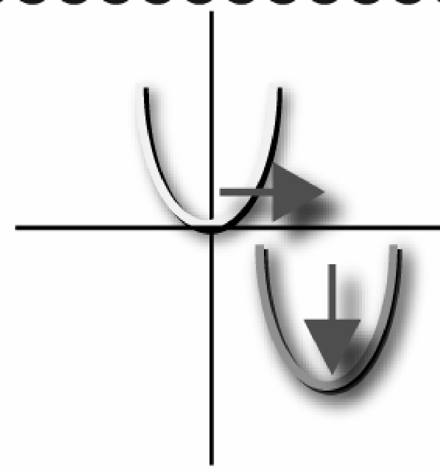
Principles of Mathematics 12

TRANSFORMATIONS



LESSON 1:

BASIC TRANSFORMATIONS

Principles of Math 12

EXPLAINED!

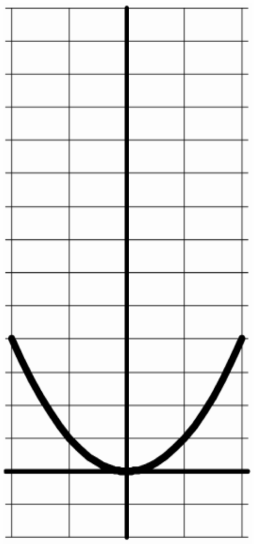
By Barry Mabillard

Transformations Lesson 1

Part I: Vertical Stretches

Vertical Stretches: **A vertical stretch is represented by the form $y = af(x)$, where a is the vertical stretch factor.**

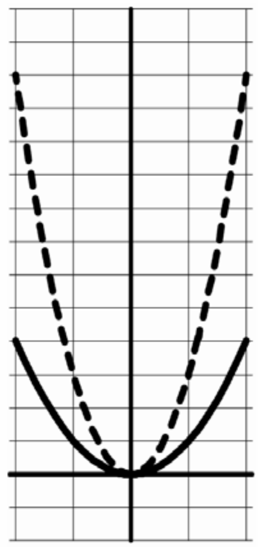
Example 1: Stretch the following graph vertically about the x-axis by a factor of 3



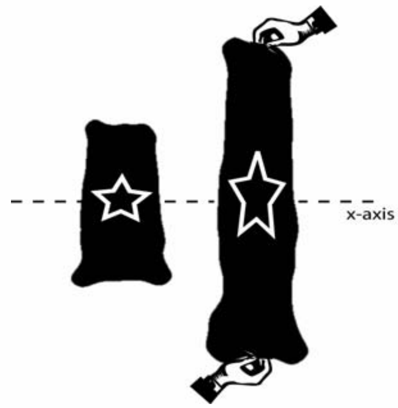
The transformation is applied by multiplying all the y-values by 3.

Since all the y-values are now higher, this has the effect of "stretching" the graph up vertically.

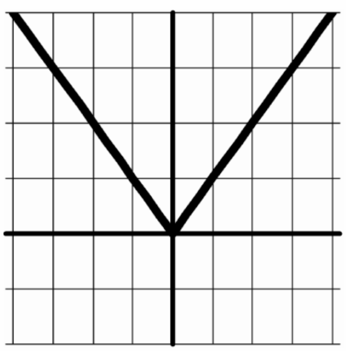
The x-intercepts will not move in a vertical stretch about the x-axis.
They are called the *invariant points.*



The phrase *"about the x-axis"* means the graph will be stretched such that the centre is the x-axis. It's the same idea as taking a stretchy cloth and then pulling it with both hands in opposite directions.

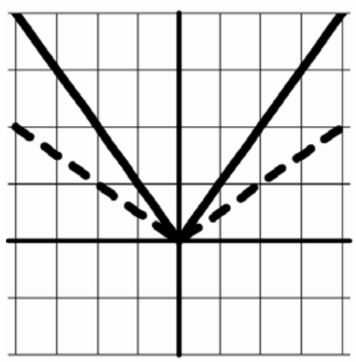


Example 2: Stretch the following graph vertically about the x-axis by a factor of $\frac{1}{2}$



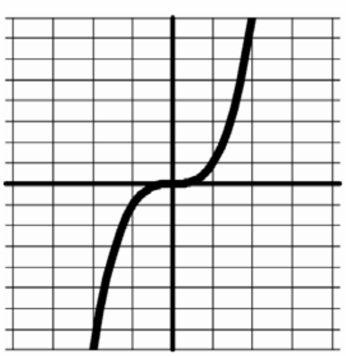
The transformation is applied by multiplying all the y-values by ½.

This has the effect of "squishing" the graph down vertically.



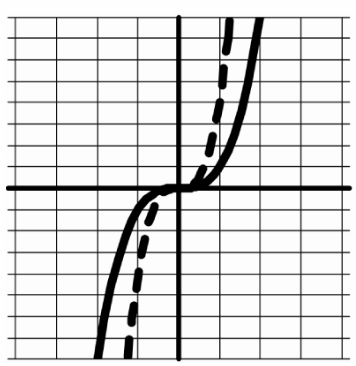
Solid = Original
Dashed = Transformed

Example 3: Draw the graph of $y = x^3$ and then vertically stretch it about the x-axis by a factor of 4.



The transformation is applied by multiplying all the y-values by 4.

This has the effect of stretching the graph vertically.



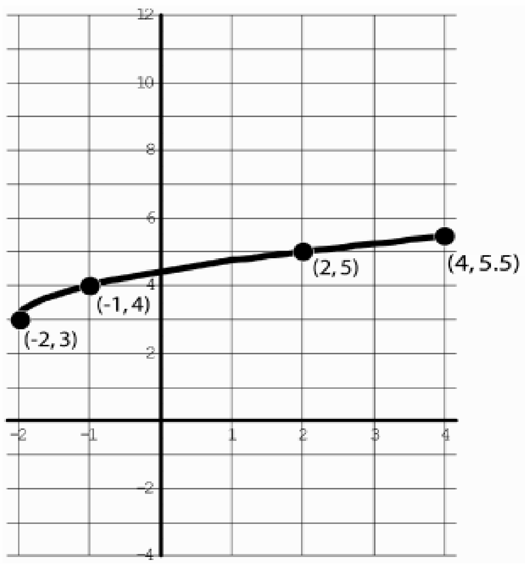
Solid = Original
Dashed = Transformed

TRANSFORMATIONS LESSON 1

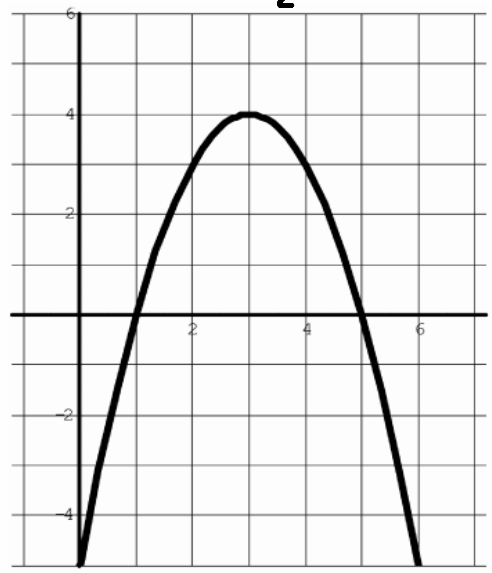
PART I: VERTICAL STRETCHES

Questions: **For each of the following graphs, draw in the vertical stretch.**

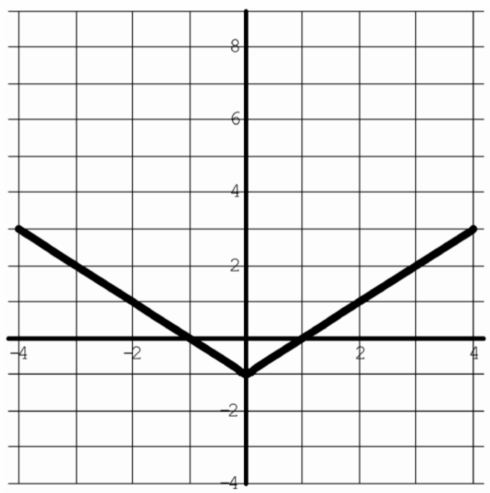
1) $y = 2f(x)$



2) $y = \frac{1}{2}f(x)$



3) $y = 3f(x)$

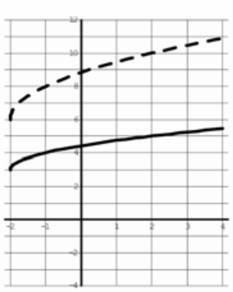


Answers:

Solid = Original

Dashed = Transformed

1.



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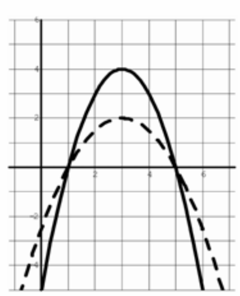
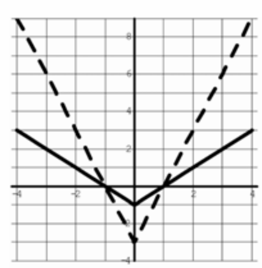
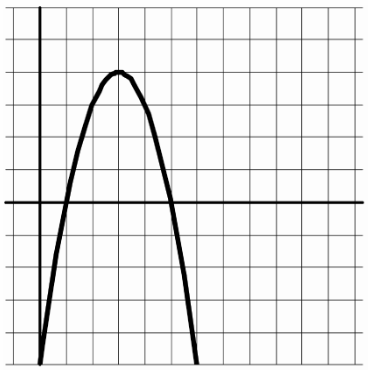


3.



Horizontal Stretches: **A horizontal stretch is represented by the form y = f(bx), where the *reciprocal* of b is the stretch factor.**

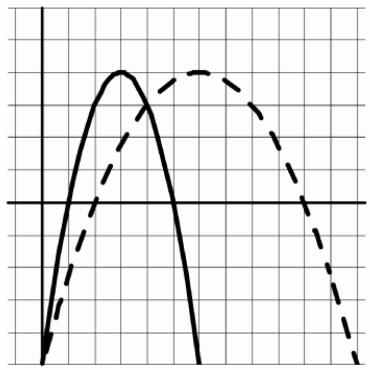
Example 1: Apply $f\left(\frac{1}{2}x\right)$ to the graph.



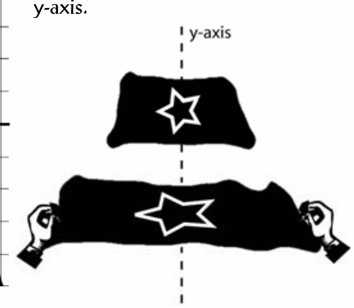
The b-value of ½ is **not** the stretch factor!

The stretch factor is the ***reciprocal*** of the b-value.

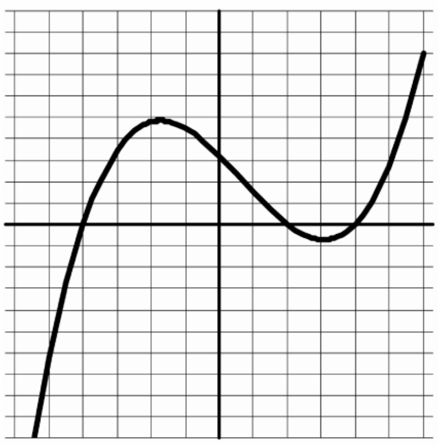
You will multiply all the x-values by **2** in order to transform the graph.



The phrase *"about the y-axis"* means the graph will be stretched horizontally such that the centre is the y-axis.



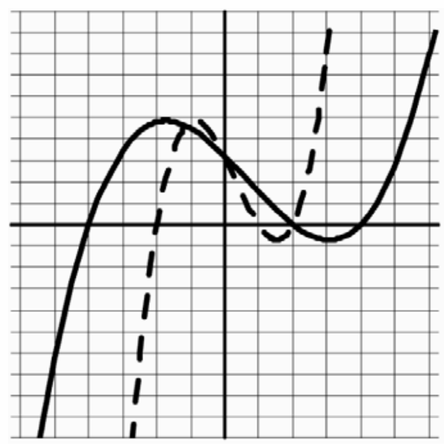
Example 2: Stretch the graph horizontally about the y-axis by a factor of 1/2



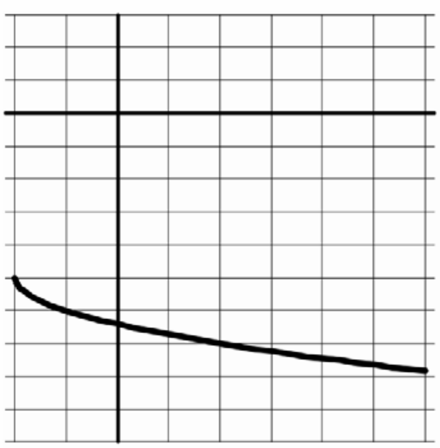
***Important Note:** When the horizontal stretch factor is given to you in a sentence, you can apply it to the graph *without* taking the reciprocal.

You only use a reciprocal when reading the stretch factor from an equation such as y = f(bx)

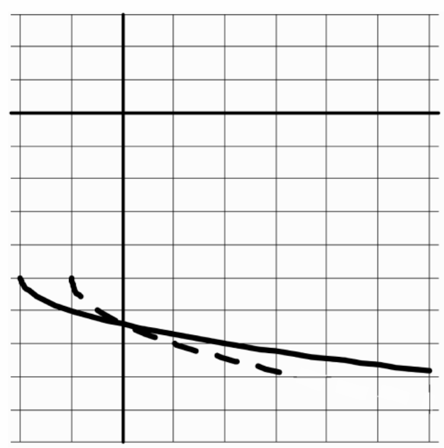
The y-intercepts do not change in a horizontal stretch about the y-axis. They are the *invariant points*.



Example 3: Apply f(2x) to the given graph.



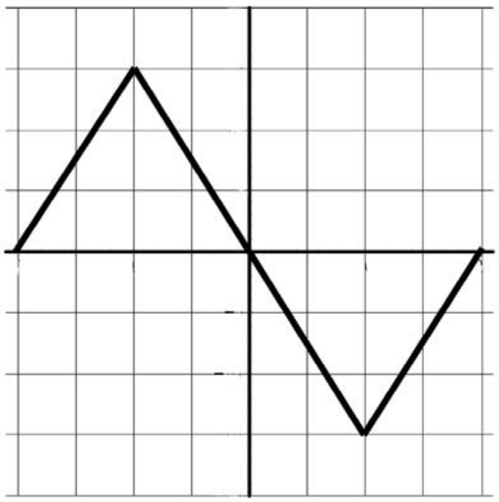
To transform the graph, multiply all the x-values by $\frac{1}{2}$



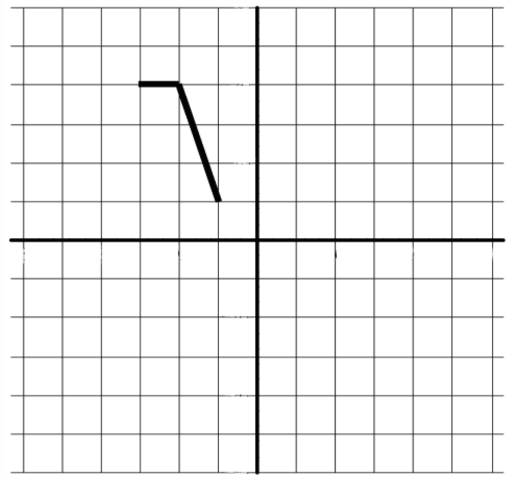
TRANSFORMATIONS LESSON 1
PART II: HORIZONTAL STRETCHES

Questions: **For each of the following graphs, draw in the horizontal stretch.**

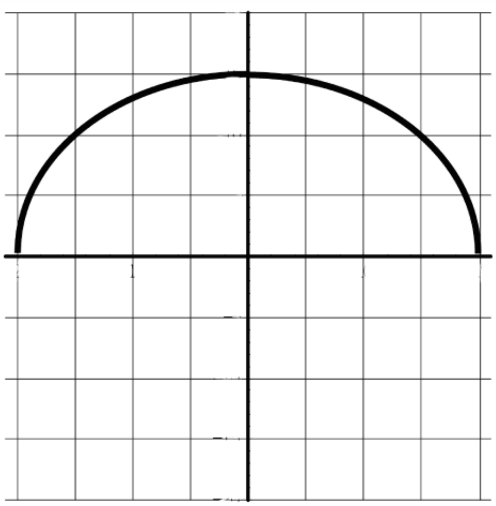
1) $y = f(2x)$



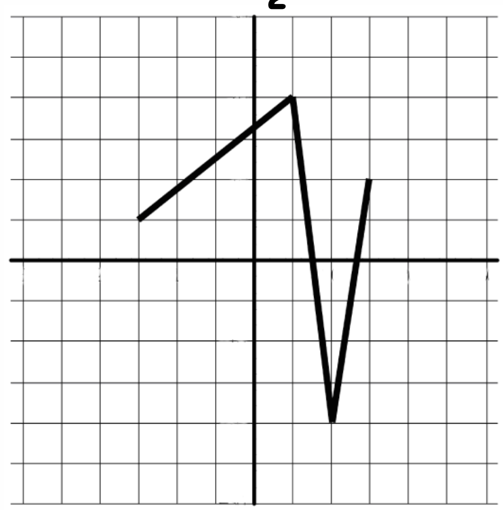
2) $y = f(\frac{1}{2}x)$



3) $y = f(4x)$

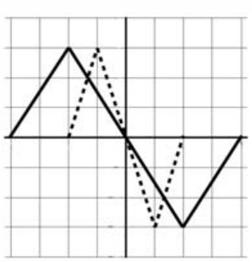


4) $y = f(\frac{1}{2}x)$



Answers:

1.



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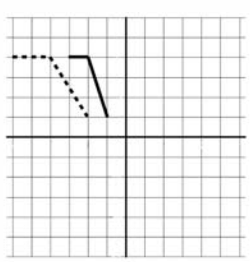

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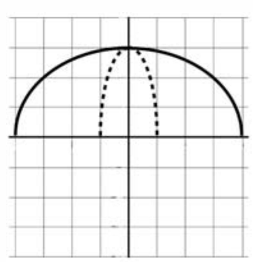

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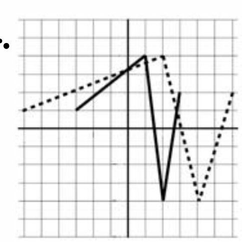
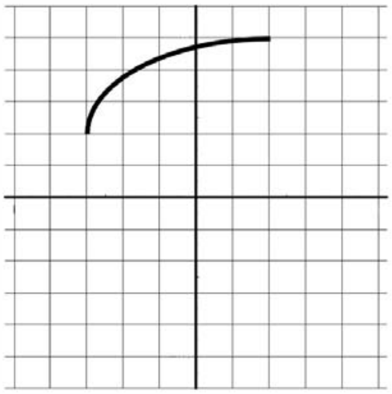


Part III: Vertical Reflections

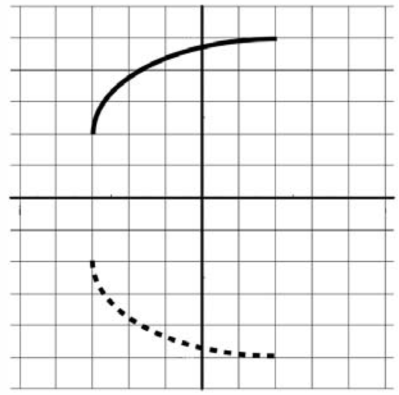
Vertical Reflections: **A vertical reflection *(about the x-axis)* is represented by the form y = -f(x)**

Example 1: Draw y = -f(x) for the following graph



A vertical reflection is done by changing the signs of all y-values. This will reflect the graph over the x-axis.

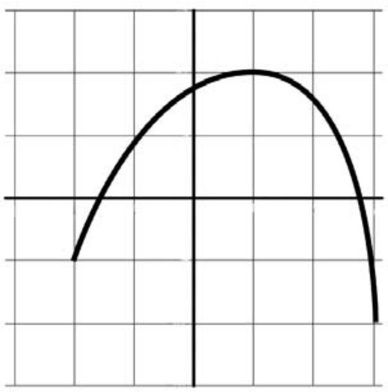
In a vertical reflection (about the x-axis), the x-intercepts are the invariant points.

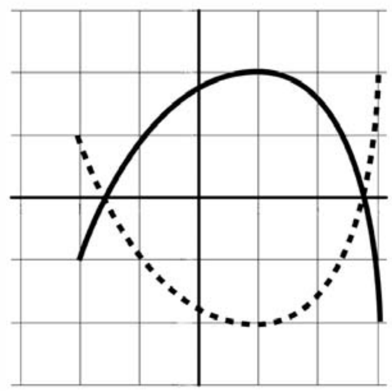


Solid = Original

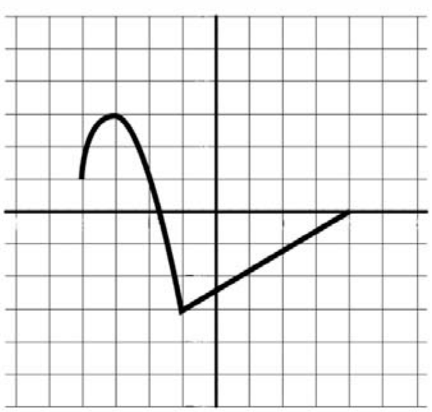
Dashed = Transformed.

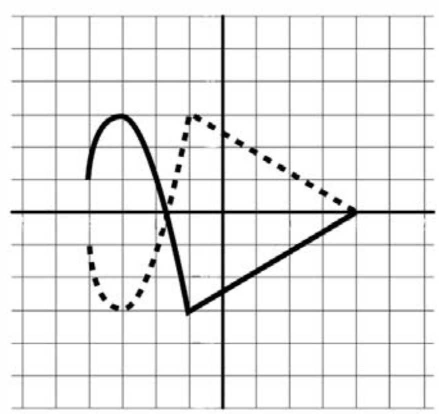
Example 2: Draw y = -f(x) for the following graph.





Example 3: Draw y = -f(x) for the following graph.



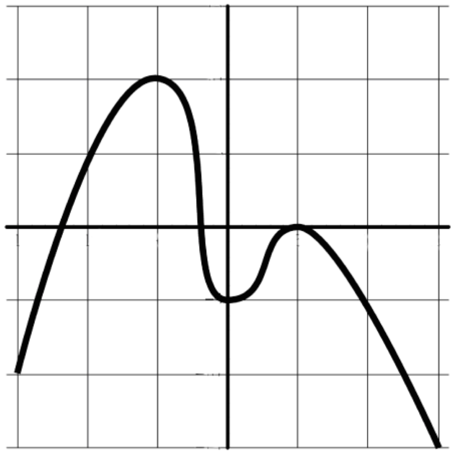


TRANSFORMATIONS LESSON 1

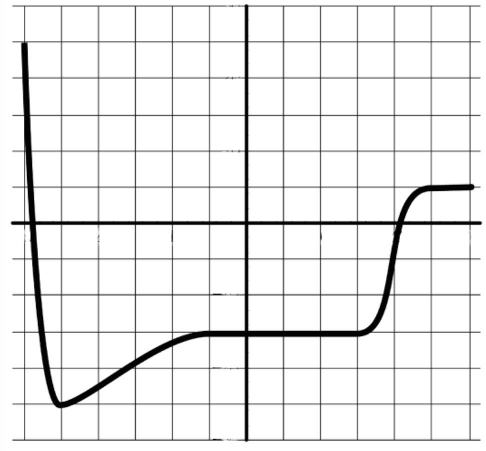
PART III: VERTICAL REFLECTIONS

Questions: **For each of the following graphs, draw in the vertical reflection.**

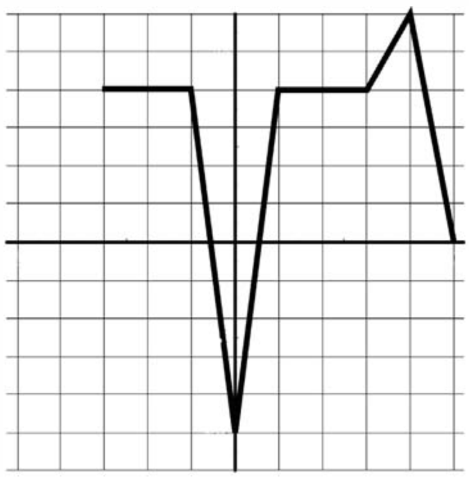
1) $y = -f(x)$



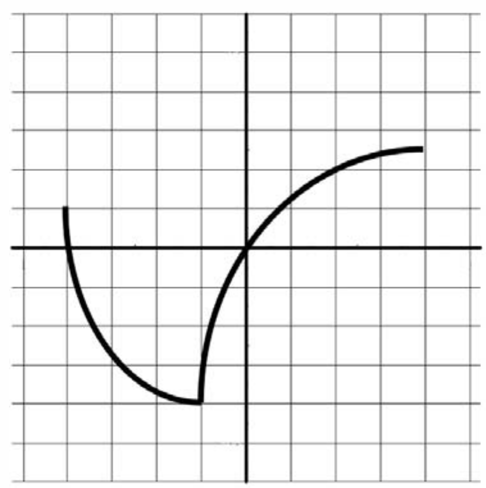
2) $y = -f(x)$



3) $y = -f(x)$

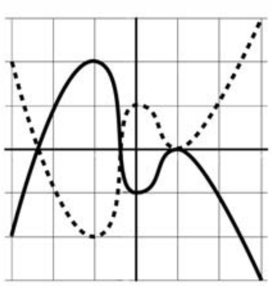


4) $y = -f(x)$



Answers:

1.



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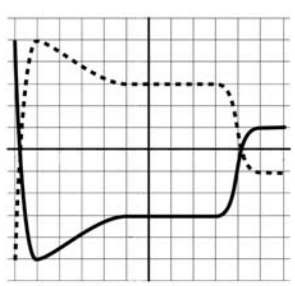

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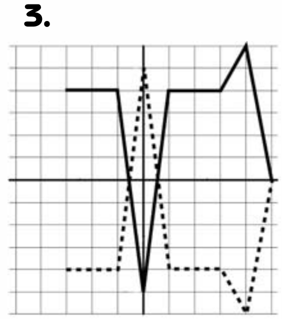

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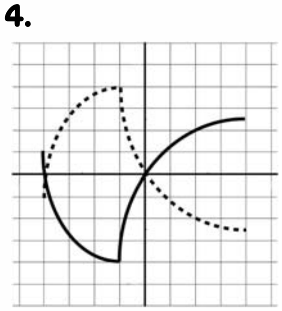
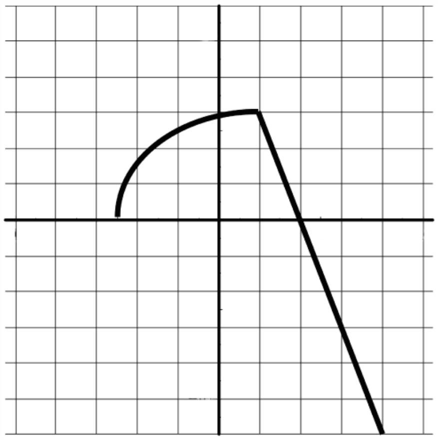


TRANSFORMATIONS LESSON 1

PART IV: HORIZONTAL REFLECTIONS

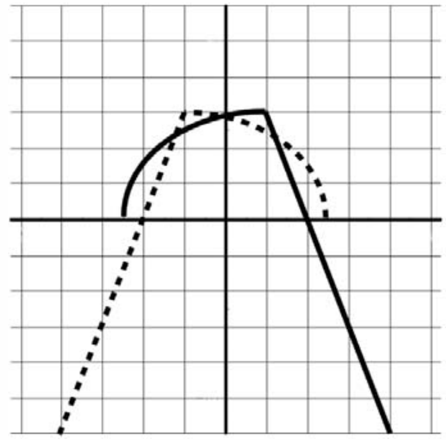
Horizontal Reflections: **A horizontal reflection *(about the y-axis)* is represented by the form $y = f(-x)$**

Example 1: **Draw $y = f(-x)$ for the following graph.**

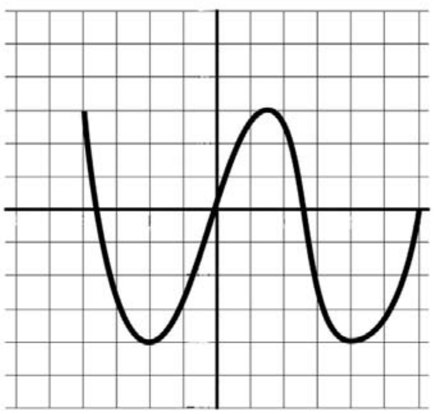


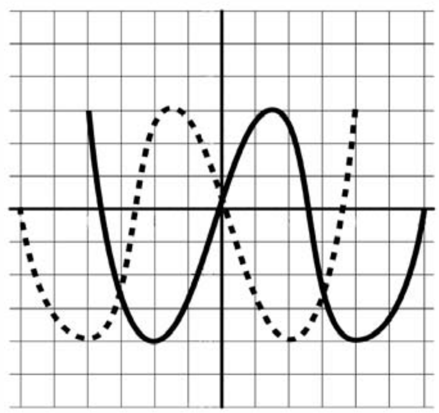
A horizontal reflection is done by changing the signs of all x-values. This will reflect the graph over the y-axis.

In a horizontal reflection (about the y-axis), the y-intercepts are the invariant points.

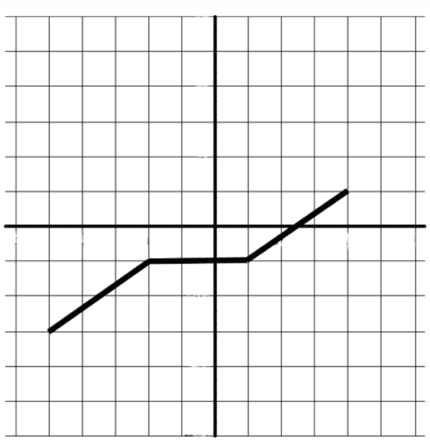


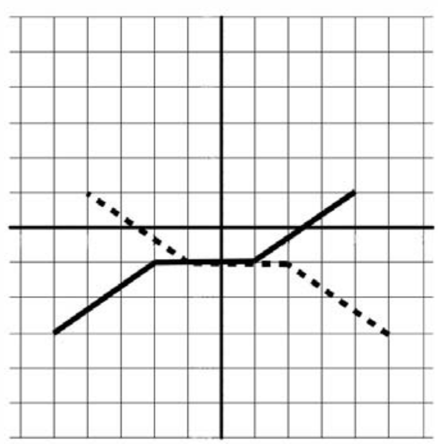
Example 2: **Draw $y = f(-x)$ for the following graph.**





Example 3: **Draw $y = f(-x)$ for the following graph.**



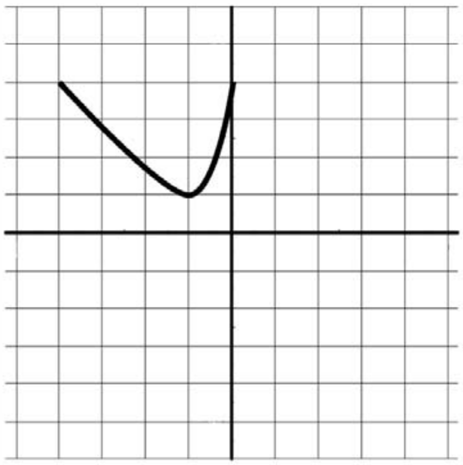


TRANSFORMATIONS LESSON 1

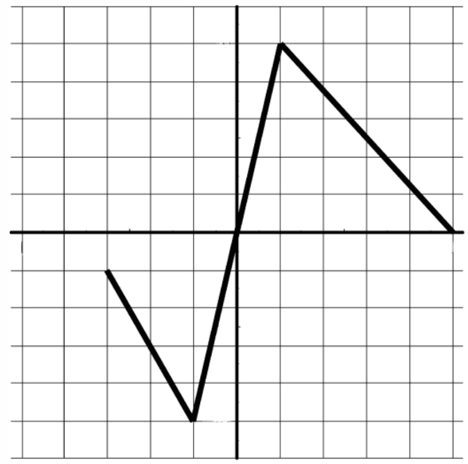
PART IV: HORIZONTAL REFLECTIONS

Questions: **For each of the following graphs, draw in the horizontal reflection.**

1) y = f(-x)

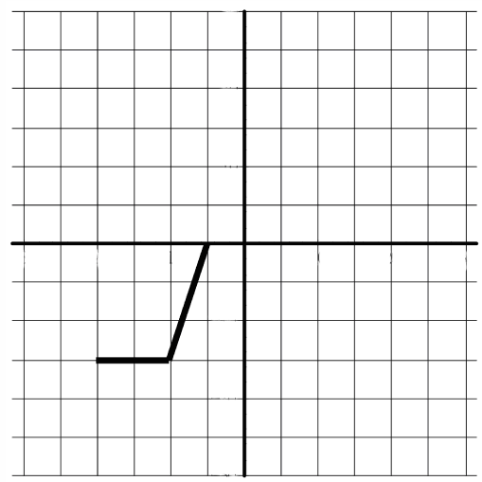


2) y = f(-x)



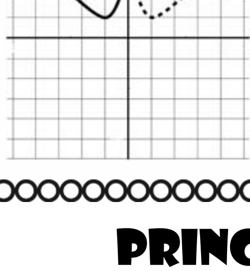
3) y = f(-x)

4) y = f(-x)

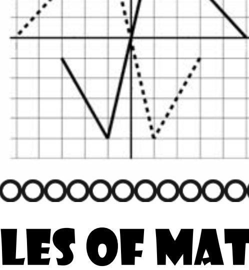


Answers:

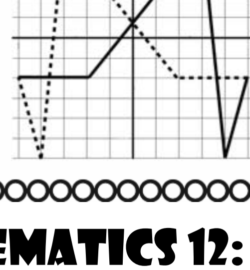
1.



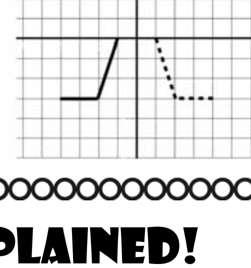
2.



3.



4.

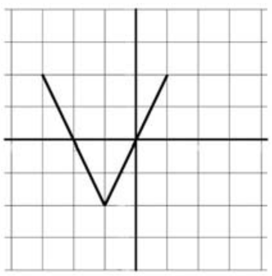


Horizontal Translation: **A horizontal translation is of the form $y = f(x - c)$**

Vertical Translation: **A vertical translation is of the form $y = f(x) + d$**

> The word ***Translation*** means to slide a graph.
>
> The word ***Transformation*** is more general, including anything you can do to a graph that moves it or changes the shape.

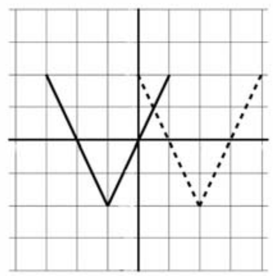
Example 1: Graph $y = f(x - 3)$



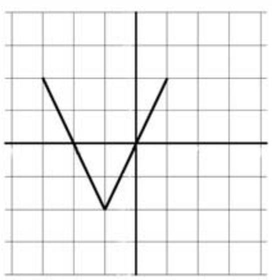
f(x-3) is telling you to move the graph 3 units to the right.

Think of it this way:

When you have a number added or subtracted from x inside brackets, do the opposite of what the sign is.



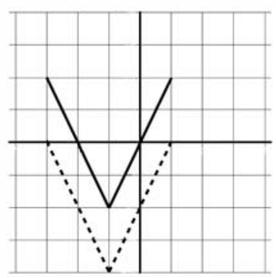
Example 2: Graph $y = f(x) - 2$



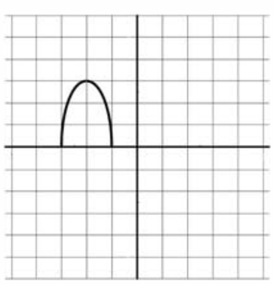
f(x) - 2 is telling you to move the graph 2 units down.

-Think of it this way:

When you have a number added or subtracted to f(x), the vertical translation is exactly the same as that number.

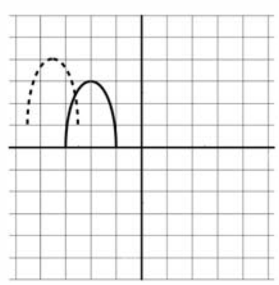


Example 3: Graph $y = f(x + \frac{3}{2}) + 1$

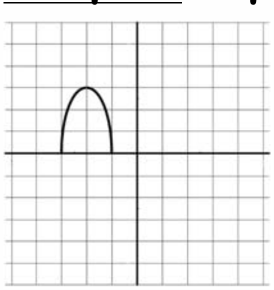


Vertical & horizontal translations can be performed in either order.

1.5 Left, 1 Up.
OR
1 Up, 1.5 Left



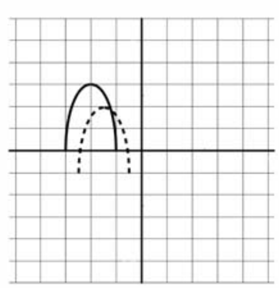
Example 4: Graph $y + 1 = f(x - \frac{1}{2})$



When presented in this form, take the 1 from the left side and put it on the other side of the equals.

Write as:
$y = f(x - 0.5) - 1$

0.5 Right, 1 Down.

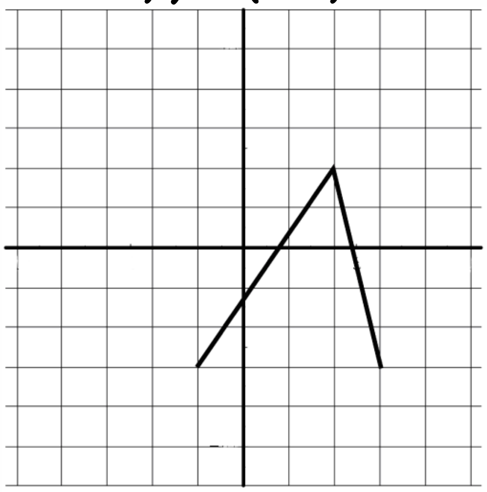


Transformations Lesson 1

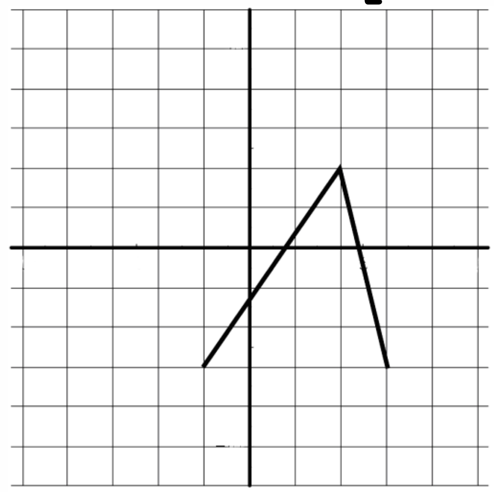
Part V: Translations

Questions: **Apply the following translations on each of the graphs.**

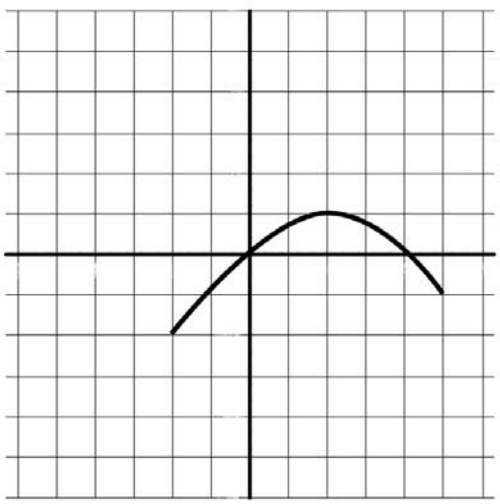
1) $y = f(x - 1)$



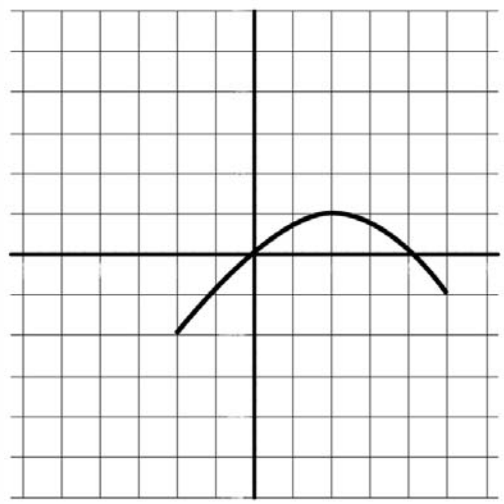
2) $y = f(x + 2) - \frac{3}{2}$



3) $y - 3 = f(x + 4)$

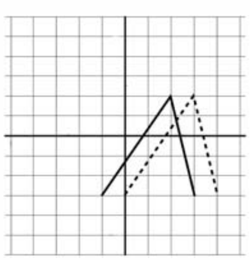


4) $y + 2 = f(x - 1)$

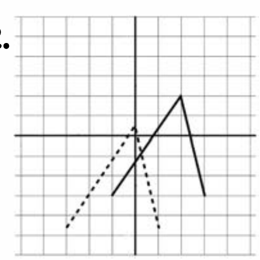


Answers:

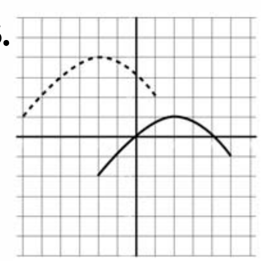
1.



2.



3.



4.