Principles of Mathematics 12

Probability

Lesson 2

Conditional Probability

Principles of Math 12 EXPLAINED!

By Barry Mabillard

Probability Lesson 2
Part 1: Conditional Probability

<u>Conditional Probability:</u> In many situations, the outcome of one event will have a direct influence on the outcome of another event.
Such events are called *dependent* events.

<u>Example 1:</u> A person draws one card from a deck of 52 cards, then draws a second card. Calculate the probability of both cards being clubs.

The probability of drawing a club is $\frac{13}{52}$

The probability of drawing a second club is $\frac{12}{51}$ since there is one less club, and consequently one less card in the deck.

The probability both cards are clubs is $\frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$

Drawing a card without replacement changes the probability for the second card.

Thus, the outcome of the second card is **<u>dependent</u>** on the outcome of the first card.

<u>Example 2:</u> A person draws one card from a deck of 52 cards, then returns it to the deck. Another card is then drawn. Calculate the probability of both cards being clubs.

The probability of drawing a club is $\frac{13}{52}$

The probability of drawing a second club is $\frac{13}{52}$ since the first card was put back in the deck.

The probability both cards are clubs is $\frac{13}{52} \times \frac{12}{51} = \frac{1}{16}$

Replacing a card returns the deck to the initial state.

Thus, the outcome of the second card is **<u>independent</u>** from the outcome of the first card.

The formula for conditional probability is:

$$\boxed{P(A \text{ and } B) = P(A) \times P(B \mid A)}$$

The vertical line you see in the expression P(B | A) is not a mathematical operation.
It is simply a reminder that you want the probability of B, given that A has occurred.
Example 1 illustrates the use of this formula.

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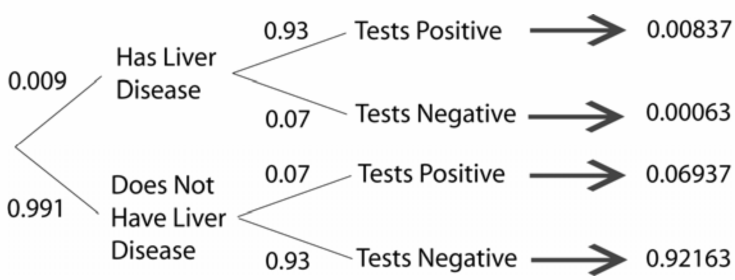
<u>Bayes' Formula:</u> The following is a manipulation of the formula introduced on the previous page. It is used to find the conditional probability of Event B occurring, given that Event A has already occurred.

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

<u>Example 3:</u> A diagnostic test for liver disease is accurate 93% of the time, and 0.9% of the population actually has liver disease.

a) Determine the probability the patient tests positive
b) Determine the probability the patient tests negative
c) Determine the probability the patient has liver disease and tests positive
d) Determine the probability the patient does not have liver disease and tests negative

First draw a tree diagram to determine all required probabilities:



Special care should be taken when writing in the probabilities, it's easy to mix up where the positive tests & negative tests go if you too fast!

a) Add the probabilities where a positive result is obtained: 0.00837 + 0.06937 = **0.07774**
b) Add the probabilities where a negative result is obtained: 0.00063 + 0.92163 = **0.92226**

c) This question is asking you to solve for P(liver disease | positive).
Use Bayes' Formula

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$P(\text{liver disease} \mid \text{positive}) = \frac{P(\text{liver disease and positive})}{P(\text{positive})}$$

$$P(\text{liver disease} \mid \text{positive}) = \frac{0.00837}{0.07774}$$

$$P(\text{liver disease} \mid \text{positive}) = \mathbf{0.10767}$$

d) This question is asking you to solve for P(no liver disease | negative).
Use Bayes' Formula

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$P(\text{no liver disease} \mid \text{negative}) = \frac{P(\text{no liver disease and negitive})}{P(\text{negative})}$$

$$P(\text{no liver disease} \mid \text{negative}) = \frac{0.92163}{0.92226}$$

$$P(\text{no liver disease} \mid \text{negative}) = \mathbf{0.99932}$$

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Questions:

1) Cards are drawn from a standard deck of 52 cards **without** replacement. Calculate the probability of obtaining:

a) A king, then another king

b) A club, then a heart

c) A black card, then another black card

d) A black card, then a heart, then a diamond

e) The seven of spades, then another spade, then a club, then a heart

2) Cards are drawn from a standard deck of 52 cards **with** replacement. Calculate the probability of obtaining:

a) A king, then another king

b) A club, then a heart

c) A black card, then another black card

d) A black card, then a heart, then a diamond

e) The seven of spades, then another spade, then a club, then a heart

3) Given P(A) = 0.34, and P(B | A) = 0.65, calculate P(A & B)

4) Given P(A) = 0.2, P(B) = 0.8, P(A & B) = 0.1, calculate

a) P(A | B)

b) P(B | A)

5) A diagnostic test for eye disease is accurate 88% of the time, and 2.4% of the population actually has eye disease.

a) Determine the probability the patient tests positive

b) Determine the probability the patient tests negative

c) Determine the probability the patient has eye disease and tests positive

d) Determine the probability the patient does not have eye disease and tests negative

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Answers:

1)

a) $\frac{4}{52} \times \frac{3}{51} = \frac{1}{21}$

b) $\frac{13}{52} \times \frac{13}{51} = \frac{13}{204}$

c) $\frac{26}{52} \times \frac{25}{51} = \frac{25}{102}$

d) $\frac{26}{52} \times \frac{13}{51} \times \frac{13}{50} = \frac{169}{5100}$

e) $\frac{1}{52} \times \frac{12}{51} \times \frac{13}{50} \times \frac{13}{49} = 0.000288$

2)

a) $\frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$

b) $\frac{13}{52} \times \frac{13}{52} = \frac{1}{16}$

c) $\frac{26}{52} \times \frac{26}{52} = \frac{1}{4}$

d) $\frac{26}{52} \times \frac{13}{52} \times \frac{13}{52} = \frac{1}{32}$

e) $\frac{1}{52} \times \frac{13}{52} \times \frac{13}{52} \times \frac{13}{52} = 0.0003$

3) Use the formula P(A & B) = P(A) x P(B | A)
P(A & B) = 0.34 x 0.65
P(A & B) = 0.221

4) Use the formula $P(A|B) = \frac{P(A\&B)}{P(B)}$

a)

$$P(A|B) = \frac{P(A\&B)}{P(B)}$$

$$P(A|B) = \frac{0.1}{0.8}$$

$$P(A|B) = \frac{1}{8}$$

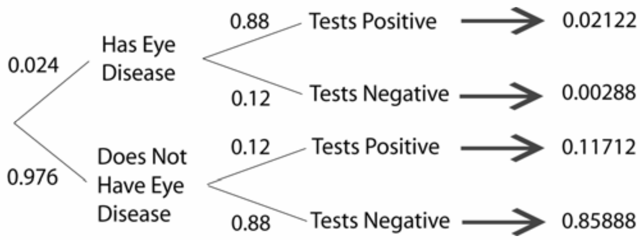
b)

$$P(B|A) = \frac{P(B\&A)}{P(A)}$$

$$P(B|A) = \frac{0.1}{0.2}$$

$$P(B|A) = \frac{1}{2}$$

5) Draw a tree diagram:



a) Add up the cases with a positive result:
0.02122 + 0.11712 = 0.13834

b) Add up the cases with a negative result:
0.00288 + 0.85888 = 0.86176

c)

$$P(\text{Eye Disease} \mid \text{Positive}) = \frac{P(\text{Eye Disease \& Positive})}{P(\text{Positive})}$$

$$P(\text{Eye Disease} \mid \text{Positive}) = \frac{0.02122}{0.13834} = 0.1534$$

d)

$$P(\text{No Disease} \mid \text{Negative}) = \frac{P(\text{No Disease \& Negative})}{P(\text{Negative})}$$

$$P(\text{Eye Disease} \mid \text{Positive}) = \frac{0.85888}{0.86176} = 0.9967$$