Principles of Mathematics 12

Practice Exam

Probability

--ANSWERS--

Probability Practice Exam - ANSWERS

ANSWERS

1. A	**12.** B	**23.** C	**34.** C
2. A	**13.** D	**24.** B	**35.** C
3. B	**14.** B	**25.** D	**36.** C
4. D	**15.** B	**26.** B	**37.** D
5. C	**16.** A	**27.** A	**38.** A
6. A	**17.** C	**28.** A	**39.** D
7. A	**18.** D	**29.** C	
8. D	**19.** A	**30.** B	
9. C	**20.** B	**31.** C	
10. A	**21.** C	**32.** A	
11. A	**22.** D	**33.** D	

1. Look at the sample space for rolling two die:

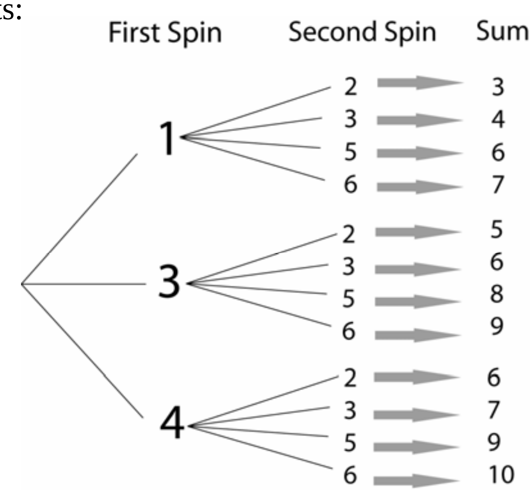
Dice 1 \ Dice 2	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

The question is saying we want the sum to be an odd number greater than 8, meaning it can be **9** or **11**. There are six outcomes that are 9 or 11.

The probability is $\frac{6}{36} \rightarrow \frac{1}{6}$

The answer is **A**

2. Draw a tree diagram illustrating the possible results:



From the diagram, it can be seen that there are six even sums, out of a possible 12 sums.

Thus, the probability of getting an even number for a sum is $\frac{6}{12} \rightarrow \frac{1}{2}$

The answer is **A**

3. The probability of pulling a chocolate chip cookie *then* an oatmeal cookie is:

$$\left(\frac{3}{x+3}\right)\left(\frac{x}{x+2}\right)$$

$\frac{\text{3 chocolate chip cookies}}{\text{chocolate chip + oatmeal cookies}}$ $\qquad$ $\frac{x \text{ oatmeal cookies}}{\text{one fewer cookie than previous term}}$

The answer is **B**

4. Use combination probability since several cards are being pulled at once.

The probability of having a hand with no clubs is $\frac{_{39}C_3}{_{52}C_3}$

The probability of having at least one club is ***1 – probability of no clubs,*** which is written as $1-\frac{_{39}C_3}{_{52}C_3}$

The answer is **D**

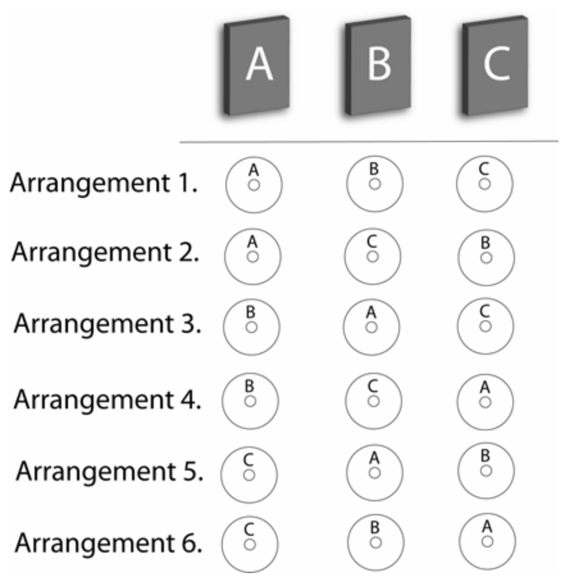
5. The probability the first toss will be a 1, 2, or a 3 is $\frac{3}{6}\rightarrow\frac{1}{2}$

The probability the second toss will be a 5 or a 6 is $\frac{2}{6}\rightarrow\frac{1}{3}$

The probability is $\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$

The answer is **C**

6. First draw out the possible cases:
There is only one favorable outcome out of six possible outcomes.



The answer is $\frac{1}{6}$

This matches with answer **A**.

7. The probability Team A wins is $\frac{3}{5}$

The probability Team C wins is $\frac{4}{7}$, so the probability it loses is the complement $\frac{3}{7}$

The probability Team A wins *and* Team C loses is $\frac{3}{5} \times \frac{3}{7} = \frac{9}{35}$

The answer is **A**

8. First determine the number of possible arrangements for the PIN number that satisfy the stated restrictions (*begins with a 7, ends with an 8*).

$$\underline{1}\ \ \underline{10}\ \ \underline{10}\ \ \underline{10}\ \ \underline{1}\ \ = 1000$$

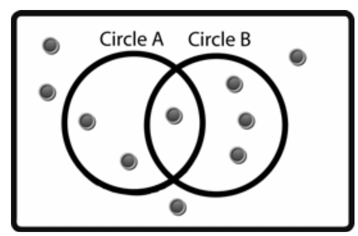
Now determine the total number of arrangements

$$\underline{9}\ \ \underline{10}\ \ \underline{10}\ \ \underline{10}\ \ \underline{10}\ \ = 90000$$

The probability is $\frac{1000}{90000} = \frac{1}{90}$

The answer is **D**.

9. The expression $P\left(\overline{A}\right)$ means ***the probability it's not A***.



From the diagram, we can see that 7 out of the 10 discs are not in circle A.
The probability is **0.7**
The answer is **C**

10. $P(B \mid A) = \frac{P(B \,\&\, A)}{P(A)} = \frac{0.1}{0.3} \to \frac{1}{3}$

The answer is **A**

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11. The probability of drawing a red face card is $\frac{6}{52}$. *(There are 12 face cards in a deck, half of which are red)* The probability of pulling a club is $\frac{13}{51}$. *(Use 51 since one card has been removed from the deck)*

Multiply the probabilities together to obtain the answer: $\frac{6}{52} \times \frac{13}{51} = \frac{78}{2652} \to \frac{1}{34}$

The answer is **A**

12. The table provided can be thought of as a sample space:

	Students who have a cell phone	**Students who do not have a cell phone**	**Total**
School A	365	156	521
School B	408	71	479
Total	773	227	1000

From this table, we can see that there are 408 students with a cell phone who also attend School B. Thus, the probability is $\frac{408}{1000} = 0.41$

The answer is **B**

13. To answer this question, we must use Bayes' formula since we want the probability a student has a cell phone ***given that*** the student attends School B.

$$P(Cell \mid B) = \frac{P(Cell \,\&\, B)}{P(B)}$$

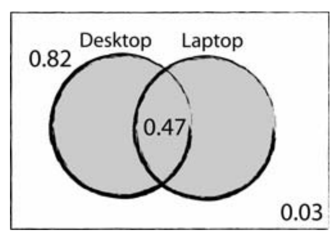
$$P(Cell \mid B) = \frac{0.408}{0.479} = 0.852$$

P(Cell & B) = 0.408

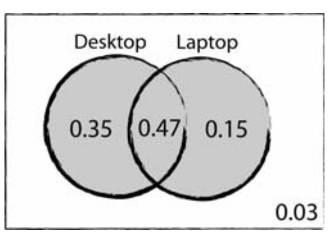
P(B) = 479/1000 = 0.479

The answer is **D**

14. Write the information you know in a Venn Diagram:



Calculate the probability of a person just having a desktop using 0.82 – 0.47 = 0.35

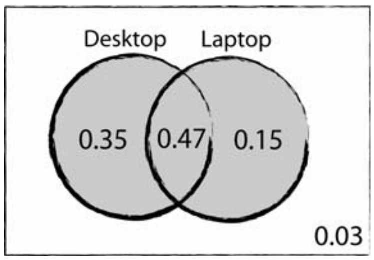


Calculate the probability of a person just having a laptop using 1 - 0.35 – 0.47 – 0.03 = 0.15

The probability of a home having a laptop is 0.47 + 0.15 = 0.62
The answer is **B**

15. Write this question as *"what is the probability a person does not have a laptop given that they have a desktop?"* Use Bayes' formula

$$P(\textit{No Laptop} \mid \textit{Have Desktop}) = \frac{P(\textit{No Laptop} \ \& \ \textit{Have Desktop})}{P(\textit{Have Desktop})}$$



From the diagram, we can see that
P(No Laptop & Have Desktop) = 0.35
P(Have Desktop) = 0.82

$$P(\textit{No Laptop} \mid \textit{Have Desktop}) = \frac{0.35}{0.82} = 0.43$$

The answer is 0.43
The answer is **B**

16. There is only one way to correctly pick the top three runners ***and*** the finishing order. There are ${}_{12}P_3 = 1320$ ways in total for 12 runners to finish. *(Use a permutation since order matters when runners finish a race.)*

The probability is $\frac{1}{1320}$

The answer is **A**

17. The probability of rolling a 1 or a 2 is $\frac{2}{6} \rightarrow \frac{1}{3}$

The probability of selecting a metal ball from Bag A is $\frac{4}{10} \rightarrow \frac{2}{5}$

The probability of rolling a 3, 4, 5, or 6 is $\frac{4}{6} \rightarrow \frac{2}{3}$

The probability of selecting a metal ball from Bag B is $\frac{5}{7}$

The probability of selecting a metal ball *(from either bag)* is: $\left(\frac{1}{3} \times \frac{2}{5}\right) + \left(\frac{2}{3} \times \frac{5}{7}\right) = \frac{64}{105}$

The answer is **C.**

18. To answer this question, we must use Bayes' formula since we want the probability the ball is from Bag B ***given that*** it is metal.

$$P(Bag\ B \mid metal) = \frac{P(Bag\ B\ \&\ metal)}{P(metal)}$$

$$P(Bag\ B \mid metal) = \frac{0.4762}{0.6095} = 0.7813$$

$$P(Bag\ B\ \&\ metal) = \frac{2}{3} \times \frac{5}{7} = \frac{10}{21} = 0.4762$$

$$\text{P}(metal) = \frac{64}{105} = 0.6095$$

The answer is 0.78
The answer is **D**

19. The probability of pulling two metal balls from Bag B is $\frac{5}{7} \times \frac{4}{6} = \frac{20}{42} \to \frac{10}{21}$

The probability of pulling two glass balls from Bag B is $\frac{2}{7} \times \frac{1}{6} = \frac{2}{42} \to \frac{1}{21}$

The probability of two metal ***or*** two glass balls may be obtained by adding the results.

$$\frac{10}{21} + \frac{1}{21} = \frac{11}{21}$$

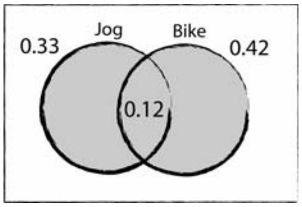
The answer is **A.**

*Since order is not important, we could also use combination probability:

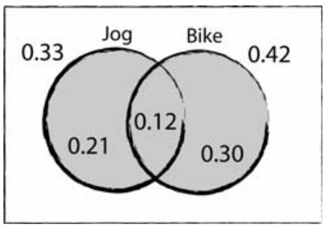
$$\frac{Ways\ to\ pick\ two\ glass\ balls\ +\ Ways\ to\ pick\ two\ metal\ balls}{Total\ ways\ to\ pick\ 2\ balls\ out\ of\ 7}$$

$$= \frac{{}_2C_2 + {}_5C_2}{{}_7C_2} = \frac{11}{21}$$

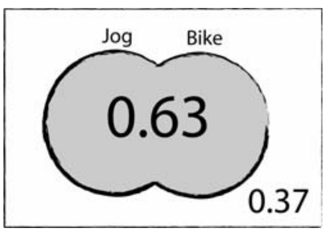
20. First draw the information you know in a Venn Diagram:



Then determine the values inside the circles: *(0.33 – 0.12 = 0.21 , 0.42 – 0.12 = 0.30)*



Determine the total probability inside the circles by adding the 3 numbers in the grey region. The region outside the circles is 1 – 0.63 = 0.37



The answer is 0.37
The answer is **B**

21. Use combination probability since we are simultaneously pulling 5 cards from the deck of 52.

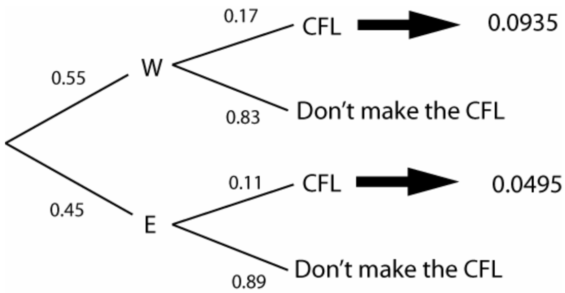
The number of ways to obtain a hand containing 2 black cards & 3 red face cards is:
${}_{26}C_2 \times {}_6C_3 = 6500$ *(Out of 26 black cards we need 2, and out of 6 red face cards we need 3)*

The number of ways to obtain a 5-card hand is ${}_{52}C_5 = 2598960$

The probability is $\dfrac{6500}{2598960} \rightarrow \dfrac{25}{9996}$

The answer is **C**.

22. Use a tree diagram to illustrate the probabilities:



To answer this question, we must use Bayes' formula since we want the probability the player is from Eastern Canada ***given that*** he is in the CFL.

$$P\left(Eastern\ Canada\,|\,CFL\right)=\frac{P\left(Eastern\ Canada\ \ \&\ \ CFL\right)}{P\left(CFL\right)}$$

$$P\left(Eastern\ Canada\,|\,CFL\right)=\frac{0.0495}{0.143}\rightarrow\frac{9}{26}$$

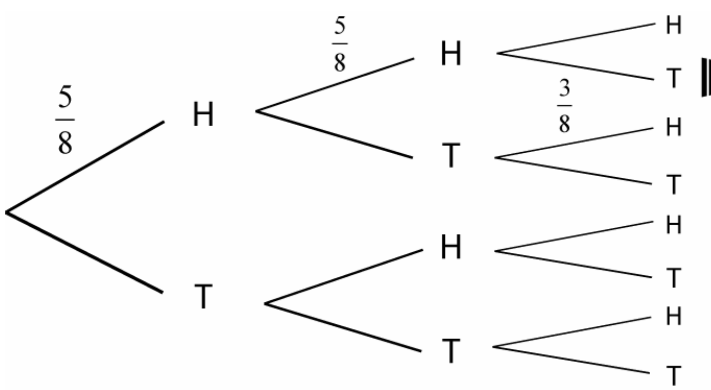
$P\left(Eastern\ Canada\,\&\,CFL\right)=0.0495$
$P(CFL)$ = 0.0935 + 0.04950 = 0.143

The answer is **D**

23. The probability of obtaining a head is $\frac{5}{8}$

The probability of obtaining a tail is the complement, $1-\frac{5}{8}=\frac{3}{8}$

Draw a tree diagram illustrating the possible results, then multiply the probabilities.



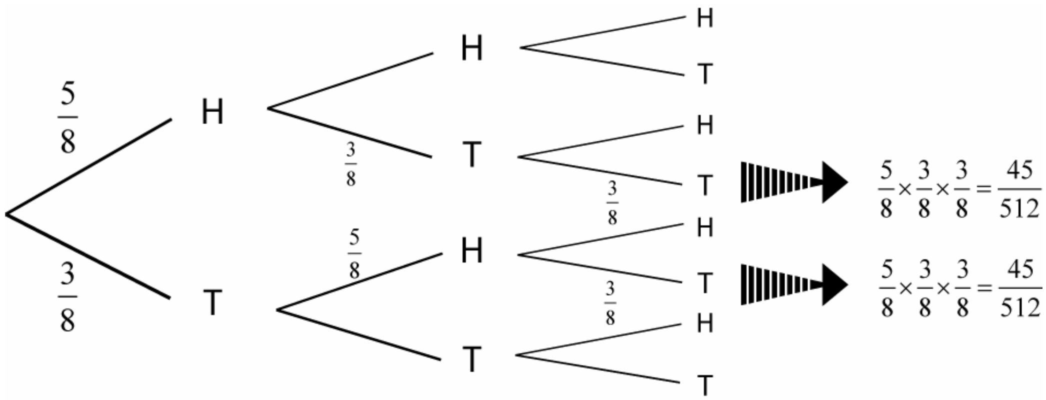
$$\frac{5}{8}\times\frac{5}{8}\times\frac{3}{8}=\frac{75}{512}$$

The answer is **C**

24. This question is saying that we want only two tails, and the second tail *must* occur on the third flip. That leads to the following outcomes:

HTT
THT

According to the tree diagram, the following outcomes are possible:



Add the results to obtain the answer: $\frac{45}{512}+\frac{45}{512}=\frac{45}{256}$

The answer is **B**

25. The probability of rolling a 1, 2 or 3 is $\frac{3}{6}\rightarrow\frac{1}{2}$

The probability of selecting a glass ball from Bag A is $\frac{6}{9}\rightarrow\frac{2}{3}$

The probability of rolling a 4, 5, or 6 is $\frac{3}{6}\rightarrow\frac{1}{2}$

The probability of selecting a glass ball from Bag B is $\frac{3}{7}$

The probability of selecting a glass ball *(from either bag)* is: $\left(\frac{1}{2}\times\frac{2}{3}\right)+\left(\frac{1}{2}\times\frac{3}{7}\right)=\frac{23}{42}$

The answer is **D.**

26. To answer this question, we must use Bayes' formula since we want the probability the ball is from Bag B ***given that*** it is glass.

$$P(Bag\ B \mid glass) = \frac{P(Bag\ B\ \&\ glass)}{P(glass)}$$

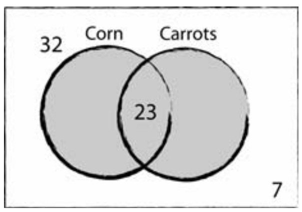
$$P(Bag\ B \mid glass) = \frac{0.2143}{0.5476} = 0.3913$$

$$P(Bag\ B\ \&\ glass) = \frac{1}{2} \times \frac{3}{7} = \frac{3}{14} = 0.2143$$

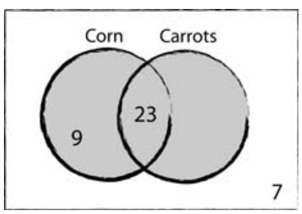
$$\text{P}(glass) = \frac{23}{42} = 0.5476$$

The answer is 0.39
The answer is **B**

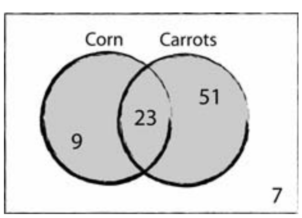
27. First draw the information you know in a Venn Diagram



Then determine the number of people who like only corn: *(32 – 23 = 9)*



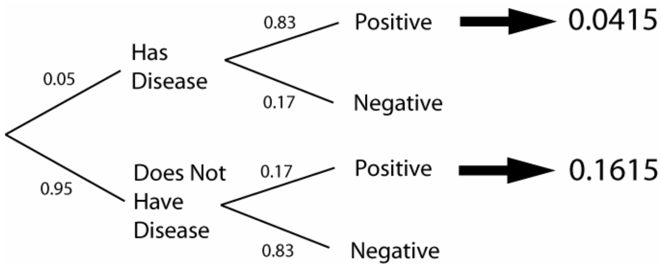
Determine the number of people who like only carrots *(90 – 23 – 9 – 7 = 51)*



The probability a person likes only carrots is $\frac{51}{90} = 0.567$

The answer is **A**

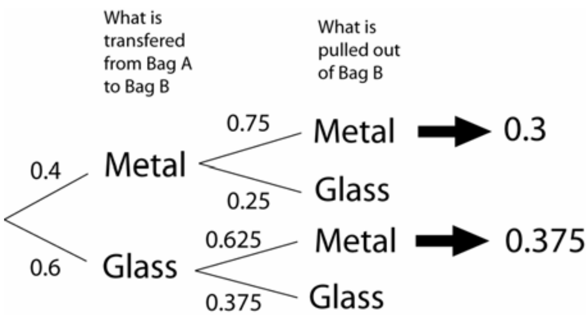
28. First draw a tree diagram to determine the probabilities



Add the results to determine the probability a person tests positive: Answer = 0.203

The answer is **A**

29. First draw a tree diagram to determine the probabilities



** To get the first set of probabilities, look at the first bag.*
There are 4 metal balls out of 10. The probability of drawing a metal ball is 0.4.
Using the complement, the probability of getting
a glass ball is 0.6

If a metal ball was transferred, it follows that there are 6 metal balls out of 8 in Bag B.
The probability of pulling a metal ball is then 6/8 = 0.75.
Likewise, there are 2 out of 8 glass balls, so the probability of pulling a glass ball is now 2/8 = 0.25.

If a glass ball was transferred, it follows that there are 5 metal balls out of 8 in Bag B.
The probability of pulling a metal ball from Bag B is 5/8 = 0.625.
Likewise, there are 3 out of 8 glass balls, so the probability of pulling a glass ball is 3/8 = 0.375.

Add the results shown in the diagram to get the probability the ball from Bag B is metal.
The answer is **C**

30. This question is asking *"What is the probability a glass ball was drawn from Bag A* ***given that*** *a metal ball is pulled out of Bag B"*

$$P\left(Glass\ from\ Bag\ A \mid Metal\ pulled\ from\ Bag\ B\right) = \frac{P\left(Glass\ from\ Bag\ A \,\&\, Metal\ pulled\ from\ Bag\ B\right)}{P\left(Metal\ pulled\ from\ Bag\ B\right)}$$

$$P\left(Glass\ from\ Bag\ A \mid Metal\ pulled\ from\ Bag\ B\right) = \frac{0.375}{0.675} = 0.556$$

The answer is 0.56
The answer is **B**

$P\left(Glass\ from\ Bag\ A \,\&\, Metal\ pulled\ from\ Bag\ B\right) = 0.6 \times 0.625 = 0.375$
$P(\,Metal\,) = 0.675$

31. The number of ways to draw 2 diamonds and 3 black cards is

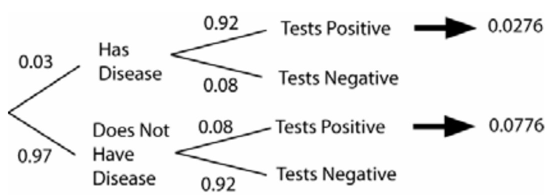
$${}_{13}C_2 \times {}_{26}C_3 = 202800$$

The number of ways to draw a five card hand is ${}_{52}C_5 = 2598960$

The probability is $\frac{202800}{2598960} \rightarrow \frac{65}{833}$

The answer is **C**

32. Draw a tree diagram to illustrate the probabilities



To answer this question, we must use Bayes' formula since we want the probability a person does not have the disease ***given that*** they test positive

$$P\left(No\ Disease \mid Positive\right) = \frac{0.0776}{0.1052} = 0.7376$$

P(No Disease & Positive) = 0.0776
P(Positive) = 0.0276+0.0776 = 0.1052

The answer is **A**

33. The number of ways to get a hand with exactly one heart is ${}_{13}C_1 \times {}_{39}C_4 = 1069263$
The number of ways to get a five card hand is ${}_{52}C_5 = 2598960$
The probability is $\dfrac{1069263}{2598960} = 0.41$
The answer is **D**

34. The number of ways to get a hand with no heart is ${}_{39}C_5 = 575757$
The number of ways to get a five card hand is ${}_{52}C_5 = 2598960$
The probability of getting no heart is $\dfrac{575757}{2598960} = 0.222$
Therefore, the probability of getting *at least one* heart is 1 – 0.222 = 0.78
The answer is **C**

35. Draw a tree diagram to illustrate the probabilities

$$P(Vendor\ A \mid Spoiled) = \frac{P(Vendor\ A \,\&\, Spoiled)}{P(Spoiled)}$$
$$P(Vendor\ A \mid Spoiled) = \frac{0.042}{0.1525}$$
$$P(Vendor\ A \mid Spoiled) = \frac{84}{305}$$
The answer is **C**

36. Exactly five males / two females can be chosen in ${}_{10}C_5 \times {}_{11}C_2$ ways
Any seven members can be chosen in ${}_{21}C_7$ ways
The probability is $\dfrac{{}_{10}C_5 \times {}_{11}C_2}{{}_{21}C_7} = 0.12$
The answer is **C**

37. We need the cases of (3 glass / 2 metal) + (4 glass / 1 metal) + (5 glass)
This is obtained with ${}_5C_3 \bullet {}_3C_2 + {}_5C_4 \bullet {}_3C_1 + {}_5C_5 = 46$
The total ways to select any 5 balls is ${}_8C_5 = 56$
The probability is $\frac{46}{56} = 0.82$
The answer is **D**

38. The first two digits must be 7, so there is only 1 possibility for each position.
The third digit must NOT be 7, so there are 9 possibilities. *(Recall that exactly two seven's means the first two digits are seven, and the third is not seven)*
The remaining digits can be anything from 0 – 9, so there are 10 possibilities.

$\underline{1}\ \ \underline{1}\ \ \underline{9}\ \ \underline{10}\ \ \underline{10}\ \ \underline{10}\ \ \underline{10}\ \ \underline{10}\ = 900\,000$

There are 10^8 = 100 000 000 arrangements without restrictions.

The probability is $\frac{900000}{100000000} = 0.009 = 0.01(\textit{Rounded})$
The answer is **A**

39. The number of ways of selecting two math books **or** two physics books is
${}_6C_2 + {}_{12}C_2 = 81$

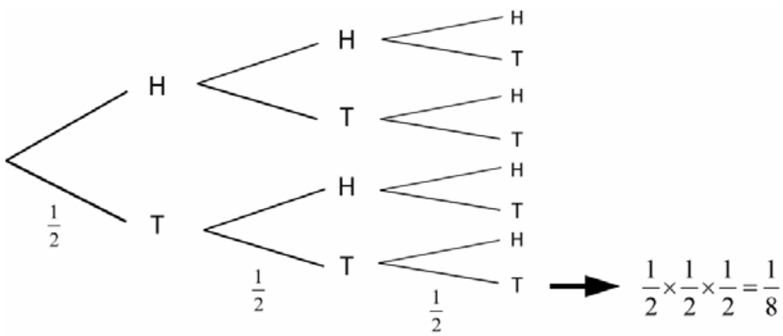
The number of ways to select any two books is ${}_{18}C_2 = 153$

The probability is $\frac{81}{153} = \frac{9}{17}$
The answer is **D**.

Written Response 1:

- According to the experiment, three tails occurs twice in 14 trials.
  The experimental probability is $\frac{2}{14} \to \frac{1}{7}$

- Draw the sample space

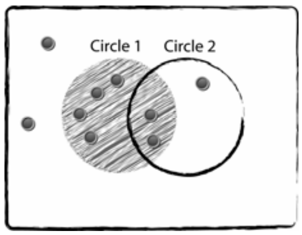


The theoretical probability from the sample space is $\frac{1}{8}$

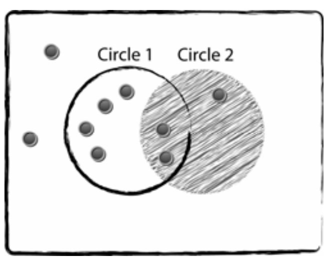
- The two answers above are different because insufficient trials were performed. Had several thousand flips been recorded, the experimental & theoretical probability would come very close to each other.

Written Response 2:

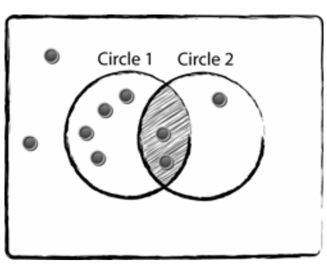
- $P\left(Circle\ 1\right) = \dfrac{favorable\ cases}{total\ cases} = \dfrac{6}{9} \rightarrow \dfrac{2}{3}$



- $P\left(Circle\ \ 2\right) = \dfrac{favorable\ \ cases}{total\ \ cases} = \dfrac{3}{9} \rightarrow \dfrac{1}{3}$



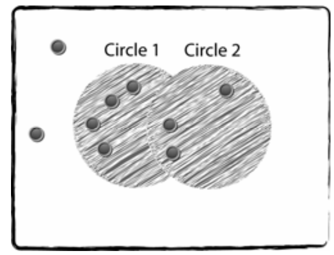
- $P\left(Circle\ \ 1\ \ and\ \ 2\right) = \dfrac{favorable\ cases}{total\ \ cases} = \dfrac{2}{9}$



- $P\left(Circle\ \ 1\ \ or\ \ 2\right) = P\left(Circle\ \ 1\right) + P\left(Circle\ \ 2\right) - P\left(Circle\ \ 1\ \ and\ \ 2\right)$

  $P\left(Circle\ \ 1\ \ or\ \ 2\right) = \dfrac{6}{9} + \dfrac{3}{9} - \dfrac{2}{9}$

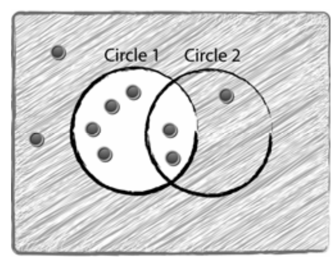
  $P\left(Circle\ \ 1\ \ or\ \ 2\right) = \dfrac{7}{9}$



- $P\left(NOT\ \ Circle\ \ 1\right) = 1\ -\ P\left(Circle\ \ 1\right)$

  $P\left(NOT\ \ Circle\ \ 1\right) = 1\ -\ \dfrac{2}{3}$

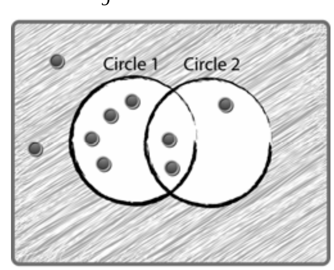
  $P\left(NOT\ \ Circle\ \ 1\right) = \dfrac{1}{3}$



- $P\left(NOT\ \ Circle\ \ 1\right) = 1\ -\ P\left(Circle\ \ 1\ \ or\ \ 2\right)$

  $P\left(NOT\ \ Circle\ \ 1\right) = 1\ -\ \dfrac{7}{9}$

  $P\left(NOT\ \ Circle\ \ 1\right) = \dfrac{2}{9}$



Written Response 3:

- Four aces can be in a hand in ${}_4C_4 \bullet {}_{48}C_1$ ways.

  Probability is $\frac{{}_4C_4 \; x \; {}_{48}C_1}{{}_{52}C_5} = \frac{48}{2598960}$ $= \boldsymbol{0.000018}$

- Three tens can be dealt in: ${}_4C_3 \bullet {}_{48}C_2$ ways.

  Probability is $\frac{{}_4C_3 \; x \; {}_{48}C_2}{{}_{52}C_5} = \frac{4512}{2598960}$ $= \boldsymbol{0.0017}$

- At most one queen means we need the cases of no queens, and one queen.
  This can be done in ${}_{48}C_5 + {}_4C_1 \bullet {}_{48}C_4$ ways.

  Probability is $\frac{{}_{48}C_5 + {}_4C_1 \bullet {}_{48}C_4}{{}_{52}C_5} = \frac{2490624}{2598960}$ $= \boldsymbol{0.96}$

- We need the cases of three black cards *(and two red)*, four black cards *(and one red)*, and five black cards.
  This can be done in ${}_{26}C_3 \; x \; {}_{26}C_2 + {}_{26}C_4 \; x \; {}_{26}C_1 + {}_{26}C_5$ ways.

  Probability is $\frac{{}_{26}C_3 x {}_{26}C_2 + {}_{26}C_4 x {}_{26}C_1 + {}_{26}C_5}{{}_{52}C_5} = \frac{1299480}{2598960}$ $= \boldsymbol{0.5}$