Principles of Mathematics 12

Probability

LESSON 3

PROBABILITY AND COMBINATORICS

Principles of Math 12 EXPLAINED!

By Barry Mabillard

Probability Lesson 3

Part 1: Basic Probability

Basic Probability: When pulling a single item from a set, use basic probability.

$$\text{Probability} = \frac{\text{Number Of Favorable Outcomes}}{\text{Total Possible Outcomes}}$$

Example 1: A bag contains three red balls and four green balls. One ball is pulled out. What is the probability of:

a) Pulling out a red ball?

The number of favorable cases is 3, since there are three red balls.
The total number of cases is 7, since there are seven balls altogether.

The probability of pulling out a red ball is $\frac{3}{7}$.

b) Pulling out a green ball?

The number of favorable cases is 4, since there are four green balls.
The total number of cases is 7, since there are seven balls altogether.

The probability of pulling out a green ball is $\frac{4}{7}$.

c) Pulling out a blue ball?

The probability is zero, since there are no blue balls.

Example 2: A letter is selected from the word **STATISTICS**. What is the probability the letter is an **S** or a **T** ?

There are three **S**'s and three **T**'s, leading to six favorable outcomes. There are ten letters altogether. The probability of pulling an **S** or a **T** is $\frac{6}{10} = \mathbf{0.60}$

Questions:

1) A jar contains 22 marbles. 4 are red, 3 are green, 7 are blue, and 8 are yellow. If a single marble is pulled out of the jar, find the probability of pulling:

a) a red marble

b) a blue or yellow marble

c) not green

d) purple

Answers:

1) a) A red marble has a probability of selection of $\frac{4}{22} = \mathbf{\textit{0.18}}$

b) A blue or yellow marble has a probability of selection of $\frac{7}{22} + \frac{8}{22} = \frac{15}{22} = \mathbf{\textit{0.68}}$

(Remember that "or" means to add probabilities together.)

c) 19 marbles are not green, so the probability of getting one of these is $\frac{19}{22} = \mathbf{\textit{0.86}}$

d) Since there are no purple marbles, the probability is **zero**.

Probability Lesson 3

Part 2: Probability With FCP & Permutations

Probability involving Fundamental Counting Principal & Permutations

Example 1: A 5 digit PIN number can begin with any digit (except zero) and the remaining digits have no restriction. If repeated digits are allowed, find the probability of the PIN code beginning with a 7 and ending with an 8.

First determine the number of possible arrangements for the PIN number that satisfy the stated restrictions.

$\underline{1}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{1} = 1000$ *There is only one possible digit for the first & last positions, and the middle positions can be any digit.*

Now determine the total number of arrangements

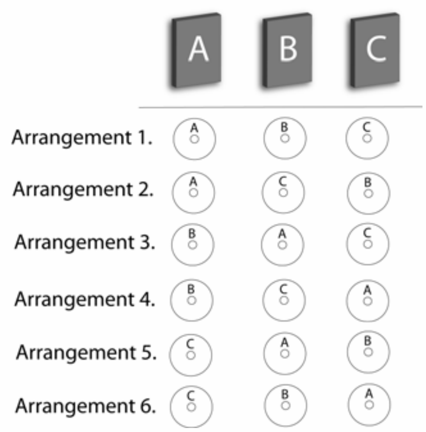
$\underline{9}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 90000$ *The first digit can't be zero*

The probability is $\frac{1000}{90000} = \frac{1}{90}$

Example 2: Three different DVD's and their corresponding DVD cases are randomly strewn about on a shelf. If a young child puts the DVD's in the cases at random, determine the probability of correctly matching all DVD's and cases.

Method 1: Draw out the possible cases:
There is only one favorable outcome out of six possible outcomes.

The answer is $\frac{1}{6}$



Method 2: There is only 1 favorable outcome out of 3! = 6 possible outcomes.

$$\frac{\text{Favorable outcomes}}{\text{Total possible outcomes}} = \frac{1}{3!} = \frac{1}{6}$$

Example 3: A security code consists of 8 digits, which may be any number from 0 to 9. Repetitions are allowed. Determine the probability a particular code begins with exactly two 7's, to the nearest hundredth.

The first two digits must each be 7, so there is only 1 possibility for each position. The third digit must **NOT** be 7, so there are 9 possibilities.
The remaining digits can be anything from 0 – 9, so there are 10 possibilities.

$$\underline{1}\ \underline{1}\ \underline{9}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10} = 900\,000$$

There are 10^8 = 100 000 000 arrangements without restrictions.

The probability is $\frac{900000}{100000000} = 0.009 = 0.01$(Rounded)

Probability Lesson 3

Part 2: Probability With FCP & Permutations

Example 4: There are 12 male athletes and 14 female athletes competing in a marathon. Find the probability of three different prizes being awarded to all males **or** all females.

The ways the top three prizes can go to men or women is ${}_{12}P_3 + {}_{14}P_3 = 3504$

(Use permutations since the prizes are different, adding since the groups are independent.)

The number of ways three prizes can be ordered randomly is ${}_{26}P_3 = 15600$

The probability is $\frac{3504}{15600} = 0.2246$

Example 5: Kevin, Rachel, and 6 other students are standing in a line. Determine the probability Kevin and Rachel are **not** standing together.

The number of lines with Kevin and Rachel **not** standing together is $8! - (7! \bullet 2!) = 30240$

The number of lines without restrictions is $8! = 40320$

The probability is $\frac{30240}{40320} = 0.75$

Questions:

1) A 4 digit PIN number can begin with any digit (except zero) and the remaining digits have no restriction. If repeated digits are allowed, find the probability of the PIN code beginning with a number greater than 7 and ending with a 3.

2) A security code consists of 6 digits, which may be any number from 0 to 9. The code can begin with any digit (except zero). No repetitions are allowed. Determine the probability a particular code begins with an even digit, to the nearest hundredth.

3) Jeff, Amy, and 5 other students are standing in a line. Determine the probability Jeff and Amy are standing together.

4) Jeff, Amy, and 5 other students are standing in a line. Determine the probability Jeff and Amy are not standing together.

5) There are 7 accountants and 4 marketing agents at a conference. Find probability of three different door prizes being awarded to all accountants **or** all marketing agents.

Answers:

1) There are 2 possibilities for the first digit. *(8 or 9)*
There are 10 possibilities for each of the middle digits *(any number)*
There is only one possibility for the final digit. *(Must be 3)*

Multiply together to get the favorable cases: 2 • 10 • 10 • 1 = 200
The total possible cases are 9 • 10 • 10 • 10 = 9 000

The probability is $\frac{200}{9000}$ = **0.022**

2) There are 4 possibilities for the first digit. *(2, 4, 6, or 8 → Remember that zero is excluded)*
There are 9 possibilities for the next digit *(since we have used up a number)* then 8 possibilities for the next digit, then 7, then 6, then 5 possibilities for the last digit.

Multiply together to get the favorable cases: 4 • 9 • 8 • 7 • 6 • 5 = 60480
The total possible cases are 9 • 9 • 8 • 7 • 6 • 5 = 136080

The probability is $\frac{60480}{136080}$ = **0.444**

3) Jeff and Amy can stand together in 2! • 6! = 1440 different ways
The number of possible lines without restrictions is 7! = 5040

The probability is $\frac{1440}{5040} = \frac{2}{7}$ = **0.29**

4) Jeff and Amy do not stand together in 7! – (2! • 6!) = 3600 different ways
The number of possible lines without restrictions is 7! = 5040

The probability is $\frac{3600}{5040} = \frac{5}{7}$ = **0.71**
(You could also use: *Probability of not standing together = 1 – probability of standing together.*)

5) The ways the top three prizes can go to accountants or marketing agents is ${}_7P_3 + {}_4P_3 = 234$
(Use permutations since the prizes are different, adding since the groups are independent.)
The number of ways three prizes can be ordered randomly is ${}_{11}P_3 = 990$

The probability is $\frac{234}{990}$ = **0.236**

Probability Lesson 3

Part 3: Probability With Combinations

Probability With Combinations: When pulling multiple items from a set, use combination probability.

Example 1: A bookcase contains 6 different math books and 12 different physics books. If a student randomly selects two of these books, determine the probability they are both math or both physics books.

The number of ways to select two math books **or** two physics books is ${}_6C_2 + {}_{12}C_2 = 81$

The number of ways to select any two books is ${}_{18}C_2 = 153$

The probability is $\frac{81}{153} = \frac{9}{17}$

Example 2: A jar contains 5 orange, 3 purple, 7 blue, and 5 green candies. If the total number of candies is 20, determine the probability that a handful of four candies contains one of each color.

The number of ways to select four candies, such that one from each color is obtained, is ${}_5C_1 \times {}_3C_1 \times {}_7C_1 \times {}_5C_1 = 525$.

(Multiply since you are creating one set using items from multiple sets)

The total number of ways to select four candies is ${}_{20}C_4 = 4845$

The probability is $\frac{{}_5C_1 \times {}_3C_1 \times {}_7C_1 \times {}_5C_1}{{}_{20}C_4} = \frac{525}{4845} = 0.108$

Example 3: A committee of 4 people is to be formed from a pool of 8 people. What is the probability James is on the committee?

If James is on the committee, this reduces the pool of people to 7, and 3 remain to be selected.

The committees with James is ${}_7C_3 = 35$, and the total possible committees is ${}_8C_4 = 70$

The probability James is on the committee is: $\frac{35}{70} = \boldsymbol{0.5}$

Example 4: Seven males and nine females are in a selection pool for a committee of 4 people. What is the probability **at most** 3 males are on the committee?

The easiest way to do this is to use: 1 – (Probability of 4 males on the committee)

4 males can be chosen in ${}_7C_4 = 35$ ways, and the total possible cases are ${}_{16}C_4 = 1820$.

Probability = $1 - \frac{35}{1820} = \frac{1785}{1820} = \boldsymbol{0.9808}$

Probability Lesson 3

Part 3: Probability With Combinations

Example 5: Lotto 7-45 has 45 numbers you can choose from, and you must pick 7. If a free ticket is given out by matching 2 correct numbers, what is the probability of getting this particular prize?

Out of the 7 winning numbers, 2 must be picked. ${}_7C_2 = 21$
Of the 38 non-winning numbers, 5 must be picked. ${}_{38}C_5 = 501942$
Number of favorable cases is ${}_7C_2 \cdot {}_{38}C_5 = 10540782$,
Total number of cases is ${}_{45}C_7 = 45379620$

The probability of winning a free lottery ticket is: $\frac{10540782}{45379620} = \boldsymbol{0.2323}$

Example 6: In the card game Bridge, 4 players are each given a hand of 13 cards.

a) What is the probability a player receives all the kings?
A hand can contain all 4 kings (*and 9 other cards*) in ${}_4C_4 \cdot {}_{48}C_9$ ways.
The total ways a 13 card hand can be dealt is ${}_{52}C_{13}$.

Probability of receiving all kings = $\frac{{}_4C_4 \cdot {}_{48}C_9}{{}_{52}C_{13}} = \frac{11677106640}{635013559600} = \boldsymbol{0.026}$

b) What is the probability a player does not have any 2's?
A hand can contain no 2's in ${}_{48}C_{13}$ ways.

Probability of receiving no 2's is $\frac{{}_{48}C_{13}}{{}_{52}C_{13}} = \frac{192928249300}{635013559600} = \boldsymbol{0.30}$

c) What is the probability a player receives at least 3 queens?
A hand can contain 3 or 4 queens in ${}_4C_3 \cdot {}_{48}C_{10} + {}_4C_4 \cdot {}_{48}C_9$ ways.

Probability = $\frac{{}_4C_3 \cdot {}_{48}C_{10} + {}_4C_4 \cdot {}_{48}C_9}{{}_{52}C_{13}} = \frac{27839970220}{635013559600} = \boldsymbol{0.044}$

d) What is the probability a player receives two 4's, four 6's, three 7's, three 10's, and an ace?
This hand can be selected in: ${}_4C_2 \cdot {}_4C_4 \cdot {}_4C_3 \cdot {}_4C_3 \cdot {}_4C_1$ ways.

Probability = $\frac{{}_4C_2 \cdot {}_4C_4 \cdot {}_4C_3 \cdot {}_4C_3 \cdot {}_4C_1}{{}_{52}C_{13}} = \frac{384}{635013559600} = \boldsymbol{0.00000000060}$

Probability Lesson 3

Part 3: Probability With Combinations

Questions:

1) A jar contains 2 white marbles and 5 green marbles. Three marbles are drawn simultaneously. What is the probability that:

a) two marbles are green?

b) at least two marbles are green?

c) no white marbles are drawn?

2) A school committee of 5 members is to be formed from a selection pool of 9 boys and 7 girls. What is the probability that:

a) all boys are on the committee?

b) there are 2 boys and 3 girls on the committee?

c) the girls form a majority?

d) Jane is on the committee, and there is no other restriction?

e) Brian is on the committee, but Nathan is not, with no other restriction?

3) Five males and six females are in a selection pool for a committee of 3 people. What is the probability **at most** 2 males are on the committee?

4) In a sample of households with a vehicle, four owned a truck and five owned a car. If three households were randomly selected from the sample, calculate the probability that two would have a car and one would have a truck

5) In a five card hand from a deck of 52 cards, what is the probability of receiving:

a) four aces?

b) three 10's

c) at most one queen?

d) at least three black cards?

Probability Lesson 3

Part 3: Probability With Combinations

Answers:

1) a) Two green marbles and one white can be selected in ${}_5C_2 \cdot {}_2C_1$ ways. Probability is $\frac{{}_5C_2 \cdot {}_2C_1}{{}_7C_3} = \frac{20}{35}$ = ***0.5714***

b) We need the cases of 2 green marbles being drawn or 3 green marbles being drawn.

This can be done in ${}_5C_2 \cdot {}_2C_1 + {}_5C_3$ ways. Probability is $\frac{{}_5C_2 \cdot {}_2C_1 + {}_5C_3}{{}_7C_3} = \frac{30}{35}$ = ***0.8571***

c) No white marbles means all green marbles, and can be drawn in ${}_5C_3$ ways. Probability is $\frac{{}_5C_3}{{}_7C_3} = \frac{10}{35}$ = ***0.2857***

2) a) all boys can be selected in ${}_9C_5$ ways. Probability is $\frac{{}_9C_5}{{}_{16}C_5} = \frac{126}{4368}$ = ***0.02885***

b) Two boys and three girls can be selected in ${}_9C_2 \times {}_7C_3$ ways. Probability is $\frac{{}_9C_2 \bullet {}_7C_3}{{}_{16}C_5} = \frac{1260}{4368}$ = ***0.2885***

c) the girls can form a majority if there are 3 girls (2 boys), 4 girls (1 boy) or 5 girls.

This can be done in ${}_7C_3 \cdot {}_9C_2 + {}_7C_4 \cdot {}_9C_1 + {}_7C_5$ ways. Probability is $\frac{{}_7C_3 \bullet {}_9C_2 + {}_7C_4 \bullet {}_9C_1 + {}_7C_5}{{}_{16}C_5} = \frac{1596}{4368}$ = ***0.3654***

d) If Jane is on the committee, we have 15 people left to select from, and 4 positions to fill. ${}_{15}C_4$

Probability is $\frac{{}_{15}C_4}{{}_{16}C_5} = \frac{1365}{4368}$ = ***0.3125***

e) If Brian is on the committee, but Nathan is not, we have 14 people left to select from, and 4 positions to fill. ${}_{14}C_4$

Probability is $\frac{{}_{14}C_4}{{}_{16}C_5} = \frac{1001}{4368}$ = ***0.2292***

3) The easiest way to do this calculation is to use:
Probability of at most 2 males = 1 – Probability of 3 males

Probability of at most 2 males = $1 - \frac{{}_5C_3}{{}_{11}C_3} = 0.9393$

4) The number of cases with two cars and one truck is ${}_5C_2 \times {}_4C_1 = 40$

The total number of cases is ${}_9C_3 = 84$

The probability of 2 cars and 1 truck is $\frac{40}{84} = 0.476$

5) a) Four aces can be in a hand in ${}_4C_4 \cdot {}_{48}C_1$ ways. Probability is $\frac{{}_4C_4 \times {}_{48}C_1}{{}_{52}C_5} = \frac{48}{2598960}$ = ***0.000018***

b) Three tens can be dealt in: ${}_4C_3 \cdot {}_{48}C_2$ ways. Probability is $\frac{{}_4C_3 \times {}_{48}C_2}{{}_{52}C_5} = \frac{4512}{2598960}$ = ***0.0017***

c) At most one queen means we need the cases of no queens, and one queen.

This can be done in ${}_{48}C_5 + {}_4C_1 \cdot {}_{48}C_4$ ways. Probability is $\frac{{}_{48}C_5 + {}_4C_1 \cdot {}_{48}C_4}{{}_{52}C_5} = \frac{2490624}{2598960}$ = ***0.96***

d) We need the cases of three black cards (and two red), four black cards (and one red), and five black cards. This can be done in ${}_{26}C_3 \times {}_{26}C_2$ **+** ${}_{26}C_4 \times {}_{26}C_1$ **+** ${}_{26}C_5$ ways.

Probability is $\frac{{}_{26}C_3 \times {}_{26}C_2 + {}_{26}C_4 \times {}_{26}C_1 + {}_{26}C_5}{{}_{52}C_5} = \frac{1299480}{2598960}$ = ***0.5***