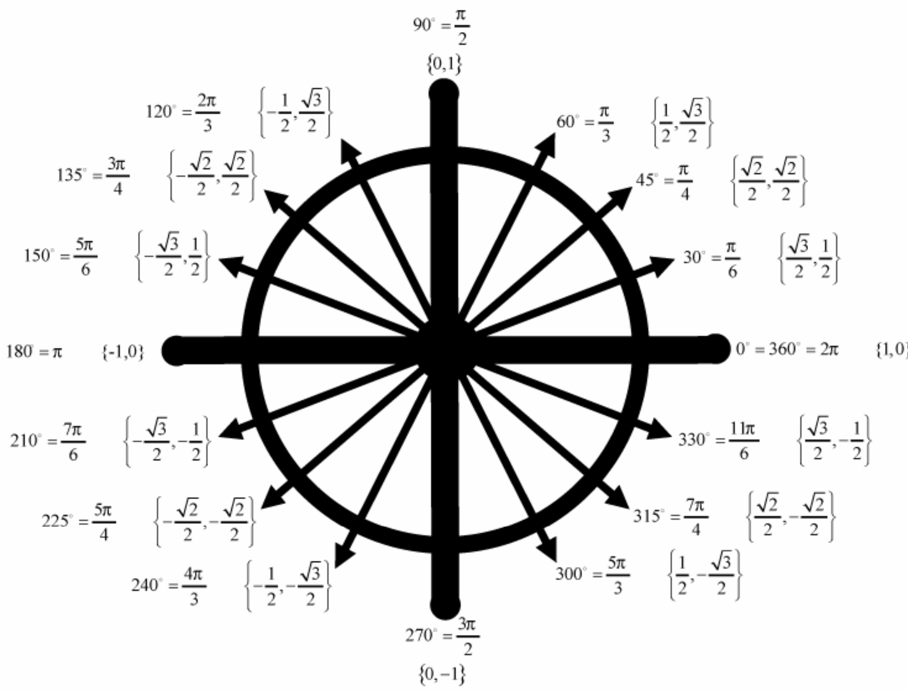
Principles of Mathematics 12

TRIGONOMETRY I



Lesson Nine

Other Trigonometric Graphs

Principles of Math 12 EXPLAINED!

By Barry Mabillard

Part I - Graphing Other Trig Functions

Graphing y = tanθ & y = cotθ

The vertical lines you see are called *asymptotes*. They are places where the graph is undefined.

Recall that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\tan\theta$ is undefined at the angles where $\cos\theta$ is equal to zero. $\left[\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right]$

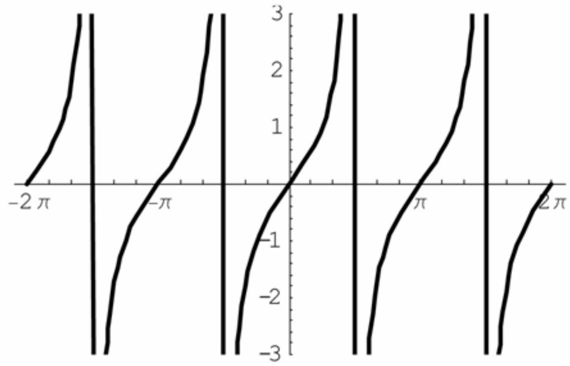
Likewise, $\cot\theta$ is undefined whenever $\sin\theta$ is equal to zero. $[0, \pi]$

> ***a-value:*** **We only use the term *amplitude* in describing the graphs of sinθ and cosθ. The other four trig graphs are not "closed in", they go up & down forever. So, we simply call the a-value the *vertical stretch.***
>
> ***b-value & period:*** IMPORTANT! **The period of a basic tanθ or cotθ graph is π, not 2π like sinθ and cosθ. Thus, we have the following formulas:**
>
> $$\text{Period} = \frac{\pi}{b} \quad \text{and} \quad b = \frac{\pi}{\text{Period}}$$
>
> ***c-value:*** **No difference from sinθ and cosθ, but remember to move your asymptotes if you shift the graph.**
>
> ***d-value:*** **No difference from sinθ and cosθ.**

We always write the general solution of $\tan\theta$ & $\cot\theta$ asymptotes in the following way:

x = Angle of first positive asymptote ± n(Period)

y = tanθ



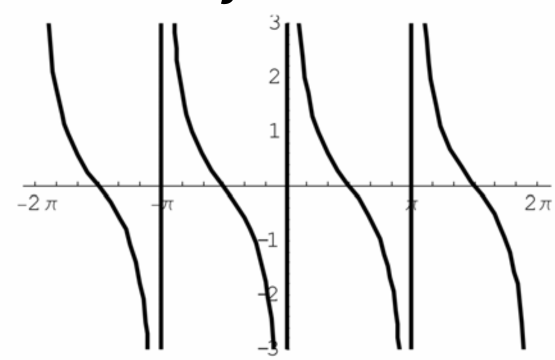
For tanθ, we can see from the graph that the first positive asymptote occurs at $\dfrac{\pi}{2}$.

All asymptotes are exactly π units away from each other.

The general equation of the asymptotes is:

$$x = \frac{\pi}{2} \pm n\pi$$

y = cotθ



For cotθ, we can see from the graph that the first positive asymptote occurs at 0, and all asymptotes are exactly π units away from each other.

The general equation of the asymptotes is:

$x = 0 \pm n\pi$, or simply, $x = \pm n\pi$

Graphing y = cscθ & y = secθ

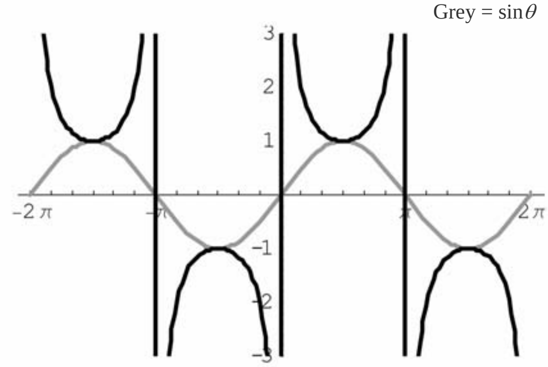
As with the previous graphs, the vertical lines you see are asymptotes. and they occur where the graph is undefined.

Recall that $\csc\theta = \frac{1}{\sin\theta}$ and $\sec\theta = \frac{1}{\cos\theta}$

At the angles where cosθ is equal to zero, the graph is undefined

Likewise, cotθ is undefined whenever sinθ is equal to zero.

y = cscθ



For cscθ, we can see from the graph that the first positive asymptote occurs at 0, and all asymptotes are exactly π units away from each other.
The general equation of the asymptotes is:
$x = 0 \pm n\pi$, or simply, $x = n\pi$

a-value: **Vertical stretch, makes graphs taller and more narrow.**

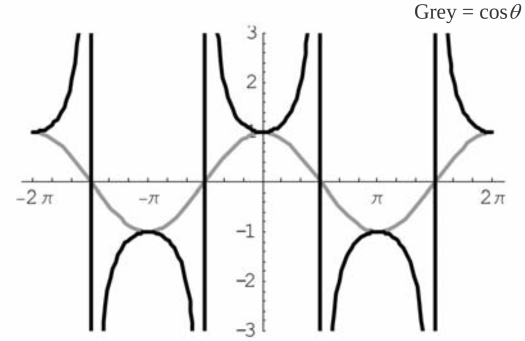
b-value & period: **For the reciprocal graphs cscθ and secθ, they have a period of 2π, so we can use the formulas we're used to:**

$$\text{Period} = \frac{2\pi}{b} \quad \text{and} \quad b = \frac{2\pi}{\text{Period}}$$

c-value: **No difference from sinθ and cosθ, but remember to move your asymptotes if you shift the graph.**

d-value: **No difference from sinθ and cosθ.**

y = secθ



For secθ, we can see from the graph that the first positive asymptote occurs at $\frac{\pi}{2}$.
All asymptotes are exactly π units away from each other.

The general equation of the asymptotes is:

$$x = \frac{\pi}{2} \pm n\pi$$

We always write the general solution of cscθ & secθ asymptotes in the following way:

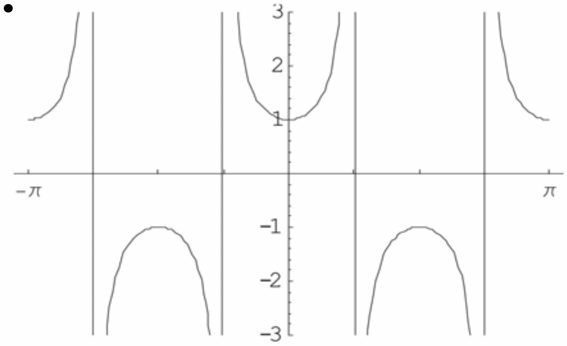
$$\mathbf{x = \text{Angle of first positive asymptote} \pm n\left(\frac{\text{Period}}{2}\right)}$$

Trigonometry Lesson 9

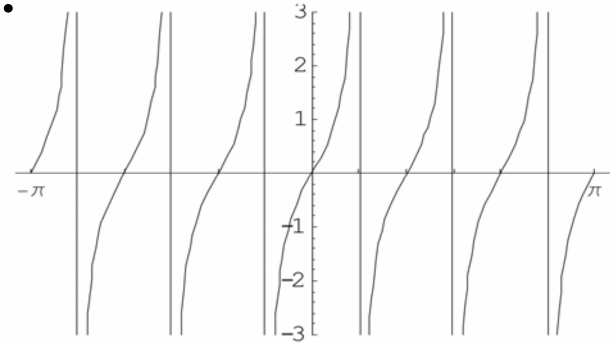
Part I - Graphing Other Trig Functions

Find the equation of each of the following graphs. Also, state the general solution of the asymptotes.

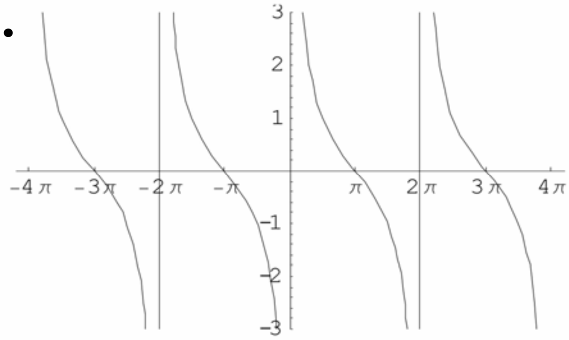
1.



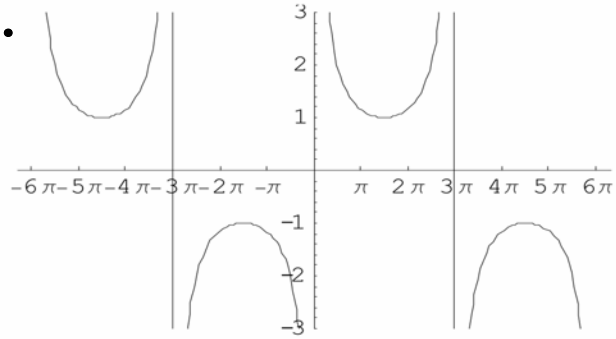
2.



3.

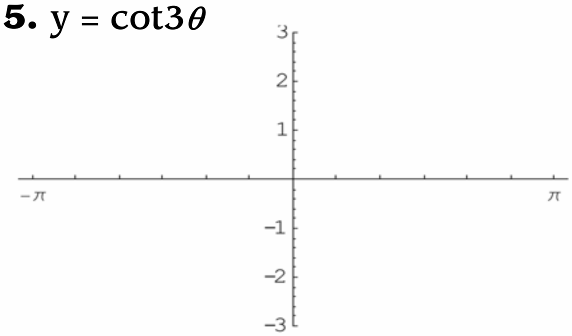


4.

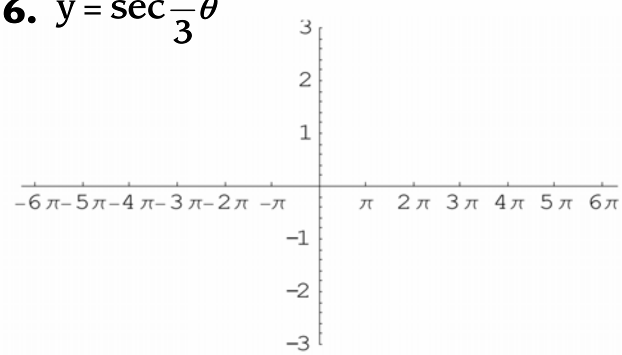


Draw the graph for each of the following equations, and state the general solution of the asymptotes.

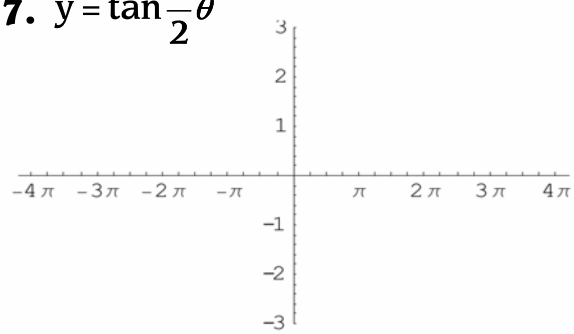
5. $y = \cot 3\theta$



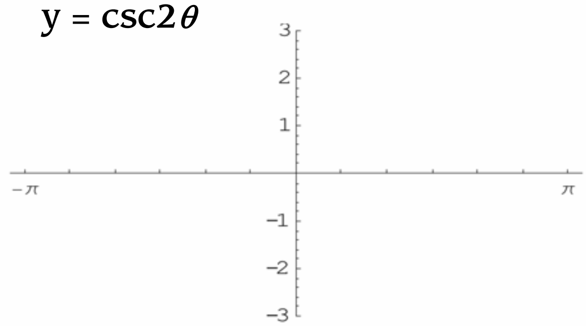
6. $y = \sec\frac{1}{3}\theta$



7. $y = \tan\frac{1}{2}\theta$



8. $y = \csc 2\theta$



Trigonometry Lesson 9

Part I - Graphing Other Trig Functions

1) The period of this $\sec\theta$ graph is π.
(The length of one complete cycle must contain both the top U and the upside-down U.)

Now find the b-value: $b = \dfrac{2\pi}{Period} = \dfrac{2\pi}{\pi} = 2$. Equation is: **y = sec2θ**.

General Solution: $x = \dfrac{\pi}{4} \pm n\dfrac{\pi}{2}$

2) The period of this $\tan\theta$ graph is $\pi/3$. We can tell since each tick is 30° and one cycle uses two ticks.

Now find the b-value: $b = \dfrac{\pi}{Period} = \dfrac{\pi}{\frac{\pi}{3}} = \pi \times \dfrac{3}{\pi} = 3$. Equation is **y = tan3$\theta$.**

General Solution: $x = \dfrac{\pi}{6} \pm n\left(\dfrac{\pi}{3}\right)$

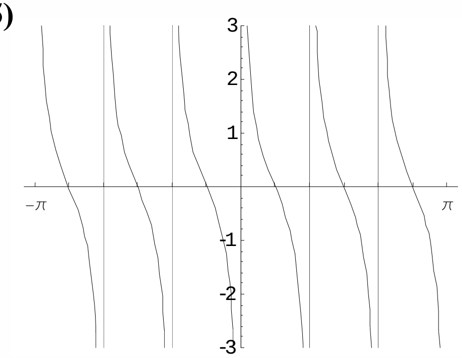
3) The period of the $\cot\theta$ graph is 2π. Now find the b-value: $b = \dfrac{\pi}{Period} = \dfrac{\pi}{2\pi} = \dfrac{1}{2}$ Equation is $\mathbf{y = \cot\frac{1}{2}\theta}$

General Solution: $x = \pm n(2\pi)$

4) The period of the $\csc\theta$ graph is 6π. Now find the b-value: $b = \dfrac{2\pi}{Period} = \dfrac{2\pi}{6\pi} = \dfrac{1}{3}$ Equation is $\mathbf{y = \csc\frac{1}{3}\theta}$

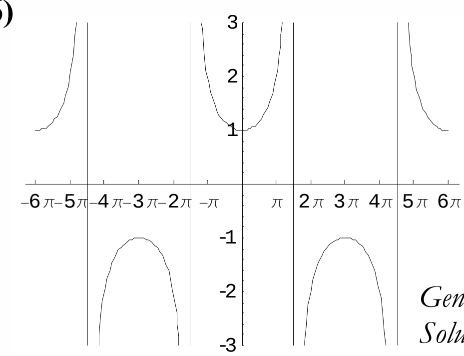
General Solution: $x = \pm n(3\pi)$

5)



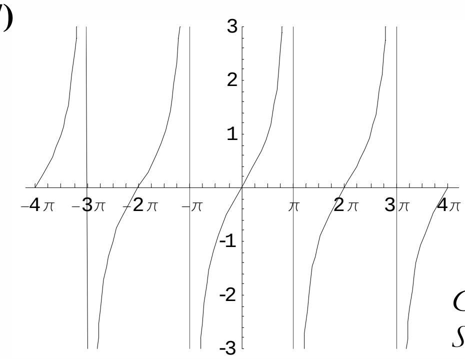
General Solution: $x = \pm n\left(\dfrac{\pi}{3}\right)$

6)



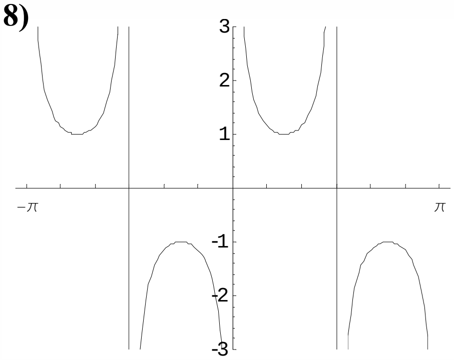
General Solution: $x = \dfrac{3\pi}{2} \pm n(3\pi)$

7)



General Solution: $x = \pi \pm n(2\pi)$

8)



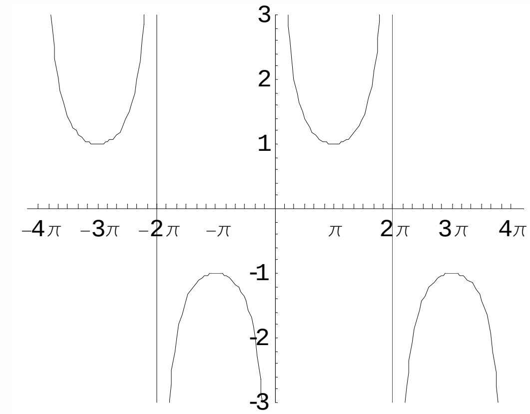
General Solution: $x = \pm\dfrac{n\pi}{2}$

Trigonometry Lesson 9

Part II - Summary of Trig Functions

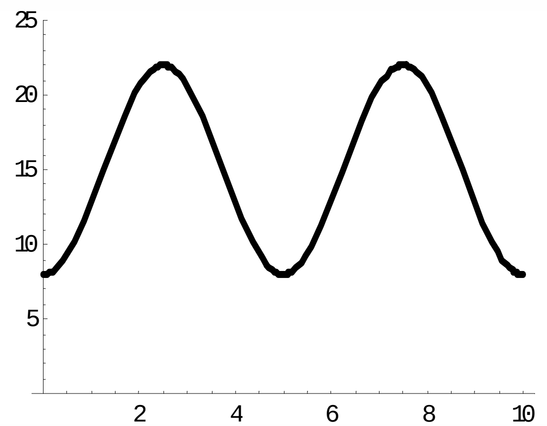
Example 1: Given the equation: $y = \csc \frac{1}{2}\theta$, find the following:

Questions:	Answers:
a) a-value	a) 1
b) b-value	b) ½
c) Period	c) $Period = \frac{2\pi}{b} = \frac{2\pi}{0.5} = 4\pi$
d) Phase shift	d) None
e) Vertical Displacement	e) None
f) Domain	f) $x \neq \pm n(2\pi)$
g) Range	g) $y \leq -1$, $y \geq 1$
h) x-intercepts	h) None
i) y-intercepts	i) None
j) Equation of asymptotes	j) $x = \pm n(2\pi)$



Example 2: Given the equation: $h(t) = 7\sin\frac{2\pi}{5}(t - 1.25) + 15$, find the following:

Questions:	Answers:
a) a-value	a) 7
b) b-value	b) $\frac{2\pi}{5}$
c) Period	c) 5
d) Phase shift	d) 1.25 right
e) Vertical Displacement	e)15 up
f) Domain	f) $t \in R$
g) Range	g) $8 \leq h(t) \leq 22$
h) x-intercepts	h) None
i) y-intercepts	i) 8
j) Equation of asymptotes	j) None



Reminder: To find x-intercepts, use
2^{nd} → Trace → Zero.
You should always state general solutions for x-intercepts.
(*unless the domain is specified*)

Find y-intercepts using:
2^{nd} → Trace → Value → x=0

Trigonometry Lesson 9

Part II - Summary of Trig Functions

	Equation	a-value	b-value	Period	Phase Shift	Vertical Displacement	Domain	Range	x-intercepts	y-intercepts	Equation Of Asymptote
1.	$y=-2\sin(4\theta-\pi)+5$										
2.	$T(d)=21\sin\frac{2\pi}{365}(d-118)+5.5$										
3.	$y=\tan\frac{1}{2}\theta$										
4.	$y=-\frac{1}{2}\cos(2\theta-90^\circ)+1$										
5.	$h(t)=18.5\cos\frac{2\pi}{365}(t-28)+4.5$										
6.	$y=\frac{1}{2}\sin(\frac{\theta}{3}+\frac{\pi}{6})+3$										
7.	$y=\sec\frac{1}{3}\theta$										
8.	$y=\cot\frac{1}{2}\theta$										
9.	$y=2\sin(\frac{\theta}{3}-\frac{\pi}{3})+2$										
10	$y=20.1\sin\frac{2\pi}{300}(t-265)+6.2$										

Trigonometry Lesson 9

Part II - Summary of Trig Functions

	a-value	b-value	Period	Phase Shift	Vertical Displacement	Domain	Range	x-int	y-int	Equation Of Asymptotes
1.	2	4	$\frac{\pi}{2}$	$\frac{\pi}{4}$ right	5 up	$\theta \in R$	$3 \le y \le 7$	None	5	None
2.	21	$\frac{2\pi}{365}$	365	118 right	5.5 up	$d \in R$	$-15.5 \le T \le 26.5$	$102^\circ \pm n(360^\circ)$ $316^\circ \pm n(360^\circ)$	-13.31	None
3.	1	$\frac{1}{2}$	2π	None	None	$x \ne \pi \pm n(2\pi)$	$y \in R$	$\pm n(2\pi)$	0	$x = \pi \pm n(2\pi)$
4.	$\frac{1}{2}$	2	π	45º right	1 up	$\theta \in R$	$0.5 \le y \le 1.5$	None	1	None
5.	18.5	$\frac{2\pi}{365}$	365	28 right	4.5 up	$t \in R$	$-14 \le h \le 23$	$134^\circ \pm n(360^\circ)$ $303^\circ \pm n(360^\circ)$	20.9	None
6.	$\frac{1}{2}$	$\frac{1}{3}$	6π	$\frac{\pi}{2}$ left	3 up	$\theta \in R$	$2.5 \le y \le 3.5$	None	3.25	None
7.	1	$\frac{1}{3}$	6π	None	None	$x \ne \frac{3\pi}{2} \pm n(3\pi)$	$y \le -1$ & $y \ge 1$	None	1	$x = \frac{3\pi}{2} \pm n(3\pi)$
8.	1	$\frac{1}{2}$	2π	None	None	$x \ne \pm n(2\pi)$	$y \in R$	$\pi \pm n(2\pi)$	None	$x = \pm n(2\pi)$
9.	2	$\frac{1}{3}$	6π	π right	2 up	$\theta \in R$	$0 \le y \le 4$	None	0.29	None
10	20.1	$\frac{2\pi}{300}$	300	265 right	6.2 up	$t \in R$	$-13.9 \le y \le 26.3$	$130^\circ \pm n(360^\circ)$ $250^\circ \pm n(360^\circ)$	19.6	None