14주 081201 . 081202.

5장 도파관과 전송선로.

< waveguide >

5.1 기초 방정식 : Maxwell's Eq 정식적 해석

* wave = Sinusoidal wave

propagation direction = $\bar{z}$

propagation constant = β

매질 loss = 0 , $\sigma = 0$

도파관 loss = 0

가정했을 때,

$\mathbb{E} = E_x \bar{x} + E_y \bar{y} + E_z \bar{z}$

$\mathbb{E} = E_0 e^{-j\mathbb{K}\cdot\mathbb{r}}$

지금까지 $\mathbb{E} = E_x \bar{x}$

$\mathbb{H} = H_y \bar{y}$

- Maxwell's Eq 식 (1-14. 15)

$$\nabla \times \mathbb{E} = -j\omega \mathbb{B} = -j\omega \mu \mathbb{H}$$

$$= \begin{vmatrix} \bar{x} & \bar{y} & \bar{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$$= -j\omega\mu \left(H_x \bar{x} + H_y \bar{y} + H_z \bar{z} \right)$$

$$\therefore \left.\begin{aligned} \partial E_z/\partial y - \partial E_y/\partial z &= -j\omega\mu H_x \\ \partial E_x/\partial z - \partial E_z/\partial x &= -j\omega\mu H_y \\ \partial E_y/\partial x - \partial E_x/\partial y &= -j\omega\mu H_z \end{aligned}\right\} \quad (5\text{-}1)$$

$$\nabla \times \mathbb{H} = j\omega \mathbb{D} = j\omega \varepsilon \mathbb{E}$$

마찬가지로,

$$\left.\begin{aligned} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= j\omega\varepsilon E_x \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= j\omega\varepsilon E_y \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= j\omega\varepsilon E_z \end{aligned}\right\} \quad (5\text{-}1)$$

where, $\mathbb{E}$, $\mathbb{H}$는 $+z$ 방향 진행.

$$\mathbb{E}(x,y,z) = \left[\underbrace{\mathbf{e}_t(x,y)}_{} + \underbrace{e_z(x,y)\,\bar{z}}_{} \right] e^{-j\beta z}$$

+z 방향과 각각 평면상의 관계.

+z 방향으로 진행하는 전계.

(5-2a)

transverse electric field

$$\left[\left(e_x(x,y)\,\bar{x} + e_y(x,y)\,\bar{y} \right) + e_z(x,y)\,\bar{z} \right] \sim$$

$$\mathbb{H}(x,y,z) = \left[\mathbf{h}_t(x,y) + h_z(x,y)\,\bar{z} \right] e^{-j\beta z} \qquad (5\text{-}2b)$$

그러므로 전계와 자계의 각 방향별 성분.

$$\left.\begin{aligned}
E_x(x,y,z) &= e_x(x,y)\, e^{-j\beta z} \\
E_y(x,y,z) &= e_y(x,y)\, e^{-j\beta z} \\
E_z(x,y,z) &= e_z(x,y)\, e^{-j\beta z} \\
\\
H_x(x,y,z) &= h_x(x,y)\, e^{-j\beta z} \\
H_y(x,y,z) &= h_y(x,y)\, e^{-j\beta z} \\
H_z(x,y,z) &= h_z(x,y)\, e^{-j\beta z}
\end{aligned}\right\} \quad (5\text{-}3)$$

(식 5-3) → (식 5-1)

$$\frac{\partial e_z(x,y)\, e^{-j\beta z}}{\partial y} - \frac{\partial e_y\, e^{-j\beta z}}{\partial z} = -j\omega\mu\, h_x\, e^{-j\beta z}$$

미분.

$$\frac{\partial e_z\, \cancel{e^{-j\beta z}}}{\partial y} - (-j\beta)\, e_y \cdot \cancel{e^{-j\beta z}} = -j\omega\mu\, h_x\, \cancel{e^{-j\beta z}}$$

$$\partial e_z/\partial y + j\beta\, e_y = -j\omega\mu\, h_x \qquad (5-4a)$$

: 무손실.

$$-j\beta\, e_x - \partial e_z/\partial x = -j\omega\mu\, h_y \qquad (5-4b)$$

$$\frac{\partial e_y}{\partial x} - \frac{\partial e_x}{\partial y} = -j\omega\mu\, h_z \qquad (5-4c)$$

$$\frac{\partial h_z}{\partial y} + j\beta\, h_y = j\omega\varepsilon\, e_x \qquad (5-4d)$$

$$-j\beta\, h_x - \partial h_z/\partial x = j\omega\varepsilon\, e_y \qquad (5-4e)$$

$$\frac{\partial h_y}{\partial x} - \partial h_x/\partial y = j\omega\varepsilon\, e_z \qquad (5-4f)$$

e_x. e_y . h_x. h_y 에 대해 정리하면,

$$\left.\begin{aligned}
&c)\quad e_x = \frac{-j}{k_c^2}\cdot\left(\beta\frac{\partial e_z}{\partial x} + \omega\mu\frac{\partial h_z}{\partial y}\right)\\
&d)\quad e_y = \frac{j}{k_c^2}\cdot\left(-\beta\frac{\partial e_z}{\partial y} + \omega\mu\frac{\partial h_z}{\partial x}\right)\\
&a)\quad h_x = \frac{j}{k_c^2}\cdot\left(\omega\varepsilon\frac{\partial e_z}{\partial y} - \beta\frac{\partial h_z}{\partial x}\right)\\
&b)\quad h_y = \frac{-j}{k_c^2}\cdot\left(\omega\varepsilon\frac{\partial e_z}{\partial y} + \beta\frac{\partial h_z}{\partial y}\right)
\end{aligned}\right\}(5-5)$$

where, $k_c^2 = k^2 - \beta^2$

$k^2 = \omega^2\varepsilon\mu$

→ (식 5-5) 에서 e_z, h_z 를 알면.

e_x, e_y, h_x h_y 를 구할 수 있다. *

e_z. h_z 를 구하기 위한 helmholtz's Eq

$$\nabla^2 e_z + k^2 e_z = 0$$

$$\frac{\partial^2 e_z}{\partial x^2} + \frac{\partial^2 e_z}{\partial y^2} + \frac{\partial^2 e_z}{\partial z^2} = -k^2 e_z$$

142p.

〈 TEM. TE. TM 〉

92.

$$\frac{\partial^2 e_z}{\partial x^2} + \frac{\partial^2 e_z}{\partial y^2} = -\frac{\partial^2 e_z}{\partial z^2} - k^2 e_z$$

$* \ \frac{\partial}{\partial z} = (-j\beta)^2 = -\beta^2$ 이용.

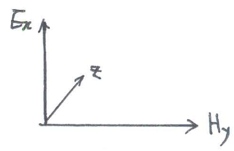
$$= \beta^2 e_z - k^2 e_z$$

$$= (\beta^2 - k^2)\, e_z$$

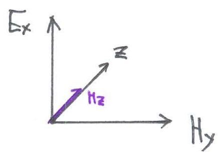
$$= -k_c^2\, e_z$$

$$\therefore \left. \begin{aligned} \frac{\partial^2 e_z}{\partial x^2} + \frac{\partial^2 e_x}{\partial y^2} &= -k_c^2 \cdot e_z \\ \frac{\partial^2 h_z}{\partial x^2} + \frac{\partial^2 h_z}{\partial y^2} &= -k_c^2 \cdot h_z \end{aligned} \right\}$$

where, ① $e_z = h_z = 0 \rightarrow$ TEM 파



② $e_z = 0$, $h_z \neq 0 \rightarrow$ TE 파. (H파)



③ $e_z \neq 0$, $h_z = 0 \rightarrow$ TM 파 (E파)