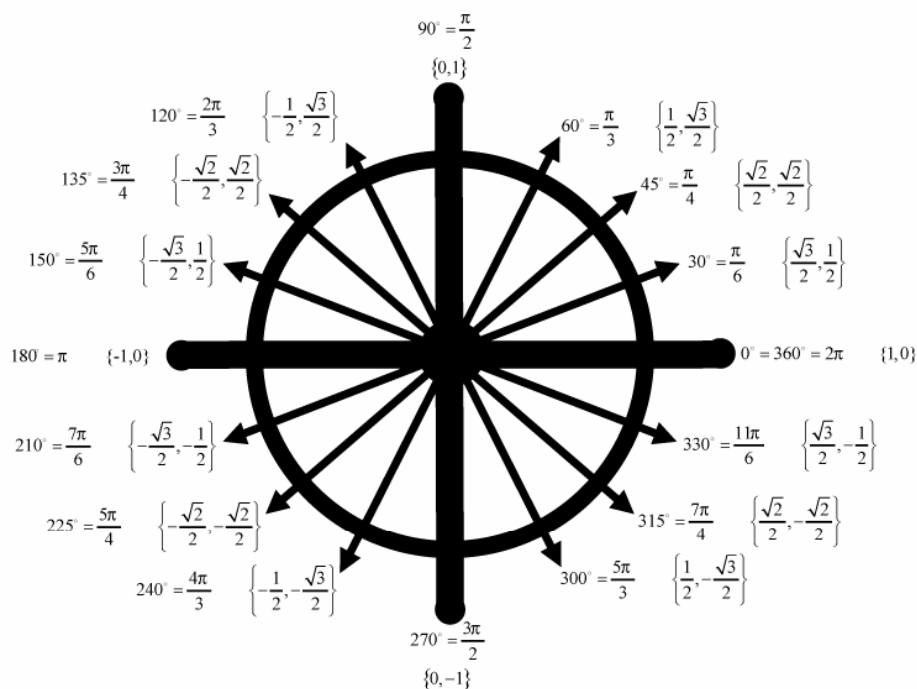


Principles of Mathematics 12

# TRIGONOMETRY I



## LESSON THREE

### Six Trigonometric Ratios

Principles of  
Math 12

**EXPLAINED!**

By  
Barry  
Mabillard

# TRIGONOMETRY LESSON 3

## PART I FINDING QUADRANTS

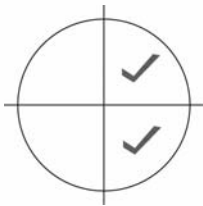
Look at the angle shown in the diagram to the right.

Is it possible to predict if  $\sin\theta$  or  $\cos\theta$  will be positive or negative?

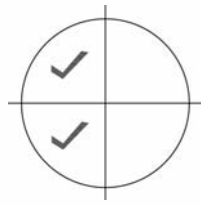
The answer is yes, and you can predict the sign by knowing which quadrant the terminal arm is located in.

Memorize the following rules:

$\cos\theta$  is positive in  
quadrants I & IV  
(where  $x$  is positive.)



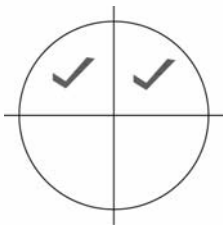
$\cos\theta$  is negative in  
quadrants II & III  
(where  $x$  is negative.)



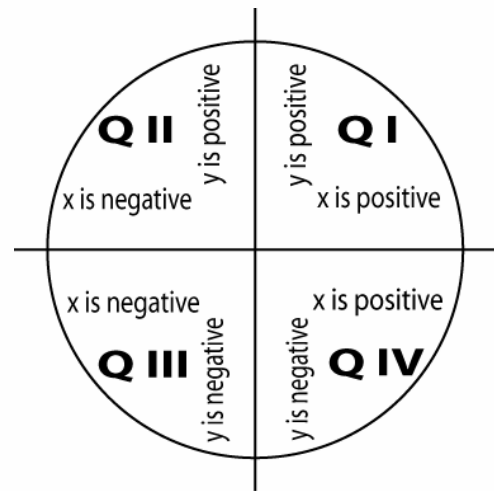
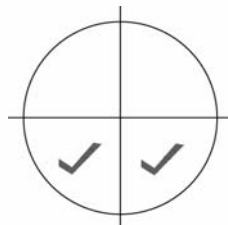
You can easily verify these rules in your calculator by taking the cosine of different angles.

QI:  $\cos 60^\circ = 0.5$   
QII:  $\cos 120^\circ = -0.5$   
QIII:  $\cos 240^\circ = -0.5$   
QIV:  $\cos 300^\circ = 0.5$

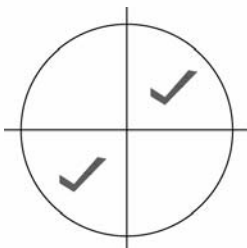
$\sin\theta$  is positive in  
quadrants I & II  
(where  $y$  is positive.)



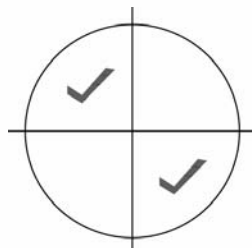
$\sin\theta$  is negative in  
quadrants III & IV  
(where  $y$  is negative.)



$\tan\theta$  is positive in  
quadrants I & III  
( $x$  &  $y$  have the same signs.)



$\tan\theta$  is negative in  
quadrants II & IV  
( $x$  &  $y$  have different signs.)



The signs of the reciprocal trig ratios can be determined by remembering the following:

$\sec\theta$  follows the same pattern as  $\cos\theta$

$\csc\theta$  follows the same pattern as  $\sin\theta$

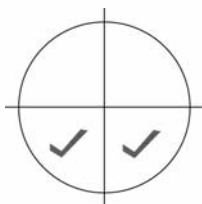
$\cot\theta$  follows the same pattern as  $\tan\theta$

# TRIGONOMETRY LESSON 3

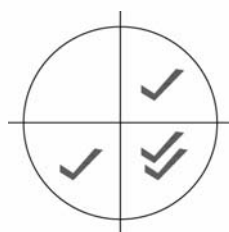
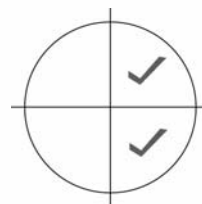
## PART 1 FINDING QUADRANTS

**Example 1:**  $\sin\theta < 0$  &  $\cos\theta > 0$ . What quadrant is the angle in?

$\sin\theta$  is negative in these quadrants



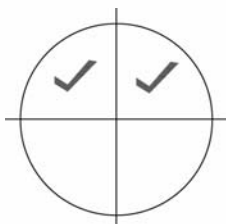
$\cos\theta$  is positive in these quadrants



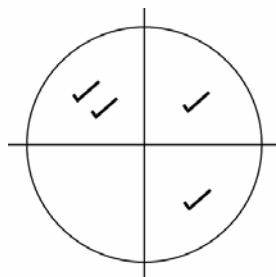
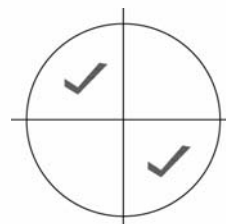
We have overlap in quadrant IV, so that is where the angle is.

**Example 2:**  $\csc\theta > 0$  &  $\cot\theta = -\frac{\sqrt{3}}{3}$ . What quadrant is the angle in?

$\csc\theta$  follows the same pattern as  $\sin\theta$ , and  $\sin\theta$  is positive in these quadrants



$\cot\theta$  follows the same pattern as  $\tan\theta$ , and  $\tan\theta$  is negative in these quadrants



We have overlap in quadrant II, so that is where the angle is.

Even though an actual value is given for  $\cot\theta$ , it is not needed if we only want to find the quadrant.

We do need the negative though, since that indicates  $\cot\theta < 0$ .

# TRIGONOMETRY LESSON 3

## PART 1 FINDING QUADRANTS

**Questions:** For each of the following, state what quadrant the angle is in:

1)  $\sin \theta > 0$  &  $\cos \theta < 0$



7)  $\sin \theta < 0$  &  $\sec \theta > 0$



2)  $\csc \theta > 0$  &  $\tan \theta > 0$



8)  $\sin \theta > 0$  &  $\tan \theta > 0$



3)  $\cot \theta < 0$  &  $\sec \theta > 0$



9)  $\cos \theta = -0.2$  &  $\cot \theta < 0$



4)  $\sec \theta = -2$  &  $\tan \theta < 0$



10)  $\cot \theta > 0$  &  $\sin \theta < 0$



5)  $\csc \theta = \frac{1}{5}$  &  $\cos \theta = -\frac{5}{6}$



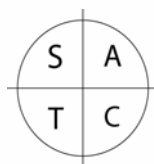
11)  $\cot \theta < 0$  &  $\csc \theta = \frac{\sqrt{3}}{3}$



6)  $\cos \theta > 0$  &  $\tan \theta < 0$



12)  $\csc \theta < 0$  &  $\sec \theta < 0$



**Answers:**

- 1) II
- 2) I
- 3) IV
- 4) II
- 5) II
- 6) IV
- 7) IV
- 8) I
- 9) II
- 10) III
- 11) II
- 12) III

An alternative way of doing these questions is to use the **CAST** rule.

**C:**  $\cos \theta$  (and  $\sec \theta$ ) is positive, everything else is negative.

**A:** all are positive

**S:**  $\sin \theta$  (and  $\csc \theta$ ) is positive, everything else is negative.

**T:**  $\tan \theta$  (and  $\cot \theta$ ) is positive, everything else is negative.

If you use this method, remember that you start in quadrant IV and spell it out counterclockwise.

# TRIGONOMETRY LESSON 3

## PART II SIX TRIGONOMETRIC RATIOS

The six trigonometric ratios are shown to the right.

Given a point, it is possible to evaluate all six ratios by creating a triangle and determining the lengths of the three sides.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\csc \theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

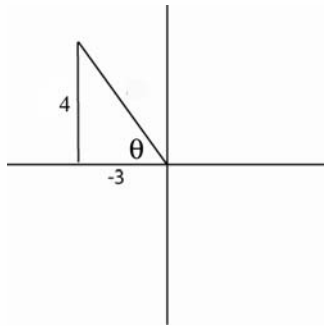
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\cot \theta = \frac{\text{adjacent}}{\text{opposite}}$$

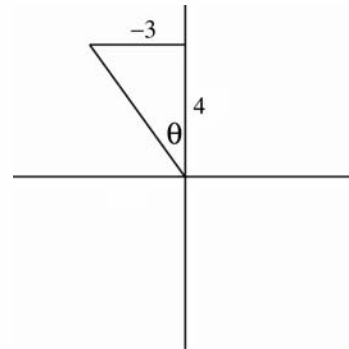
**Example 1:** Given the point P(-3, 4), find the six trigonometric ratios.

1) Draw your terminal arm from the origin to the given point. Form a triangle by drawing a line from the point to the closest x-axis!

**Draw like this.**



**NOT like this**



2) Use Pythagoras to find the unknown side:

$$a^2 + b^2 = c^2$$

$$(-3)^2 + 4^2 = c^2$$

$$9 + 16 = c^2$$

$$25 = c^2$$

$$c = 5$$

3) Now that all three sides are known, list the trigonometric ratios.

$$\sin \theta = \frac{4}{5}$$

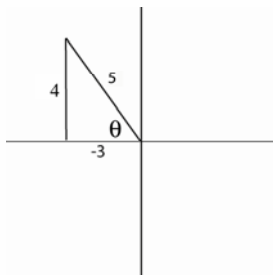
$$\csc \theta = \frac{5}{4}$$

$$\cos \theta = \frac{-3}{5}$$

$$\sec \theta = \frac{5}{-3}$$

$$\tan \theta = \frac{4}{-3}$$

$$\cot \theta = \frac{-3}{4}$$



\*The hypotenuse is **ALWAYS** positive!

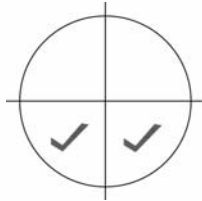
# TRIGONOMETRY LESSON 3

## PART II SIX TRIGONOMETRIC RATIOS

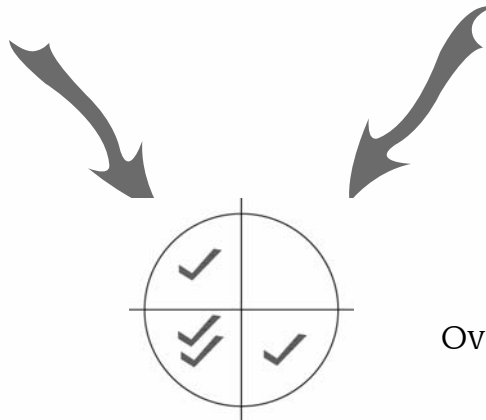
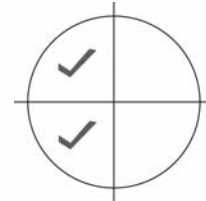
**Example 2:** If  $\csc \theta = -\frac{13}{5}$  and  $\cos \theta < 0$ , find the six trigonometric ratios.

1) First find the quadrant the angle is in:

$\csc \theta$  follows the same pattern as  $\sin \theta$ ,  
and  $\sin \theta$  is negative in these quadrants

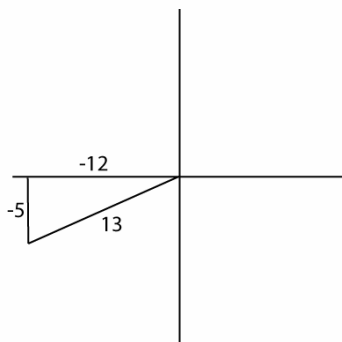


$\cos \theta$  is negative in  
these quadrants



Overlap in quadrant III.

2) Draw the triangle, and find the unknown side using Pythagoras:



$$a^2 + b^2 = c^2$$

$$(a)^2 + (-5)^2 = 13^2$$

$$a^2 + 25 = 169$$

$$a^2 = 144$$

$$a = 12$$

3) Now find the six trigonometric ratios.

$$\sin \theta = \frac{-5}{13}$$

$$\csc \theta = \frac{13}{-5}$$

$$\cos \theta = \frac{-12}{13}$$

$$\sec \theta = \frac{13}{-12}$$

$$\tan \theta = \frac{-5}{-12} = \frac{5}{12}$$

$$\cot \theta = \frac{12}{5}$$

Even though the answer of 12 from Pythagoras is positive, you must include a minus sign since the number is being placed on the negative x-axis. Remember to label all triangle sides with the correct sign!

# TRIGONOMETRY LESSON 3

## PART II SIX TRIGONOMETRIC RATIOS

**Questions:** For each of the following questions, state the six trigonometric ratios:

1)  $P(\sqrt{3}, -1)$

\*Don't forget to rationalize the denominator when there is a radical on the bottom!

2)  $P(-2, \sqrt{5})$

6)  $\csc \theta = -\frac{5}{4}$  &  $\cot \theta > 0$

3)  $P(-2, -3)$

7)  $\cot \theta = \frac{2}{5}$  &  $\csc \theta < 0$

4)  $\cos \theta = -\frac{1}{3}$  &  $\sin \theta > 0$

8)  $\sin \theta = \frac{1}{2}$  &  $\sec \theta > 0$

5)  $\sec \theta = -\frac{5}{2}$  &  $\tan \theta < 0$

9)  $\tan \theta = \sqrt{3}$  &  $\cos \theta < 0$

# TRIGONOMETRY LESSON 3

## PART II SIX TRIGONOMETRIC RATIOS

$$\sin \theta = \frac{-1}{2}$$

$$\csc \theta = -2$$

$$1) \quad \cos \theta = \frac{\sqrt{3}}{2}$$

$$\sec \theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\tan \theta = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\cot \theta = -\sqrt{3}$$

$$\sin \theta = \frac{\sqrt{5}}{3}$$

$$\csc \theta = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

$$2) \quad \cos \theta = \frac{-2}{3}$$

$$\sec \theta = \frac{3}{-2}$$

$$\tan \theta = \frac{\sqrt{5}}{-2}$$

$$\cot \theta = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$$

$$\sin \theta = \frac{-3}{\sqrt{13}} = \frac{-3\sqrt{13}}{13}$$

$$\csc \theta = \frac{\sqrt{13}}{-3}$$

$$3) \quad \cos \theta = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{13}$$

$$\sec \theta = \frac{\sqrt{13}}{-2}$$

$$\tan \theta = \frac{-3}{-2} = \frac{3}{2}$$

$$\cot \theta = \frac{2}{3}$$

4) The angle is in quadrant II

$$\sin \theta = \frac{2\sqrt{2}}{3}$$

$$\csc \theta = \frac{3}{2\sqrt{2}} = \frac{3\sqrt{2}}{4}$$

$$\cos \theta = \frac{-1}{3}$$

$$\sec \theta = -3$$

$$\tan \theta = \frac{2\sqrt{2}}{-1} = -2\sqrt{2}$$

$$\cot \theta = \frac{-1}{2\sqrt{2}} = \frac{-\sqrt{2}}{4}$$

5) The angle is in quadrant II

$$\sin \theta = \frac{\sqrt{21}}{5}$$

$$\csc \theta = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$$

$$\cos \theta = \frac{-2}{5}$$

$$\sec \theta = \frac{5}{-2}$$

$$\tan \theta = \frac{\sqrt{21}}{-2}$$

$$\cot \theta = \frac{-2}{\sqrt{21}} = \frac{-2\sqrt{21}}{21}$$

6) The angle is in quadrant II

$$\sin \theta = \frac{-4}{5}$$

$$\csc \theta = \frac{5}{-4}$$

$$\cos \theta = \frac{-3}{5}$$

$$\sec \theta = \frac{5}{-3}$$

$$\tan \theta = \frac{-4}{-3} = \frac{4}{3}$$

$$\cot \theta = \frac{3}{4}$$



# TRIGONOMETRY LESSON 3

## PART II SIX TRIGONOMETRIC RATIOS

**7)** The angle is in quadrant III

$$\sin \theta = \frac{-5}{\sqrt{29}} = \frac{-5\sqrt{29}}{29} \quad \csc \theta = \frac{\sqrt{29}}{-5}$$

$$\cos \theta = \frac{-2}{\sqrt{29}} \quad \sec \theta = \frac{-2\sqrt{29}}{29}$$

$$\tan \theta = \frac{-5}{-2} = \frac{5}{2} \quad \cot \theta = \frac{2}{5}$$

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**8)** The angle is in quadrant I

$$\sin \theta = \frac{1}{2} \quad \csc \theta = 2$$

$$\cos \theta = \frac{\sqrt{3}}{2} \quad \sec \theta = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\tan \theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \quad \cot \theta = \frac{\sqrt{3}}{1} = \sqrt{3}$$

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**9)** The angle is in quadrant III

$$\sin \theta = \frac{-\sqrt{3}}{2} \quad \csc \theta = \frac{2}{-\sqrt{3}} = \frac{2\sqrt{3}}{-3}$$

$$\cos \theta = \frac{-1}{2} \quad \sec \theta = -2$$

$$\tan \theta = \frac{-\sqrt{3}}{-1} = \sqrt{3} \quad \cot \theta = \frac{-1}{-\sqrt{3}} = \frac{\sqrt{3}}{3}$$