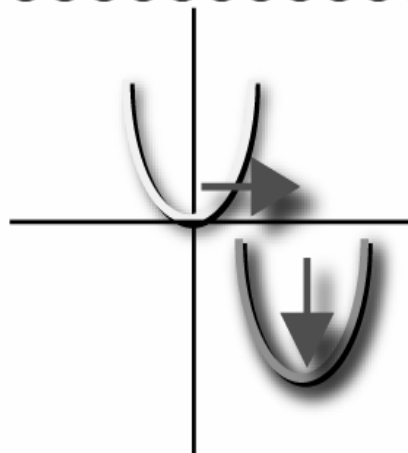


Principles of Mathematics 12

TRANSFORMATIONS



LESSON 2.

Combining Transformations

Principles of
Math 12

EXPLAINED!

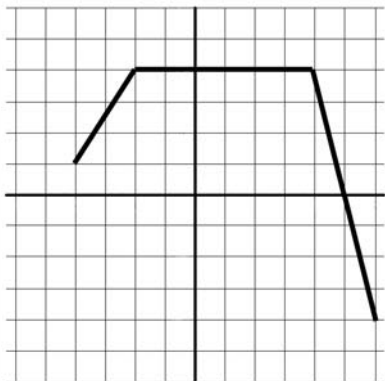
By
Barry
Mabillard

TRANSFORMATIONS LESSON 2

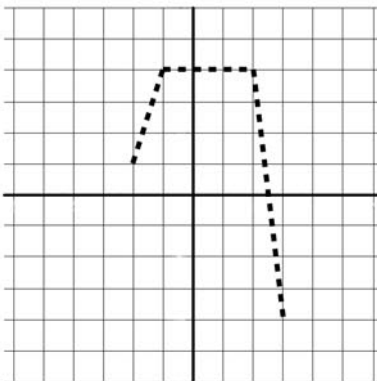
Part I: Combining Stretches and Reflections

Combining Stretches & Reflections: These can be done in either order and you'll still get the correct graph.

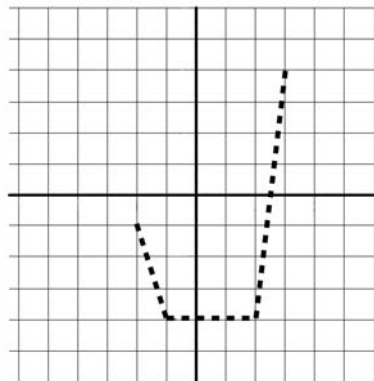
Example 1: Given the following graph, draw in the graph of $y = -f(2x)$



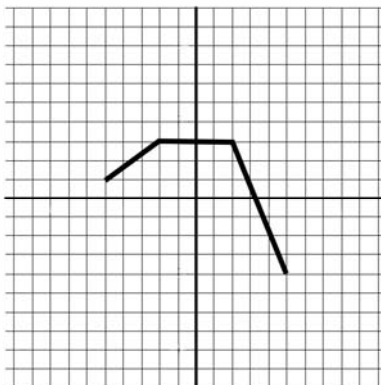
1) Multiply all x-values by $\frac{1}{2}$.
Remember to use reciprocal!



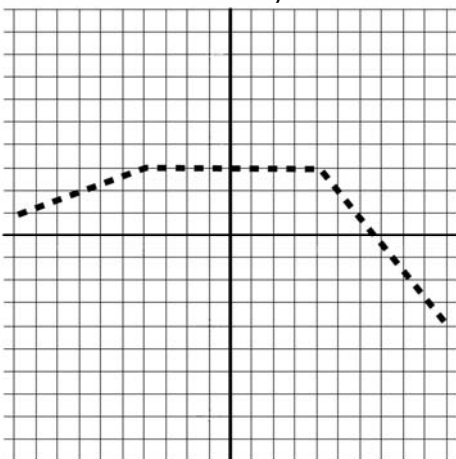
2) Reflect about the x-axis



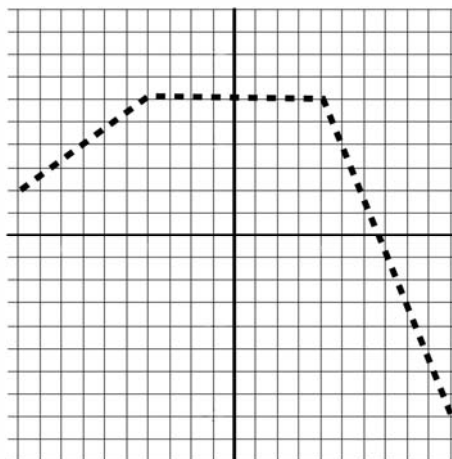
Example 2: Given the graph on the right, draw the graph of $y = 2f\left(\frac{1}{2}x\right)$



1) Multiply all x-values by 2.
Remember to use reciprocal!



2) Multiply all y-values by 2.



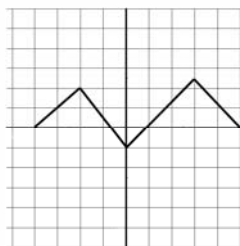
These transformations may be performed in any order:

- Horizontal stretch about the y-axis.
- Vertical stretch about the x-axis
- Horizontal reflection about the y-axis
- Vertical reflection about the x-axis.

TRANSFORMATIONS LESSON 2

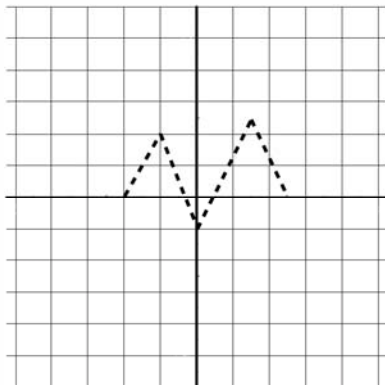
Part I: Combining Stretches and Reflections

Example 3: Given the graph on the right, draw the graph of $y = 2f(-2x)$

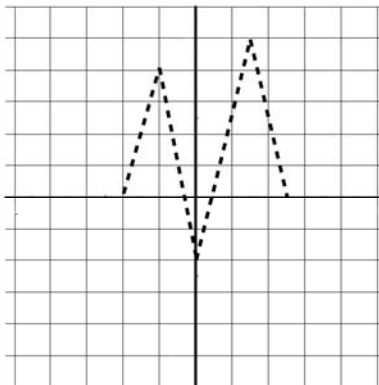


Don't forget to use the reciprocal for the horizontal stretch!

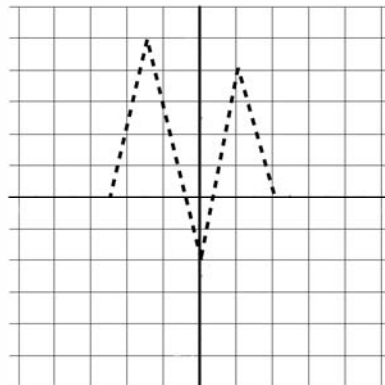
1) Multiply the x-values by $\frac{1}{2}$.



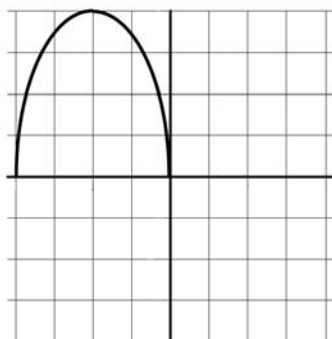
2) Multiply the y-values by 2.



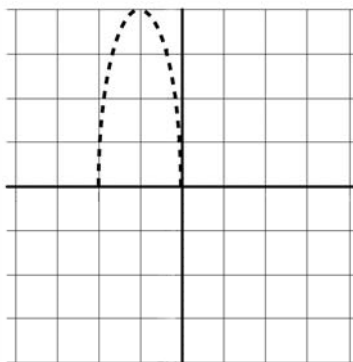
3) Reflect about the y-axis.



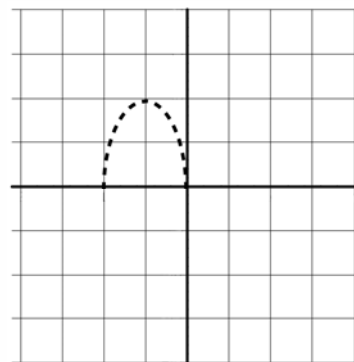
Example 4: Given the graph below, draw the graph of $y = \frac{1}{2}f(-2x)$



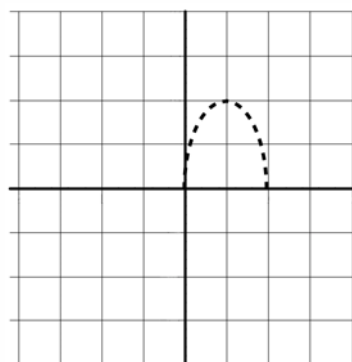
1) Multiply the x-values by $\frac{1}{2}$.



2) Multiply the y-values by $\frac{1}{2}$.



3) Reflect in the y-axis



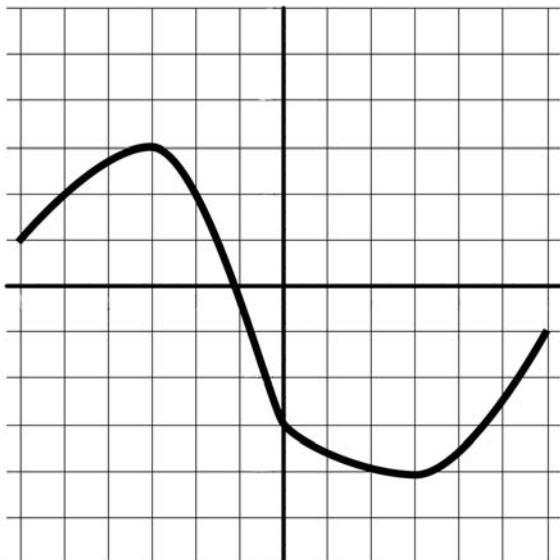
Multiple transformations can be drawn with different colored felts to keep track of what you are doing.

TRANSFORMATIONS LESSON 2

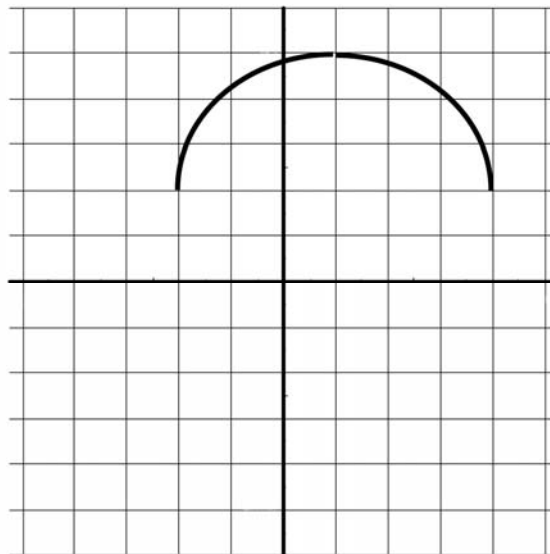
Part I: Combining Stretches and Reflections

Questions: Apply the following transformations to each of the graphs.

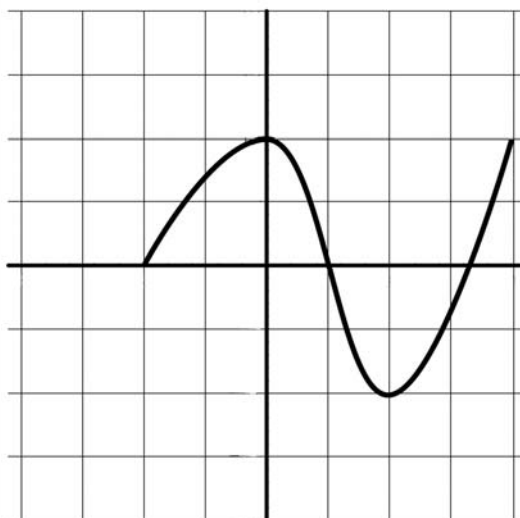
1) $y = f(-3x)$



2) $y = -\frac{1}{2}f(x)$

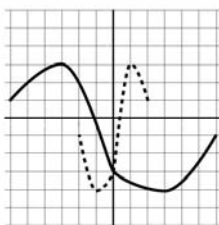


3) $y = \frac{1}{2}f(-x)$

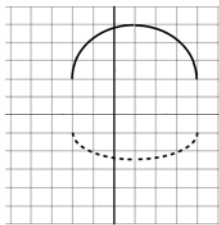


Answers:

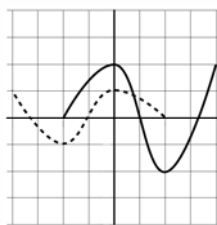
1.



2.



3.



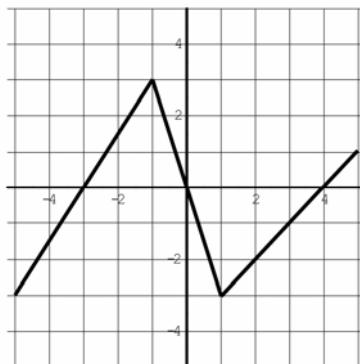
TRANSFORMATIONS LESSON 2

Part II: Combining All Transformations

Combining Stretches, Reflections, & Translations:

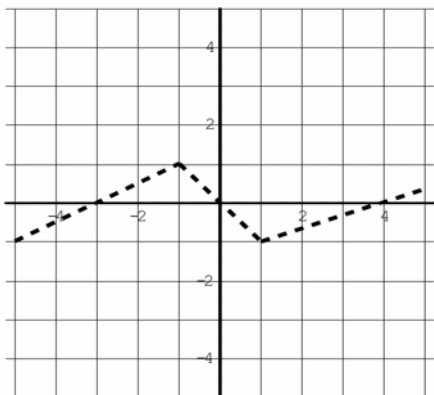
Stretches & reflections must be done first → **translations last!**

Example 1: Given the graph on the right, sketch $y = \frac{1}{3}f(2x) + 1$



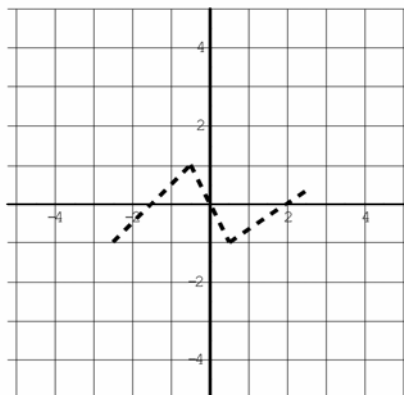
Step 1: First do $y = \frac{1}{3}f(x)$

by multiplying all y-values by $\frac{1}{3}$



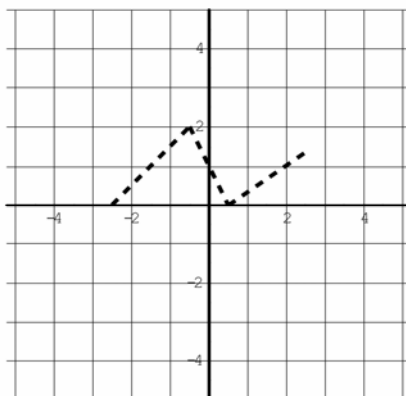
Step 2: Then do $y = \frac{1}{3}f(2x)$ by

multiplying all x-values by $\frac{1}{2}$



Step 3: Finish off with $y = \frac{1}{3}f(2x) + 1$

by moving the entire graph up one unit.



TRANSFORMATIONS LESSON 2

Part II: Combining All Transformations

Watch out for these types of questions!

If x has a coefficient attached to it, that coefficient must be factored out of the brackets before proceeding.

Consider the transformation $y = f(5x - 15)$.

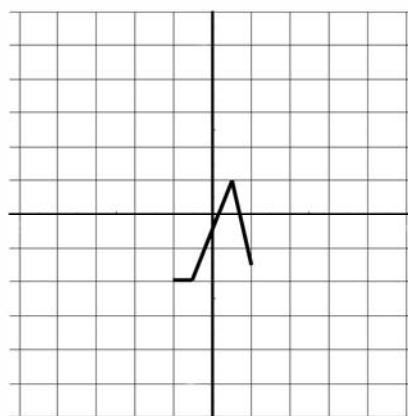
This can be simplified to $y = f[5(x-3)]$ by factoring out the 5.

It is now easy to see that there is a horizontal stretch by a factor of $1/5$, and a translation of 3 units right.

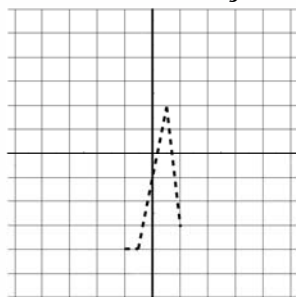
Example 2: Given the graph on the

right, sketch $y = -2f\left[\frac{1}{2}x + 1\right]$

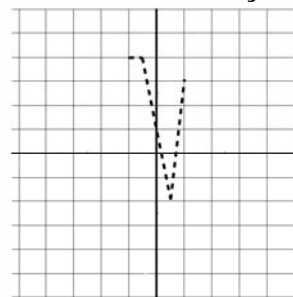
→ Rewrite as $y = -2f\left[\frac{1}{2}(x + 2)\right]$



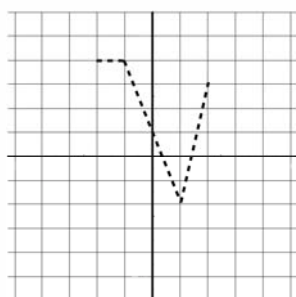
Step 1: First sketch $y = 2f(x)$



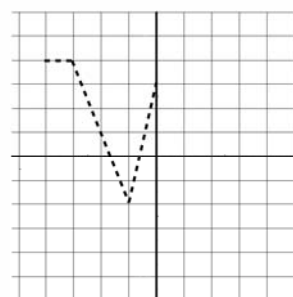
Step 2: Then sketch $y = -2f(x)$



Step 3: Now sketch $y = -2f\left(\frac{1}{2}x\right)$



Step 4: Finally sketch $y = -2f\left[\frac{1}{2}(x + 2)\right]$

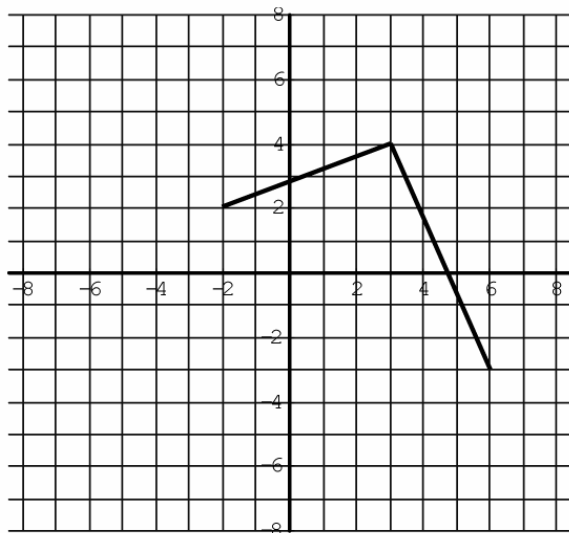


TRANSFORMATIONS LESSON 2

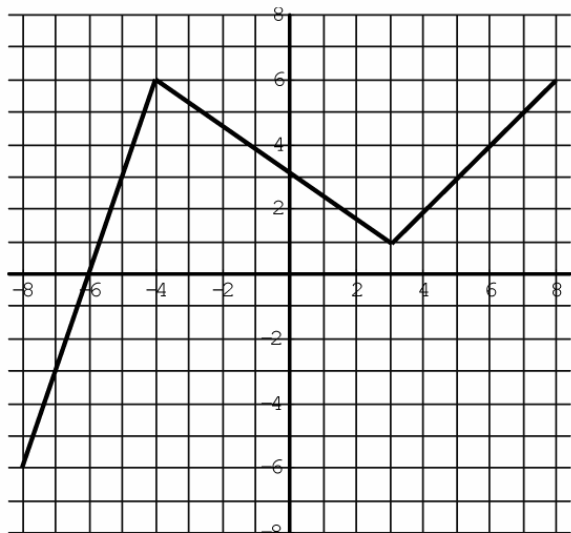
Part II: Combining All Transformations

Questions: For each of the graphs, apply the transformation:

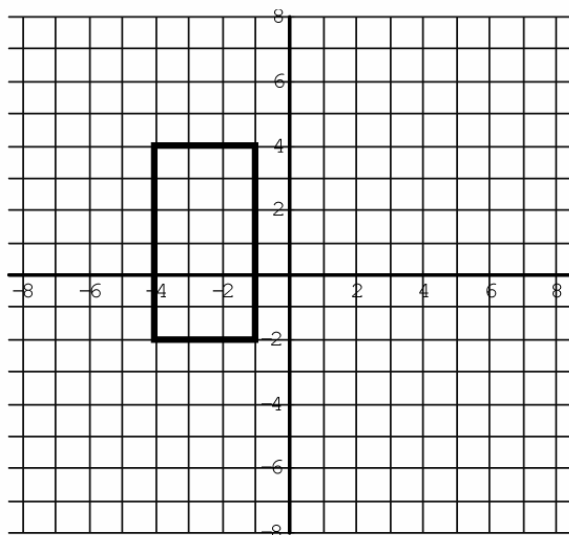
1) $y = -f(2x + 4)$



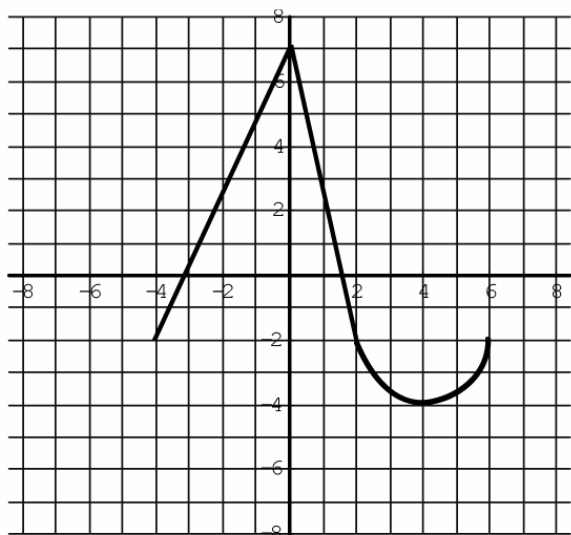
2) $y = -\frac{1}{2}f(-2x)$



3) $y = 2f\left(\frac{1}{2}x\right) - 2$



4) $y = -\frac{1}{2}f(2x)$



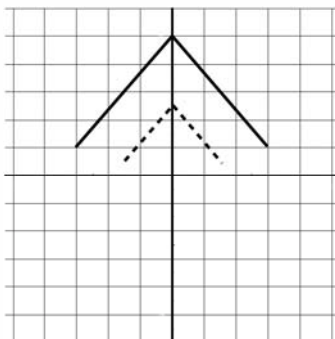
TRANSFORMATIONS LESSON 2

Part II: Combining All Transformations

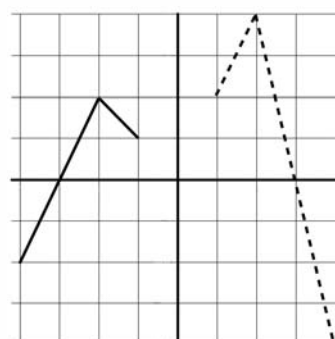
Questions: For each of the following graphs, write the equation of the transformation:

Solid = Original
Dashed = Transformed

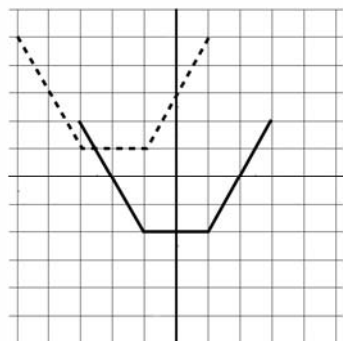
5)



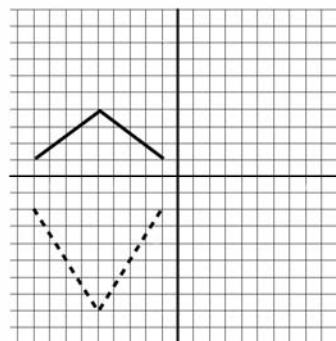
6)



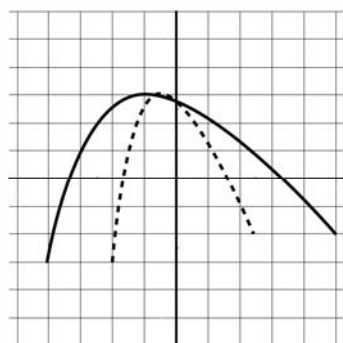
7)



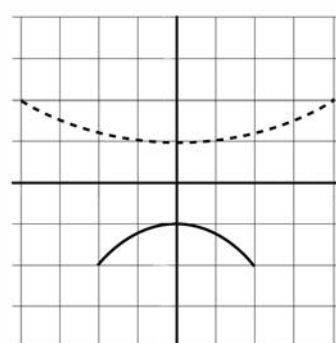
8)



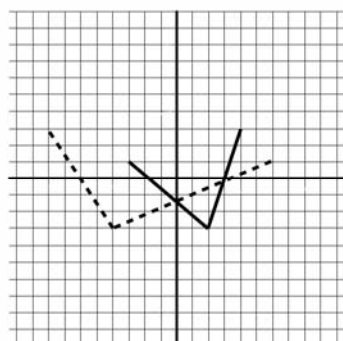
9)



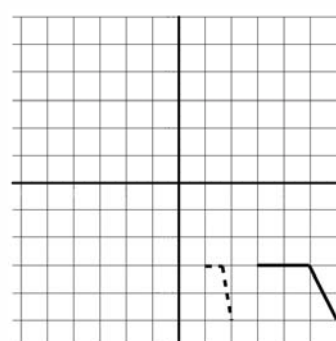
10)



11)



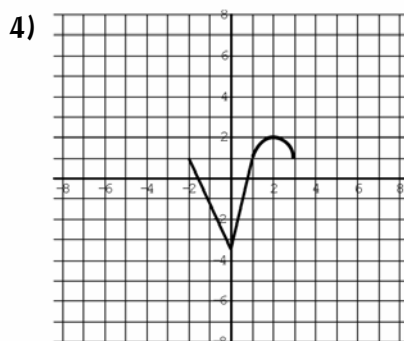
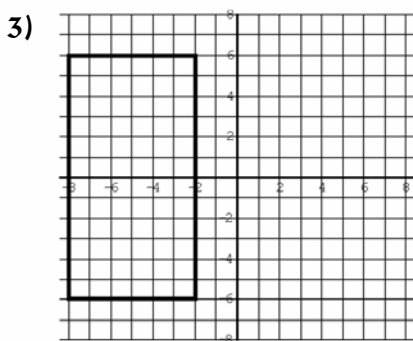
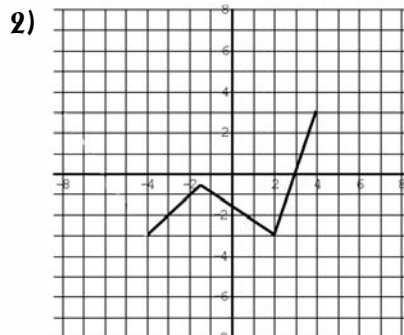
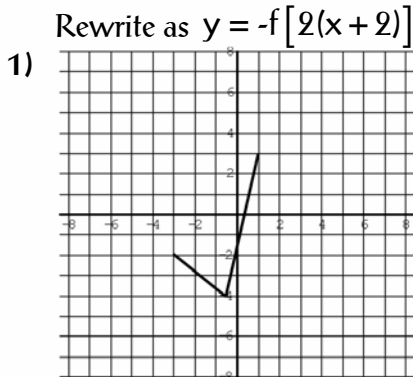
12)



TRANSFORMATIONS LESSON 2

Part II: Combining All Transformations

Answers:



5) $y = \frac{1}{2}f(2x)$

6) $y = 2f(-x)$

7) $y = f(x + 2) + 3$

8) $y = -2f(x)$

9) $y = f(2x)$

10) $y = -f\left(\frac{1}{2}x\right)$

11) $y = f\left(-\frac{1}{2}x\right)$

12) $y = f(3x)$