

Pure Math 30

EXPLAINED!

*Diploma Style
Practice Exam*

**TRIGONOMETRY II
-- ANSWERS--**

Trigonometry II Practice Exam - ANSWERS

These are the formulas for Trig II you will be given on your diploma.

$$a = r\theta$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

ANSWERS

- | | | | |
|------------|-----------|------------|---------|
| 1. C | NR 2) 28 | 19. B | 28. D |
| 2. D | NR 3) 1 | 20. D | NR 7. 4 |
| NR 1) 1.50 | 11. C | 21. D | 29. B |
| 3. A | 12. C | NR 5. 2413 | 30. C |
| 4. A | 13. A | 22. A | 31. C |
| 5. D | 14. C | 23. C | 32. A |
| 6. A | NR 4) 0.7 | 24. B | 33. B |
| 7. D | 15. A | 25. B | |
| 8. A | 16. C | NR 6. 0.5 | |
| 9. C | 17. B | 26. A | |
| 10. B | 18. A | 27. D | |

1. Analyze each of the possible answers to see which one gives back $\sin 75^\circ$

Look at $\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$.

Comparing this expression with the identity $\sin(A + B) = \sin A \cos B + \cos A \sin B$, it can be seen that the value of A is 45° , and B is 30°

Therefore, $\sin(A + B) = \sin(45^\circ + 30^\circ) = \sin 75^\circ$

The answer is **C**.

2. The graph of $\tan 5x$ can be obtained by horizontally stretching the original graph by a factor of $\frac{1}{5}$. This will make the arms of the graph much closer to each other.

Multiply the original general solution by $\frac{1}{5}$ to obtain the new general solution.

$$\frac{1}{5} \left(-\frac{\pi}{4} + n\pi \right) = -\frac{\pi}{20} + \frac{n\pi}{5}$$

The answer is **D**.

NR #1 Substitute 2.1 radians into the right side $\frac{\cos^2 x + \sin x}{\sin^2 x}$. (You will get the same answer if you use the left side, but since the right side is already in terms of $\sin x$ and $\cos x$, it's easier to use.)

$$\frac{\cos^2 2.1 + \sin 2.1}{\sin^2 2.1} = 1.50$$

*Make sure you have your calculator in radian mode.

**Type in $\cos^2 2.1$ as $(\cos(2.1))^2$

The answer is **1.50**

3. Substitute the height of 14 m for $h(t)$, then graph both sides and find the point of intersection. The x -value will give the time 14 m is first reached.

$$14 = -6 \cos \frac{\pi}{20} t + 9$$

$$t = 16.3 \text{ s}$$

The answer is **A**.

4. Use the identity $\cot^2 x = \csc^2 x - 1$

$$\cot^2 x + \csc x - 4$$

$$(\csc^2 x - 1) + \csc x - 4$$

$$\csc^2 x + \csc x - 5$$

The answer is **A**.

5. The equation $\cos^2 x + \cos x + 1 = 3$ can be transformed into the equation

$$\cos^2\left(\frac{x}{3}\right) + \cos\left(\frac{x}{3}\right) + 1 = 3 \text{ using a horizontal stretch by a factor of 3.}$$

(Remember that the reciprocal is what goes inside the brackets.)

The new general solution is found by multiplying the original general solution by 3.

$$3(2n\pi) = 6n\pi$$

What this means is that the new solutions will be $6\pi, 12\pi, 18\pi\ldots$

The answer is **D**.

6. Start with the relationship $\tan A = \frac{\sin A}{\cos A}$

$$\tan A = \frac{\sin A}{\cos A}$$

$$\tan A \cos A = \sin A$$

$$\cos A = \frac{\sin A}{\tan A}$$

Now plug in the corresponding expressions $\sin A = \frac{m}{n}$ and $\tan A = \frac{m^2}{n^3}$

$$\cos A = \frac{\sin A}{\tan A}$$

$$\cos A = \frac{\frac{m}{n}}{\frac{m^2}{n^3}}$$

$$\cos A = \frac{m}{n} \times \frac{n^3}{m^2}$$

$$\cos A = \frac{n^2}{m}$$

The answer is **A**.

7. Use the identities $1 + \tan^2 x = \sec^2 x$ and $\cos^2 x = 1 - \sin^2 x$

$$\begin{aligned} & \sqrt{\frac{1 + \tan^2 x}{1 - \sin^2 x}} \\ & \sqrt{\frac{\sec^2 x}{\cos^2 x}} \\ & \sqrt{\frac{1}{\frac{\cos^2 x}{\cos^2 x}}} \\ & \sqrt{\frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x}} \\ & \sqrt{\frac{1}{\cos^4 x}} \\ & \frac{1}{\cos^2 x} \\ & \sec^2 x \end{aligned}$$

The answer is **D**.

8. Use the identity $1 + \tan^2 x = \sec^2 x \rightarrow$ substitute $\frac{5}{7}$ for $\tan^2 x$

$$1 + \frac{5}{7} = \sec^2 x$$

$$\frac{7}{7} + \frac{5}{7} = \sec^2 x$$

$$\sec^2 x = \frac{12}{7}$$

The answer is **A**.

9. Use the identity $\cos(2A) = \cos^2 A - \sin^2 A$

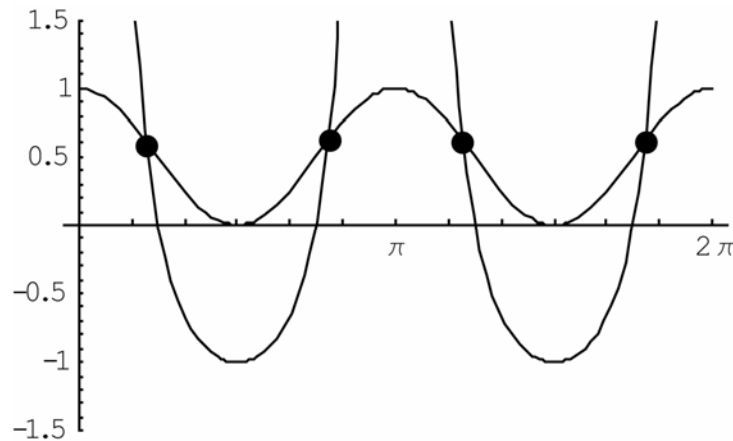
From the given expression $\cos^2(4\pi) - \sin^2(4\pi)$, we can see the A - value is 4π

Now replace 4π for A on the left side: $\cos(2A) = \cos[2(4\pi)] = \cos(8\pi)$

The answer is **C**.

10. When the domain is changed to $0 < x < 2\pi$, the solutions on the extreme ends are no longer included, so there are fewer solutions.
The answer is **B**.

NR #2 Graph the equation in your calculator. The interval $0 \leq x \leq 14\pi$ is too crowded to see clearly, so use the interval $0 < x < 2\pi$ instead.



If there are 4 solutions in the interval $0 \leq x \leq 2\pi$, there are 28 in the interval $0 \leq x \leq 14\pi$ since that interval is 7 times bigger. (We can do this since there are no points of intersection occurring at the extreme ends.)

The answer is **28**

NR #3 The expression $(\sin x + \cos x)^2$ must be foiled to get
 $(\sin x + \cos x)(\sin x + \cos x) = \sin^2 x + 2 \sin x \cos x + \cos^2 x$.

The error occurs in Step I. All other steps would be correct if the error did not occur.
The answer is **1**

11.

$$\frac{m \sin x \cot x}{4 \csc x \tan x} = 8$$

$$m \sin x \cot x = 32 \csc x \tan x$$

$$m \sin x \frac{\cos x}{\sin x} = 32 \frac{1}{\sin x} \frac{\sin x}{\cos x}$$

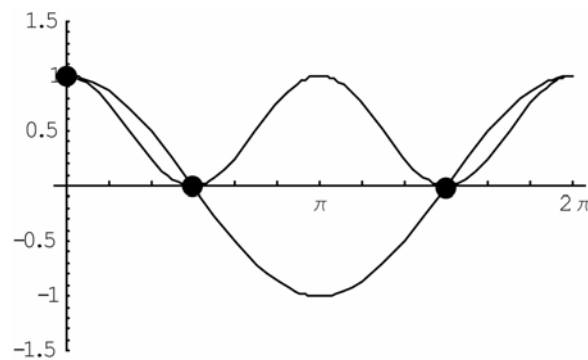
$$m \cos x = \frac{32}{\cos x}$$

$$m = \frac{32}{\cos^2 x}$$

$$m = 32 \sec^2 x$$

The answer is **C**.

12. Solve on your graphing calculator



Don't include the solution at 2π since it's not included in the domain.

The answer is **C**.

13. $\frac{\cos x}{1 - 2 \sin x}$ is undefined when the denominator becomes zero

$$1 - 2 \sin x = 0$$

$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

The general solutions for the asymptotes are $\frac{\pi}{6} \pm 2n\pi, \frac{5\pi}{6} \pm 2n\pi$

The answer is **A**.

14.

$$\sec\left(x - \frac{\pi}{2}\right)$$

$$= \frac{1}{\cos\left(x - \frac{\pi}{2}\right)}$$

$$= \frac{1}{\cos x \cos \frac{\pi}{2} + \sin x \sin \frac{\pi}{2}}$$

$$= \frac{1}{\cos x(0) + \sin x(1)}$$

$$= \frac{1}{\sin x}$$

$$= \csc x$$

The answer is **C**.

NR #4) First simplify the expression using the identity $1 + \cot^2 x = \csc^2 x$

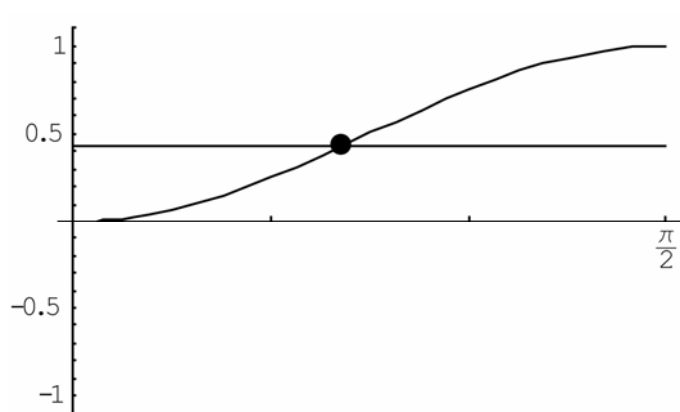
$$\frac{1}{1 + \cot^2 x} = 0.43$$

$$\frac{1}{\csc^2 x} = 0.43$$

$$\frac{1}{\frac{1}{\sin^2 x}}$$

$$\sin^2 x = 0.43$$

Then solve by graphing over the interval $0 \leq x < \frac{\pi}{2}$ (Make sure you're in radian mode)



The answer is **0.7**

15.

$$\frac{\sin x}{\tan x} + \frac{1}{\sec x}$$

$$\frac{\sin x}{\frac{\sin x}{\cos x}} + \frac{1}{\frac{1}{\cos x}}$$

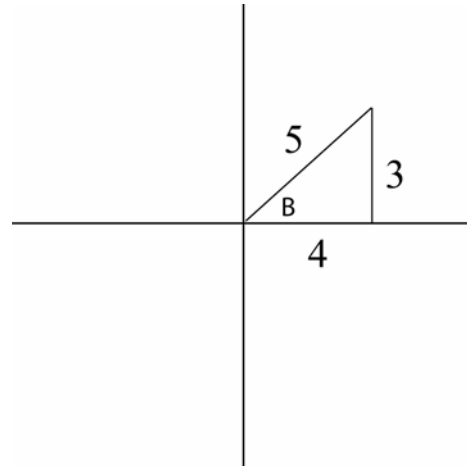
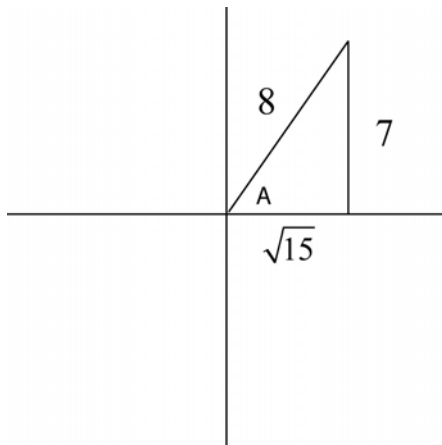
$$\sin x \bullet \frac{\cos x}{\sin x} + \cos x$$

$$\cos x + \cos x$$

$$2 \cos x$$

The answer is **A**

16. First draw out the two triangles, then use Pythagoras to determine the unknown side.



Now state what you know:

$$\sin A = \frac{7}{8} \qquad \sin B = \frac{3}{5}$$

$$\cos A = \frac{\sqrt{15}}{8} \qquad \cos B = \frac{4}{5}$$

Plug all of these into $\cos(A - B) = \cos A \cos B + \sin A \sin B$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{\sqrt{15}}{8} \cdot \frac{4}{5} + \frac{7}{8} \cdot \frac{3}{5}$$

$$= \frac{4\sqrt{15}}{40} + \frac{21}{40}$$

$$= \frac{4\sqrt{15} + 21}{40}$$

The answer is **C**.

17. First use the identity $1 + \cot^2 x = \csc^2 x$

$$-5\csc^2 x + 12\cot^2 x - 9 = 0$$

$$-5(1 + \cot^2 x) + 12\cot^2 x - 9 = 0$$

$$-5 - 5\cot^2 x + 12\cot^2 x - 9 = 0$$

$$7\cot^2 x - 14 = 0$$

$$7\cot^2 x = 14$$

$$\cot^2 x = 2 \qquad \text{The answer is } \mathbf{B}.$$

18. Expand $\cos(x-y) - \cos(x+y)$ using the sum & difference identities.

$$\begin{aligned} & \cos x \cos y + \sin x \sin y - (\cos x \cos y - \sin x \sin y) \\ &= \cos x \cos y + \sin x \sin y - \cos x \cos y + \sin x \sin y \\ &= 2 \sin x \sin y \end{aligned}$$

The answer is **A**.

19. When $\cos^2 x - \sin x$ intersects the line $y = \frac{1}{2}$, we can set the two equations equal to each other, then simplify.

$$\begin{aligned} \cos^2 x - \sin x &= \frac{1}{2} \\ 2 \cos^2 x - 2 \sin x &= 1 \\ 2 \cos^2 x - 2 \sin x - 1 &= 0 \end{aligned}$$

The answer is **B**.

20. The expression $\sin\left(\frac{\theta}{5}\right)\cos\left(\frac{2\theta}{7}\right) - \cos\left(\frac{\theta}{5}\right)\sin\left(\frac{2\theta}{7}\right)$ is generated using the identity $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$\begin{aligned} \sin(A-B) &= \sin\left(\frac{\theta}{5}\right)\cos\left(\frac{2\theta}{7}\right) - \cos\left(\frac{\theta}{5}\right)\sin\left(\frac{2\theta}{7}\right) \\ A &= \frac{\theta}{5} \quad B = \frac{2\theta}{7} \end{aligned}$$

Now replace A & B on the *left* side:

$$\begin{aligned} \sin(A-B) &= \sin\left(\frac{\theta}{5} - \frac{2\theta}{7}\right) \\ &= \sin\left(\frac{7\theta}{35} - \frac{10\theta}{35}\right) \\ &= \sin\left(-\frac{3\theta}{35}\right) \end{aligned}$$

The answer is **D**.

21. Simplify the identity so it can be typed into the TI-83

$$\frac{\csc 2x}{\sec 2x} = \sqrt{5}$$

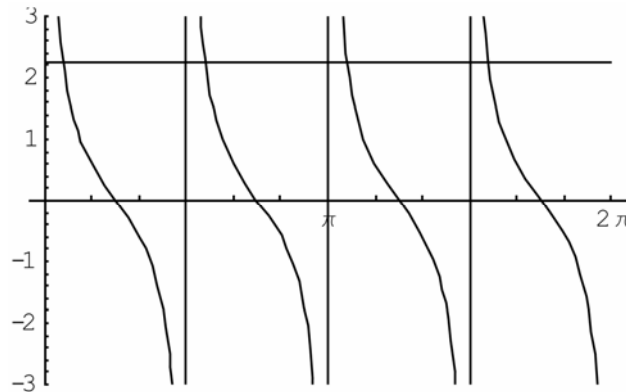
$$\frac{1}{\frac{\sin 2x}{1}} = \sqrt{5}$$

$$\cos 2x$$

$$\frac{1}{\sin 2x} \bullet \cos 2x = \sqrt{5}$$

$$\frac{\cos 2x}{\sin 2x} = \sqrt{5}$$

Graph in your TI-83 to get the following:



The first point of intersection occurs at 0.21 radians, and there is a period of $\frac{\pi}{2} = 1.57$

The answer is **D**.

NR #5. Simplify each of the following

$$\text{I} \quad \frac{1}{9} \sec x \cos x = \frac{1}{9} \bullet \frac{1}{\cos x} \bullet \cos x = \frac{1}{9}$$

$$\text{II} \quad \cot^2 x - \csc^2 x = \cot^2 x - (1 + \cot^2 x) = \cot^2 x - 1 - \cot^2 x = -1$$

$$\text{III} \quad 2\cos^2 x + 2\sin^2 x = 2(\cos^2 x + \sin^2 x) = 2(1) = 2$$

$$\text{IV} \quad 2\cot x - \frac{2\cos x}{\sin x} = \frac{2\cos x}{\sin x} - \frac{2\cos x}{\sin x} = 0$$

The answer is **2413**

22. Use the identity $\cos 2x = \cos^2 x - \sin^2 x$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= (1 - \sin^2 x) - \sin^2 x$$

$$= 1 - 2\sin^2 x$$

$$= 1 - 2m^2$$

The answer is **A**.

23.

$$\frac{1 + \csc x}{\sin x} = \frac{1 + \frac{1}{\sin x}}{\sin x}$$

$$= \frac{\frac{\sin x}{\sin x} + \frac{1}{\sin x}}{\sin x}$$

$$= \frac{\sin x + 1}{\sin x}$$

$$= \frac{\sin x + 1}{\sin x} \cdot \frac{1}{\sin x}$$

$$= \frac{\sin x + 1}{\sin^2 x}$$

The answer is **C**.

24.

$$\frac{\cos(x + y)}{\cos y} = \frac{\cos x \cos y - \sin x \sin y}{\cos y}$$

$$\frac{\cos 45^\circ \cos y - \sin 45^\circ \sin y}{\cos y}$$

$$\frac{\frac{\sqrt{2}}{2} \cos y - \frac{\sqrt{2}}{2} \sin y}{\cos y}$$

$$\frac{\frac{\sqrt{2}}{2} \cos y}{\cos y} - \frac{\frac{\sqrt{2}}{2} \sin y}{\cos y}$$

$$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \tan y$$

$$\frac{\sqrt{2}}{2} (1 - \tan y)$$

The answer is **B**.

25. First determine the angle as a degree to make things easier: $\sec\left(-\frac{\pi}{12}\right) = \sec(-15^\circ)$

Then find two angles which give -15° : $\sec(-15^\circ) = \sec(30^\circ - 45^\circ)$

$$\sec(30^\circ - 45^\circ)$$

$$= \frac{1}{\cos(30^\circ - 45^\circ)}$$

$$= \frac{1}{\cos 30^\circ \cos 45^\circ + \sin 30^\circ \sin 45^\circ}$$

$$= \frac{1}{\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}}$$

$$= \frac{1}{\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}}$$

$$= \frac{1}{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

$$= \frac{4}{\sqrt{6} + \sqrt{2}}$$

Now rationalize the denominator

$$= \frac{4}{\sqrt{6} + \sqrt{2}} \cdot \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}$$

$$= \frac{4(\sqrt{6} - \sqrt{2})}{(\sqrt{6} + \sqrt{2})(\sqrt{6} - \sqrt{2})}$$

$$= \frac{4(\sqrt{6} - \sqrt{2})}{6 - 2}$$

$$= \frac{4(\sqrt{6} - \sqrt{2})}{4}$$

$$= \sqrt{6} - \sqrt{2}$$

The answer is **B**.

NR 6. Compare $\cos^2 kx - \sin^2 kx$ with the identity $\cos(2A) = \cos^2 A - \sin^2 A$

In this identity, the right-side variables will be half the left-side variable.

Example: $\cos(8x) = \cos^2 4x - \sin^2 4x$

So, following that logic, $\cos(1x) = \cos^2 \frac{1}{2}x - \sin^2 \frac{1}{2}x$

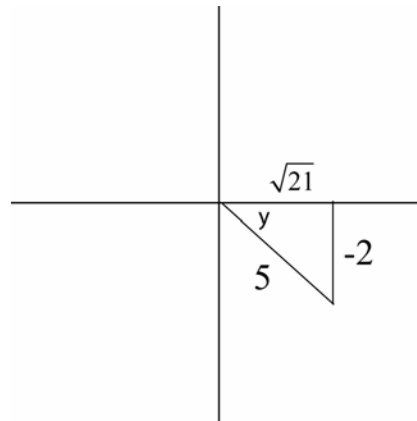
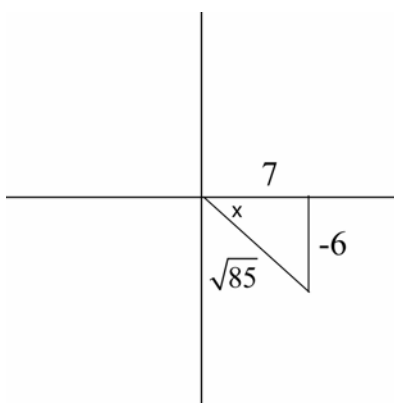
The answer is **0.5**

26.

$$\begin{aligned} &\cos^2 x - \sin^2 x - 1 + 2 \sin x \\ &(1 - \sin^2 x) - \sin^2 x - 1 + 2 \sin x \\ &2 \sin x - 2 \sin^2 x \\ &2 \sin x (1 - \sin x) \end{aligned}$$

The answer is **A**.

27. Draw in the triangles and use Pythagoras to solve for the unknown side



Now state what you know:

$$\begin{aligned} \sin x &= \frac{-6}{\sqrt{85}} & \sin y &= \frac{-2}{5} \\ \cos x &= \frac{7}{\sqrt{85}} & \cos y &= \frac{\sqrt{21}}{5} \end{aligned}$$

$$\sec(x+y) = \frac{1}{\cos(x+y)}$$

$$\frac{1}{\cos(x+y)} = \frac{1}{\cos x \cos y - \sin x \sin y}$$

$$= \frac{1}{\frac{7}{\sqrt{85}} \cdot \frac{\sqrt{21}}{5} - \frac{-6}{\sqrt{85}} \cdot \frac{-2}{5}}$$

$$= \frac{1}{\frac{7\sqrt{21}}{5\sqrt{85}} - \frac{12}{5\sqrt{85}}}$$

$$\begin{aligned} &= \frac{1}{\frac{7\sqrt{21}-12}{5\sqrt{85}}} \\ &= \frac{5\sqrt{85}}{7\sqrt{21}-12} \end{aligned}$$

The answer is **D**

28.

$$\csc x - \sin x$$

$$\frac{1}{\sin x} - \sin x$$

$$\frac{1}{\sin x} - \left(\frac{\sin^2 x}{\sin x} \right) \quad \text{Get a common denominator}$$

$$\frac{1 - \sin^2 x}{\sin x}$$

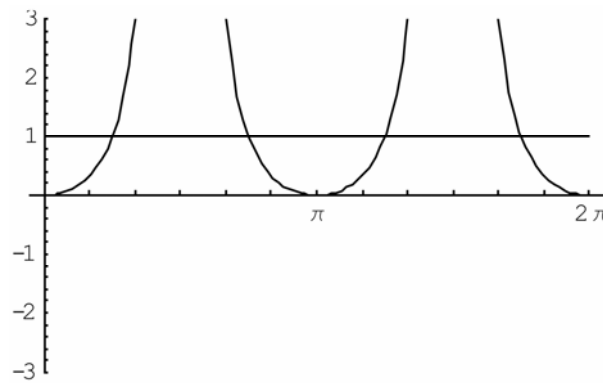
$$\frac{\cos^2 x}{\sin x}$$

$$\cos x \left(\frac{\cos x}{\sin x} \right) \quad \text{Separate out to finish the question}$$

$$\cos x \cot x$$

The answer is **D**.

NR 7. Graph the equation



The answer is **4**.

29.

$$\begin{aligned}& \frac{\sin x + \tan x}{\cos x + 1} \\& \frac{\sin x + \frac{\sin x}{\cos x}}{\cos x + 1} \\& \frac{\frac{\sin x \cos x}{\cos x} + \frac{\sin x}{\cos x}}{\cos x + 1} \\& \frac{\sin x \cos x + \sin x}{\cos x} \\& \frac{\sin x \cos x + \sin x}{\cos x} \cdot \frac{1}{\cos x + 1} \\& \frac{\sin x (\cos x + 1)}{\cos x} \cdot \frac{1}{\cos x + 1} \\& = \frac{\sin x}{\cos x} \\& = \tan x\end{aligned}$$

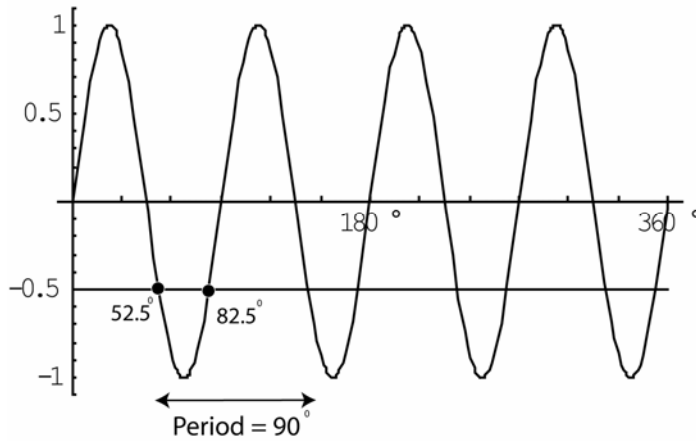
The answer is **B**.

30.

$$\begin{aligned}& \csc^4 x - 1 \\& (\csc^2 x - 1)(\csc^2 x + 1) \\& \cot^2 x (\csc^2 x + 1)\end{aligned}$$

The answer is **C**.

31. Graph $\sin 4x = -\frac{1}{2}$ in degree mode



$$52.5^\circ \bullet \frac{\pi}{180^\circ} = \frac{7\pi}{24}$$

$$82.5^\circ \bullet \frac{\pi}{180^\circ} = \frac{11\pi}{24}$$

The general solutions are $\frac{7\pi}{24} \pm \frac{n\pi}{2}, \frac{11\pi}{24} \pm \frac{n\pi}{2}$

The answer is **C**.

32.

$\sec x = \frac{1}{\cos x}$ is undefined at $\frac{\pi}{2}, \frac{3\pi}{2}, \dots$ since the denominator becomes zero at these values.

$\sec 2x = \frac{1}{\cos 2x}$ is undefined at $\frac{\pi}{4}, \frac{3\pi}{4}, \dots$, since these angles will result in the denominator becoming zero.

The answer is **A**.

33. The reason no answer can be found is because the range of $y = \cos x$ exists only between -1 and 1, and there are no intersection points in the equations $\cos x = \sqrt{2}$ and $\cos x = -\sqrt{2}$.

The answer is **B**.

Written Response #1

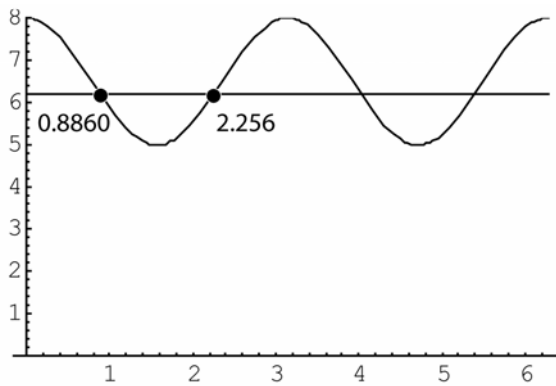
- The equation $f(x) = 8 - 3\sin^2 x$ should be typed into the graphing calculator using $y_1 = 8 - 3(\sin(x))^2$

Make sure calculator is in radian mode.

Window Settings: Domain $[-2\pi, 2\pi, \frac{\pi}{2}]$ Range $[0, 8, 1]$

*There are multiple answers for the range, as long as the bottom and top of the graph can be seen clearly.

- Draw the graph $6.2 = 8 - 3(\sin(x))^2$ and find the intersection points.



The period is $\pi = 3.14$ radians.

The general solutions are

$$0.89 + n\pi, \quad n \in I$$

$$2.26 + n\pi, \quad n \in I$$

- State what you know from the graph:

$$a = \frac{|\max - \min|}{2} = \frac{|8 - 5|}{2} = \frac{3}{2} = 1.5$$

$$b = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$$

$$c = 0$$

$$d = \frac{|\min + \max|}{2} = \frac{|5 + 8|}{2} = \frac{13}{2} = 6.5$$

The equation is $g(x) = 1.5\cos 2x + 6.5$

$$7 + \sin^2 x = 8 - 3\sin^2 x$$

$$4\sin^2 x = 1$$

$$\sin^2 x = \frac{1}{4}$$

$$\sqrt{\sin^2 x} = \sqrt{\frac{1}{4}}$$

$$\sin x = \pm \frac{1}{2}$$

Use the unit circle to determine the correct angles.

$$\sin x = \frac{1}{2}$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

Written Response #2

$\frac{\cos x}{1 - \sin x}$ $\frac{\cos \frac{\pi}{6}}{1 - \sin \frac{\pi}{6}}$ $\frac{\frac{\sqrt{3}}{2}}{1 - \frac{1}{2}}$ $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$ $\frac{\sqrt{3}}{2} \cdot \cancel{2}$	$\frac{1 + \sin x}{\cos x}$ $\frac{1 + \sin \frac{\pi}{6}}{\cos \frac{\pi}{6}}$ $\frac{1 + \frac{1}{2}}{\frac{\sqrt{3}}{2}}$ $\frac{\frac{3}{2}}{\frac{\sqrt{3}}{2}}$ $\frac{3}{2} \cdot \cancel{\frac{2}{\sqrt{3}}}$	$\frac{\cos x}{1 - \sin x} + \frac{1 + \sin x}{\cos x}$ $\frac{\cos x(\cos x)}{(1 - \sin x)(\cos x)} + \frac{(1 + \sin x)(1 - \sin x)}{\cos x(1 - \sin x)}$ $\frac{\cos^2 x}{\cos x(1 - \sin x)} + \frac{1 - \sin^2 x}{\cos x(1 - \sin x)}$ $\frac{\cos^2 x}{\cos x(1 - \sin x)} + \frac{\cos^2 x}{\cos x(1 - \sin x)}$ $\frac{2\cos^2 x}{\cos x(1 - \sin x)}$ $\frac{2\cos x}{(1 - \sin x)}$
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- The graphs are not identical because the graph of $y_1 = \frac{\cos x}{1 - \sin x}$ exists at $\frac{3\pi}{2}$, but the graph of $y_2 = \frac{1 + \sin x}{\cos x}$ is undefined at $\frac{3\pi}{2}$. This point discontinuity is not visible when you use the calculator.

If you go 2nd → Trace → Value → $x = \frac{3\pi}{2}$, notice that you get zero for y_1 , and no solution for y_2

- $$\begin{aligned} & \frac{\cos x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} \quad \text{Use conjugate} \\ &= \frac{\cos x(1 + \sin x)}{(1 - \sin x)(1 + \sin x)} \\ &= \frac{\cos x(1 + \sin x)}{1 - \sin^2 x} \\ &= \frac{\cos x(1 + \sin x)}{\cos^2 x} \\ &= \frac{1 + \sin x}{\cos x} \end{aligned}$$

- $$\begin{aligned} & \frac{\cos x}{1 - \sin x} + \frac{1 + \sin x}{\cos x} \\ &= \frac{\cos x(\cos x)}{(1 - \sin x)(\cos x)} + \frac{(1 + \sin x)(1 - \sin x)}{\cos x(1 - \sin x)} \\ &= \frac{\cos^2 x}{\cos x(1 - \sin x)} + \frac{1 - \sin^2 x}{\cos x(1 - \sin x)} \\ &= \frac{\cos^2 x}{\cos x(1 - \sin x)} + \frac{\cos^2 x}{\cos x(1 - \sin x)} \\ &= \frac{2\cos^2 x}{\cos x(1 - \sin x)} \\ &= \frac{2\cos x}{(1 - \sin x)} \end{aligned}$$

Written Response #3

$$\begin{aligned} & \frac{1 + \cos 2x}{\sin 2x} \\ &= \frac{1 + (2\cos^2 x - 1)}{2\sin x \cos x} \\ &= \frac{2\cos^2 x}{2\sin x \cos x} \\ &= \frac{\cos x}{\sin x} \\ &= \cot x \end{aligned}$$

$$\begin{aligned} & (\sin x + \cos x)^2 \\ &= \sin^2 x + 2\sin x \cos x + \cos^2 x \\ &= \sin^2 x + \cos^2 x + 2\sin x \cos x \\ &= 1 + 2\sin x \cos x \end{aligned}$$

$$\begin{aligned} & \sin 2x = \sin(x + x) \\ & \sin(x + x) = \sin x \cos x + \cos x \sin x \\ &= 2\sin x \cos x \end{aligned}$$

$$\begin{aligned} & 2\sin x \cos x - \cos x = 0 \\ & \cos x(2\sin x - 1) = 0 \\ & x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2} \end{aligned}$$

$$\begin{aligned} & 3\sin x = 2\sin x \\ & 3\sin x - 2\sin x = 0 \\ & \sin x = 0 \\ & x = 0, \pi \end{aligned}$$

$$\begin{aligned} & 15\left(\frac{\csc x}{5} + \frac{\csc x}{3}\right) = 15\left(\frac{16}{15}\right) \\ & 3\csc x + 5\csc x = 16 \\ & 8\csc x = 16 \\ & \csc x = 2 \\ & \sin x = \frac{1}{2} \\ & x = \frac{\pi}{6}, \frac{5\pi}{6} \end{aligned}$$