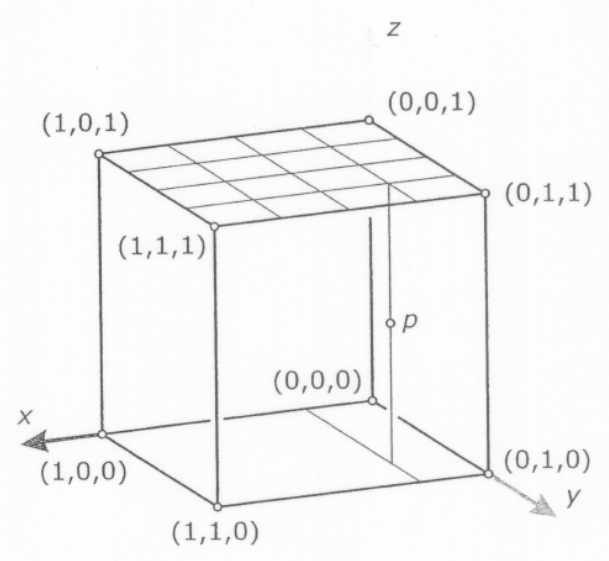


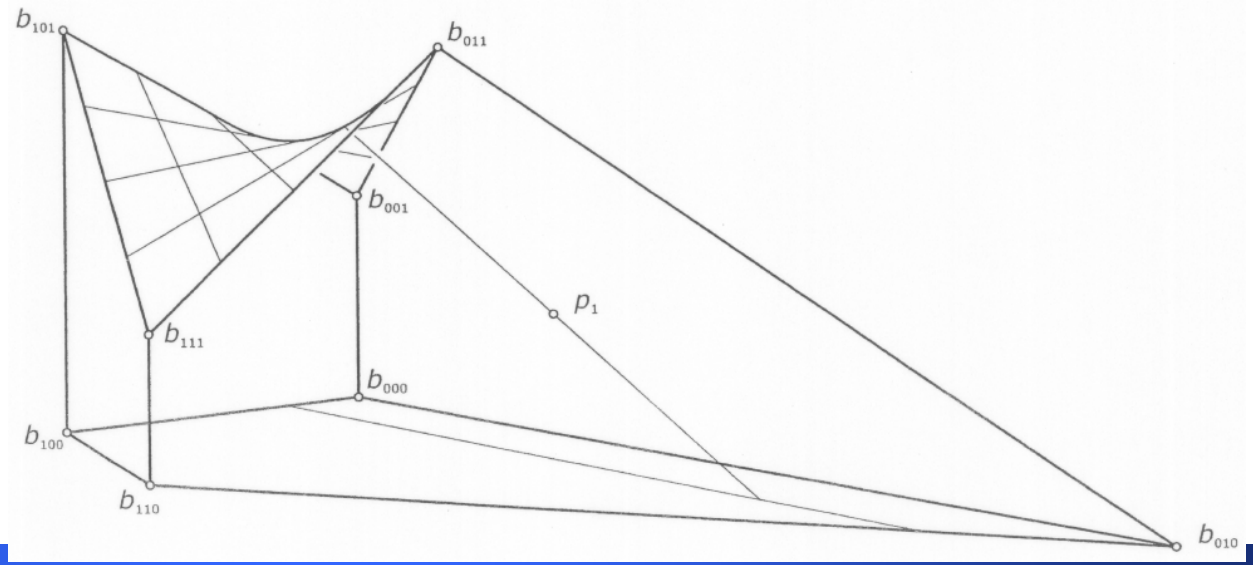
Bézier deformations



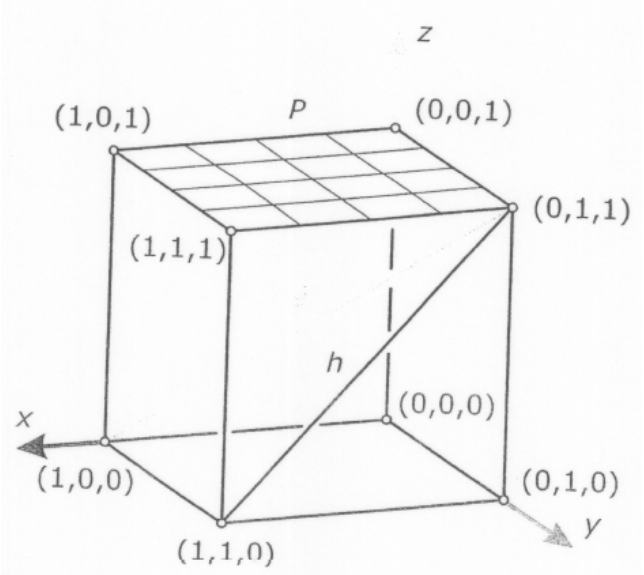
- **Deformations in 3D**
  - **Parameter domain: a cube**
  - **Deform a cube S into a solid S1**
  - **Trilinear**
  - **Straight line → cubic curve**

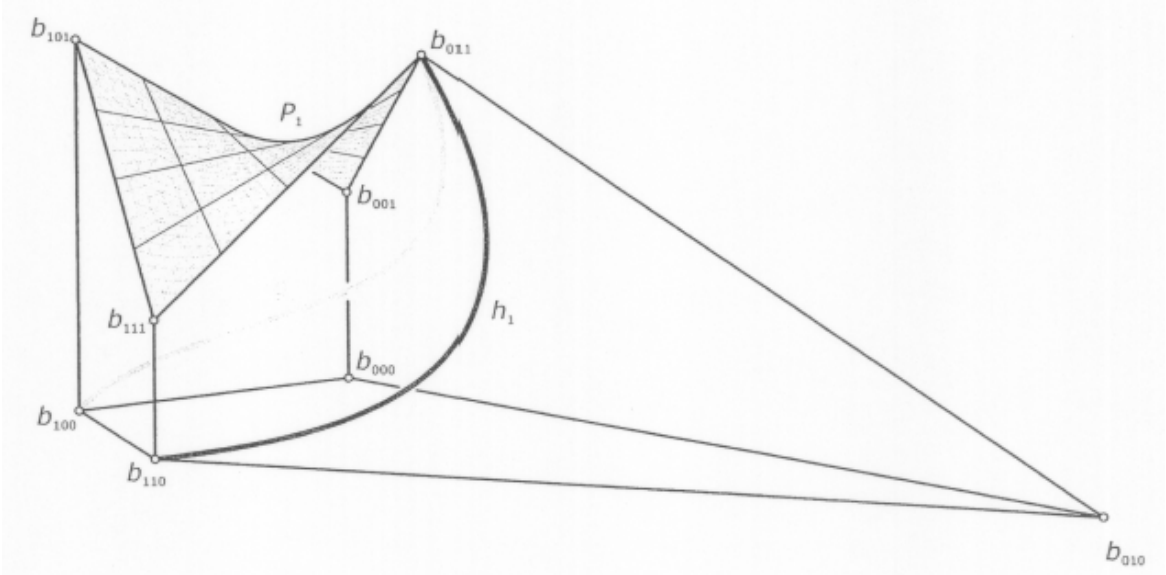
$$(x_1,y_1,z_1) = (1-x)(1-y)(1-z)b_{000} + (1-x)(1-y)zb_{001} + (1-x)y(1-z)b_{010} + (1-x)yzb_{011} + x(1-y)(1-z)b_{100} + x(1-y)zb_{101} + xy(1-z)b_{110} + xyzb_{111}$$

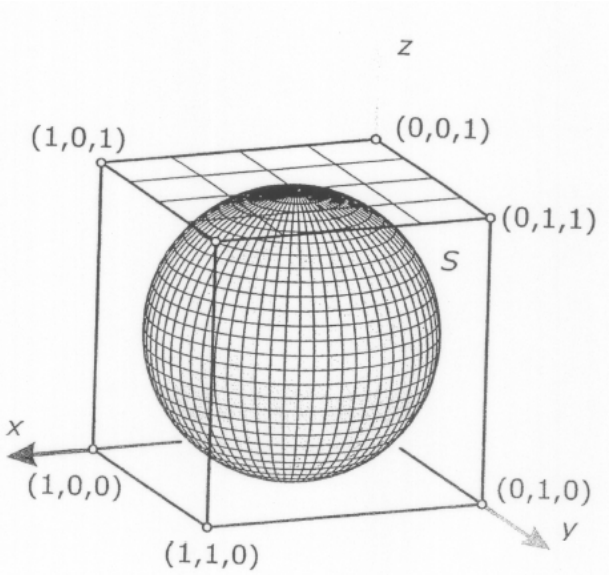


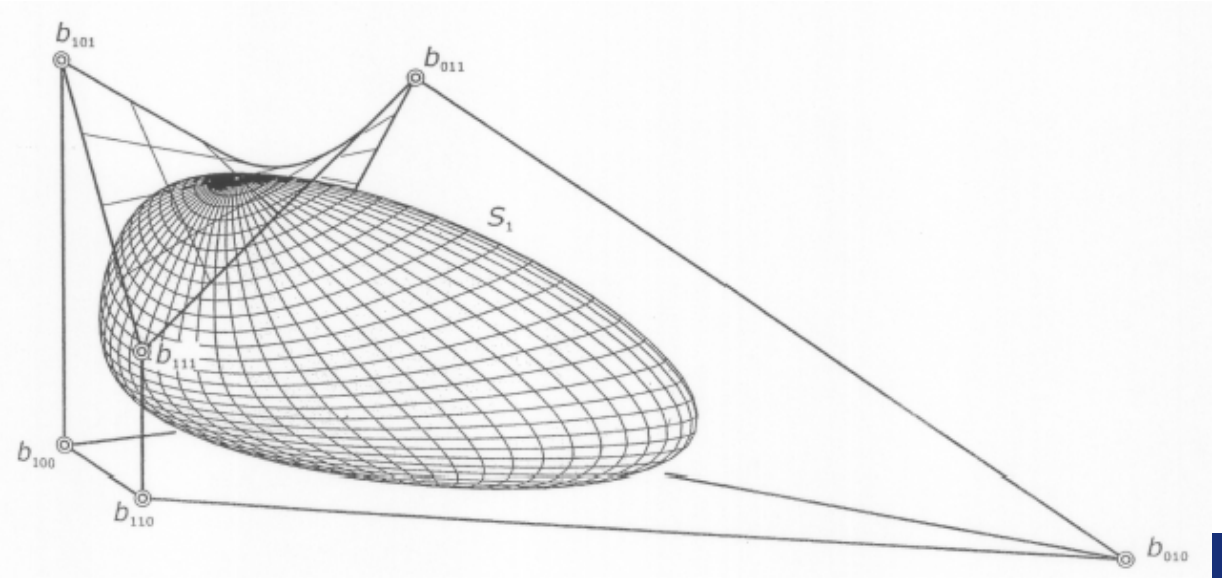


Bézier deformations

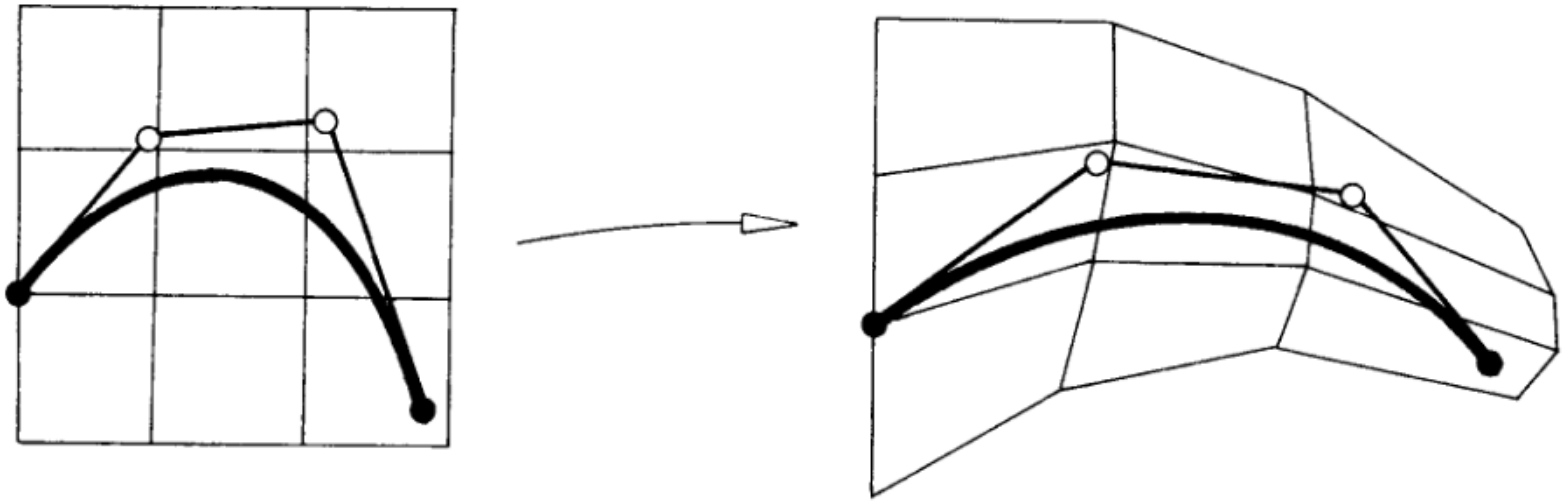


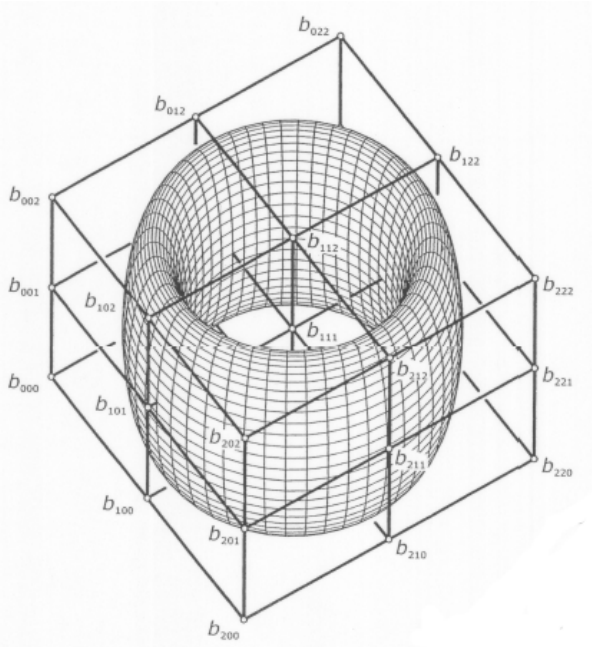






Bézier deformations with higher degree





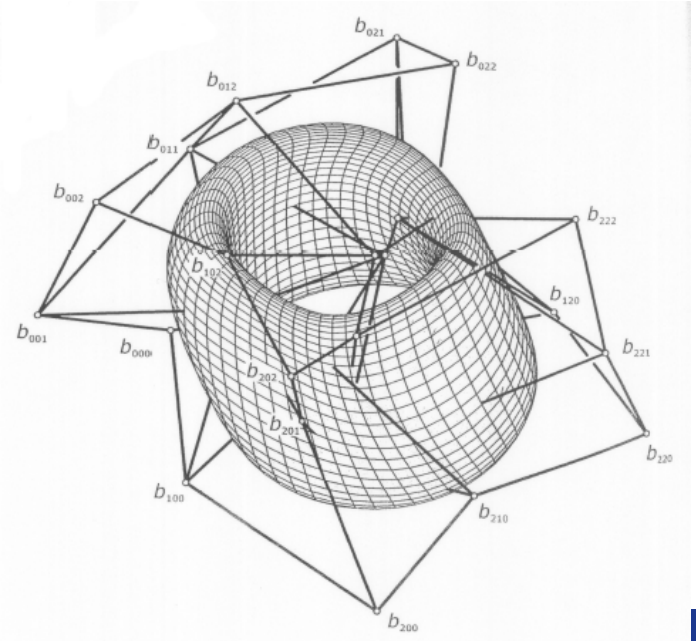
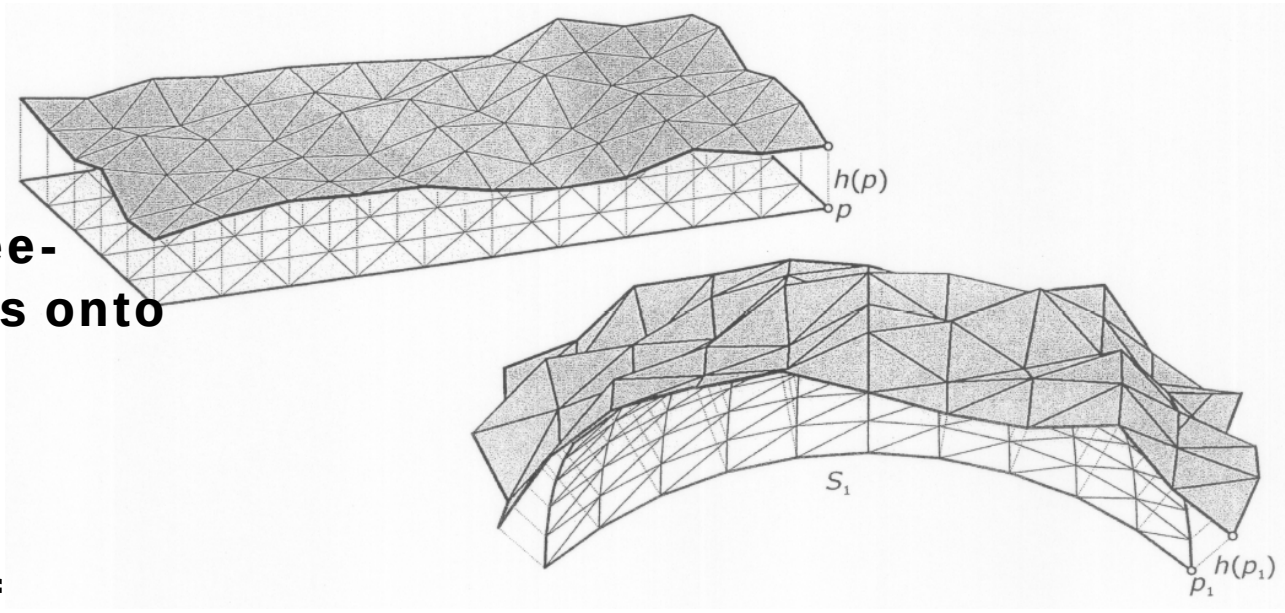


Diagram Res

Three-dimensional textures



- The mapping of three-dimensional textures onto a smooth surface

- Self-intersections of textures