"mode" 형태. 종류.

1) TEM (Transverse Electric and Magnetic)

$$e_z = h_z = 0 \quad \rightarrow \quad (5\text{-}5 \ c \ 4)$$

$$e_x = \frac{-j}{k_c^2}\left(\beta \frac{\partial 0}{\partial x} + \omega\mu \frac{\partial 0}{\partial y}\right)$$

$\therefore \ k_c^2 = 0$ 이어야

$$= k^2 - \beta^2$$

$$k^2 = \beta^2 = \omega^2 \varepsilon\mu$$

$k = \omega\sqrt{\varepsilon\mu} = \beta = \frac{2\pi}{\lambda}$ 무손실인 경우.

$$k = j\gamma$$

$$V_p = \omega/\beta = \omega/k = 1/\sqrt{\varepsilon\mu} = c$$

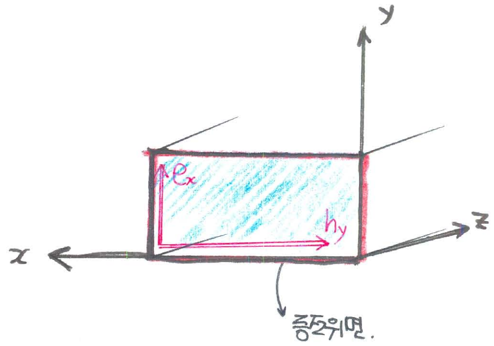
한편, $\nabla \cdot \mathbb{E} = \rho/\varepsilon \rightarrow \nabla \cdot \mathbb{E} = 0$

$$= \frac{\partial^2 E_x}{\partial x^2} - \frac{\partial^2 E_y}{\partial y^2}$$

$$\mathbb{E} = -\nabla V = -\left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}\right)$$

$$= 0$$

즉, TEM 모드의 전계 분포 → 정전계 전계 분포.



그러므로 waveguide에

TEM 전송

↳ 도파관 (벽) ⇒ 등전위면 형성

→ energy 전송불가.

(2) TE (Transverse Electric) : H wave

$e_z = 0$, $h_z \neq 0$ → (식 5-5)

$$
\star \left\{
\begin{aligned}
&\left.
\begin{aligned}
e_x &= -\frac{j}{k_c^2} \cdot \omega\mu \frac{\partial h_z}{\partial y} \\
e_y &= \frac{j}{k_c^2} \cdot \omega\mu \frac{\partial h_z}{\partial z}
\end{aligned}
\right\} \\
&\left.
\begin{aligned}
h_x &= \frac{-j}{k_c^2} \beta \frac{\partial h_z}{\partial x} \\
h_y &= -j/k_c^2 \cdot \beta \cdot \frac{\partial h_z}{\partial y}
\end{aligned}
\right\}
\end{aligned}
\right.
$$

(식 5-5) 이용.

where, $\frac{e_x}{h_y} = \omega\mu/\beta = Z_{TE}$

↓

TE모드 임피던스

* TE파 파동임피던스.

$$= -e_y/h_x = \frac{j\omega\mu}{\gamma}$$

(식 5-25)

(3) TM mode (Transverse Magnetic) : E wave

$e_z \neq 0$ 이고 $h_z = 0$ 일때, $\rightarrow$ (식 5-5)

$$
\begin{cases}
\left.\begin{aligned}
e_x &= \frac{-j}{k_c^2}\,\beta\,\frac{\partial e_z}{\partial x} \\
e_y &= \frac{-j}{k_c^2}\,\beta\,\frac{\partial e_z}{\partial y}
\end{aligned}\right\} \\
\\
\left.\begin{aligned}
h_x &= \frac{j}{k_c^2}\,\omega\varepsilon\,\frac{\partial e_z}{\partial y} \\
h_y &= \frac{-j}{k_c^2}\,\omega\varepsilon\,\frac{\partial e_z}{\partial x}
\end{aligned}\right\}
\end{cases}
\qquad (식\ 5\text{-}5'')
$$

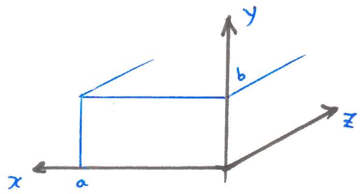
where, $\dfrac{e_x}{h_y} = \dfrac{\beta}{\omega\varepsilon} = Z_{TM}$

$$= -\frac{e_y}{h_x} = \frac{\gamma}{j\omega\mu}$$

wave impedence.

5-2 Rectangular waveguide

(1) TE mode : $e_z = 0$



(식 5-5')

$$e_x = \frac{-j}{k_c^2}\,\omega\mu\,\frac{\partial h_z}{\partial y}$$

$$e_y = \frac{j}{k_c^2}\,\omega\mu\,\frac{\partial h_z}{\partial x}$$

$$h_x = \frac{-j}{k_c^2}\,\beta\,\frac{\partial h_z}{\partial x}$$

$$h_y = \frac{-j}{k_c^2}\,\beta\,\frac{\partial h_z}{\partial y}$$

where, $k_c^2 = k^2 - \beta^2$

$h_z(x, y)$ 는 helmholtz's Eq (2-8)을 만족해야 함.

$$\frac{\partial^2 h_z}{\partial x^2} + \frac{\partial^2 h_z}{\partial y^2} = -k_c^2 h_z \qquad (5\text{-}12)$$

where $h_z(x, y) = X(x)\,Y(y)$

$$X''Y + XY'' = -k_c^2 XY$$

양변을 XY로 나누면,

$$\underbrace{\frac{X''}{X}}_{X\text{ 만의 함수}} + \underbrace{\frac{Y''}{Y}}_{Y\text{ 만의 함수}} = \underbrace{-k_c^2}_{\text{상수값}} \qquad (5\text{-}14)$$

$$\begin{cases} \dfrac{X''}{X} = -k_x^2 & : x \text{ 방향 파수} \\[2em] \dfrac{Y''}{Y} = -k_y^2 & : y \text{ 방향 파수} \end{cases}$$

$$\therefore \quad k_x^2 + k_y^2 = -k_c^2 \qquad (5\text{-}16)$$

(책 145p)

* $\dfrac{1}{X}\dfrac{d^2X}{dx^2} = -k_x^2$ 의 해

$$\frac{d^2X}{dx^2} + k_x^2 X = 0$$

$$\therefore \quad X = A\, e^{jk_x x} + B\, e^{-jk_x x}$$

$$= A \cos k_x \cdot x + B \sin k_x \cdot x$$

마찬가지로,

$$Y = Ce^{jk_y\cdot y} + De^{-jk_y\cdot y}$$

$$= C\cos k_y\cdot y + D\sin k_y\cdot y$$

(5-5′)에 의한

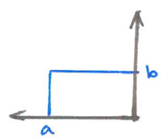
$e_x \propto \frac{\partial h_z}{\partial y}$ 이므로 $\frac{\partial h_z}{\partial y} = \frac{\partial}{\partial y}(X\cdot Y)$

$$= \frac{\partial}{\partial y}\left[(A\cos k_x\cdot x + B\sin k_x\cdot x)(C\cos k_y y + D\sin k_y\cdot y)\right]$$

(5-18a) $$= (A\cos k_x\cdot x + B\sin k_x\cdot x)(-k_y\cdot C\sin k_y y + k_y\cdot D\cos k_y y)$$

where 경계조건.

$e_x(x, y) = 0$; $y=0$, b 에서



① $y = 0$ 일때, $e_x = 0$ 이므로.

$$-k_y C\sin(k_y\cdot 0) = 0$$

$$k_y D\cos(k_y\cdot 0) = k_y D = 0$$

$\therefore D = 0$

② $y=b$ 일때, $e_x = 0$

$$-k_y \cdot C \cdot \sin(k_y \cdot b) = 0$$

$$k_y \cdot b = n\pi \qquad (n = 0,\ 1,\ 2 \cdots)$$

$$\Downarrow$$

$$k_y = \frac{n\pi}{b}$$

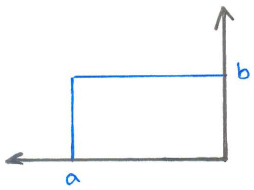
마찬가지로.

$$e_y \propto \frac{\partial h_z}{\partial x} \quad \text{이므로,}$$

$$\frac{\partial h_x}{\partial x} = \frac{\partial}{\partial x}(XY)$$

$$= \left(-k_x A \sin(k_x \cdot x) + k_x B \cos(k_x \cdot x)\right) \cdot \left(C \cos(k_y \cdot y) + D \sin(k_y \cdot y)\right)$$

(h-18b)



where, 경계조건 $e_y(x, y) = 0$. $(x = 0, \ a)$ 에서

① $x = 0$ 일 때,

$$-k_x \cdot A \cdot \sin(k_x \cdot 0) = 0$$

$$k_x \cdot B \cdot \cos(k_x \cdot 0) = k_x \cdot B = 0$$

$$\therefore B = 0$$

② $x = a$ 일 때,

$$-k_x \cdot A \cdot \sin(k_x \cdot a) = 0$$

$$k_x \cdot a = m\pi \quad (m\pi = 0, 1, 2 \cdots)$$

$$\therefore k_a = m\pi / a$$

그러므로 경계조건에 맞는 해는

$$\begin{cases} X = A\cos(k_x \cdot x) \\ Y = C\cos(k_y \cdot y) \end{cases}$$

where, $k_x = {}^{m\pi}/_{a}$ $\quad$ $(m, n = 0.1.2\cdots)$

$k_y = {}^{n\pi}/_{b}$

$$\therefore h_z(x.y) = X_{(x)} \cdot Y_{(y)}$$

$$= AC\cos(k_x \cdot x) \cdot \cos k_y y$$

$$\therefore h_z(x.y)e^{-j\beta z}$$

$$= H_{mn} \cdot \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y \cdot e^{-j\beta z} \qquad (\text{5-20식})$$

$$= H(x.y.z)$$

If, $m = n = 0$ 일때,

$h_z = H_{mn}$ = Constant (상수값)

$e_x = e_y = h_x = h_y = 0$: 무의미.

∴ TE_{00} mode는 없다.

↳ TE mode는 $\underbrace{TE_{01} . TE_{10}}_{\text{기저모드}}$ mode부터 시작

식 (5-5), 식 (5-20)을 이용하여 TE_{mn} 모드의

횡전자계 성분을 구함.

↳ 식 (5-21) * 노트필기.

↓

(5-21 a) $$E_x = \frac{j\omega\mu \, n\pi}{k_c^2 \, b} H_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}$$

(5-21 b) $$E_y = \frac{-j\omega\mu \, m\pi}{k_c^2 \, a} H_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}$$

(5-21 c) $$H_x = \frac{j\beta \, m\pi}{k_c^2 \, a} H_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}$$

(5-21 d) $$H_y = \frac{j\beta \, n\pi}{k_c^2 \, b} H_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}$$

(식 5-22) β값.

$$\beta = \sqrt{k^2 - k_c^2} = \sqrt{k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

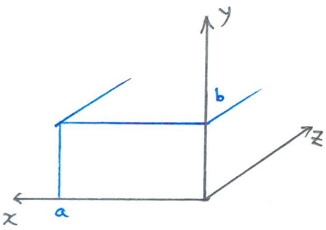
if. 매질 loss ≠ 0

$$e^{-j\beta_{mn} z} \rightarrow e^{-\gamma_{mn} z} \qquad (\gamma = \alpha + j\beta)$$

Report

(5-40) 유도 *

* 접나 어렵네ㅋ.



[2] TM mode : E mode

* $e_z \neq 0$ 이고, $h_z = 0$ 일때,

* 식 5-5'')

* 교수님 필기 부분 *

$$\left.\begin{aligned} e_x &= \frac{-j}{k_c^2} \cdot \beta \frac{\partial e_z}{\partial x} \\ e_y &= -j/k_c^2 \cdot \beta \frac{\partial e_z}{\partial y} \\ h_x &= j/k_c^2 \; \omega\varepsilon \frac{\partial e_z}{\partial y} \\ h_y &= -j/k_c^2 \; \omega\varepsilon \frac{\partial e_z}{\partial x} \end{aligned}\right\} \text{(식 5-5'')}$$

where, $k_c^2 = k^2 - \beta^2$

$e_z\ (x, y)$ 는 $\underbrace{\text{helmholtz's Eq}\ (2\text{-}6)}$ 을 만족해야함.

$$\nabla^2 \mathbb{E} + k^2 \mathbb{E} = 0$$

$$\frac{\partial^2 e_z}{\partial x^2} + \frac{\partial^2 e_z}{\partial y^2} = -k_c^2 e_z$$ (5-36 변형식)

where, $e_z\ (x, y) = X_{(x)} Y_{(y)}$

$$X''Y + XY'' = -k_c^2 XY$$

양변을 XY 로 나누면,

$$\frac{X''}{X} + \frac{Y''}{Y} = -k_c^2$$

$$\begin{cases} \dfrac{X''}{X} = -k_x^2 & : x \text{ 방향 파수} \\[2em] \dfrac{Y''}{Y} = -k_y^2 & : y \text{ 방향 파수} \end{cases}$$

$$\therefore \underbrace{k_x^2 + k_y^2 = -k_c^2}_{*}$$

$$\frac{1}{X}\frac{d^2X}{dx^2} = -k_x^2$$ 의 해

$$\frac{d^2X}{dx^2} + k_x \cdot X = 0$$

미.방.

$$\underline{X} = A \cdot e^{jk_x x} + B \cdot e^{-jk_x x}$$

삼각함수.

$$\underline{= A \cdot \cos k_x x + B \sin k_x x}$$

같은 방법으로 , $\frac{1}{Y} \cdot \frac{d^2Y}{dy^2} = -k_y^2$ 를 풀면.

$$\underline{Y = C \cos k_y y + D \sin k_y y}$$

$$\therefore e_z(x, y) = X_{(x)} Y_{(y)}$$

$$= \underline{(A \cos k_x x + B \sin k_x x) \cdot (C \cos k_y y + D \sin k_y y)}$$

*

(식 ㅎ-ㅎ") 에 의해

Ⓐ $h_x \propto \frac{\partial e_z}{\partial y}$ 이므로,

$$\frac{\partial e_z}{\partial y} = \frac{\partial}{\partial y}\left\{ (A\cos k_x x + B\sin k_x x)\cdot(C\cos k_y y + D\sin k_y y)\right\}$$

y에 대해 편미분.

$$= \underline{(A\cos k_x x + B\sin k_x x)}(-k_y\cdot C\cdot\sin k_y y + k_y\cdot D\cdot\cos k_y y)$$

① $x = 0$ 일때

$$\begin{cases} A\cdot\underbrace{\cos k_x 0}_{=1} = 0 \\ B\cdot\underbrace{\sin k_x 0}_{=0} = 0 \end{cases}$$

$A\cdot 1 = 0$, $\therefore A = 0$

$B = ?$

② $x = a$ 일때,

$B\sin(k_x\cdot a) = 0$ 이므로

$\sin k_x a = 0$, $\underline{k_x\cdot a = m\pi}$ $(m = 1, 2, 3\cdots)$

정리하면. $A = 0$, $k_x = \frac{m\pi}{a}$

Ⓑ $h_y \propto \frac{\partial e_z}{\partial x}$ 이므로,

$$\frac{\partial e_z}{\partial x} = \frac{\partial}{\partial x}\left(A\cos k_x x + B\sin k_x x\right)\cdot\left(C\cos k_y y + D\sin k_y y\right)$$

x에 미분.

$$= \left(-k_x A\sin k_x x + k_x B\cos k_x x\right)\cdot\left(C\cos k_y y + D\sin k_y y\right)$$

① $y = 0$ 일때

$C\cdot\cos k_y 0 = 0$ $\quad\therefore\ C = 0$

$D\ \sin k_y 0 = 0$

② $y = b$ 일때

$D\cdot\sin \underbrace{k_y b}_{n\pi} = 0$

$\therefore\ k_y\cdot b = n\pi \quad (n = 1.2.3\cdots)$

정리하면, $C = 0,\quad k_y = \frac{n\pi}{b}$

정리하면,

$$\begin{cases} A = 0 & k_x = {}^{m\pi}/_{a} & (m = 1, 2, 3 \cdots) \\ C = 0 & k_y = {}^{n\pi}/_{b} & (n = 1, 2, 3 \cdots) \end{cases}$$

$$\therefore \; e_z(x, y) = B \sin \frac{m\pi}{a} x \cdot D \sin \frac{n\pi}{b} y$$

$$= BD \sin \frac{m\pi}{a} x \cdot \sin \frac{n\pi}{b} y$$

$$\therefore \; e_z(x, y)\, e^{-j\beta z}$$

$$= E_{mn} \sin \frac{m\pi x}{a} \cdot \sin \frac{n\pi y}{b} \cdot e^{-j\beta z} \qquad (\hbar\text{-}394)$$

$$= E(x, y, z)$$

where, $E_{mn} = B, D$의

진폭 상수.

(5-5)와 (5-39) 을 통해

$$\left\langle E_{mn} \sin\frac{m\pi x}{a} \cdot \sin\frac{n\pi y}{b}\, e^{-j\beta z} \right\rangle$$

5 - 40 식을 얻을 수 있다.

$$(5\text{-}40)\quad \begin{cases} E_x = \dfrac{-j\beta m\pi}{a\,k_c^2} E_{mn} \cos\dfrac{m\pi x}{a} \sin\dfrac{n\pi y}{b}\, e^{-j\beta z} \\[2ex] E_y = \dfrac{-j\beta n\pi}{b\,k_c^2} E_{mn} \sin\dfrac{m\pi x}{a} \cos\dfrac{n\pi y}{b}\, e^{-j\beta z} \\[2ex] H_x = \dfrac{j\omega\varepsilon\, n\pi}{b\,k_c^2} E_{mn} \sin\dfrac{m\pi x}{a} \cos\dfrac{n\pi y}{b}\, e^{-j\beta z} \\[2ex] H_y = \dfrac{-j\omega\varepsilon\, m\pi}{a\,k_c^2} E_{mn} \cos\dfrac{m\pi x}{a} \sin\dfrac{n\pi y}{b}\, e^{-j\beta z} \end{cases}$$

교재 155p.

[3] 구형 도파관의 특징.

(1) 모드의 직교성.

TE mode (식 5-21)
TM mode (식 5-40)
} 삼각함수 (직교성)

직교성. $f(x)$ $g(x)$ 가 구간 T 에서

$\int_T f(x)g(x)\,dx = 0$ 이면 $f(x) \perp g(x)$

단, $f(x) \neq g(x)$

ex) $$\int_0^a \sin\left(\frac{s\pi x}{a}\right) \cdot \sin\left(\frac{t\pi x}{b}\right) dx$$

$$= \begin{cases} 0 & s \neq t \quad (\text{직교}) \\ \frac{a}{2} & s = t \end{cases}$$

(식 5-21) (5-40) waveguide = 완전도체 : 직교성

≠ 완전도체 : not 직교성

(2) Cut off frequency and Wave length

무손실 이므로, $\gamma = \alpha + j\beta$ = 전파상수

$= j\beta$ = 위상정수

$$\therefore \beta = \sqrt{k^2 - k_c^2}$$

where, • $k > k_c$

$\downarrow$

β = 실수 → propagation.

• $k = k_c$

$\downarrow$

$\beta = 0$ → Cut off

• $k < k_c$

$\downarrow$

β = 허수 → $\beta' = j\beta$

; $e^{-j\beta z}$ → $e^{j\beta z}$ 이므로,

⇒ $-z$ 방향

⇒ no propagation.

$$\beta = 0 = \sqrt{k^2 - k_c^2}$$

$$k^2 = k_c^2 = \omega^2 \varepsilon\mu = (2\pi f_c)^2 \varepsilon\mu$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\therefore f_c = \frac{1}{2\pi\sqrt{\varepsilon\mu}} \cdot \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

(5-46식)

Cut off frequency

↓

이 주파수보다 높아야

통과한다는 말~!

그러므로, $f > f_c$ 인 경우에만

"Propagation."

= High Pass Filter.

(고역통과 필터)

차단 파장, $$\lambda_c = c/f_c = \frac{1}{\sqrt{\varepsilon\mu}} \bigg/ f_c$$

$$= \frac{2}{\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

$$= \frac{2\pi}{k_c}$$

예제 5-1) 함 풀어보세요!