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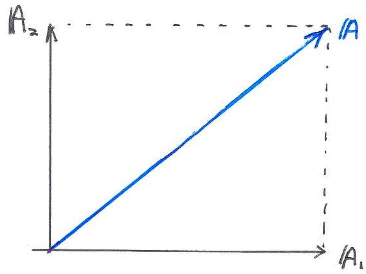
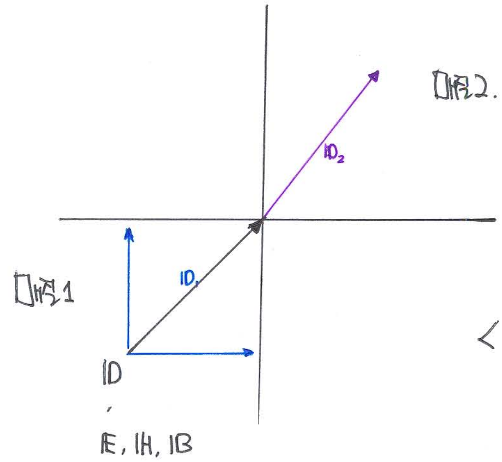
1.6 경계 조건.




$\langle \mathbb{D}_1 = \mathbb{D}_2 \rangle$

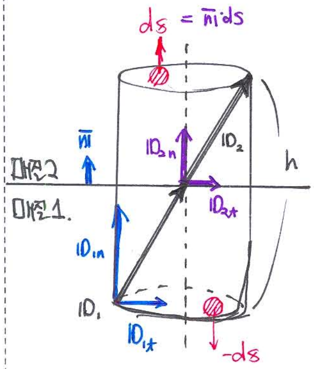
"[접선] 성분과 법선 성분이 같다"

## (1) 법선 성분.

* 전속밀도 ( $\mathbb{D}$ )의 경계조건.

$$\underbrace{\text{Maxwell's Eq} \quad (1-17.b)}$$

$$\nabla \cdot \mathbb{D} = \rho$$




위식 양변에 체적 적분.

$$\int \nabla \cdot \mathbb{D} \, dV = \int \rho \, dV$$

$$\Big\downarrow \text{ 발산정리.}$$

$$\int \mathbb{D} \cdot d\mathbf{s} = \int \rho \, dV = Q$$

$$\oint \mathbb{D} \cdot d\mathbf{s} = \int_{\underset{\text{윗면}}{us}} \mathbb{D} \cdot d\mathbf{s}_{\text{윗면}} + \int_{\underset{\text{사이드면}}{ss}} \mathbb{D} \cdot d\mathbf{s}_{\text{사이드}} + \int_{\underset{\text{아랫면}}{Ls}} \mathbb{D} \cdot d\mathbf{s}_{\text{아래}}$$

where, 경계면에서 계산 해야 하므로 $h \rightarrow 0$ 으로 수렴할 때,

$$h \rightarrow 0 \quad \int_{ss} \mathbb{D} \cdot d\mathbf{s} = 0$$

$$\therefore \oint \mathbb{D} \cdot d\mathbf{s} = \int_{us} \mathbb{D} \cdot d\mathbf{s}_2 + \int_{Ls} \mathbb{D} \cdot d\mathbf{s}_1$$

$$= \int_{\substack{us \\ \text{윗}}} \mathbb{D} \cdot \bar{n}\, ds_2 - \int_{\substack{ls \\ \text{아랫}}} \mathbb{D} \cdot \bar{n}\, ds_1$$

$$= \mathbb{D}_2 (\bar{n} \cdot \Delta s) - \mathbb{D}_1 (\bar{n} \cdot \Delta s) = Q$$

where, $Q = \Delta s \cdot \rho_s + \frac{1}{2}(\rho_1 + \rho_2)\,\Delta s \cdot \underbrace{\Delta h}_{=0\ (\text{극한})}$

$$\therefore \quad \bar{n} \cdot (\mathbb{D}_2 - \mathbb{D}_1) = \underline{\rho_s}$$

면전하

or $\underbrace{\text{“}D_2 - D_1 = \rho_s\text{”}}$

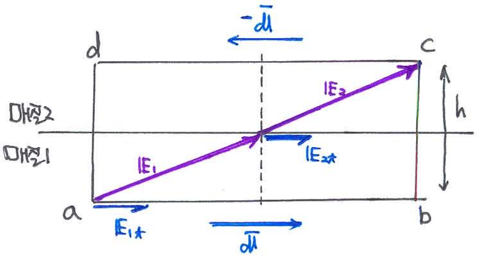
$$\bar{n} \cdot (\varepsilon_2 \mathbb{E}_2 - \varepsilon_1 \mathbb{E}_1) = \rho_s$$

$$\begin{cases} \mathbb{D} = \varepsilon \mathbb{E} \\ \mathbb{E} = \mathbb{D}/\varepsilon \end{cases}$$

* $\nabla \cdot \mathbb{B} = 0$ , $\bar{n} \cdot (\mathbb{B}_2 - \mathbb{B}_1) = 0$

$$\bar{n} \cdot (\mu_2 \mathbb{H}_2 - \mu_1 \mathbb{H}_1) = 0$$

(2) 접선 성분.




$$\oint \mathbb{E}\cdot\overline{dl} = 0$$

$$\begin{cases} \nabla\times\mathbb{E} = -\dfrac{\partial\mathbb{B}}{\partial t} \\ \nabla\times\mathbb{E} = -j\omega\mu\mathbb{H} \end{cases}$$

양면적

$$\int \mathbb{E}\cdot\overline{dl} = -\frac{\partial}{\partial t}\int \mathbb{B}\cdot ds$$

좌변 stoke 정리.

$$\oint \mathbb{E}\cdot\overline{dl} = -j\omega\mu\int \mathbb{H}\cdot ds$$

$$\oint \mathbb{E} \cdot \bar{dl} = \int_a^b \mathbb{E} \cdot \bar{dl} + \int_b^c \sim + \int_c^d \sim + \int_d^a \sim$$

$$= \int_a^b \mathbb{E} \cdot \bar{dl} - \int_c^d \mathbb{E} \cdot \bar{dl}$$

$$= \int_a^b \mathbb{E}_1 \cdot \bar{dl} - \int_c^d \mathbb{E}_2 \cdot \bar{dl}$$

$$= \int_a^b \mathbb{E}_{1t} \cdot dl - \int_c^d \mathbb{E}_{2t} \cdot dl$$

$$= E_{1t} \Delta l - E_{2t} \Delta l = 0$$

$$\therefore \; E_{2t} = E_{1t} \qquad \text{or} \qquad (\mathbb{E}_2 - \mathbb{E}_1) \times \bar{n} = 0$$

$$\underbrace{\bar{n} \times (\mathbb{E}_2 - \mathbb{E}_1) = 0}_{\text{접선성분.}}$$

$$\downarrow$$

$$\bar{n} \times \left( \frac{\mathbb{D}_2}{\varepsilon_2} - \frac{\mathbb{D}_1}{\varepsilon_1} \right) = 0$$

자계. $\quad \bar{n} \times (\mathbb{H}_2 - \mathbb{H}_1) = \mathbb{J}_s$

(1-33 식)

$$\begin{cases} \mathbf{n} \cdot \mathbf{D}_1 = \mathbf{n} \cdot \mathbf{D}_2 \\ \mathbf{n} \cdot \mathbf{B}_1 = \mathbf{n} \cdot \mathbf{B}_2 \\ \mathbf{n} \times \mathbf{E}_1 = \mathbf{n} \times \mathbf{E}_2 \\ \mathbf{n} \times \mathbf{H}_1 = \mathbf{n} \times \mathbf{H}_2 \end{cases}$$

경계조건들은 $\mathbf{D} = \varepsilon \mathbf{E}$ 와 $\mathbf{B} = \mu \mathbf{H}$ 를 사용하여 구할 수 있다.

$\mathbf{D}$ 와 $\mathbf{B}$ 의 법선 성분은 연속이고, $\mathbf{E}$ 와 $\mathbf{H}$ 의 접선 성분도 연속이다.

이 밖의 성분들은 불연속이다.

## (4) 완전도체의 경계조건.

무손실 조건

- Static 경우. (정적)
- Time Varies 경우, (시변)

※ Static 경우,

$$
\begin{cases}
\bar{n} \cdot D_2 = \rho_s \\
\bar{n} \cdot B_1 = \bar{n} \cdot B_2 \\
\bar{n} \times E_2 = 0 \\
\bar{n} \times H_1 = \bar{n} \times H_2
\end{cases}
$$

※ time Varies 경우.

$$
\begin{cases}
\bar{n} \cdot D_2 = \rho_s \\
\bar{n} \cdot B_2 = 0 \\
\bar{n} \times E_2 = 0 \\
\bar{n} \times H_2 = J_s
\end{cases}
$$

081104 Tu.

# 2장 파동방정식과 평면파의 전파.

## 2-1. 파동방정식.

: 손실, 무손실 매질에서 파동방정식을 살피고,

손실, 무손실 매질에서 평면파에 대해 고찰.

- 먼저 $\underbrace{\text{Source 로부터 충분히 떨어지고}}_{\rho=0}$

손실이 있는 ($\sigma \neq 0$ 인 $\underbrace{\text{단순매질}}_{\text{선형성, 등방성}}$) 에서

Maxwell's eq (식 2-1, 2-2)

$$\begin{cases} \nabla \times \mathbb{E} = -\dfrac{\partial \mathbb{B}}{\partial t} = -\mu \dfrac{\partial \mathbb{H}}{\partial t} & -(식\ 2\text{-}1) \\ \nabla \times \mathbb{H} = \mathbb{J} + \dfrac{\partial \mathbb{D}}{\partial t} = \sigma \mathbb{E} + \varepsilon \dfrac{\partial \mathbb{E}}{\partial t} & -(식\ 2\text{-}2) \end{cases}$$

(식 2-1) 에 양변에 Curl 을 취하면.

$$\nabla \times \nabla \times \mathbb{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \mathbb{H}) \qquad -(식\ 2\text{-}3)$$

(식2-2)의 우변을 대입.

$$\nabla\times\nabla\times\mathbb{E} = -\mu\frac{\partial}{\partial t}\left(\sigma\mathbb{E} + \varepsilon\frac{\partial\mathbb{E}}{\partial t}\right)$$

$$= -\mu\sigma\frac{\partial\mathbb{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\mathbb{E}}{\partial t^2} \quad \text{-(식2-4)}$$

Where, $\nabla\times\nabla\times\mathbb{E} = \nabla\cdot(\nabla\cdot\mathbb{E}) - \nabla^2\mathbb{E}$

(where, $\nabla\cdot\mathbb{E} = \rho/\varepsilon = 0$)

$$\nabla^2\mathbb{E} = \mu\sigma\frac{\partial\mathbb{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\mathbb{E}}{\partial t^2} \quad \text{(식2-5a)}$$

동일한 방법에 IH에 대해.

$$\nabla\times\nabla\times\mathbb{H} = \sigma(\nabla\times\mathbb{E}) + \varepsilon\frac{\partial}{\partial t}(\nabla\times\mathbb{E})$$

$$= \left(\sigma + \varepsilon\frac{\partial}{\partial t}\right)(\nabla\times\mathbb{E})$$

$$= \left(\sigma + \varepsilon\,\partial/\partial t\right)\left(-\mu\frac{\partial\mathbb{H}}{\partial t}\right)$$

$$= -\mu\sigma\frac{\partial\mathbb{H}}{\partial t} - \mu\varepsilon\,\partial^2\mathbb{H}/\partial t^2$$

Where, $\nabla\times\nabla\times\mathbb{H} = \nabla(\nabla\cdot\mathbb{H}) - \nabla^2\mathbb{H}$

($\nabla\cdot\mathbb{H} = \nabla\cdot(\mathbb{B}/\mu) = 0$)

$$\therefore \quad \nabla^2 \mathbb{H} = \mu\sigma \frac{\partial \mathbb{H}}{\partial t} + \mu\varepsilon \frac{\partial^2 \mathbb{H}}{\partial t^2} \qquad (식 2-5 b)$$

순시식.

(식 2-5 a)를 phasor로 표현하면,

$$\nabla^2 \mathbb{E} = j\omega\mu\sigma \,\mathbb{E} - \omega^2\mu\varepsilon \,\mathbb{E}$$

$$= (j\omega\mu\sigma - \omega^2\mu\varepsilon)\,\mathbb{E}$$

$$\therefore \quad \nabla^2 \mathbb{E} - (j\omega\mu\sigma - \omega^2\mu\varepsilon)\,\mathbb{E} = 0$$

$$\begin{cases} \nabla^2 \mathbb{E} + \omega^2\mu\varepsilon \left(1 - j\sigma/\omega\varepsilon\right)\mathbb{E} = 0 \\ \nabla^2 \mathbb{E} + \omega^2\mu\varepsilon \underbrace{\left(1 - \frac{\sigma}{j\omega\varepsilon}\right)}_{\varepsilon_c}\mathbb{E} = 0 \end{cases}$$

$$\begin{cases} \nabla^2 \mathbb{E} + \omega^2\mu\,\varepsilon_c\,\mathbb{E} = 0 \\ \nabla^2 \mathbb{E} + k_c^2\,\mathbb{E} = 0 \qquad (식 2-6) \end{cases}$$

where, $\varepsilon_c = \varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)$ (*복소유전율)

$$k_c^2 = \omega^2\mu\varepsilon - j\omega\mu\sigma \qquad -(식 2-7)$$

$$= \omega^2\mu\varepsilon_c$$

(식 2-6) "Wave Equation"

= Helmholtz's Eq.

= Homogenous Electromagnetic Wave Eq.

= 동차 방정식

(식 2-7) : $k_c$ = $\sigma \neq 0$ 인 매질에서

⇒ wave number (파수)

⇒ propagation constant

※ 같은 방법을 이용하여 유도해보세요.

$$\nabla^2 \mathbb{H} + \underbrace{k_c^2}_{\gamma} \mathbb{H} = 0 \qquad (식\ 2\text{-}8)$$

$\sigma \neq 0$ 인 매질에서 wave number

- 무손실 $(\sigma=0)$인 단순매질에서

$$\therefore \nabla^2 \mathbb{E} + k^2 \mathbb{E} = 0 \qquad (식\ 2\text{-}9a)$$

$$\nabla^2 \mathbb{H} + k^2 \mathbb{H} = 0 \qquad (식\ 2\text{-}9b)$$

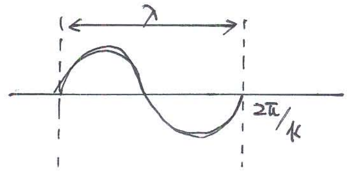
where, $k^2 = \omega^2 \mu\varepsilon$

$$k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{v} = \frac{2\pi f}{f\lambda} = \frac{2\pi}{\lambda}$$

wave number

= propagation constant

= 위상정수 (무손실)




cf) $\gamma = \sqrt{ZY}$

전송선로. $= \underbrace{\alpha}_{\text{"0"}} + j\beta$ ($\alpha$ : 감쇠. $\beta$ : 위상)

$= j\beta = j\beta\sqrt{LC}$ , $\beta = 2\pi/\lambda$

전자파 $\beta = 2\pi/\lambda = \omega\sqrt{LC}$

[I] 무손실 매질에서 평면파.

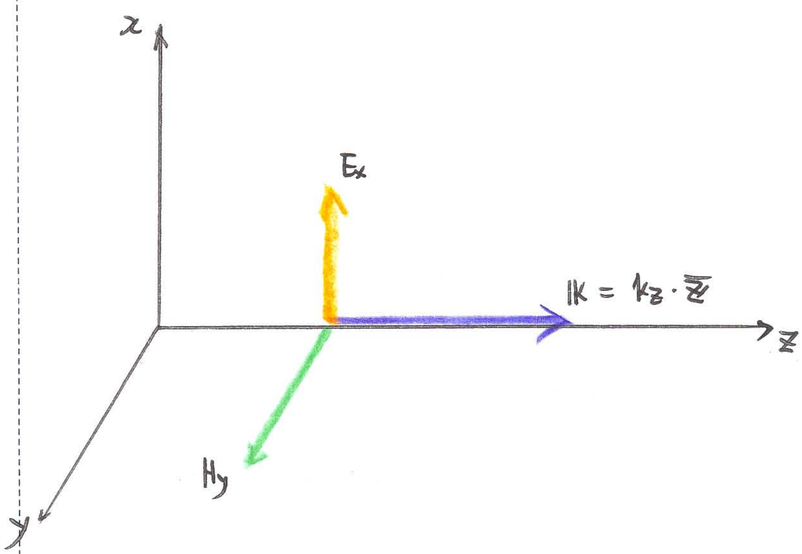
식 (2-9a)를 직각 좌표계에서 표현

$$\frac{\partial^2 \mathbb{E}}{\partial x^2} + \frac{\partial^2 \mathbb{E}}{\partial y^2} + \frac{\partial^2 \mathbb{E}}{\partial z^2} + k^2 \mathbb{E} = 0$$

$$\mathbb{E} = E_x \bar{x} + E_y \bar{y} + E_z \bar{z}$$

$$\begin{cases} \dfrac{\partial^2 E_x}{\partial x^2} + \dfrac{\partial^2 E_x}{\partial y^2} + \dfrac{\partial^2 E_x}{\partial z^2} + k^2 E_x = 0 & \text{(식 2-10a)} \\[2ex] \dfrac{\partial^2 E_y}{\partial x^2} + \dfrac{\partial^2 E_y}{\partial y^2} + \dfrac{\partial^2 E_y}{\partial z^2} + k^2 E_y = 0 & \text{(식 2-10b)} \\[2ex] \dfrac{\partial^2 E_z}{\partial x^2} + \dfrac{\partial^2 E_z}{\partial y^2} + \dfrac{\partial^2 E_z}{\partial z^2} + k^2 E_z = 0 & \text{(식 2-10c)} \end{cases}$$

우선 $E_x$ 만 고려.




$$\begin{cases} E_x \neq 0 \neq H_y \\ E_y = E_z = 0 = H_x = H_z \end{cases}$$

$$\left( \frac{\partial E_x}{\partial x} = \frac{\partial E_x}{\partial y} = \frac{\partial H_y}{\partial x} = \frac{\partial H_y}{\partial y} = 0 \right)_{z \text{ 방향.}}$$

( 식 2-10a )

$$\begin{cases} \dfrac{\partial^2 E_x}{\partial z^2} = -k^2 E_x = \underbrace{-\omega^2 \mu \varepsilon E_x}_{\text{phasor}} = \mu\varepsilon \dfrac{\partial^2 E_x}{\partial t^2} \\ \dfrac{\partial^2 H_y}{\partial z^2} = -k^2 H_y = -\omega^2 \mu \varepsilon H_y = \mu\varepsilon \dfrac{\partial^2 H_y}{\partial t^2} \end{cases}$$

$$\therefore \quad \partial^2 E_x / \partial z^2 + k^2 E_x = 0 \qquad (2\text{-}13a)$$

$$\partial^2 H_y / \partial z^2 + k^2 H_y = 0 \qquad (2\text{-}13b)$$

where, $k_z$ : $z$ 방향 wave number

$$d^2 V(z) / dz^2 - \gamma^2 V(z) = 0$$

$$\therefore \begin{cases} E_x(z) = E_{xo}\, e^{-k_z \cdot z} \\ H_y(z) = H_{yo}\, e^{-k_z \cdot z} \end{cases}$$

$+z$ 방향만 고려.

$$\therefore \begin{cases} \mathbb{E}(z) = E_{xo}\, e^{-jk_z \cdot z}\, \bar{x} \\ \mathbb{H}(z) = H_{yo}\, e^{-jk_z \cdot z}\, \bar{y} \end{cases} \qquad (2\text{-}14\ 식)$$

※ $\mathbb{E} = E_x \bar{x} + E_y \bar{y} + E_z \bar{z}$

where, $H_{y0}$ 를 $E_{x0}$ 로 대치하면,

by
$$\begin{cases} \nabla \times \mathbb{E} = -j\omega\mu \mathbb{H} \\ \nabla \times \mathbb{E}_x = -j\omega\mu \mathbb{H}_y \end{cases}$$

$$\mathbb{H}_y = \frac{1}{-j\omega\mu} \cdot \nabla \times \mathbb{E}_x$$

$$= \frac{1}{-j\omega\mu} \begin{vmatrix} \bar{x} & \bar{y} & \bar{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix}$$

where, $E_x = E_{x0}\, e^{-jk_z \cdot z}$

$$= \frac{1}{-j\omega\mu} \left[ \frac{\partial}{\partial z} \left( E_{x0}\, e^{-jk_z \cdot z} \right) \bar{y} - \frac{\partial}{\partial y} \left( E_{x0}\, e^{-jk_z \cdot z} \right) \bar{z} \right]$$

$$= -\frac{1}{j\omega\mu} (-jk_z)\, E_{x0}\, e^{-jk_z \cdot z}\, \bar{y}$$

$k = \sqrt{\varepsilon\mu}$

$$= \frac{\omega\sqrt{\varepsilon\mu}}{\omega\mu} \cdot E_{x0}\, e^{-jk_z \cdot z}\, \bar{y}$$

$$= \sqrt{\frac{\varepsilon}{\mu}}\, E_{x0} \cdot e^{-jk_z \cdot z}\, \bar{y}$$

( where, $k_z = \omega\sqrt{\varepsilon\mu}$

$\eta = \sqrt{\mu/\varepsilon}$ 고유임피던스. )

$$\therefore \ \mathbb{H}_y = \frac{1}{\eta} E_{x0}\, e^{-jk_z \cdot z} \cdot \bar{y} \qquad (\text{식 } 2\text{-}15)$$

where. 무손실 매질 에서

= 자유공간 $(\rho = 0)$

$$\mathbb{H}_y(z) = {}^{1}\!/_{\eta_0} \cdot E_{x0} \cdot e^{-jk_z \cdot z} \cdot \bar{y}$$

where, $\eta_0 = \sqrt{\mu_0 / \varepsilon_0} \doteqdot 377\ \Omega$

= 자유공간에서의 임피던스.

- 순시값을 보면,

$$\mathbb{E}(z,t) = Re\left[ E_{x0} \cdot e^{-jk_z \cdot z} \cdot e^{j\omega t} \cdot \bar{x} \right]$$

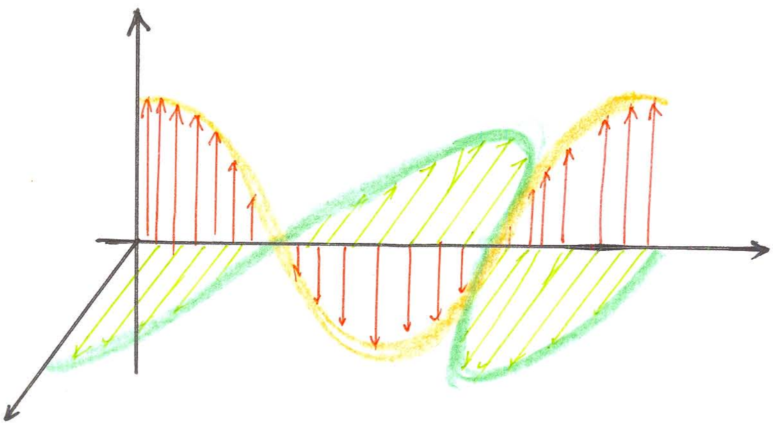
$$= E_{x0} \cos(\omega t - k_z \cdot z)\, \bar{x}$$

where, $E_{x0} = |E_{x0}| e^{j\theta_x}$

$$= |E_{x0}| \cos(\omega t - k_z \cdot z + \theta_x)\, \bar{x} \quad (식2\text{-}16)$$

또한

$$\mathbb{H}(z,t) = \frac{|E_{x0}|}{\eta} \cos(\omega t - k_z \cdot z + \theta_x)\, \bar{y} \quad (식2\text{-}17)$$



이제 "$-z$" 방향 까지 고려,

$$\begin{cases} \mathbb{E}_x = \left( E_o^+ e^{-jk_z \cdot z} + E_o^- e^{+jk_z \cdot z} \right) \bar{x} & (\text{식}2\text{-}18) \\ \mathbb{H}_y = \frac{1}{\eta} \left( E_o^+ e^{-jk_z \cdot z} + E_o^- e^{+jk_z \cdot z} \right) \bar{y} & (\text{식}2\text{-}19) \end{cases}$$

순시식 $\theta = 0$ 경우,

$$\mathbb{E}(z,t) = \Big[ \underbrace{E_o^+ \cos(\omega t - k_z \cdot z)}_{+z \text{ 방향}} + \underbrace{E_o^- \cos(\omega t + k_z \cdot z)}_{-z \text{ 방향}} \Big] \bar{x}$$

* wave 들이 일정한 점을 유지하기 위해서 시간에 따라 $\pm z$ 방향으로 이동해야 함.
곧, 위상 = 일정 $\overline{\omega}$ = Constant

즉, "$\omega t - k_z \cdot z = C$" 양변을 시간 $t$에 미분.

$$\frac{d}{dt}(\omega t - kz) = 0 \quad , \quad \omega = k \cdot \frac{dz}{dt}$$

$$V_p = dz/dt = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\varepsilon\mu}} = 1/\sqrt{\varepsilon\mu}$$

위상속도.

$$\therefore V_p = \omega/k_z = 1/\sqrt{\varepsilon_o\mu_o}$$

= phasor Velocity

$$= 2.988 \times 10^8 \ [m/sec]$$

$$(\omega t - k \cdot z) - (\omega t - k(z + \lambda)) = 2\pi$$

$$\therefore \lambda = 2\pi/k_z = 2\pi/\omega\sqrt{\varepsilon_o\mu_o}$$

$$= \frac{2\pi \cdot V_p}{\omega} = V_p/f$$

(식 2-21)