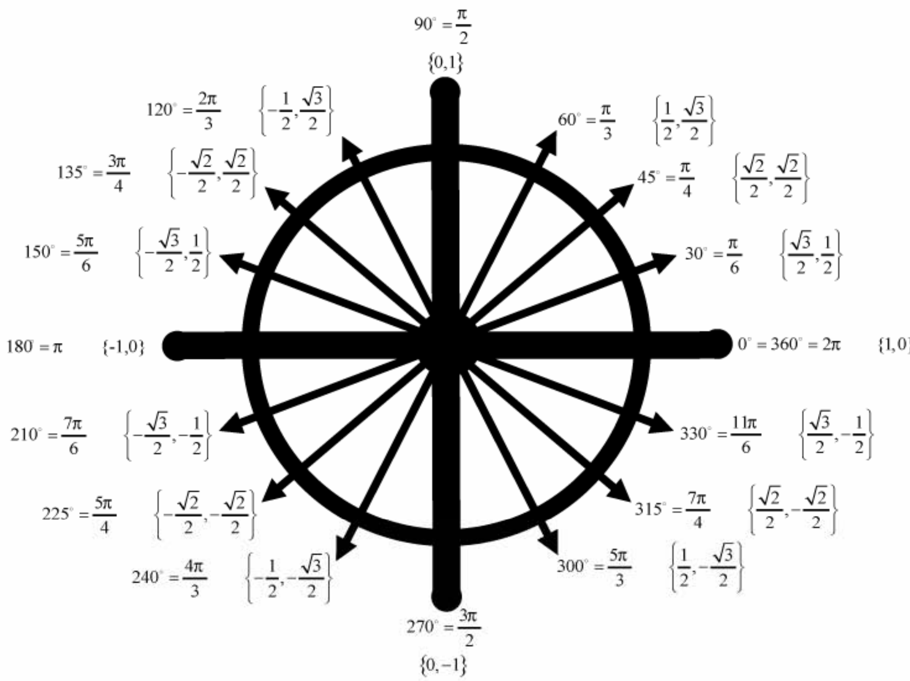
Principles of Mathematics 12

TRIGONOMETRY I



LESSON FIVE

Graphing b & c

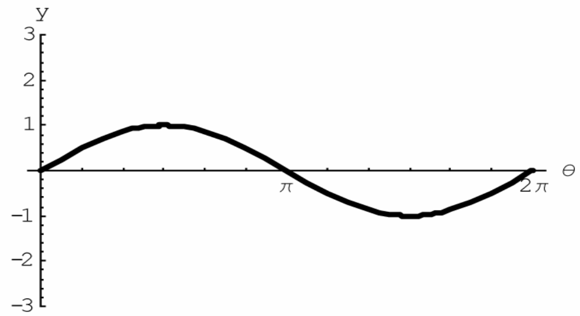
Principles of Math 12 EXPLAINED!

By Barry Mabillard

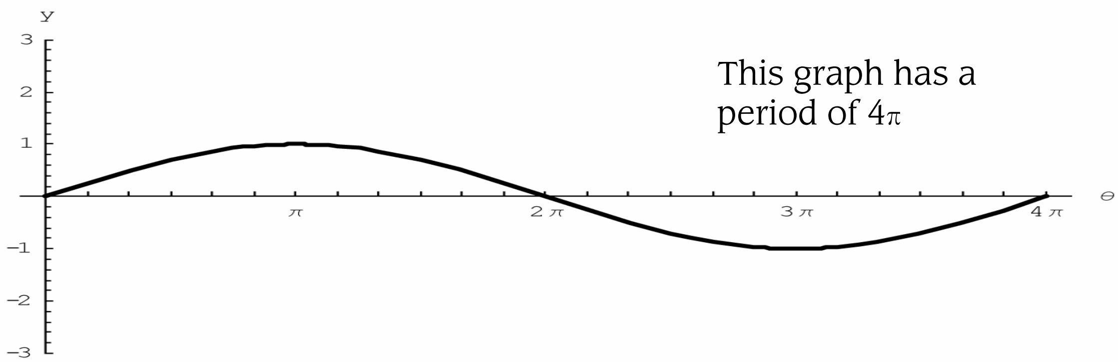
TRIGONOMETRY LESSON FIVE

PART I - PERIOD

The period of a graph is defined as the length of one complete cycle.

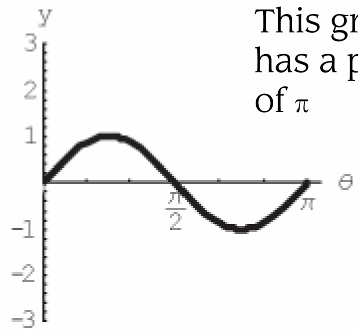


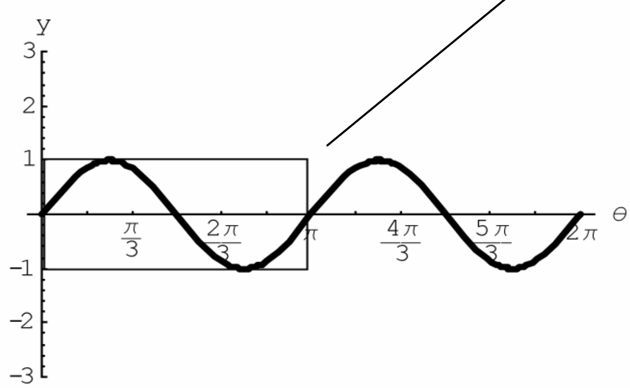
This graph has a period of 2π



This graph has a period of 4π

This graph has a period of π





Most graphs given to you won't be as simple as the first three. In trig graphs that are continuous, you will have to first identify a sine or cosine pattern before you can determine the period.

The easiest way to do this is to draw a square around either pattern and look at the length.

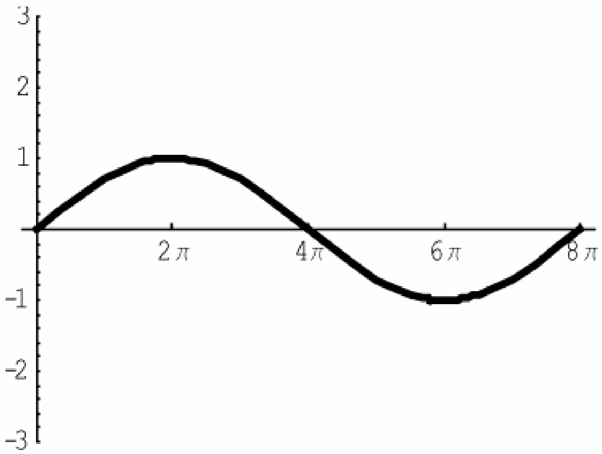
The graph on the left has a period of π

TRIGONOMETRY LESSON FIVE

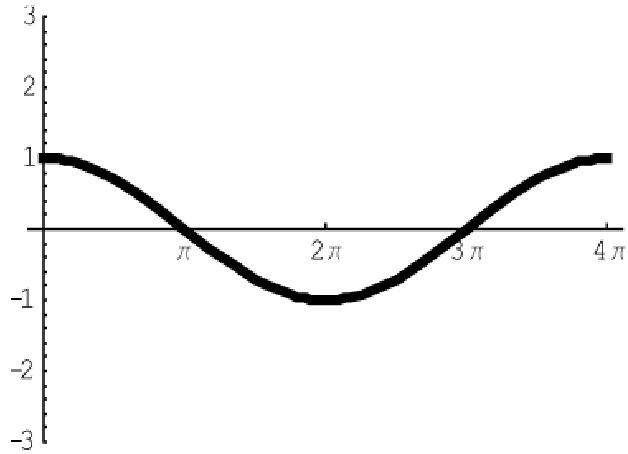
PART I - PERIOD

***Questions:** For each of the following graphs, draw a rectangle around the indicated pattern and state the period.*

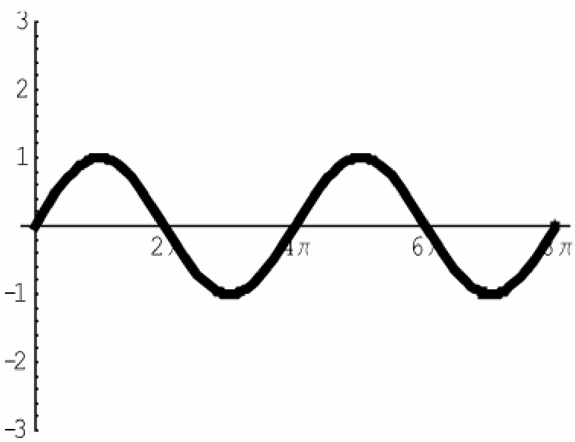
1) Draw a rectangle around a **sine** pattern.



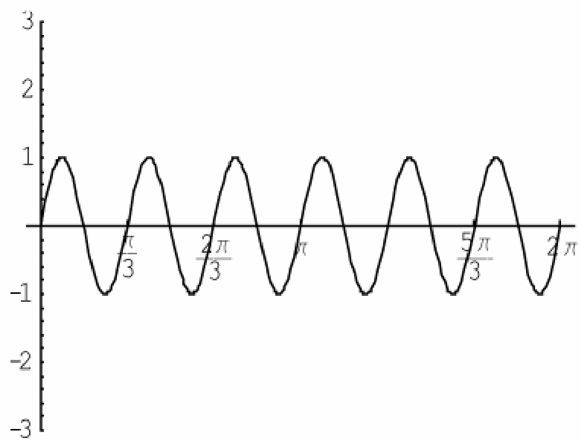
2) Draw a rectangle around a **cosine** pattern.



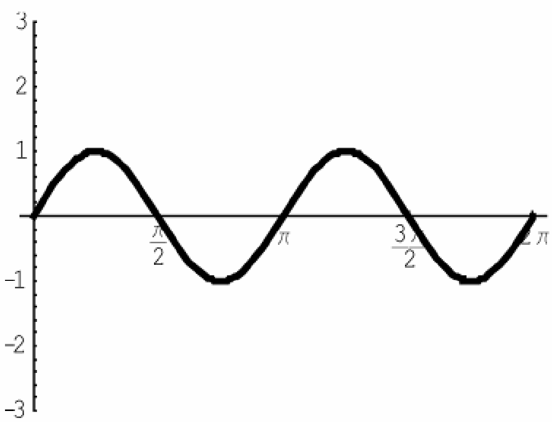
3) Draw a rectangle around a **sine** pattern.



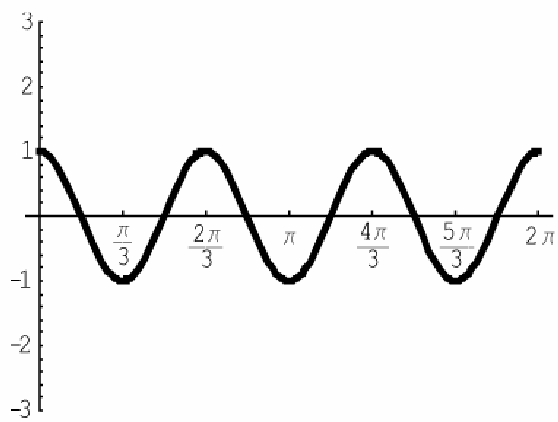
4) Draw a rectangle around a **sine** pattern.



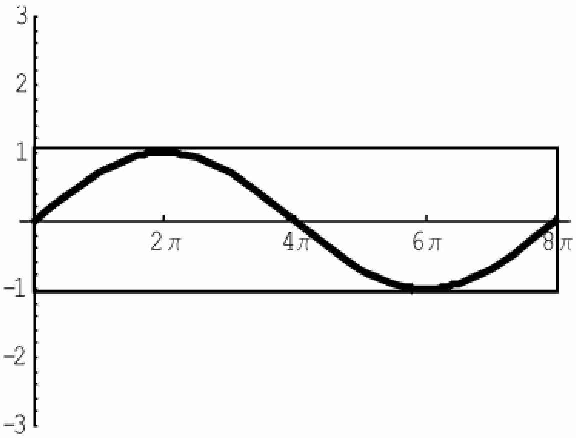
5) Draw a rectangle around a **sine** pattern.



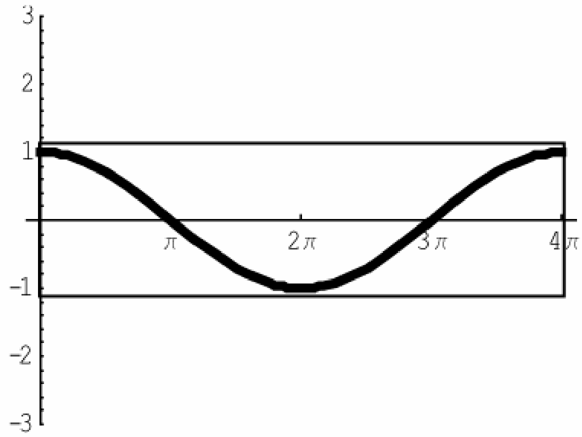
6) Draw a rectangle around a **cosine** pattern.



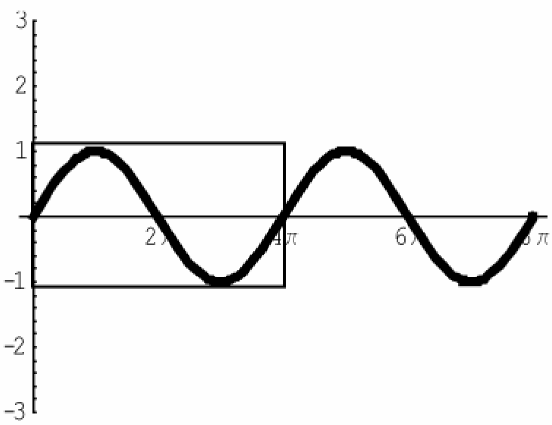
ANSWERS: **1)** Period = 8π



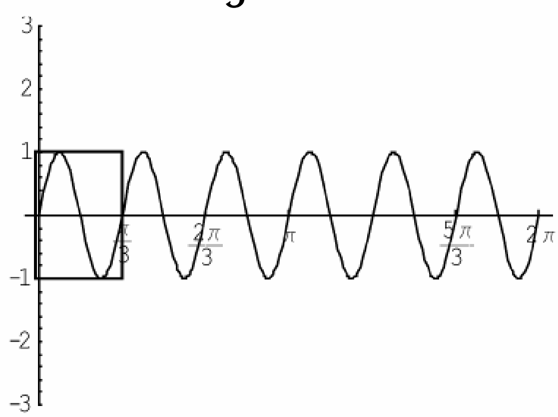
2) Period = 4π



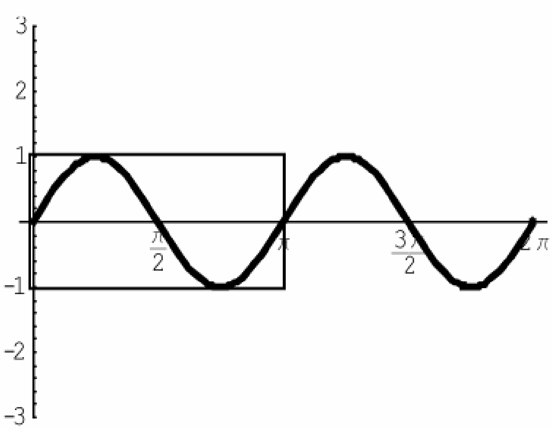
3) Period = 4π



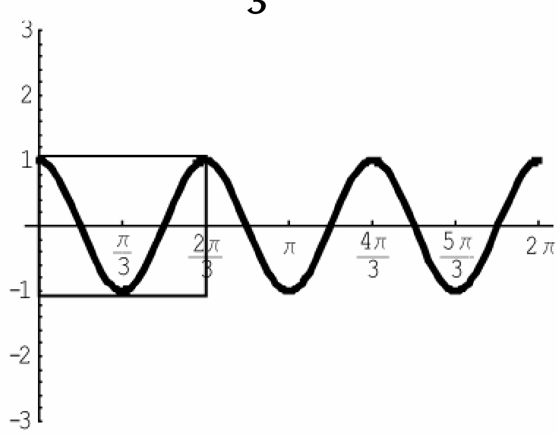
4) Period = $\frac{\pi}{3}$



5) Period = π



6) Period = $\frac{2\pi}{3}$



TRIGONOMETRY LESSON FIVE

PART II - THE B VALUE

The "b" value represents the number of cycles a trig graph has within a span of 2π.
It is the number that you see in a trig function right beside θ. ($y = \sin b\theta$)
The b value is <u>NOT</u> the period.

The b-value and period (for radians) are related by the formula: $\text{Period} = \dfrac{2\pi}{b}$ *or* $b = \dfrac{2\pi}{\text{Period}}$

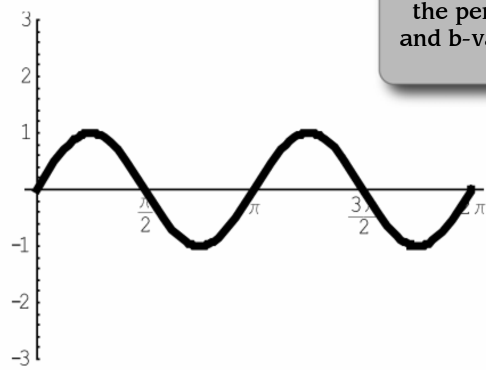
The b-value and period (for degrees) are related by the formula: $\text{Period} = \dfrac{360^\circ}{b}$ *or* $b = \dfrac{360^\circ}{\text{Period}}$

Note that $\tan\theta$ graphs do not use these equations for the period and b-value.

<u>Example 1:</u> Draw the graph of $y = \sin 2\theta \quad (0 \le \theta \le 2\pi)$

The first step in graphing this trig function is to find the period.

$$\text{Period} = \frac{2\pi}{b}$$
$$\text{Period} = \frac{2\pi}{2}$$
$$\text{Period} = \pi$$

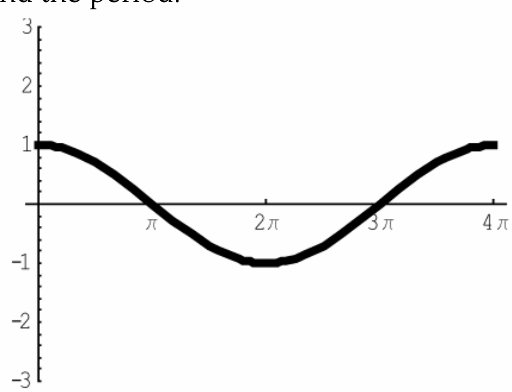


Once we know the period, draw the graph from 0 to 2π, since that is the specified domain.

<u>Example 2:</u> Draw the graph of $y = \cos\dfrac{1}{2}\theta \quad (0 \le \theta \le 4\pi)$

The first step in graphing this trig function is to find the period.

$$\text{Period} = \frac{2\pi}{b}$$
$$\text{Period} = \frac{2\pi}{0.5}$$
$$\text{Period} = 4\pi$$



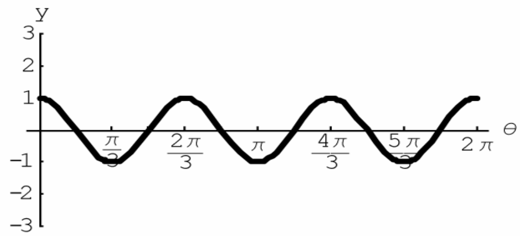
Once we know the period, draw the graph from 0 to 4π since that is the specified domain.

TRIGONOMETRY LESSON FIVE

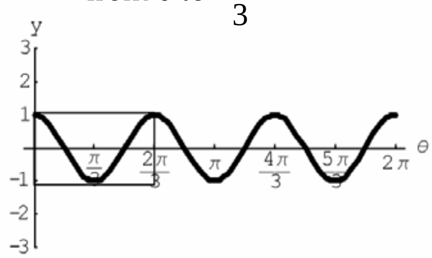
PART II - THE B VALUE

Given a graph, you must find the b value before you can write the equation.

Example 1: Find the cosine equation of the following graph:



Step 1: First you need to draw a rectangle around the cosine pattern. In this graph, we can easily see a cosine patten going from 0 to $\frac{2\pi}{3}$



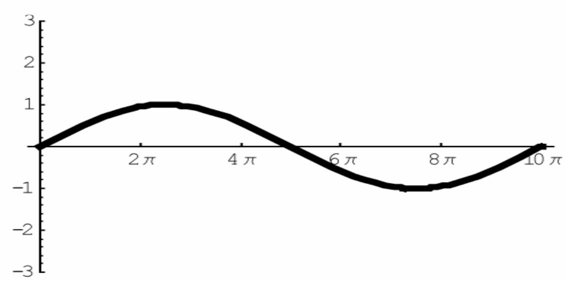
Step 2: Once you identify the period, find *b* by performing the following calculation:

$$b = \frac{2\pi}{\text{Period}}$$
$$b = \frac{2\pi}{\frac{2\pi}{3}}$$
$$b = 2\cancel{\pi} \times \frac{3}{2\cancel{\pi}}$$
$$b = 3$$

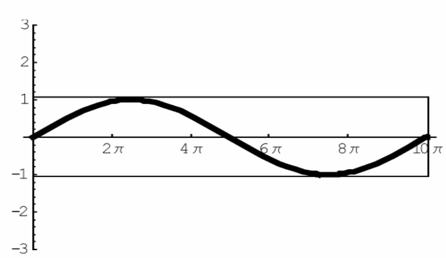
Step 3: Now that we have the *b* value, and a cosine pattern is identified, we can write the equation :

$$y = \cos 3\theta$$

Example 2: Find the sine equation of the following graph:



Step 1: First you need to draw a rectangle around the sine pattern you want to use. In this graph, we can easily see a sine pattern going from 0 to 10π



Step 2: Once you identify the period, find *b* by performing the following calculation:

$$b = \frac{2\pi}{\text{Period}}$$
$$b = \frac{2\cancel{\pi}}{10\cancel{\pi}}$$
$$b = \frac{1}{5}$$

Step 3: Now that we have the *b* value, and we identified a sine pattern, we can write the equation:

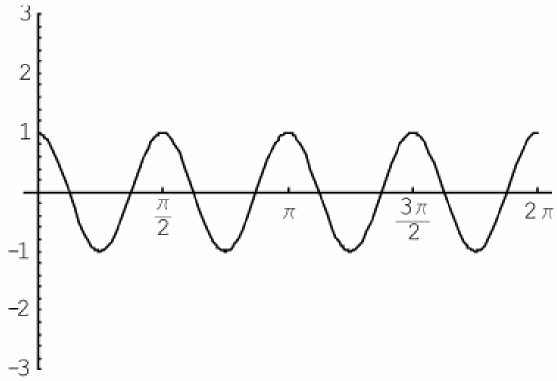
$$y = \sin\frac{1}{5}\theta$$

TRIGONOMETRY LESSON FIVE

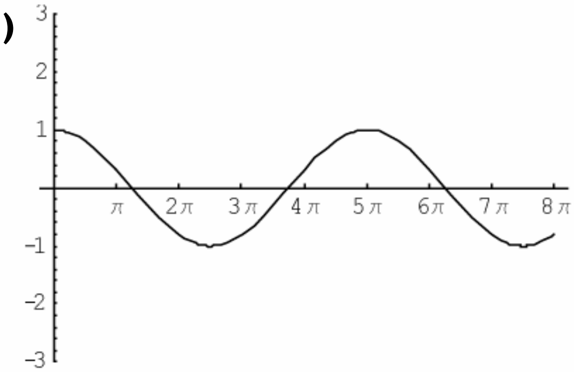
PART II - THE B VALUE

Questions: For each of the following graphs, write the equation:

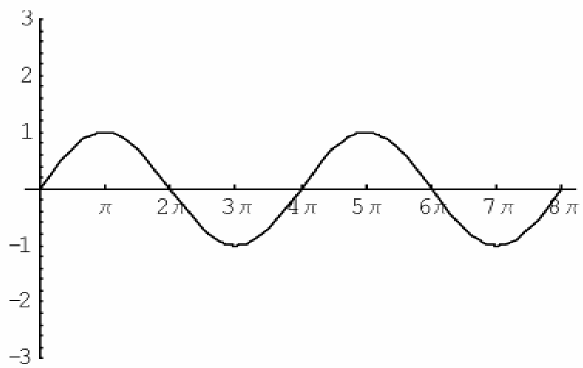
1)



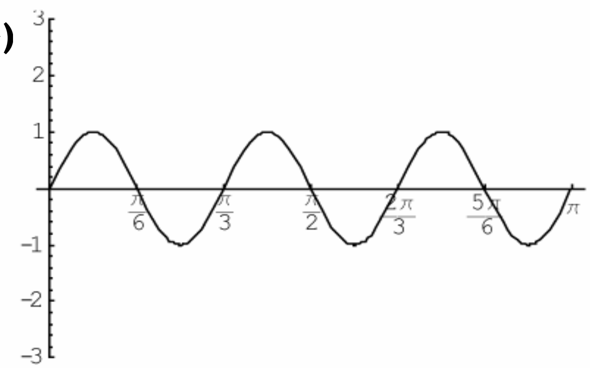
2)



3)

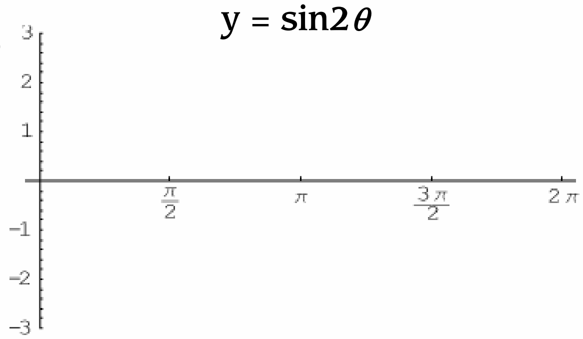


4)

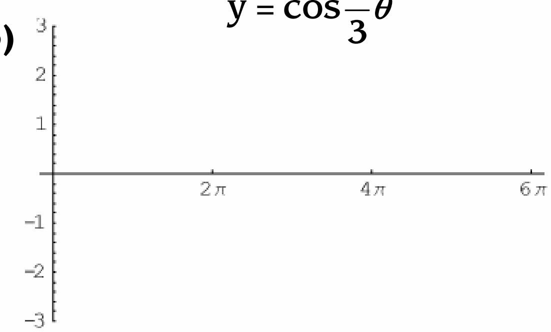


For each of the following equations, draw the graph:

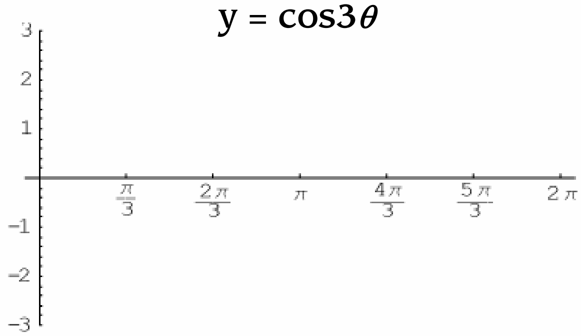
5) $y = \sin 2\theta$



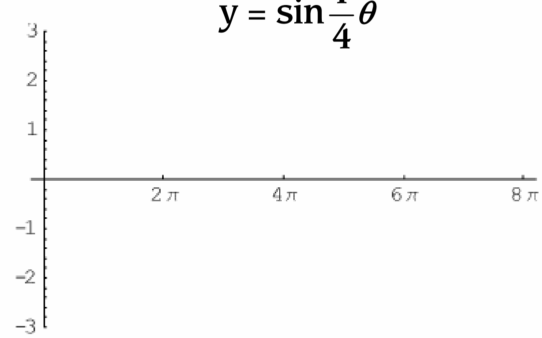
6) $y = \cos\frac{1}{3}\theta$



7) $y = \cos 3\theta$

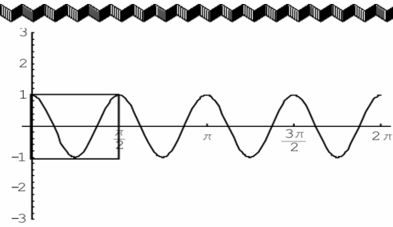


8) $y = \sin\frac{1}{4}\theta$



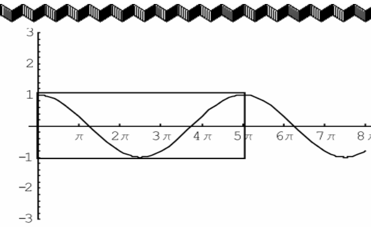
Answers:

1) $\cos 4\theta$



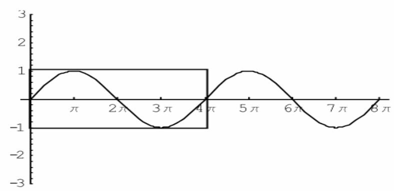
$$b = \frac{2\pi}{P} = \frac{2\pi}{\frac{\pi}{2}} = 2\cancel{\pi} \times \frac{2}{\cancel{\pi}} = 4$$

2) $\cos \frac{2}{5}\theta$



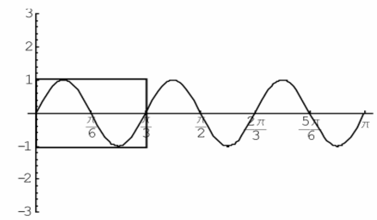
$$b = \frac{2\pi}{P} = \frac{2\cancel{\pi}}{5\cancel{\pi}} = \frac{2}{5}$$

3) $\sin \frac{1}{2}\theta$



$$b = \frac{2\pi}{P} = \frac{2\cancel{\pi}}{4\cancel{\pi}} = \frac{1}{2}$$

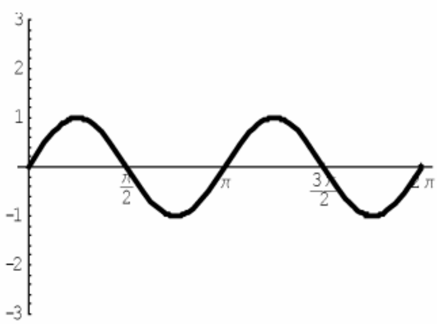
4) $\sin 6\theta$



$$b = \frac{2\pi}{P} = \frac{2\pi}{\frac{\pi}{3}} = 2\cancel{\pi} \times \frac{3}{\cancel{\pi}} = 6$$

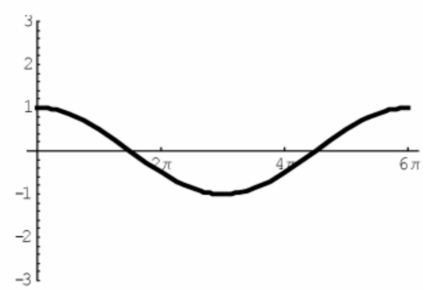
5)

$$P = \frac{2\pi}{b}$$
$$P = \frac{\cancel{2}\pi}{\cancel{2}}$$
$$P = \pi$$



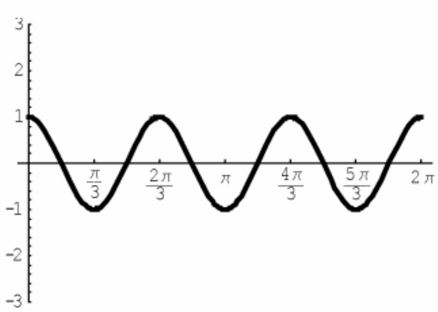
6)

$$P = \frac{2\pi}{b}$$
$$P = \frac{2\pi}{\frac{1}{3}}$$
$$P = 2\pi \times \frac{3}{1}$$
$$P = 6\pi$$



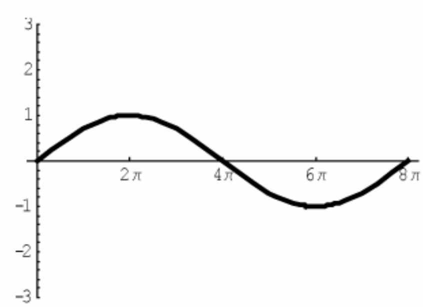
7)

$$P = \frac{2\pi}{b}$$
$$P = \frac{2\pi}{3}$$



8)

$$P = \frac{2\pi}{b}$$
$$P = \frac{2\pi}{\frac{1}{4}}$$
$$P = 2\pi \times \frac{4}{1}$$
$$P = 8\pi$$

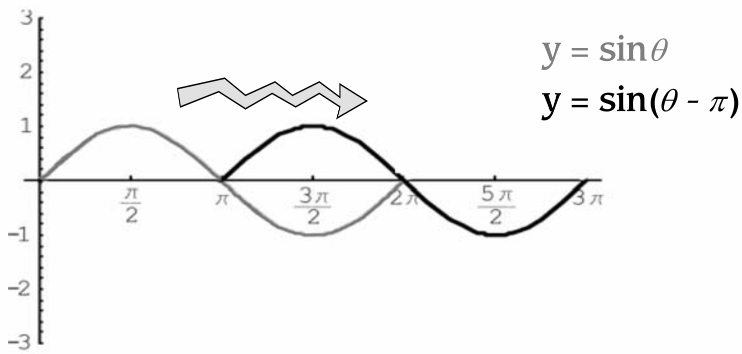


TRIGONOMETRY LESSON FIVE
PART III - THE C VALUE

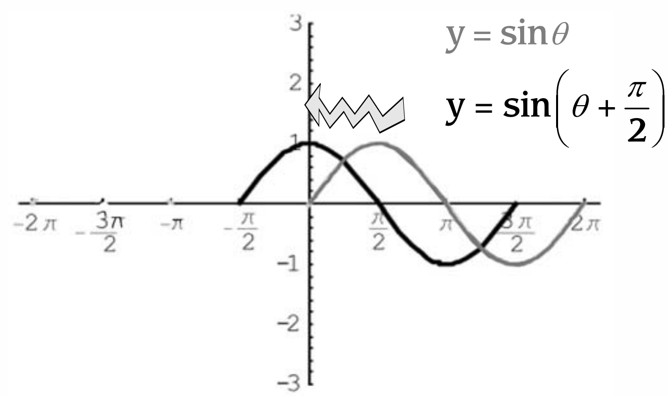
The phase shift is the horizontal translation applied to a trig graph. It is the number added or subtracted to θ inside the equation.

Phase shift is represented by the letter "c" in $y = \sin(\theta \pm c)$

Notice in the following graphs that you will do the opposite of what the sign is. The + will move the graph *left*, and the – will move the graph *right*.

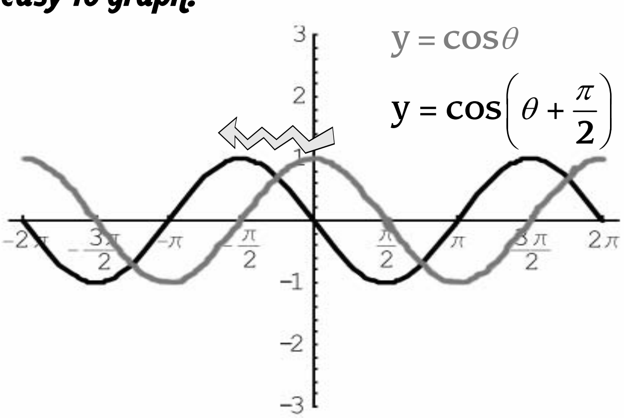


The $-\pi$ means we move the graph ***right*** by π units.



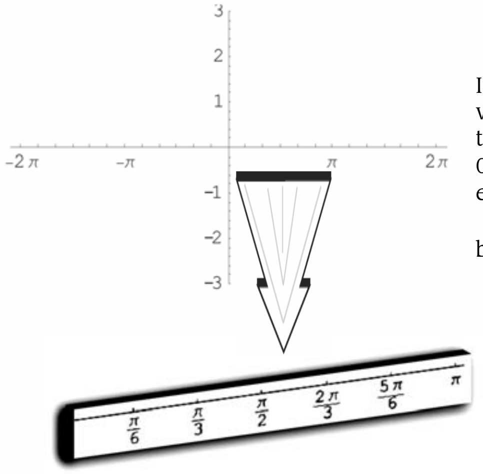
The $+\frac{\pi}{2}$ means we move the graph ***left*** by $\frac{\pi}{2}$ units.

Not all graphs are going to be given as one cycle, since trig graphs can go forever in both directions! A phase shift will shift everything horizontally by the same amount, so it's still easy to graph.

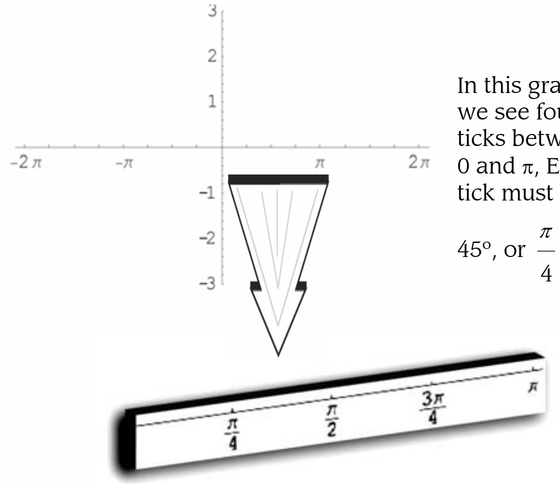


The $+\frac{\pi}{2}$ means we move the graph ***left*** by $\frac{\pi}{2}$ units.

Quite often, a graph will be given with ticks where no radian measure is indicated. In these questions, we need to figure out what the exact value of each tick is first.

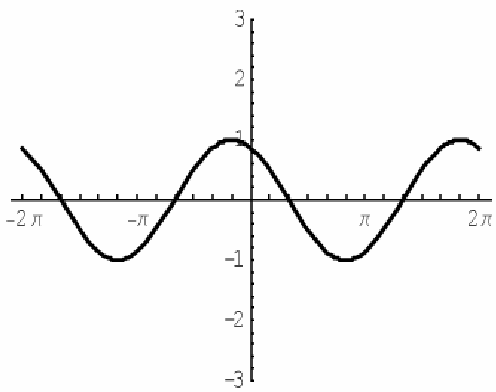


In this graph, we see six ticks between 0 and π, so each tick must be 30° or $\frac{\pi}{6}$



In this graph, we see four ticks between 0 and π, Each tick must be 45°, or $\frac{\pi}{4}$

It is always possible to write at least one sine equation and one cosine equation for the same trig graph.

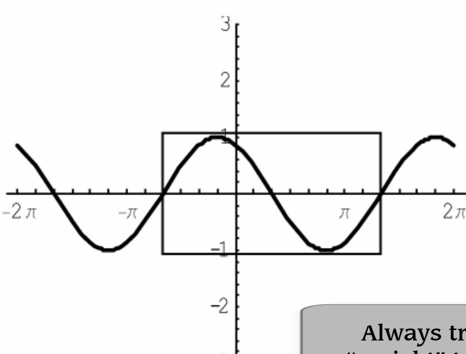


Notice how ticks are given in the graph between -π and 0. Think in terms of degrees for a moment. If we have 180° and 6 ticks, that makes each one 30°. So, if our sine pattern starts at the fourth tick back, that would be -120°, or in radians, $-\frac{2\pi}{3}$. The sine equation is $y = \sin\left(\theta + \frac{2\pi}{3}\right)$.

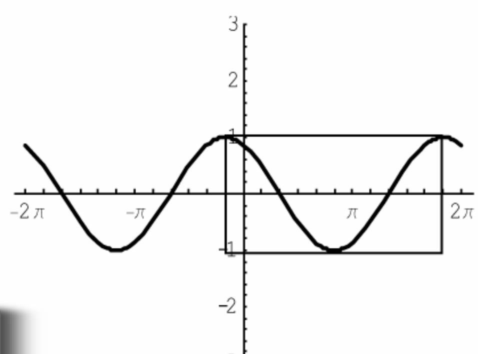
Likewise, we can see that if we were thinking in terms of cosine, the cosine pattern starts one tick back, at -30 °

The cosine equation would be $y = \cos\left(\theta + \frac{\pi}{6}\right)$

$$y = \sin\left(\theta + \frac{2\pi}{3}\right) \qquad\qquad y = \cos\left(\theta + \frac{\pi}{6}\right)$$



OR



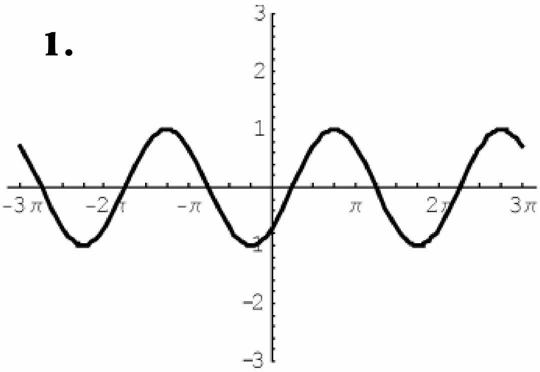
Always try to find an "upright" trig pattern to derive the equation.

TRIGONOMETRY LESSON FIVE

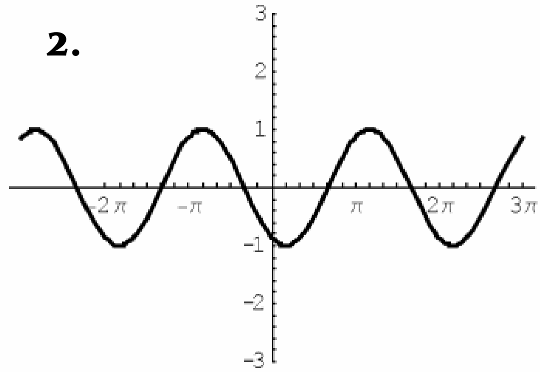
PART III - THE C VALUE

Questions: For 1 & 2, write the __sine__ equation. For 3 & 4, write the __cosine__ equation.

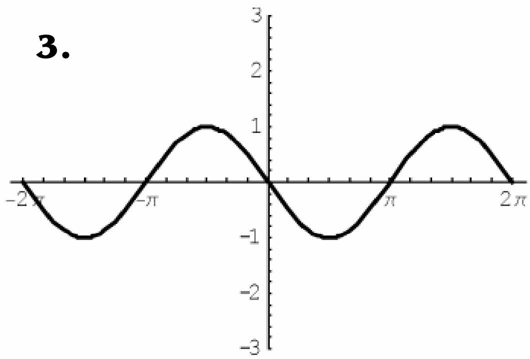
1.



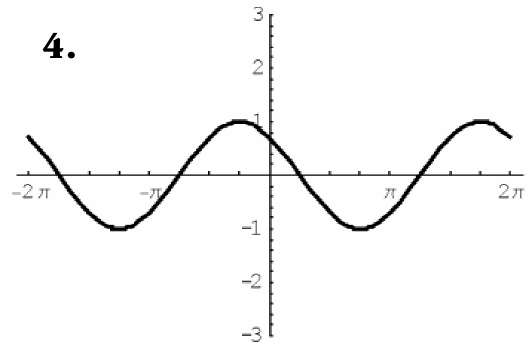
2.



3.

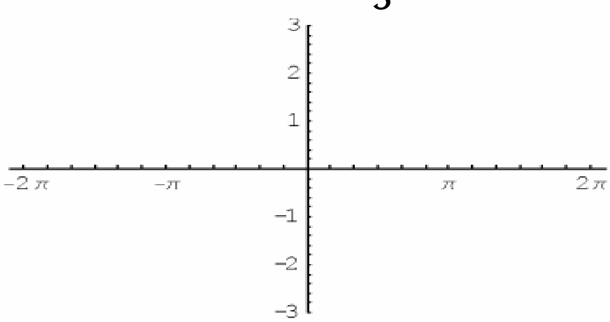


4.

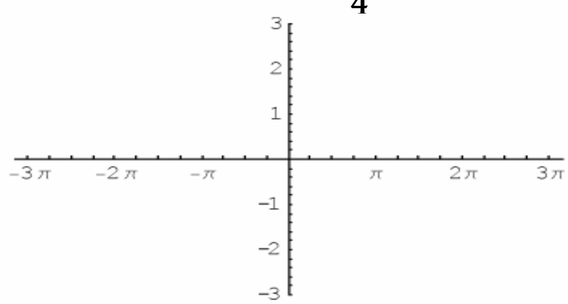


For 5 & 6, draw the __sine__ graph. For 7 & 8, draw the __cosine__ graph.

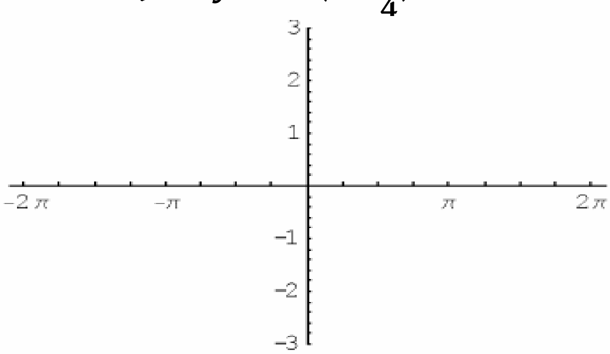
5. $y = \sin(\theta + \frac{\pi}{3})$



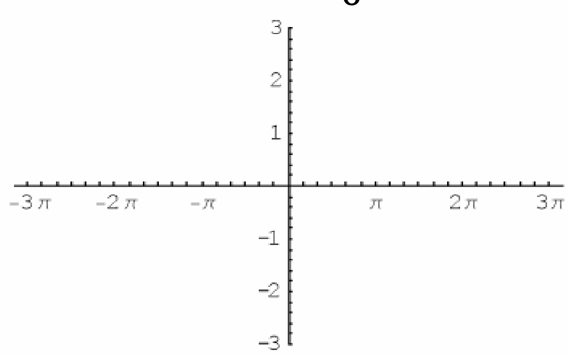
6. $y = \sin(\theta - \frac{\pi}{4})$



7. $y = \cos(\theta + \frac{\pi}{4})$

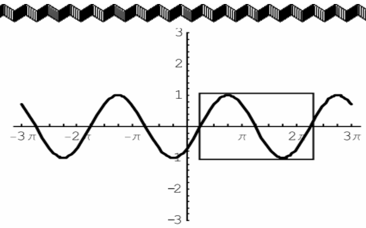


8. $y = \cos(\theta - \frac{5\pi}{6})$



Answers: **1.**

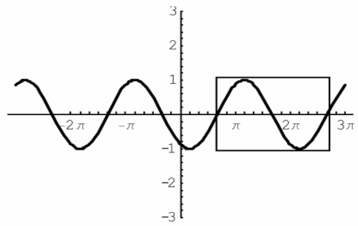
$\sin(\theta - \frac{\pi}{4})$



We can draw a rectangle around the sine pattern closest to the origin. There are four ticks between 0 and π, so each one is 45°. Since the sine pattern starts on the first tick to the right, the equation is $y = \sin(\theta - \frac{\pi}{4})$

2.

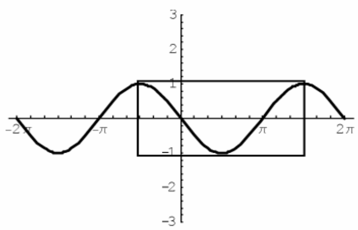
$\sin(\theta - \frac{2\pi}{3})$



We can draw a rectangle around the sine pattern closest to the origin. There are six ticks between 0 and π, so each one is 30°. Since the sine pattern starts on the fourth tick to the right, which is 120°, the equation is $y = \sin(\theta - \frac{2\pi}{3})$

3.

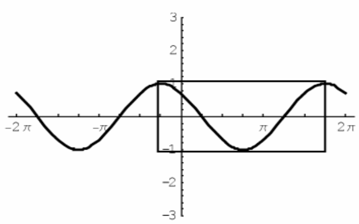
$\cos(\theta + \frac{\pi}{2})$



We can draw a rectangle around the cosine pattern closest to the origin. There are six ticks between 0 and π, so each one is 30°. Since the cosine pattern starts on the third tick to the left, which is -90°, the equation is $y = \cos(\theta + \frac{\pi}{2})$

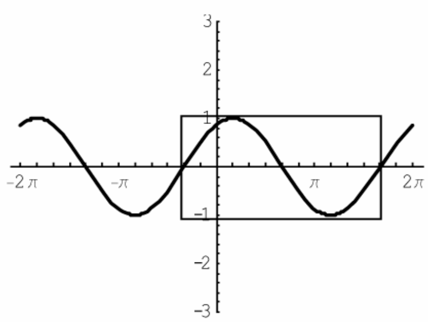
4.

$\cos(\theta + \frac{\pi}{4})$

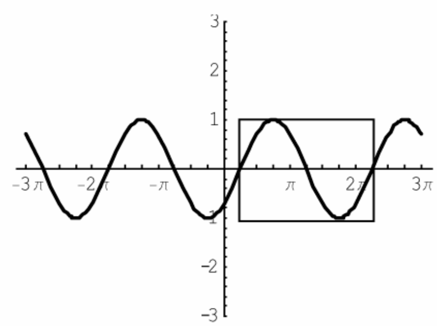


We can draw a rectangle around the cosine pattern closest to the origin. There are four ticks between 0 and π, so each one is 45°. Since the cosine pattern starts on the first tick to the left, which is -45°, the equation is $y = \cos(\theta + \frac{\pi}{4})$

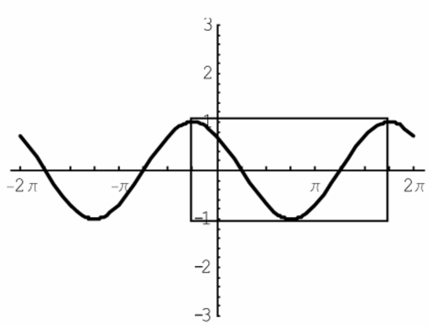
5.



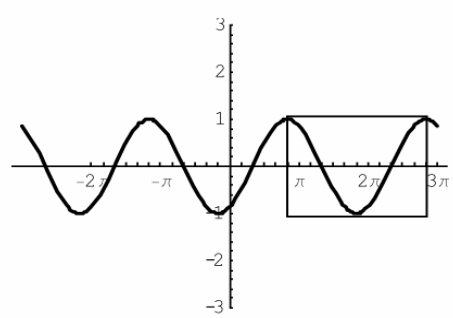
6.



7.



8.



TRIGONOMETRY LESSON FIVE

PART IV - GRAPHING B AND C

We will now look at trig graphs with the form: $y = \sin b(\theta + c)$

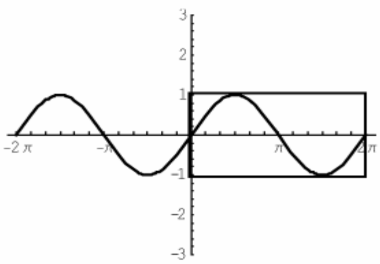
"b" is used to find the period using the formula: Period $= \dfrac{2\pi}{b}$

"c" is the letter used to represent phase shift.

When combining b & c, we should follow a particular order.
First apply the period, then the phase shift.

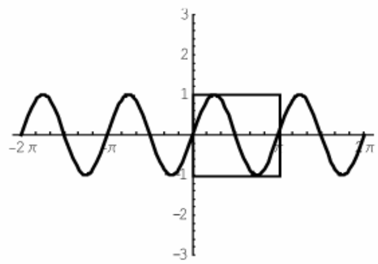
Example 1: Graph $y = \sin 2(\theta - \frac{\pi}{3})$:

$y = \sin\theta$



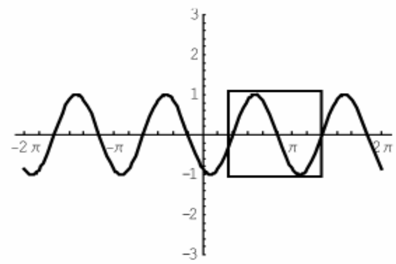
$y = \sin 2\theta$

(Remember to use your formula to find the period, which is π)



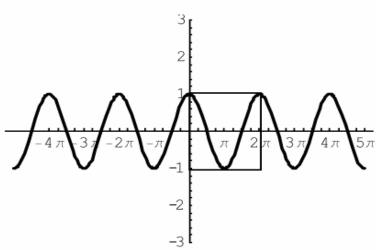
$y = \sin 2(\theta - \frac{\pi}{3})$

(Move the graph two ticks right, since each tick is 30° and we want 60°)



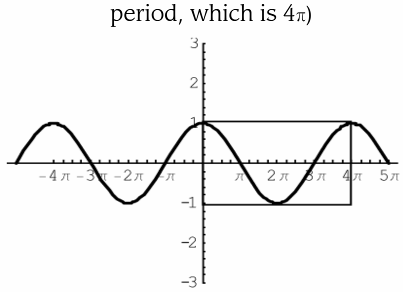
Example 2: Graph $y = \cos\frac{1}{2}(\theta + \frac{\pi}{4})$:

$y = \cos\theta$



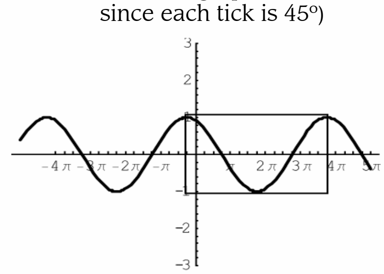
$y = \cos\frac{1}{2}\theta$

(Remember to use the formula to find period, which is 4π)



$y = \cos\frac{1}{2}(\theta + \frac{\pi}{4})$

(Now move the graph one tick left, since each tick is 45°)



---

*Sometimes, the b-value is attached to θ inside the brackets. In the equation $y = \sin(2\theta - \frac{\pi}{3})$, we **MUST** factor out the 2 before graphing. The reason for doing this is that we can now easily read off the phase shift.*

$y = \sin(2\theta - \frac{\pi}{3})$

$y = \sin 2(\theta - \frac{\pi}{6})$

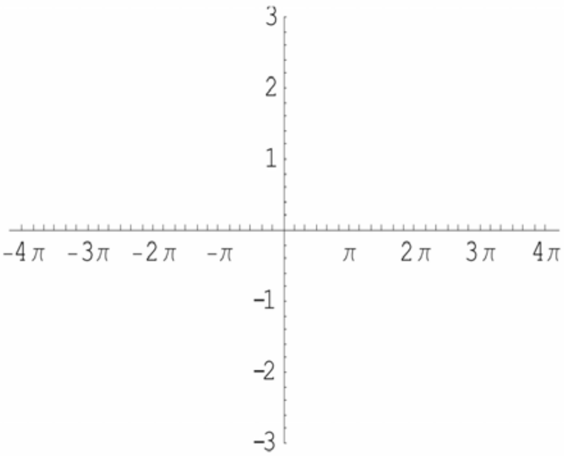
When you pull out the 2, divide each term in the original brackets by 2.

TRIGONOMETRY LESSON FIVE

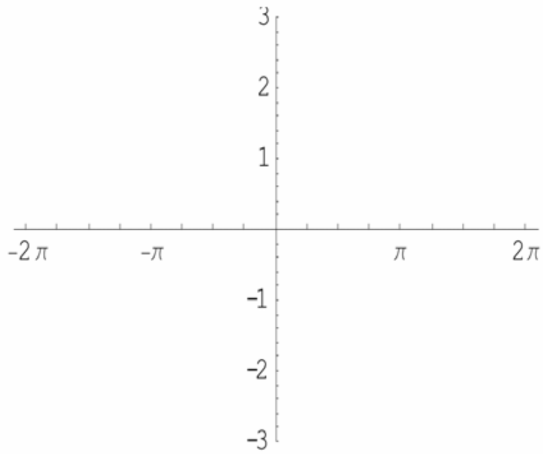
PART IV - GRAPHING B AND C

Questions: Graph the following equations:

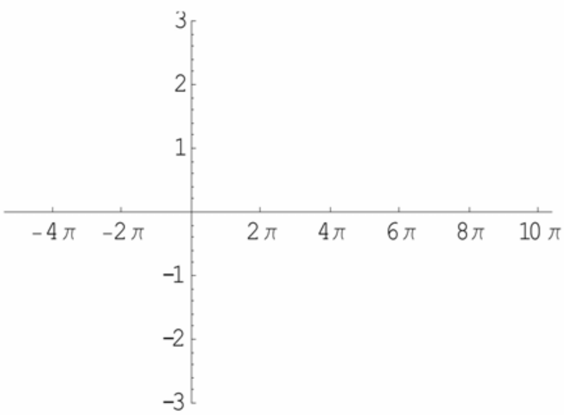
1) $y = \sin\frac{2}{3}(\theta - \frac{\pi}{2})$



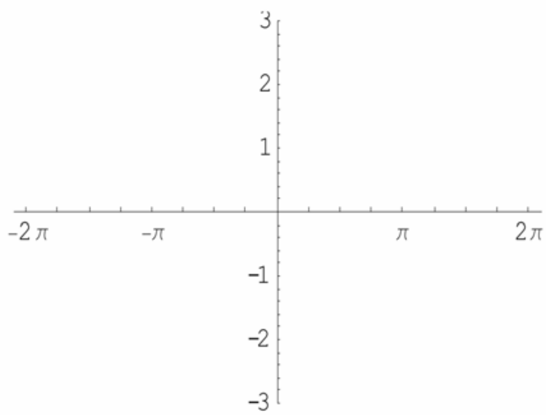
2) $y = \sin 2(\theta - \frac{\pi}{4})$



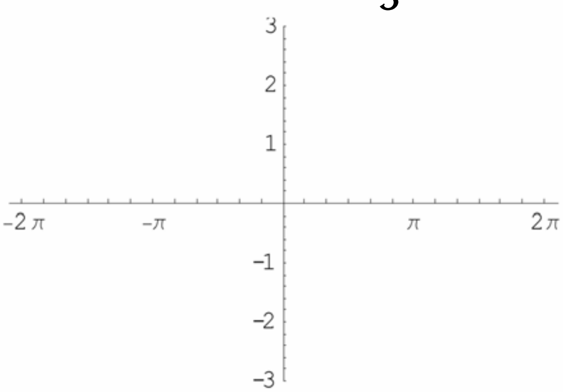
3) $y = \cos\frac{1}{3}(x - \pi)$



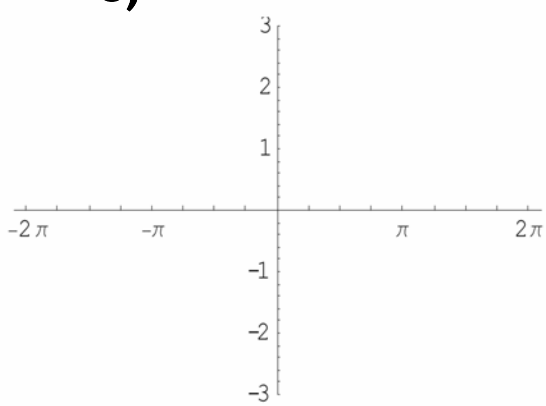
4) $y = \cos(2\theta - \pi)$



5) $y = \sin(2\theta - \frac{\pi}{3})$



6) $y = \cos(4\theta + \pi)$



TRIGONOMETRY LESSON FIVE

PART IV - GRAPHING B AND C

1)

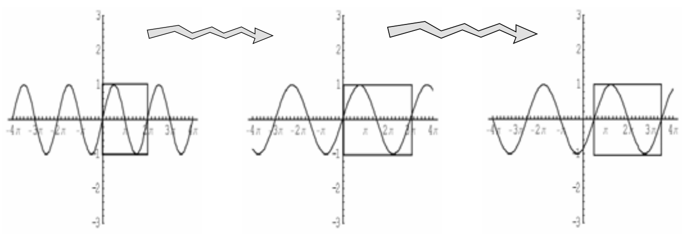
First graph
y = sinθ

Now use the formula

$$P = \frac{2\pi}{b}$$

to find the period, which is **3π**

Move the graph three ticks right, since each tick is 30° and we want a shift of **90°**

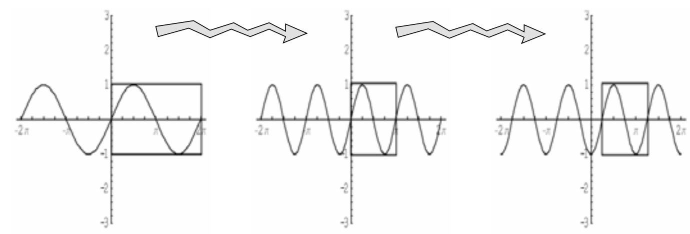


2)

First graph
y = sinθ

Now use your formula to find the period, which is **π.**

Finally move the graph one tick to the right, since each tick is 45° and we want a shift of **45°**

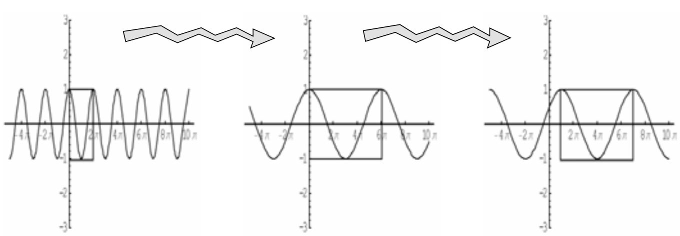


3)

First graph
y = cosθ

The period is **6π**

Move the graph one tick right to **π.**

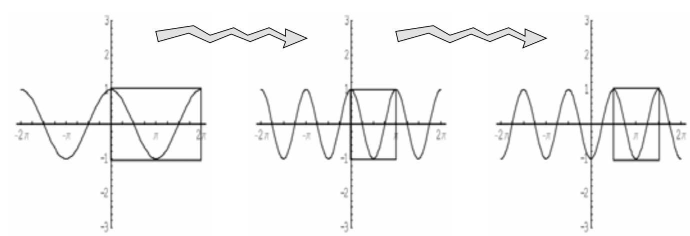


4)

First graph
y = cosθ

The period is **π**
(Don't forget to factor out the 2)

Move the graph right two units to 90°, or **$\pi/2$**

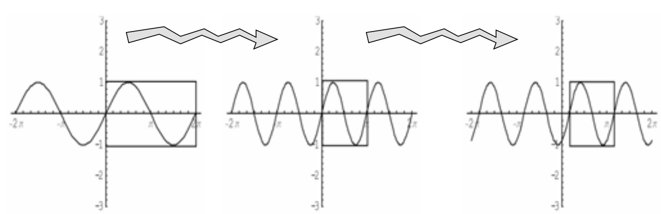


5)

First graph
y = sinθ

The period is **π**
(Don't forget to factor out the 2)

Move the graph right one unit to 30°, or **$\pi/6$**



6)

First graph
y = cosθ

The period is **$\pi/2$**
(Don't forget to factor out the 4)

Move the graph left one unit to -45°, or **$-\pi/4$**