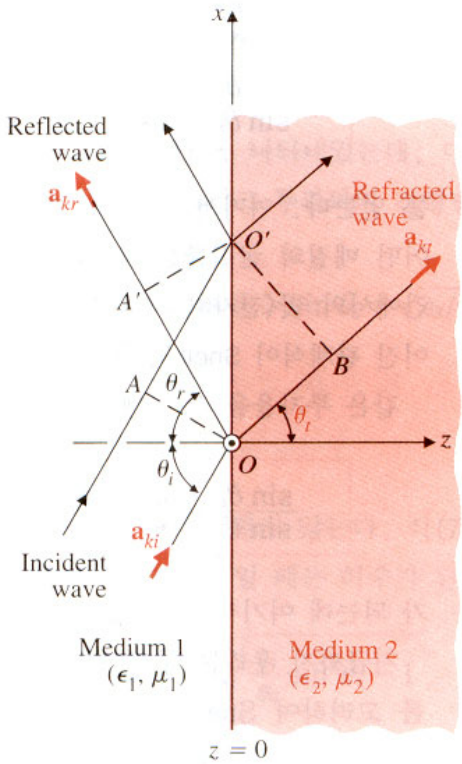
2.4 평면 유전체 경계면에 경사되게 입사하는 경우

전자기학 2(cheng 7-6, 7-7)에서 강의하였음(beam project 강의)

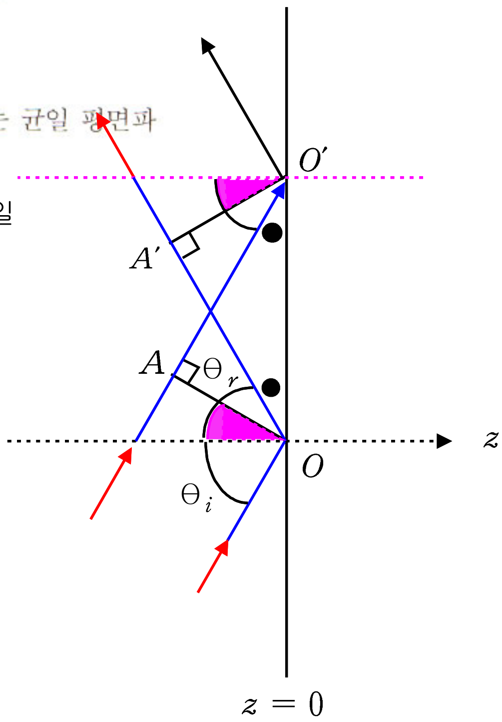
평면경계면에서 평면파의 경사 입사

(Oblique incidence of Plane waves at Plane Boundaries)



- 입사평면 :k를 포함하고 경계면에 수직인 평면
- $\overline{AO}$: 입사파 동위상면 (파면;wavefront)
- $\overline{A'O'}$: 반사파 동위상면 (파면;wavefront)
- $\overline{BO'}$: 투과파 동위상면 (파면;wavefront)

그림 7-10 평면 유전체 경계면에 경사각을 갖고 입사하는 균일 평면파



◉ 매질 1에서 입사파, 반사파, 위상속도 u_{p_1}은 동일

$$\therefore \quad \overline{OA'} = \overline{O'A}$$

옆 그림에서 의해서

$$\overline{OA'} = \sin(\theta_i - ◤ + •)\overline{OO'}$$

$$\overline{O'A} = \sin(\theta_r - ◺ + •)\overline{OO'}$$

$$\therefore \quad \sin\theta_i \overline{OO'} = \sin\theta_r \overline{OO'}$$

$$\therefore \quad \theta_i = \theta_r \qquad (7\text{-}110)$$

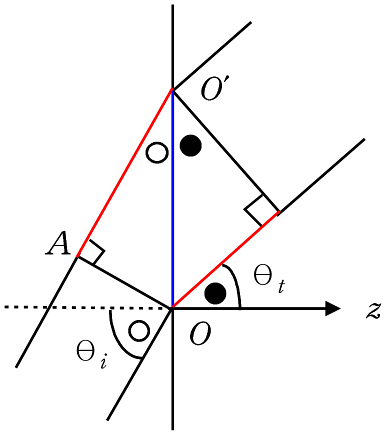
∴ Snell's Law

◉ 매질 2에서

$\overline{OB}$ 진행시간 $=$ $\overline{AO'}$ 진행시간(∴ 동위상면)

$$\frac{\overline{OB}}{u_{p_2}} = \frac{\overline{AO'}}{u_{p_1}} \quad (\because \text{거리/속도})$$

$$\therefore \frac{\overline{OB}}{\overline{AO'}} = $$



$$= \frac{\overline{OO'}\ \sin\Theta_t}{\overline{OO'}\ \sin\Theta_i} = \frac{u_{p_2}}{u_{p1}}$$

$$u_p = \frac{\omega}{k} = \frac{1}{\sqrt{\epsilon\mu}} \quad (7-10)$$
$$= \frac{\omega}{\beta} \quad (7-50)$$

$$\therefore \frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p_2}}{u_{p_1}} = \frac{\omega/\beta_2}{\omega/\beta_1} = \frac{\beta_1}{\beta_2}$$

where, $\frac{c}{u_p} = \frac{\text{자유공간 광속}}{\text{매질 전파속도}} = n$: *Refraction Index*

$$\therefore u_p = \frac{c}{n}$$

$$\therefore \frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p_2}}{u_{p_1}} = \frac{\beta_1}{\beta_2} = \frac{c/n_2}{c/n_1} = \frac{n_1}{n_2}$$

; Snell's Law of Refraction (7-117)

; Polarization과 무관

한편, 매질(유전체)에서는 $\mu_1 = \mu_2$ 이므로

$$\therefore \frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \frac{\beta_1}{\beta_2} = \frac{u_{p_2}}{u_{p_1}} = \frac{1/\sqrt{\epsilon_2\mu_2}}{1/\sqrt{\epsilon_1\mu_1}} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}} \quad (7\text{-}118)$$

$$= \frac{\eta_2}{\eta_1}$$

문 7-26) 그림 7-16에 보인바와 같이, 어떤 빛이 공기로부터 굴절률이 n 이고 두께가 d 인 투명한 판 위로 경사각을 갖고 입사하였다. 입사각을 Θ_i 라고 할 때 (a) Θ_t, (b) 빛이 투명판을 떠나는 점까지의 거리 l_1, 그리고 (c) 이 빛이 이 판을 떠날 때의 측면 거리 l_2 을 계산하라.

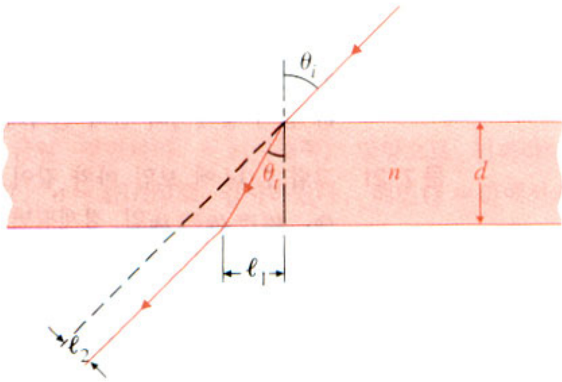


그림 7-16 굴절률이 n 인 투명판에 경사각을 갖고 입사하는 광선(문 7-26)

sol)

(a) by Snell's Law (7-117)

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p_2}}{u_{p_1}} = \frac{\beta_1}{\beta_2} = \frac{n_1}{n_2} \qquad (7\text{-}117)$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} \rightarrow \frac{1}{n}$$

$$\therefore \quad \theta_t = \sin t^{-1}\left(\frac{1}{n}\sin\theta_i\right)$$

(b) $l_1 = d\tan\theta_t = d\,\dfrac{\sin\theta_t}{\cos\theta_t}$

where,

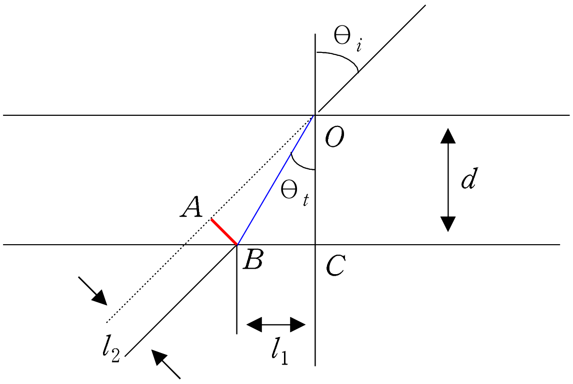
$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1-\left(\frac{1}{n}\sin\theta_i\right)^2}$$

$$\sin\theta_t = \frac{1}{n}\sin\theta_i$$

$$\therefore \quad l_1 = \frac{d\,\dfrac{1}{n}\sin\theta_i}{\sqrt{1-\left(\dfrac{1}{n}\sin\theta_i\right)^2}} = \frac{d\sin\theta_i}{n\sqrt{1-\dfrac{1}{n^2}\sin^2\theta_i}}$$

$$= \frac{d\sin\theta_i}{\sqrt{n^2-\sin^2\theta_i}}$$

(c)



$$l_2 = \overline{AB} = \overline{OB}\sin(\theta_i - \theta_t)$$

where,

$$\overline{OB} = \frac{d}{\cos\theta_t}$$

$$l_2 = \frac{d}{\cos\theta_t}\left(\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t\right)$$

$$= d\left(\sin\theta_i - \frac{\cos\theta_i\sin\theta_t}{\cos\theta_t}\right)$$

$$= d\left(\sin\theta_i - \cos\theta_i\tan\theta_t\right)$$

where,

$$\cos\theta_t = \sqrt{1 - \left(\frac{1}{n}\sin\theta_i\right)^2}$$

$$\sin\theta_t = \frac{1}{n}\sin\theta_i$$

$$\tan\theta_t = \frac{\sin\theta_i}{\sqrt{n^2 - \sin^2\theta_i}}$$

$$\therefore \quad l_2 = d\sin\theta_i\left(1 - \frac{\cos\theta_i}{\sqrt{n^2 - \sin^2\theta_i}}\right)$$

※ Total Reflection

· 입사파, 매질 1 → 매질 2, 입사

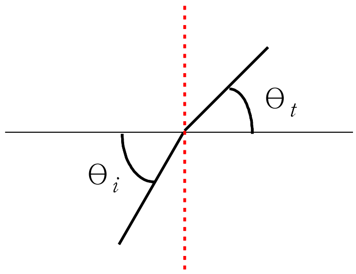
$\epsilon_1 \rightarrow \epsilon_2$, $(\mu_1 = \mu_2)$

$\epsilon_1 > \epsilon_2$ 인 경우만 전반사 발생

(밀도가 높다) (밀도가 낮다)

by) 식 (7-118)

$$\therefore \quad \frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} = \frac{\beta_1}{\beta_2} = \frac{u_{p_2}}{u_{p_1}} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{\eta_2}{\eta_1} \qquad (7\text{-}118)$$



$\epsilon_1 > \epsilon_2 \quad \rightarrow \quad \theta_t > \theta_i$

θ_i : 증가 → θ_t : 증가

θ_i : 증가 $\begin{cases} \rightarrow \ \theta_t = \frac{\pi}{2} \\ \rightarrow \text{ 굴절파는 경계면으로 진행} \end{cases}$

θ_i : 더욱 증가 $\begin{cases} \rightarrow \text{ 굴절파} = 0\text{, 반사파만 존재} \\ \rightarrow \ \textit{Total Refelction} \end{cases}$

where, $\theta_t = \frac{\pi}{2}$ 일 때 $\theta_i = \theta_c$: critical angle

by (7-118) : $\dfrac{\sin\frac{\pi}{2}}{\sin\theta_i} = \dfrac{\sin\frac{\pi}{2}}{\sin\theta_c} = \dfrac{1}{\sin\theta_c} = \sqrt{\dfrac{\epsilon_1}{\epsilon_2}} = \dfrac{n_1}{n_2}$

$$\therefore \sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \frac{n_2}{n_1} \qquad (7\text{-}119)$$

$$\therefore \ \theta_c = \sin t^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = \sin^{-1}\left(\frac{n_2}{n_1}\right) \qquad (7\text{-}119)$$

단, $\mu_1 = \mu_2$

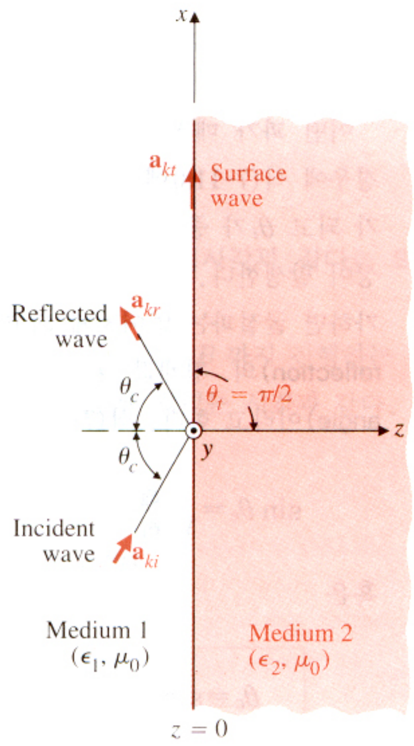


그림 7-11 $\epsilon_1 > \epsilon_2$인 두 매질의 경계면으로 경사각을 갖고 입사한 평면파.

· $\theta_i > \theta_c$ 인 경우

즉, $\sin\theta_i > \sin\theta_c = \sqrt{\dfrac{\epsilon_2}{\epsilon_1}}$ 인 경우

by (7-118) : $\dfrac{\sin\theta_t}{\sin\theta_i} = \sqrt{\dfrac{\epsilon_1}{\epsilon_2}}$ $\quad (\epsilon_1 > \epsilon_2 \rightarrow \theta_t > \theta_i)$

$$\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\,\sin\theta_i$$

$$= \sqrt{\frac{\epsilon_1 \rightarrow 大}{\epsilon_2 \rightarrow 小}}\,\sin(\theta_i \rightarrow 90^0) \;>\; 1$$

→ Θ_t에 대한 Real Solution은 존재하지 않음

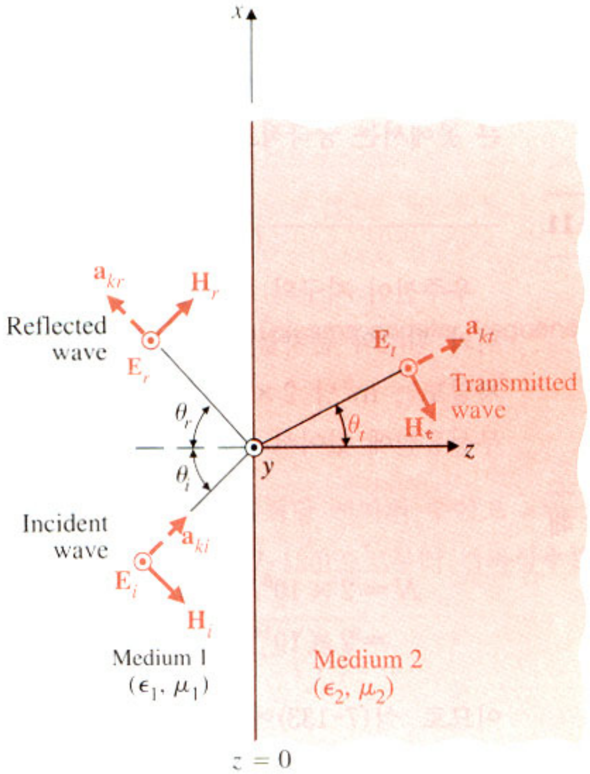
− $\cos\theta_t$ 는 $\sin\theta_t > 1$ 일 때 허수

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \pm\sqrt{\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_t - 1} \qquad (7\text{-}22)$$

식 (7-22)에서 식 (7-25)까지는 생략

· $\theta_i > \theta_t$: 매질 2에서 z 방향으로 지수적으로 감쇄되는 evanescent wave(소멸파)가 x 방향으로 존재 → Surface wave(표면파)

※ 수직편파(Perpendicular Polarization)



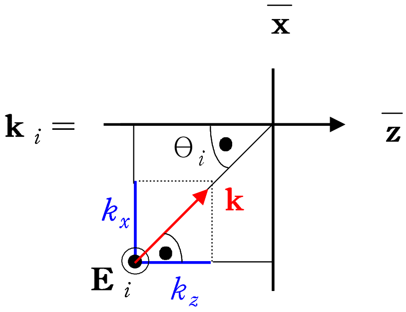


그림 7-14 유전체 경계평면에 경사각을 갖고 입사하는 평면파 (수직편파)

$\mathbf{E}_i$는 by 식 (7-23)

$$E_i(x,\ z) = E_{i0}\, e^{-j(k_i \cdot R)}\ \bar{y} \qquad (7\text{-}23)$$

where,

$$k_i \cdot R = k_1 \bar{k}_i \cdot R$$

$$= k_1 (k_x \bar{x} + k_z \bar{z}) \cdot (x\bar{x} + y\bar{y} + z\bar{z})$$

$$= k_1 (\sin\theta_i \bar{x} + \cos\theta_i \bar{z}) \cdot (x\bar{x} + y\bar{y} + z\bar{z})$$

$$= \beta_1 (x \sin\theta_i + z\cos\theta_i)$$

$$\therefore \quad E_i(x,\ z) = E_{i0}\, e^{-j\beta_1 (x\sin\theta_i + z\cos\theta_i)}\ \bar{y} \qquad (7\text{-}135)$$

한편, 자계H_i는 by (7-25)

$$H(x,\ z) = \frac{1}{\eta} \bar{k} \times E \qquad (7\text{-}25)$$

$$\therefore \quad \mathrm{H}_i(x, z) = \frac{1}{\eta_1}(k_x\bar{\mathrm{x}} + k_z\bar{\mathrm{z}}) \times \mathrm{E}_i(x, z)$$
$$= \frac{1}{\eta_1}(k_x\bar{\mathrm{x}} + k_z\bar{\mathrm{z}}) \times E_{i0}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}\bar{\mathrm{y}}$$
$$= \frac{1}{\eta_1}E_{i0}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}(k_x\bar{\mathrm{x}} + k_z\bar{\mathrm{z}}) \times \bar{\mathrm{y}}$$
$$= \frac{1}{\eta_1}E_{i0}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}(k_x\bar{\mathrm{z}} - k_z\bar{\mathrm{x}})$$
$$= \frac{1}{\eta_1}E_{i0}\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)}(\sin\theta_i\bar{\mathrm{z}} - \cos\theta_i\bar{\mathrm{x}})$$
$$= \frac{1}{\eta_1}E_{i0}(\sin\theta_i\bar{\mathrm{z}} - \cos\theta_i\bar{\mathrm{x}})\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \qquad (7\text{-}136)$$

· 반사파는

$$\mathrm{E}_r(x, z) = E_{r0}\, e^{-j(\mathrm{k}_r \cdot \mathrm{R})}\bar{\mathrm{y}}$$

where,

$$\mathrm{k}_r \cdot \mathrm{R} = k_1\bar{\mathrm{k}}_r \cdot \mathrm{R}$$
$$= k_1(k_x\bar{\mathrm{x}} - k_z\bar{\mathrm{z}}) \cdot (x\bar{\mathrm{x}} + y\bar{\mathrm{y}} + z\bar{\mathrm{z}})$$
$$= k_1(\sin\theta_r\bar{\mathrm{x}} - \cos\theta_r\bar{\mathrm{z}}) \cdot (x\bar{\mathrm{x}} + y\bar{\mathrm{y}} + z\bar{\mathrm{z}})$$
$$= \beta_1(x\sin\theta_r - z\cos\theta_r)$$

$$\therefore \quad \mathrm{E}_r(x, z) = E_{r0}\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\bar{\mathrm{y}} \qquad (7\text{-}138)$$

한편, 자계 $\mathbf{H}_i$는 by (7-25)

$$\mathrm{H}(x, z) = \frac{1}{\eta}\bar{\mathrm{k}} \times \mathrm{E} \qquad (7\text{-}25)$$

$$\therefore \quad \mathrm{H}_r(x, z) = \frac{1}{\eta_1}(k_x\bar{\mathrm{x}} - k_z\bar{\mathrm{z}}) \times \mathrm{E}_r(x, z)$$
$$= \frac{1}{\eta_1}(k_x\bar{\mathrm{x}} - k_z\bar{\mathrm{z}}) \times E_{r0}\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\bar{\mathrm{y}}$$
$$= \frac{1}{\eta_1}E_{r0}\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}(k_x\bar{\mathrm{x}} - k_z\bar{\mathrm{z}}) \times \bar{\mathrm{y}}$$
$$= \frac{1}{\eta_1}E_{r0}\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}(k_x\bar{\mathrm{z}} + k_z\bar{\mathrm{x}})$$
$$= \frac{1}{\eta_1}E_{r0}\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}(\sin\theta_r\bar{\mathrm{z}} + \cos\theta_r\bar{\mathrm{x}})$$
$$= \frac{1}{\eta_1}E_{r0}(\cos\theta_r\bar{\mathrm{x}} + \sin\theta_r\bar{\mathrm{z}})\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \qquad (7\text{-}139)$$

· 투사파는

$$\mathrm{E}_t(x,\ z) = E_{t0}\, e^{-j(\mathrm{k}_t \cdot \mathrm{R})}\ \bar{\mathrm{y}}$$

where,

$$\mathrm{k}_t \cdot \mathrm{R} = k_2 \bar{\mathrm{k}}_t \cdot \mathrm{R}$$

$$= k_2 (k_x \bar{\mathrm{x}} + k_z \bar{\mathrm{z}}) \cdot (x\bar{\mathrm{x}} + y\bar{\mathrm{y}} + z\bar{\mathrm{z}})$$

$$= k_2 (\sin\theta_t \bar{\mathrm{x}} + \cos\theta_t \bar{\mathrm{z}}) \cdot (x\bar{\mathrm{x}} + y\bar{\mathrm{y}} + z\bar{\mathrm{z}})$$

$$= \beta_2 (x \sin\theta_t + z\cos\theta_t)$$

$$\therefore \quad \mathrm{E}_t(x,\ z) = E_{t0}\, e^{-j\,\beta_2 (x\sin\theta_t + z\cos\theta_t)}\ \bar{\mathrm{y}} \qquad (7\text{-}141)$$

한편, 자계H_i는 by (7-25)

$$\mathrm{H}(x,\ z) = \frac{1}{\eta} \bar{\mathrm{k}} \times \mathrm{E} \qquad (7\text{-}25)$$

$$\therefore \quad \mathrm{H}_t(x,\ z) = \frac{1}{\eta_2}(k_x \bar{\mathrm{x}} + k_z \bar{\mathrm{z}}) \times \mathrm{E}_t(x,\ z)$$

$$= \frac{1}{\eta_2}(k_x \bar{\mathrm{x}} + k_z \bar{\mathrm{z}}) \times E_{t0}\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\bar{\mathrm{y}}$$

$$= \frac{1}{\eta_2} E_{t0}\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}(k_x \bar{\mathrm{x}} + k_z \bar{\mathrm{z}}) \times \bar{\mathrm{y}}$$

$$= \frac{1}{\eta_2} E_{t0}\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}(k_x \bar{\mathrm{z}} - k_z \bar{\mathrm{x}})$$

$$= \frac{1}{\eta_2} E_{t0}\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}(\sin\theta_t \bar{\mathrm{z}} - \cos\theta_t \bar{\mathrm{x}})$$

$$= \frac{1}{\eta_2} E_{t0} (\sin\theta_t \bar{\mathrm{z}} - \cos\theta_t \bar{\mathrm{x}})\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \qquad (7\text{-}142)$$

· 전계에 대해 고찰하면

– 식 (7-135), (7-138), (7-141)에서 미지수 E_{r0}, E_{t0}, Θ_r, Θ_t가 존재

이를 구하기 위하여, 경계조건을 동원

즉, 전계 $\mathbf{E}$ 와 자계 $\mathbf{H}$ 는 경계면 ($z = 0$) 에서

접선성문은 항상 연속 ($E_{iy} + E_{ry} = E_{ty}$)

$$\therefore \quad (7-135) = \mathrm{E}_i(x,\ z) = E_{i0}\, e^{-j\beta_1(x\sin\theta_i)}\ \bar{\mathrm{y}}$$

$$(7-138) = \mathrm{E}_r(x,\ z) = E_{r0}\, e^{-j\beta_1(x\sin\theta_r)}\ \bar{\mathrm{y}}$$

$$(7-141) = \mathbf{E}_t(x,\ z) = E_{t0}\, e^{-j\beta_2(x\sin\Theta_t)}\ \bar{\mathbf{y}}$$

$$\therefore\ E_{i0}\, e^{-j\beta_1(x\sin\theta_i)} + E_{r0}\, e^{-j\beta_1(x\sin\theta_r)} = E_{t0}\, e^{-j\beta_2(x\sin\theta_t)} \tag{7-143}$$

한편, 자계성분은 식 (7-136), (7-139), (7-142)에 의해

$$\therefore\ (7-136) = \mathrm{H}_i(x,z) = \frac{E_{i0}}{\eta_1}(-\cos\theta_i\,\overline{\mathrm{x}})\, e^{-j\beta_1(x\sin\theta_i)}$$

$$(7-139) = \mathrm{H}_r(x,z) = \frac{E_{r0}}{\eta_1}(\cos\theta_r\,\overline{\mathrm{x}})\, e^{-j\beta_1(x\sin\theta_r)}$$

$$(7-142) = \mathrm{H}_t(x,z) = \frac{E_{t0}}{\eta_2}(-\cos\theta_t\,\overline{\mathrm{x}})\, e^{-j\beta_2(x\sin\theta_t)}$$

$$\therefore\ \frac{1}{\eta_1}\left(-E_{i0}\cos\theta_i\, e^{-j\beta_1 x\sin\theta_i} + E_{r0}\cos\theta_r\, e^{-j\beta_1 x\sin\theta_r}\right) \tag{7-144}$$

$$= -\frac{E_{t0}}{\eta_2}\cos\theta_t\, e^{-j\beta_2 x\sin\theta_t}$$

식 (7-143), (7-144)는 모든 x 에 대해 연속이기 위해서는

모든 지수함수의 인자는 같아야 함.(위상 정합조건)

$$\beta_1\, x\sin\theta_i = \beta_1\, x\sin\theta_r = \beta_2\, x\sin\theta_t$$

$\therefore\ \beta_1 \sin\theta_i = \beta_1 \sin\theta_r = \beta_2 \sin\theta_t$; Phase Matching Condition

$$\therefore\ \theta_i = \theta_r \tag{7-116}$$

또한 $\beta_1 \sin\theta_i = \beta_2\, x \sin\theta_t$; Snell's Law

$$\therefore\ \frac{\sin\theta_t}{\sin\theta_i} = \frac{\beta_1}{\beta_2} = \frac{n_1}{n_2} = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \tag{7-118}$$

한편, 식 (7-143)과 식 (7-144)를 정리하면,

$$E_{i0} + E_{r0} = E_{t0} \tag{7-145}$$

$$\frac{1}{\eta_1}\left(-E_{i0}\cos\theta_i + E_{r0}\cos\theta_r\right) = -\frac{E_{t0}}{\eta_2}\cos\theta_t \tag{7-146}$$

· 여기서, 반사계수 $\Gamma_\perp$

$$\Gamma_\perp = \frac{E_{r0}}{E_i 0} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \tag{7-147}$$

Where, $\Gamma_\perp = 0$ 가 되는 $\begin{cases}\theta_i \\ \eta\end{cases}$ 값은 없음

· 투과계수 $\tau_\perp$

$$\tau_\perp = \frac{E_{t0}}{E_i 0} = \frac{2\,\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \qquad \text{(7-148)}$$

· 직각 입사 → $\theta_i = 0^0 \rightarrow \theta_r = 0^0 = \theta_t \Rightarrow \begin{cases} (7-147) = (7-94) \\ (7-148) = (7-95) \end{cases}$

$$\Rightarrow 1 + \Gamma_\perp = \tau_\perp \qquad \text{(7-149)}$$

· 매질 2 = Conductor, $\eta_2 \rightarrow 0$

$\Gamma_\perp = -1$: 완전반사, 위상 반전

$\tau_\perp = 0$: 투과파 없음 ; (접선성분 = 0)

ex 7-12

※ 수평편파(Parrel Polarization)

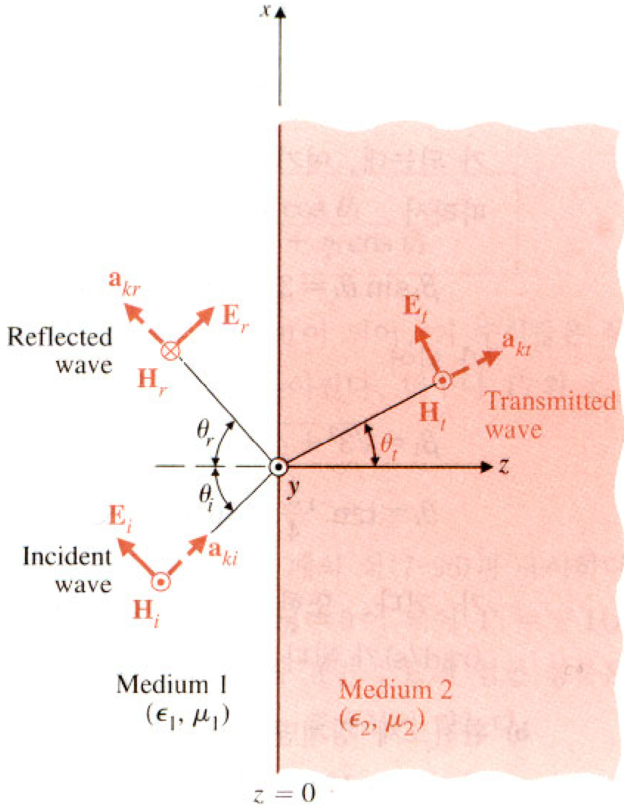


그림 7-15 유전체 경계평면에 경사각을 갖고 입사하는 평면파 (평행편파)

교재 설명