[2] 손실 매질에서 평면파(Plane Wave)

손실 매질 = $\sigma \neq 0$

※ $\sigma \neq 0$ 인 매질에서

- Maxwell's Eq

$$\nabla \times \mathrm{E} = -j\omega\mu\mathrm{H} \qquad (2\text{-}26)$$
$$\nabla \times \mathrm{H} = \sigma\mathrm{E} + j\omega\epsilon\mathrm{E}$$

- 위식에 대한 Helmholtz's Eq

$$\nabla^2\mathrm{E} + k_c^2\mathrm{E} = 0 \qquad (2\text{-}5')$$

where, $k_c^2 = \omega^2\mu\epsilon_c$

$$\epsilon_c = \epsilon\left(1 + \frac{\sigma}{j\omega\epsilon}\right)$$

※ $\sigma = 0$ (무손실)인 매질에서

- Helmholtz's Eq

$$\nabla^2\mathrm{E} + k^2\mathrm{E} = 0$$

where, $k^2 = \omega^2\mu\epsilon$

$$\therefore \quad k^2 = k_c^2 \qquad (2\text{-}7)$$

= Complex propagation constant

= $-\gamma^2$ (전송선로에서 (3-21)과 같이)

$$\therefore \quad \gamma = \alpha + j\beta = j\omega\sqrt{\epsilon\mu}\sqrt{1 - j\frac{\sigma}{\omega\epsilon}} \qquad (2\text{-}28)$$

여기서, 매질 = 손실매질이므로

$\epsilon = \epsilon' - j\epsilon''$ 대입

$$\begin{aligned}\gamma &= j\omega\sqrt{(\epsilon' - j\epsilon'')\mu}\sqrt{1 - j\frac{\sigma}{\omega(\epsilon' - j\epsilon'')}} \\ &= j\omega\sqrt{(\epsilon' - j\epsilon'')\mu - j\frac{(\epsilon' - j\epsilon'')\mu\sigma}{\omega(\epsilon' - j\epsilon'')}} \\ &= j\omega\sqrt{\mu\epsilon' - j\mu\epsilon'' - j\frac{\mu\sigma}{\omega}} \\ &= j\omega\sqrt{\mu\epsilon'\left(1 - j\frac{\epsilon''}{\epsilon'} - j\frac{\sigma}{\omega\epsilon'}\right)} \\ &= j\omega\sqrt{\mu\epsilon'\left[1 - j\left(\frac{\epsilon''}{\epsilon'} + \frac{\sigma}{\omega\epsilon'}\right)\right]}\end{aligned}$$

$$\therefore \quad \gamma = j\omega \sqrt{\mu\epsilon'\left[1 - j\left(\frac{\omega\epsilon'' + \sigma}{\omega\epsilon'}\right)\right]}$$

where, $\dfrac{\omega\epsilon'' + \sigma}{\omega\epsilon'}$ = 식 (1-56)

= Loss Tangent δ

$$\therefore \quad \gamma = j\omega\sqrt{\mu\epsilon'(1 - j\tan\delta)}$$

if), f → 高 = $\mu-$ wave, $\omega\epsilon'' \gg \delta$

- $\tan\delta = \dfrac{\omega\epsilon'' + \delta}{\omega\epsilon'} = \dfrac{\omega\epsilon''}{\omega\epsilon'} = \dfrac{\epsilon''}{\epsilon'} = \dfrac{\text{허수}}{\text{실수}}$

  = 식 (2-28)에서 = $\dfrac{\sigma}{\omega\epsilon}$

여기서, $\epsilon' \gg \epsilon''$: $\tan\delta = \dfrac{\epsilon''}{\epsilon'} \to 0$: "good dielectric material"

또는, $\sigma \ll \omega\epsilon$: $\tan\delta = \dfrac{\sigma}{\omega\epsilon}$: 손실은 분극 전류에 의한 것

: "good dielectric material"

$\sigma \gg \omega\epsilon$: $\tan\delta = \dfrac{\sigma}{\omega\epsilon}$: 손실은 도전 전류에 의한 것

: "good Conductor"

그러므로 손실이 있는 매질에서 Helmhltz's Eq. by(2-6)

$$\nabla^2\mathrm{E} - \gamma^2\mathrm{E} = 0 \qquad (2\text{-}29)$$
$$\nabla^2\mathrm{H} - \gamma^2\mathrm{H} = 0$$

if) $\mathrm{E} = E_x\overline{\mathrm{x}}$

식(2-29)$= \dfrac{d^2E_x}{dz^2} - \gamma^2E_x = 0$

$$\therefore \quad E_x(z) = E_0^+e^{-\gamma z} + E_0^-e^{\gamma z} \qquad (2\text{-}31)$$
$$= E_0^+e^{-\alpha z}e^{-\beta z} + E_0^-e^{\alpha z}e^{\beta z}$$

시간영역(순시치)에서는

$$E_x(z,t) = \Re e[\,E_0^+ e^{-\alpha z} e^{-\beta z} e^{j\omega t} + E_0^- e^{\alpha z} e^{\beta z} e^{j\omega t}]$$
$$= E_0^+ \underline{e^{-\alpha z}} \cos(\omega t - \beta z) + E_0^- \underline{e^{\alpha z}} \cos(\omega t + \beta z)$$

Attenuation

by (2-12a)

$$H_y = \frac{j}{\omega\mu}\frac{dE_x}{dz} = \frac{-j\gamma}{\omega\mu}\left(E_0^+ e^{-\gamma z} - E_0^- e^{\gamma z}\right) \qquad (2\text{-}33)$$

where, $\dfrac{E_x}{H_y} = \dfrac{j\omega\mu}{\gamma} = \eta$ (2-34)

$\gamma = jk$

- $\sigma = 0$, $\alpha = 0$

$\therefore\ \gamma = j\beta = j\dfrac{2\pi}{\lambda} = jk$, 즉, $\beta = k = \gamma$

phase constant = wave number = propagation constant

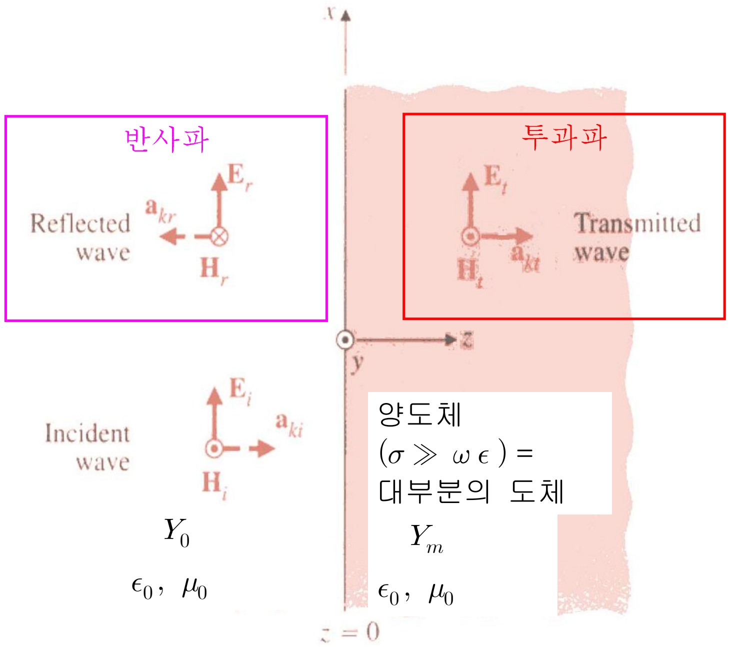
[3] 양도체내의 평면파(Plane Wave)

※ 도체면에서 TEM파의 반사, 투과(경계면에 수직으로 입사하는 경우)

1. 우선 E_i 에 의해서 E_r, E_t 가 존재하는 것으로 가정하고 계산한 후

2. 경계 조건으로 결정

3. 도전전류 >> 변위전류 $\left[\sigma \gg \omega\,\epsilon\,,\quad \text{or}\quad \dfrac{\sigma}{\omega\,\epsilon} \gg 1\right]$



(1) 입사파 : $\mathrm{E}_i\,(z) = E_{i0}\,e^{-jk_z z}\,\overline{\mathrm{x}}$

$$\mathrm{H}_i\,(z) = \frac{E_{i0}}{\eta_0}\,e^{-jk_z z}\,\overline{\mathrm{y}}$$

반사파 : $\mathrm{E}_r\,(z) = E_{r0}\,e^{jk_z z}\,\overline{\mathrm{x}} = |\Gamma|\,E_{i0}\,e^{jk_z z}\,\overline{\mathrm{x}}$

$$\mathrm{H_r}\,(z) = \frac{1}{\eta_0}\,\overline{\mathrm{k}} \times \mathrm{E}_r\,(z)$$

$$= \frac{E_{ro}}{\eta_0}\,e^{j\beta_1 z}\,(-\overline{\mathrm{y}}) = -\frac{1}{\eta_0}|\Gamma|\,E_{i0}\,e^{jk_z z}\,\overline{\mathrm{y}}$$

(2) 도체내에서 Helmholtz's Eq.

by(2-5a): $\nabla^2 \mathrm{E} = \mu\sigma \frac{\partial \mathrm{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \mathrm{E}}{\partial t^2}$

→ 손실 매질($\sigma \neq 0$), 도체내이면 $\epsilon = 0$이므로

$$\nabla^2 \mathrm{E} - \mu\sigma \frac{\partial \mathrm{E}}{\partial t} = 0 \quad ; \text{순시식}$$

$$\nabla^2 \mathrm{E} - j\omega\, \mu\sigma \mathrm{E} = 0 \quad ; \text{phasor}$$

$$\therefore \quad \nabla^2 \mathrm{E}_t - j\omega\, \mu\sigma \mathrm{E}_t = 0 \qquad (2\text{-}46)$$

where, E_t는 x 성분을 갖고 z 방향으로 진행

$$\therefore \quad \frac{\partial^2 \mathrm{E}_t}{\partial z^2} - j\,\omega\,\mu\,\sigma\, \mathrm{E}_t = 0$$

$$\mathrm{E}_t(z) = E_{t0}\, e^{-\gamma z}\, \overline{\mathrm{x}}$$

$$\boxed{\begin{aligned} &\frac{\partial^2 E}{\partial z^2} - \gamma^2 E = 0 \\ &\gamma = j\omega\sqrt{\epsilon\,\mu} \end{aligned}}$$

(3) where, by(2-28) : ;양도체($\sigma \gg \omega\,\epsilon$)

$$\gamma = (j\,\omega\,\mu\,\sigma)^{1/2} = \sqrt{j}\ \sqrt{\omega\,\mu\sigma}$$

where, $\sqrt{j} = \sqrt{\frac{2j}{2}} = \sqrt{\frac{1+2j-1}{2}} = \sqrt{\frac{(1+j)^2}{2}} = \frac{1+j}{\sqrt{2}}$

$$= \frac{1+j}{\sqrt{2}}\ \sqrt{\omega\,\mu\,\sigma} \qquad (2\text{-}35)$$

$$= (1+j)\sqrt{\frac{\omega\,\mu\,\sigma}{2}} = (1+j)\ \sqrt{\pi\,f\,\mu\,\sigma} = \alpha + j\beta$$

$$\therefore \quad \alpha = \sqrt{\pi\,f\,\mu\,\sigma} = \frac{1}{\delta}\,[\mathrm{Np/m}] = \beta$$

$$\therefore \quad \gamma = \frac{1+j}{\delta}$$

(4) where, $\delta = \frac{1}{\sqrt{\pi\,f\,\mu\,\sigma}}$ ="skin depth"

= "characteristic depth of penetration(투과의 특성 두께)

$$= \frac{1}{\alpha} \qquad (2\text{-}37)$$

여기서, $z = \delta$이면

$$e^{-\alpha z} = e^{-\alpha\delta} = e^{-\alpha\frac{1}{\alpha}} = e^{-1} = 0.368$$

즉, 크기가 e^{-1} or 0.368만큼 감쇠했을 때 두께(=깊이)

→ "skin depth"

= "characteristic depth of penetration(투과의 특성 두께)

ex 2-3 주파수 $f = 10[\mathrm{GHz}]$에서 알루미늄(aluminum), 동(copper), 금(gold) 및 은(silver)의 표피두께를 구하여라.

sol) 양도체에서 $\mu = \mu_0 \mu_r = \mu_0$ 즉, $\mu_r = 1$

부록 F(p421)를 참조

$$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} = \sqrt{\frac{1}{\pi \times 10^{10} \times (4\pi \times 10^{-7})}} \sqrt{\frac{1}{\sigma}} = 5.03 \times 10^{-3} \sqrt{\frac{1}{\sigma}}$$

알루미늄($\sigma = 3.816 \times 10^7 [\mathrm{S/m}]$): $\delta = 5.03 \times 10^{-3} \sqrt{\frac{1}{3.816 \times 10^7}} = 8.14 \times 10^{-7} [\mathrm{m}]$

동 ($\sigma = 5.813 \times 10^7 [\mathrm{S/m}]$): $\delta = 5.03 \times 10^{-3} \sqrt{\frac{1}{5.813 \times 10^7}} = 6.60 \times 10^{-7} [\mathrm{m}]$

금 ($\sigma = 4.098 \times 10^7 [\mathrm{S/m}]$): $\delta = 5.03 \times 10^{-3} \sqrt{\frac{1}{4.098 \times 10^7}} = 7.86 \times 10^{-7} [\mathrm{m}]$

은 ($\sigma = 6.173 \times 10^7 [\mathrm{S/m}]$): $\delta = 5.03 \times 10^{-3} \sqrt{\frac{1}{6.173 \times 10^7}} = 6.40 \times 10^{-7} [\mathrm{m}]$

⇒ 양도체에 흐르는 전류는 대부분 도체 표면을 따라 흐름.

(5) $v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\pi f \mu \sigma}} = \omega \delta = \sqrt{\frac{(2\pi f)^2}{\pi f \mu \sigma}} = \sqrt{2} \sqrt{\frac{\omega}{\mu \sigma}}$

ex) Cu에서 $\sigma = 5.80 \times 10^7 [\mathrm{S/m}]$이고, $f = 3[\mathrm{MHz}]$ 에서 전파속도?

sol) $\mu = \mu_0 = 4\pi \times 10^{-7}$ 이므로

$$v_p = \sqrt{2} \sqrt{\frac{\omega}{\mu \sigma}} = \sqrt{2} \sqrt{\frac{2\pi \times 3 \times 10^6}{4\pi \times 10^{-7} \times 5.80 \times 10^7}} = 720 [\mathrm{m/s}] \ll c$$

(6) $\lambda = \dfrac{2\pi}{\beta} = \dfrac{2\pi}{\sqrt{\pi f \mu\sigma}} = \sqrt{\dfrac{2\pi^2}{\pi f \mu\sigma}} = 2\sqrt{\dfrac{\pi}{f\mu\sigma}}\,[\mathrm{m}]$

ex) Cu- 3[MHz] → $\lambda = 0.25[\mathrm{mm}]$

공기- 3[MHz] → $\lambda = 100[\mathrm{m}]$

(7) 한편 자계 $\mathrm{H}_t(z)$

$\nabla \times \mathrm{E} = -j\omega\mu\mathrm{H}$ 에서

$$\mathrm{H}_t(z) = -\frac{1}{j\omega\mu}\nabla \times \mathrm{E}_t(z)$$
$$= -\frac{1}{j\omega\mu}\begin{vmatrix} \bar{\mathrm{x}} & \bar{\mathrm{y}} & \bar{\mathrm{z}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ E_{t0}e^{-\gamma z} & 0 & 0 \end{vmatrix}$$
$$= -\frac{1}{j\omega\mu}\left[\frac{\partial E_{t0}e^{-\gamma z}}{\partial z}\bar{\mathrm{y}} - \frac{\partial E_{t0}e^{-\gamma z}}{\partial y}\bar{\mathrm{z}}\right]$$
$$= -\frac{1}{j\omega\mu}(-\gamma)\,E_{t0}\,e^{-\gamma z}\,\bar{\mathrm{y}}$$
$$= \frac{\gamma}{j\omega\mu}\,E_{t0}\,e^{-\gamma z}\,\bar{\mathrm{y}}$$

[8] 도체내에서

$$\frac{E_t}{H_t} = \eta_m = \frac{j\omega\mu}{\gamma} \tag{2-34}$$

$$= \frac{j\omega\mu}{\sqrt{j\omega\mu\sigma}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \frac{1+j}{\sigma\delta} = \frac{\sqrt{2}}{\sigma\delta}\,e^{j\frac{\pi}{4}} \tag{2-36}$$

; 도체 고유 Impedance ⇒ 전계에 대하여 자계는 45^0 지연

$Y_m = \dfrac{1}{\eta_m}$: 도체 고유 admittance

ex) Cu, $f = 10[\mathrm{GHz}]$ 일 때 $\eta_m \approx 0.026(1+j)[\Omega]$

∴ $\mu-$wave 영역 : $\eta_m \ll \eta_0$

- 경계면에서 전계, 자계 접선 성분은 연속이어야 하므로

$$E_i + E_r = E_i + \Gamma E_i = (1+\Gamma)E_i = t\,E_i = E_t$$

$$H_i + H_r = H_i - \Gamma H_i = (1-\Gamma)H_i = (1-\Gamma)\frac{1}{\eta_0}\,E_i = H_t$$

$$= \frac{1}{\eta_m}\,E_t = \frac{1}{\eta_m}\,t\,E_i$$

where, $\begin{cases} t = 1+\Gamma = \dfrac{2\,\eta_m}{\eta_m + \eta_0} \\ \eta_m \ll \eta_0 \end{cases}$ 이므로

- $\mu-$ wave 영역에서 $t=0$, $\Gamma = -1$ 일 때

경계면에서 전계 접선, $E_t = t\,E_i$ → zero

자계 접선, $H_t = (1-\Gamma)\,Y_0\,E_i = 2\,Y_0\,E_i$
$= 2\,H_i$

⇒ “도체내 전류밀도 생성”

- 도체내의 전류밀도

$$\mathrm{J} = \sigma \mathrm{E}$$
$$= \sigma \mathrm{E}_t$$
$$= \sigma\,t\,E_i\,e^{-\gamma z}\,\overline{\mathrm{x}}$$

→ $\overline{\mathrm{x}}$ 방향으로 변화하면서 $\overline{z}$ 방향으로 진행

$$\mathrm{J}_s = \int_0^\infty \mathrm{J}\,dz = \sigma\,t\,E_i \int_0^\infty e^{-\gamma z}\,dz\,\overline{\mathrm{x}}$$: “ 표면전류밀도”

$$= \frac{\sigma\,t}{\gamma}\,E_i\,\overline{\mathrm{x}}$$

where,

$$t = \frac{2\,\eta_m}{\eta_m + \eta_0} \simeq \frac{2\,\eta_m}{\eta_0} = \frac{2}{\eta_0}\cdot\frac{1+j}{\sigma\,\delta}$$

$$\gamma = \frac{1+j}{\delta}$$

$$\therefore\quad \mathrm{J}_s = \frac{\sigma\cdot\dfrac{2}{\eta_0}\cdot\dfrac{1+j}{\sigma\delta}}{\dfrac{1+j}{\delta}}\,E_i\,\overline{\mathrm{x}}$$

$$= \frac{2\,E_i}{\eta_0}\,\overline{\mathrm{x}} = 2\,Y_0\,E_i\,\overline{\mathrm{x}} = 2\,H_i\,\overline{\mathrm{x}}$$

• 전력 전송(평균전력)

$$P_{av} = \frac{1}{2}\,\Re e[\mathrm{E}\times\mathrm{H}^{*}\cdot\bar{\mathrm{z}}\,]$$

where, 벡터 등식

$$\bar{\mathrm{z}}\cdot(\mathrm{E}\times\mathrm{H}^{*}) = (\bar{\mathrm{z}}\times\mathrm{E}\,)\cdot\mathrm{H}^{*} = \eta\,\mathrm{H}\cdot\mathrm{H}^{*}$$

$$P_{av} = \frac{1}{2}\,\Re e[\eta_m\,\mathrm{H}\cdot\mathrm{H}^{*}\,]$$

$$= \frac{1}{2}\,\Re e[\eta_m\,|\mathrm{H}|^2]$$

where, $\eta_m = \dfrac{1+j}{\sigma\delta}$

$$= \frac{1}{2}\,\Re e\left[\frac{1+j}{\sigma\,\delta}\ |\mathrm{H}|^2\right]$$

$$= \frac{1}{2}\,\frac{1}{\sigma\,\delta}\,|\mathrm{H}|^2$$

$$= \frac{1}{2}\,R_s\,H^2\,, \qquad R_s = \text{표면저항} = \frac{1}{\sigma\delta} = \rho\,\frac{1}{\delta}\ [\Omega/\mathrm{m}^2] \qquad (2\text{-}38)$$

$$= \frac{1}{2}\,R_s\,J_s^{\,2}\,, \qquad J_s = \text{표면전류} \qquad (2\text{-}39)$$

※ 각종 매질에서 평면파의 전파 특성을 정리하면 → 표 2-1

구분	매질형태		
	무손실 ($\epsilon'' = \sigma = 0$)	일반적손실 ($\epsilon'' \ll \epsilon'$)	양도체 ($\epsilon'' \gg \epsilon'$ or $\sigma \gg \omega\epsilon'$)
복소전파상수	$\gamma = j\,\omega\,\sqrt{\mu\,\epsilon}$	$\gamma = j\omega\sqrt{\mu\epsilon}$ $= j\omega\sqrt{\mu\epsilon'}\sqrt{1+\dfrac{\sigma}{j\omega\epsilon'}}$	$\gamma = (1+j)\sqrt{\pi f\mu\sigma}$
파 수 (wave number)	$\beta = k = \omega\,\sqrt{\mu\epsilon}$	$\beta = Im(\gamma)$	$\beta = Im(\gamma) = \sqrt{\pi f\mu\sigma}$
감 쇠 상 수	$\alpha = 0$	$\alpha = \Re e(\gamma)$	$\alpha = \Re e(\gamma) = \sqrt{\pi f\mu\sigma}$
파동임피던스	$\eta = \sqrt{\dfrac{\mu}{\epsilon}} = \dfrac{\omega\mu}{k}$	$\eta = \dfrac{j\omega\mu}{\gamma}$	$\eta = (1+j)\sqrt{\dfrac{\omega\mu}{2\sigma}}$
표 피 두 께	$\delta = \infty$	$\delta = \dfrac{1}{\alpha}$	$\delta = \sqrt{\dfrac{1}{\pi f\mu\sigma}}$
파 장	$\lambda = \dfrac{2\pi}{\beta}$	$\lambda = \dfrac{2\pi}{\beta}$	$\lambda = \dfrac{2\pi}{\beta}$
위 상 속 도	$v_p = \dfrac{\omega}{\beta}$	$v_p = \dfrac{\omega}{\beta}$	$v_p = \dfrac{\omega}{\beta}$

ex pozar p35

평면파가 구리로 만들어진 안테나에 수직으로 입사하였다. 평면파의 주파수 $f = 1[\mathrm{GHz}]$ 이고, 구리의 도전률은 $\sigma = 5.813 \times 10^7 [\mathrm{S/m}]$ 이다. 안테나에서 전파상수, 특성임피던스, 표피두께(투과두께), 위상속도, 파장, 반사계수를 구하라.

sol)

우선

표피두께 δ를 구하여 propagation constant, characteristic Impedance, Reflection coefficient를 구한다.

① skim depth δ는 식 (2-37)에서

$$\begin{aligned}\delta &= \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}, \mu = \mu_0 \mu_r = \mu_0 \\ &= \frac{1}{(3.14 \times 1 \times 10^9 \times 4\pi \times 10^{-7} \times 5.813 \times 10^7)^{1/2}} \\ &= 2.088 \times 10^{-6} [\mathrm{m}] \\ &= 2.088 [\mu\mathrm{m}]\end{aligned}$$

② Propagation constant γ는 식 (2-35)로부터

$$\begin{aligned}\gamma &= (1+j)\sqrt{\pi f \mu \sigma} = (1+j)\alpha = \frac{1+j}{\delta} \\ &= \frac{1+j}{2.088 \times 10^{-6} [\mathrm{m}]} = (4.789 + j\,4.489) \times 10^5 [\mathrm{m}^{-1}]\end{aligned}$$

③ Characteristic Impedance η_m 은 식 (2-36)으로부터

$$\begin{aligned}\eta_m &= \frac{E}{H} = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega\mu\sigma}} = \frac{1+j}{\sigma\delta} \\ &= \frac{1+j}{5.813 \times 10^7 [\mathrm{S/m}])(2.088 \times 10^{-6} [\mathrm{m}])} \\ &= (8.239 + j\,8.239) \times 10^{-3} [\Omega]\end{aligned}$$

= 저항 + coil 성분

= 평면파 크기 감쇠 발생하고, 복소수이므로 전계(전압), 자계(전류) 위상차 존재

$\ll \eta_0 (= 377\,\Omega)$

④ Phase velocity v_p 는 식 (2-21)으로부터

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\pi f \mu \sigma}} = \omega \frac{1}{\sqrt{\pi f \mu \sigma}} = \omega \delta$$
$$= 2\pi \times 1 \times 10^9 \times 2.088 \times 10^{-6}$$
$$= 1.31 \times 10^4 \, [\mathrm{m/s}]$$
$$\ll c(3 \times 10^8 \, \mathrm{m/s}])$$

⑤ 구리 안테나 내에서 wavelength λ 는

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\pi f \mu \sigma}} = 2\pi\delta$$
$$= 2\,\pi \times 2.088 \times 10^{-6}$$
$$= 1.31 tmes 10^{-5} [\mathrm{m}]$$
$$= 0.131 [\mu\mathrm{m}]$$
$$\ll \text{공기중에서 파장} = 0.1[\mathrm{m}] = 10[\mathrm{cm}]$$

⑥ Reflection coefficient Γ 는

$$\Gamma = \frac{\eta_m - \eta_0}{\eta_m + \eta_0} = \frac{(8.239 + j8.239) - 377}{(8.239 + j8.239) + 377}$$
$$\fallingdotseq -0.957 + j0.042$$
$$= \sqrt{(0.957)^2 + (0.042)^2} \angle \tan\theta = \frac{0.042}{-0.957}$$
$$\fallingdotseq 0.96 \angle 177.5^0$$

report cheng 문 7-8

다음 주파수 (a) $1[\mathrm{MHz}]$와 (b) $1[\mathrm{GHz}]$에서 구리(copper)$[\sigma_{cu}=5.8\times10^{7}[\mathrm{S/m}]]$과 황동(brass)$[1.59\times10^{7}[\mathrm{S/m}]]$의 특성임피던스, 감쇄상수([Np/m]와 [dB] 모두의 단위로), 그리고 표피두께를 구하라.

report cheng 문 7-9

$100[\mathrm{MHz}]$에서 흑연의 표피두께는 $0.16[\mathrm{mm}]$이라 할 때 (a) 흑연의 도전율과 (b) $1[\mathrm{GHz}]$의 파가 흑연내를 전파할 때 그 전계강도가 $30[\mathrm{dB}]$만큼 줄어드는 거리를 계산하라.

2.2 Poynting Vector와 전력

• 평면파는 $E_x \overline{\mathrm{x}} \times H_y \overline{\mathrm{y}} = P\overline{\mathrm{z}}$ 형태, $(= E_x\ H_y \sin 90^0\ \overline{\mathrm{z}}\ )$

즉, P는 평면을 뚫고 나가는 형태이므로

$\nabla \cdot (\mathrm{E}\times\mathrm{H})$를 계산해 보자

$\nabla \cdot (\mathrm{E}\times\mathrm{H}) =$ by vector 등식

$$= \mathrm{H} \cdot (\nabla\times\mathrm{E}) - \mathrm{E} \cdot (\nabla\times\mathrm{H})$$

where,

$$\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial t} = -\mu\frac{\partial \mathrm{H}}{\partial t}$$

$$\nabla \times \mathrm{H} = \mathrm{J} + \frac{\partial \mathrm{D}}{\partial t} = \mathrm{J} + \epsilon\frac{\partial \mathrm{E}}{\partial t} \quad : \textit{source} \text{ 포함} \qquad \nabla\times\mathrm{H} = \mathrm{J}$$

$$= \mathrm{H} \cdot \left(-\mu\frac{\partial \mathrm{H}}{\partial t}\right) - \mathrm{E} \cdot \left(\mathrm{J} + \epsilon\frac{\partial \mathrm{E}}{\partial t}\right)$$

$$= -\mu\mathrm{H} \cdot \frac{\partial \mathrm{H}}{\partial t} - \mathrm{E}\cdot\mathrm{J} - \epsilon\,\mathrm{E} \cdot \frac{\partial \mathrm{E}}{\partial t}$$

where,

$$\begin{aligned}\frac{\partial}{\partial t}(\mu\mathrm{H}\cdot\mathrm{H}) &= \mu\frac{\partial}{\partial t}(\mathrm{H}\cdot\mathrm{H}) \\ &= \mu\left[\frac{\partial \mathrm{H}}{\partial t}\cdot\mathrm{H} + \mathrm{H}\cdot\frac{\partial \mathrm{H}}{\partial t}\right] \\ &= 2\mu\frac{\partial \mathrm{H}}{\partial t}\cdot\mathrm{H}\end{aligned}$$

그러므로 우변 첫 번째 항

$$\therefore \quad \mu\frac{\partial \mathrm{H}}{\partial t}\cdot\mathrm{H} = \frac{1}{2}\frac{\partial}{\partial t}(\mu\mathrm{H}\cdot\mathrm{H}) = \frac{1}{2}\frac{\partial}{\partial t}(\mu\mathrm{H}^2)$$

두 번째 항

$$\begin{aligned}\mathrm{E}\cdot\mathrm{J} &= \mathrm{E}\cdot(\mathrm{J_s}+\mathrm{J_c})\,; \textit{source current} + \textit{conduction current} \\ &= \mathrm{E}\cdot(\mathrm{J_s}+\sigma\mathrm{E}) \\ &= \mathrm{E}\cdot\mathrm{J_s}+\sigma\mathrm{E}^2\end{aligned}$$

세 번째 항

$$\therefore \quad \epsilon\frac{\partial \mathrm{E}}{\partial t}\cdot\mathrm{E} = \frac{1}{2}\frac{\partial}{\partial t}(\epsilon\,\mathrm{E}^2)$$

$$\therefore \quad \nabla\cdot(\mathrm{E}\times\mathrm{H}) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon\,\mathrm{E}^2 + \frac{1}{2}\mu\mathrm{H}^2\right) - \mathrm{E}\cdot\mathrm{J_s} - \sigma\mathrm{E}^2 \qquad (2\text{-}48)$$

식 (2-48) 양변을 체적 적분하면

$$\int \nabla \cdot (\mathrm{E} \times \mathrm{H})\, dv = -\frac{\partial}{\partial t}\int \left(\frac{1}{2}\epsilon\, \mathrm{E}^2 + \frac{1}{2}\mu\, \mathrm{H}^2\right) dv - \int \mathrm{E} \cdot \mathrm{J}_s\, dv - \int \sigma\, \mathrm{E}^2 dv$$

by Divergence

$$\int (\mathrm{E} \times \mathrm{H}) \cdot d\mathrm{S} = -\frac{\partial}{\partial t}\int \left(\frac{1}{2}\epsilon\, E^2 + \frac{1}{2}\mu\, H^2\right) dv - \int \mathrm{E} \cdot \mathrm{J}_s\, dv - \int \sigma\, \mathrm{E}^2 dv$$

V 체적 밖으로 송출되는 전력	V 체적내에 축적된 E, H 에너지의 시간 변화율	전원전류 J_s 에 의한 공급전력	Joule 열 손실

$$\therefore\quad -\int \mathrm{E} \cdot \mathrm{J}_s\, dv = \frac{\partial}{\partial t}\int \left(\frac{1}{2}\epsilon\, E^2 + \frac{1}{2}\mu\, H^2\right) dv + \int \sigma\, \mathrm{E}^2 dv + \int (\mathrm{E} \times \mathrm{H}) \cdot d\mathrm{S} \qquad (2\text{-}49)$$

즉, 식 (2-49)는 Closed Surface 내에 source가 있는 경우임.

• Source가 Closed surface 밖에 있으면

$$\int (\mathrm{E} \times \mathrm{H}) \cdot d\mathrm{S} = -\frac{\partial}{\partial t}\int \left(\frac{1}{2}\epsilon\, E^2 + \frac{1}{2}\mu\, H^2\right) dv - \int \sigma\, \mathrm{E}^2 dv$$

where,

$$\begin{aligned} \mathrm{E} \times \mathrm{H} &= \mathrm{P}\ [\mathrm{W/m^2}], \mathrm{S} \\ &= \textit{J.H. Poynting} \\ &= \textit{Poynting Vector} \\ &= \textit{Poynting Theorem} \end{aligned}$$

$$\int \mathrm{P} \cdot d\mathrm{S} = -\frac{\partial}{\partial t}\int \left(\frac{1}{2}\epsilon\, E^2 + \frac{1}{2}\mu\, H^2\right) dv - \int \sigma\, \mathrm{E}^2 dv$$

식 (2-49)에서

$\mathrm{E},\ \mathrm{H},\ \mathrm{J}_s = \textit{Sinusoidal signals} \Rightarrow \textit{Phasor}$(1-4절 참조)

식 (2-49)는

$$-\int \mathrm{E} \cdot \mathrm{J}_s^{\,*} dv = \frac{1}{2} j\omega \int (\epsilon \mathrm{E} \cdot \mathrm{E}^* + \mu\, \mathrm{H} \cdot \mathrm{H}^*) dv + \int \sigma\, \mathrm{E} \cdot \mathrm{E}^* dv + \int (\mathrm{E} \times \mathrm{H}^*) \cdot d\mathrm{S}$$

또한 시간 평균값이 필요하므로

$$\begin{aligned} -\frac{1}{2}\int \mathrm{E} \cdot \mathrm{J}_s^{\,*} dv = {} & \frac{1}{2} j\omega \int \left(\frac{1}{2}\epsilon \mathrm{E} \cdot \mathrm{E}^* + \frac{1}{2}\mu\, \mathrm{H} \cdot \mathrm{H}^*\right) dv \\ & + \frac{1}{2}\int \sigma\, \mathrm{E} \cdot \mathrm{E}^* dv + \frac{1}{2}\int (\mathrm{E} \times \mathrm{H}^*) \cdot d\mathrm{S} \end{aligned} \qquad (2\text{-}51)$$

where,

$$-\frac{1}{2}\int \mathrm{E} \cdot \mathrm{J_s}^{*} dv = P_s$$: 전원 전력

$$\frac{\epsilon}{4}\int \mathrm{E} \cdot \mathrm{E}^{*}\, dv = W_e$$: 전계 energy

$$\frac{\mu}{4}\int \mathrm{H} \cdot \mathrm{H}^{*}\, dv = W_m$$: 자계 energy

$$\frac{\sigma}{2}\int \mathrm{E} \cdot \mathrm{E}^{*}\, dv = P_{loss}$$: 저항성 전력

$$\frac{1}{2}\int \mathrm{E} \times \mathrm{H}^{*} \cdot d\mathrm{S} = P_0$$: 이동 전력(by wave)

∴ 식 (2-51)

$$P_s = 2j\omega(W_m + W_e) + P_{loss} + P_0 \tag{2-53}$$