

# Pure Math 30

## EXPLAINED!

*Diploma Style  
Practice Exam*

**TRIGONOMETRY II  
-- QUESTIONS--**

## Formulas

*These are the formulas for Trig II you will be given on your diploma.*

$$a = r\theta$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = \cos^2 A - \sin^2 A$$

|  |
|--|
| <i>Use this sheet to record your answers</i> |
|--|

- |       |       |       |       |
|-------|-------|-------|-------|
| 1.    | NR 2. | 19.   | 28.   |
| 2.    | NR 3. | 20.   | NR 7. |
| NR 1. | 11.   | 21.   | 29.   |
| 3.    | 12.   | NR 5. | 30.   |
| 4.    | 13.   | 22.   | 31.   |
| 5.    | 14.   | 23.   | 32.   |
| 6.    | NR 4. | 24.   | 33.   |
| 7.    | 15.   | 25.   |       |
| 8.    | 16.   | NR 6. |       |
| 9.    | 17.   | 26.   |       |
| 10.   | 18.   | 27.   |       |

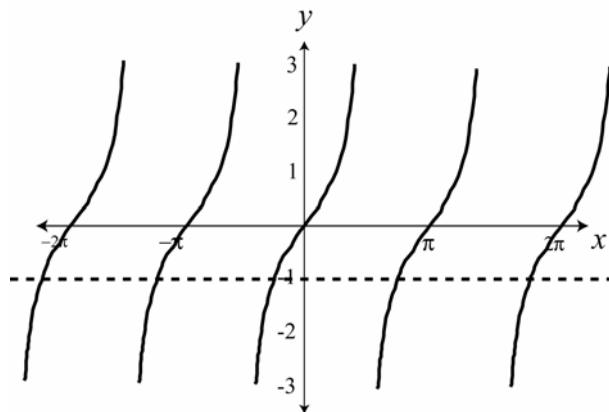
## Trigonometry II Diploma Style Practice Exam

1. The exact value of  $\sin 75^\circ$  can be determined using the expression

- A.  $\sin 90^\circ - \sin 15^\circ$
- B.  $\sin 45^\circ + \sin 30^\circ$
- C.  $\sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$
- D.  $\cos 45^\circ \cos 30^\circ + \cos 45^\circ \cos 30^\circ$

*Use the following information to answer the next question.*

A student solves the equation  $\tan x = -1$  in their graphing calculator as shown in the diagram below.



The student determines the general solution of this graph is  $-\frac{\pi}{4} + n\pi, n \in I$

2. The general solution to the equation  $\tan(5x) = -1$  is

- A.  $-\frac{\pi}{3} + \frac{n\pi}{10}, n \in I$
- B.  $-\frac{\pi}{4} + n\pi, n \in I$
- C.  $-\frac{5\pi}{4} + 5n\pi, n \in I$
- D.  $-\frac{\pi}{20} + \frac{n\pi}{5}, n \in I$

### Numerical Response

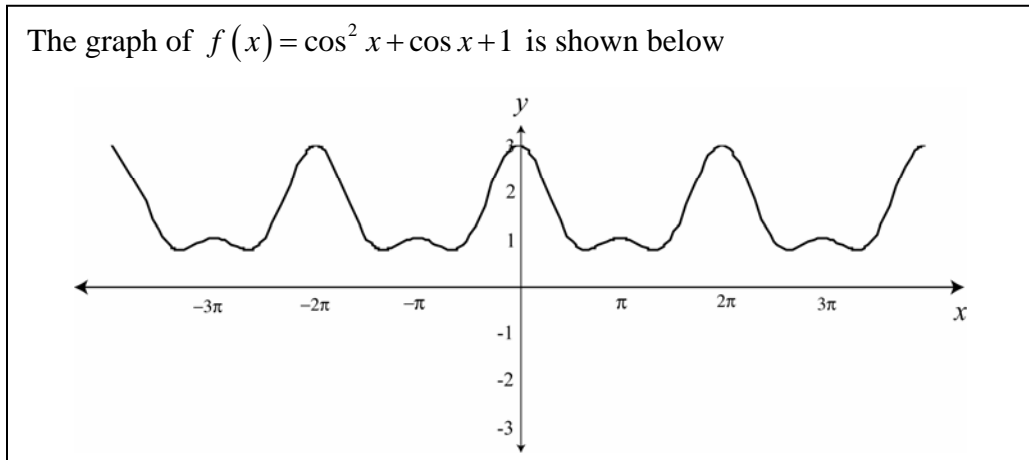
1. The identity  $\cot^2 x + \csc x = \frac{\cos^2 x + \sin x}{\sin^2 x}$  may be verified by substituting 2.1 rad for  $x$  on each side. When this substitution is made, the numerical value of each side, to the nearest hundredth, is \_\_\_\_\_.

*Use the following information to answer the next question.*

A Ferris wheel at an amusement park, with a diameter of 12 m, can be modeled using the equation  $h(t) = -6 \cos \frac{\pi}{20} t + 9$ , where  $h(t)$  is the height above the ground in metres, and  $t$  is the time in seconds.

3. The number of seconds required for a rider to reach a height of 14 m for the first time is, to the nearest tenth,
- A. 16.3 s  
B. 16.5 s  
C. 20.4 s  
D. 932 s
- 
4. The expression  $\cot^2 x + \csc x - 4$  is equivalent to
- A.  $\csc^2 x + \csc x - 5$   
B.  $\csc^2 x + \csc x - 3$   
C.  $\frac{\cos^2 x}{\sin^2 x} + \frac{1}{\cos x} - 4$   
D. -4

Use the following information to answer the next question.

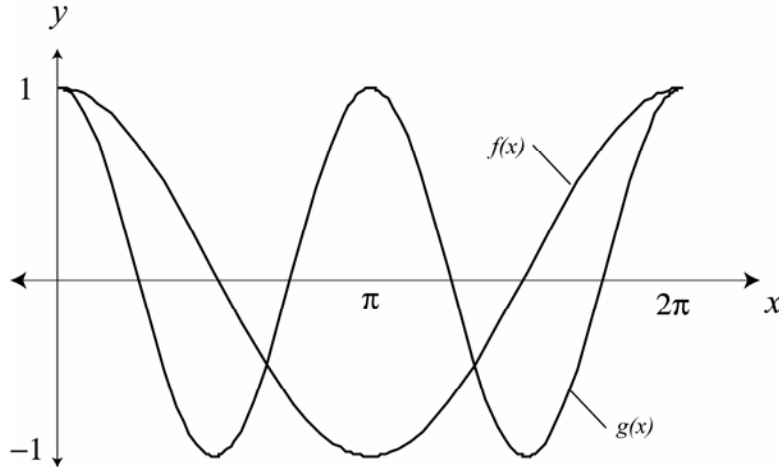


5. If the equation  $\cos^2 x + \cos x + 1 = 3$  has the general solution  $2n\pi$ ,  $n \in I$ , then a possible solution to the equation  $\cos^2\left(\frac{x}{3}\right) + \cos\left(\frac{x}{3}\right) + 1 = 3$  is
- A.  $2\pi$
  - B.  $3\pi$
  - C.  $9\pi$
  - D.  $12\pi$
6. If  $\sin A = \frac{m}{n}$  and  $\tan A = \frac{m^2}{n^3}$ , where  $m, n \neq 0$ , then  $\cos A$  is equivalent to
- A.  $\frac{n^2}{m}$
  - B.  $\frac{m^3}{n^4}$
  - C.  $mn^2$
  - D.  $\frac{1}{mn^2}$

7. The expression  $\sqrt{\frac{1 + \tan^2 x}{1 - \sin^2 x}}$ , is equivalent to
- A.  $\sqrt{\frac{(1 + \tan x)(1 - \tan x)}{(1 + \sin x)(1 - \sin x)}}$   
B. 1  
C.  $\sec x$   
D.  $\sec^2 x$
8. If  $\tan^2 x = \frac{5}{7}$ , then  $\sec^2 x$  is equivalent to
- A.  $\frac{12}{7}$   
B.  $\frac{7}{5}$   
C.  $\frac{5\sqrt{74}}{74}$   
D.  $\frac{\sqrt{74}}{7}$
9. The expression  $\cos^2(4\pi) - \sin^2(4\pi)$  is equivalent to
- A.  $\cos^2(4\pi)$   
B.  $\sin^2(8\pi)$   
C.  $\cos(8\pi)$   
D.  $\cos(4\pi)\sin(4\pi)$

Use the following information to answer the next question.

The graphs of  $f(x) = \cos x$  and  $g(x) = \cos(2x)$  are shown below



The graphs intersect four times on the interval  $0 \leq x \leq 2\pi$

10. If the domain is changed to  $0 < x < 2\pi$ , (*the equality has been removed*) a correct statement is
- A. There are more solutions
  - B. There are fewer solutions
  - C. There are the same number of solutions
  - D. There is no change in the number of solutions

### Numerical Response

2. The equation  $\csc^2 x - 2 = \cos^2 x$  has four solutions in the interval  $0 < x < 2\pi$ . The number of solutions for  $x$  in the interval  $0 < x < 14\pi$  is \_\_\_\_\_.

Use the following information to answer the next question.

The steps used by a student to simplify the expression  $(\sin x + \cos x)^2$  are shown below

**Step 1:**  $\sin^2 x + \cos^2 x$

**Step 2:**  $\sin^2 x + (1 - \sin^2 x)$

**Step 3:**  $(1 - \cos^2 x) + (1 - \sin^2 x)$

**Step 4:**  $2 - \sin^2 x - \cos^2 x$

### Numerical Response

3. The step which contains a mathematical error is step \_\_\_\_\_.

11. The value of  $m$  in the equation  $\frac{m \sin x \cot x}{4 \csc x \tan x} = 8$  is

- A.  $\frac{\sin x \cos x}{32}$
- B.  $\frac{2 \sin x \cos x}{\tan x}$
- C.  $32 \sec^2 x$
- D.  $32 \sec x \csc x$

12. The solutions to the equation  $\cos^2 x = \cos x$ , where  $0 \leq x < 2\pi$  are

- A.  $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$
- B.  $\frac{\pi}{2}, \frac{3\pi}{2}$
- C.  $0, \frac{\pi}{2}, \frac{3\pi}{2}$
- D.  $0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$



13. The expression  $\frac{\cos x}{1 - 2 \sin x}$  is undefined when the values of  $x$  are

- A.  $\frac{\pi}{6} \pm 2n\pi, \frac{5\pi}{6} \pm 2n\pi$
- B.  $\frac{\pi}{6} \pm n\pi$
- C.  $\frac{\pi}{6} \pm 2n\pi, \frac{5\pi}{6} \pm 2n\pi, \frac{\pi}{2} \pm n\pi$
- D.  $\frac{n\pi}{2}$

14. The expression  $\sec\left(x - \frac{\pi}{2}\right)$  is equivalent to

- A.  $\sec x - \sec \frac{\pi}{2}$
- B.  $\cos\left(x - \frac{\pi}{2}\right)$
- C.  $\csc x$
- D.  $-\sin\left(\frac{\pi}{2} - x\right)$

### Numerical Response

4. If  $\frac{1}{1 + \cot^2 x} = 0.43$ , and  $0 \leq x < \frac{\pi}{2}$ , then the value of  $x$  in radians, to the nearest tenth, is \_\_\_\_\_.

15. The expression  $\frac{\sin x}{\tan x} + \frac{1}{\sec x}$  is equivalent to

- A.  $2 \cos x$
- B.  $2 \sec x$
- C.  $\frac{\sin x + 1}{\tan x + \sec x}$
- D.  $\frac{\sin x}{\tan x \sec x}$

16. Given  $\sin A = \frac{7}{8}$  and  $\cos B = \frac{4}{5}$ , where  $A$  and  $B$  are acute angles, the value of  $\cos(A - B)$  is equal to
- A.  $\frac{3}{8}$   
 B.  $\frac{16}{25} + \frac{49}{64}$   
 C.  $\frac{4\sqrt{15} + 21}{40}$   
 D.  $\frac{28 - 3\sqrt{15}}{40}$
17. If the equation  $-5\csc^2 x + 12\cot^2 x - 9 = 0$  is simplified using the identity  $1 + \cot^2 x = \csc^2 x$ , the resulting equation is
- A.  $-5\tan^2 x + 12\sec^2 x - 9 = 0$   
 B.  $\cot^2 x = 2$   
 C.  $12\cot^2 x - 9 - 5\sec^2 x = 0$   
 D.  $\sec 2x(1 - \tan^2 x) = 6$
18. The expression  $\cos(x - y) - \cos(x + y)$  is equivalent to
- A.  $2\sin x \sin y$   
 B.  $0$   
 C.  $-2\cos y$   
 D.  $\cos\left(\frac{x - y}{x + y}\right)$
19. The line  $y = \frac{1}{2}$  intersects the graph of  $\cos^2 x - \sin x$  twice in the interval  $0 \leq x < 2\pi$ . An equation that can be used to solve for  $x$  is
- A.  $\cos^2 x = \sin x$   
 B.  $2\cos^2 x - 2\sin x - 1 = 0$   
 C.  $\sin x - \cos^2 x = 2$   
 D.  $2\cos^2 x + 2\sin x - 1 = 0$

20. The expression  $\sin\left(\frac{\theta}{5}\right)\cos\left(\frac{2\theta}{7}\right) - \cos\left(\frac{\theta}{5}\right)\sin\left(\frac{2\theta}{7}\right)$  is equivalent to

A.  $\cos\left(\frac{17\theta}{35}\right)$

B.  $\sin\left(\frac{17\theta}{35}\right)$

C.  $\sin\left(\frac{3\theta}{35}\right)$

D.  $\sin\left(\frac{-3\theta}{35}\right)$

21. If  $\frac{\csc 2x}{\sec 2x} = \sqrt{5}$ , then the value of  $x$ , to the nearest hundredth of a radian is

A.  $1.15 + 3.14n, n \in I$

B.  $0.42 + 3.14n, n \in I$

C.  $0.54 + 1.57n, n \in I$

D.  $0.21 + 1.57n, n \in I$

*Use the following information to answer the next question.*

A student is given four different trigonometric expressions

I  $\frac{1}{9} \sec x \cos x$

II  $\cot^2 x - \csc^2 x$

III  $2\cos^2 x + 2\sin^2 x$

IV  $2\cot x - \frac{2\cos x}{\sin x}$

### Numerical Response

5. If the expressions are simplified are ranked, from smallest to largest, the correct order is \_\_\_\_\_.

22. Given  $\sin x = m$ , an expression for  $\cos 2x$ , in terms of  $m$ , is

- A.  $1 - 2m^2$
- B.  $1 - 2m$
- C.  $2m^2 - 1$
- D.  $2m - 1$

23. The expression  $\frac{1 + \csc x}{\sin x}$  is equivalent to

- A.  $\csc x + \sin x$
- B.  $\frac{\sin x + 1}{\cos^2 x + 1}$
- C.  $\frac{\sin x + 1}{\sin^2 x}$
- D. 1

24. Given  $x = 45^\circ$ , an equivalent expression to  $\frac{\cos(x + y)}{\cos y}$  is

- A.  $\cos\left(\frac{x}{y}\right) + 1$
- B.  $\frac{\sqrt{2}}{2}(1 - \tan y)$
- C.  $\cos x$
- D.  $\frac{2 + \sqrt{2} \cos y}{2 \cos y}$

25. The exact value of  $\sec\left(-\frac{\pi}{12}\right)$  is

- A.  $\frac{4}{\sqrt{2} - \sqrt{6}}$
- B.  $\sqrt{6} - \sqrt{2}$
- C.  $-75^\circ$
- D.  $\frac{11\pi}{12}$

### Numerical Response

6. The expression  $\cos x$  may be written as  $\cos^2 kx - \sin^2 kx$ . The value of  $k$ , to the nearest tenth, is \_\_\_\_\_.
26. Using the identity  $\cos^2 x = 1 - \sin^2 x$ , the expression  $\cos^2 x - \sin^2 x - 1 + 2 \sin x$  can be simplified to
- A.  $2 \sin x(1 - \sin x)$
  - B.  $\sin x(1 - 2 \sin x)$
  - C.  $\sin 2x + 2 \sin x$
  - D.  $2 \sin 2x + 1$
27. If  $\tan x = -\frac{6}{7}$  and  $\sin y = -\frac{2}{5}$ , the exact value of  $\sec(x + y)$ , given that  $\frac{3\pi}{2} \leq x < 2\pi$ ,  $\frac{3\pi}{2} \leq y < 2\pi$ , is
- A.  $\sqrt{21} - 5$
  - B.  $5 - \sqrt{21}$
  - C.  $\frac{7\sqrt{21}}{12\sqrt{85} - 5}$
  - D.  $\frac{5\sqrt{85}}{7\sqrt{21} - 12}$
28. The expression  $\csc x - \sin x$  is equivalent to
- A.  $\frac{1}{\sin^2 x}$
  - B. 1
  - C.  $\frac{\sin x}{\cos^2 x}$
  - D.  $\cot x \cos x$

### Numerical Response

7. The number of solutions in the equation  $\tan^2 x = 1$ , where  $0 \leq x < 2\pi$ , is \_\_\_\_\_.
29. The expression  $\frac{\sin x + \tan x}{\cos x + 1}$  is equivalent to
- A.  $\csc^2 x$
  - B.  $\tan x$
  - C.  $\frac{2 \sin x}{\cos x + 1}$
  - D.  $2 \tan x$
30. The expression  $\csc^4 x - 1$  is equivalent to
- A.  $\frac{\csc^4 x}{\sec^4 x}$
  - B.  $\cot^4 x$
  - C.  $\cot^2 x (\csc^2 x + 1)$
  - D.  $\cot^2 x (\sec^2 x + 1)$
31. The general solution to the equation  $\sin 4x = -\frac{1}{2}$  is
- A.  $\frac{7\pi}{24} \pm \frac{n\pi}{2}$
  - B.  $\frac{5\pi}{12} \pm \frac{n\pi}{4}, \frac{3\pi}{12} \pm \frac{n\pi}{4}$
  - C.  $\frac{7\pi}{24} \pm \frac{n\pi}{2}, \frac{11\pi}{24} \pm \frac{n\pi}{2}$
  - D.  $\frac{3\pi}{12} \pm \frac{n\pi}{4}$

32. The expression  $\sec 2x$  is undefined when  $x$  is the angle

- A.  $\frac{\pi}{4}$
- B.  $\frac{\pi}{2}$
- C.  $\pi$
- D.  $2\pi$

*Use the following information to answer the next question.*

A student solves the equation  $\cos^2 x - 2 = 0$  algebraically, using the steps shown below

$$(\cos x - \sqrt{2})(\cos x + \sqrt{2}) = 0$$

$$\cos x - \sqrt{2} = 0 \rightarrow x \text{ has no solution.}$$

$$\cos x + \sqrt{2} = 0 \rightarrow x \text{ has no solution.}$$

33. The reason why  $\cos^2 x - 2 = 0$  has no solution is because

- A.  $\cos x$  is undefined for  $x = \sqrt{2} \pm 2n\pi$
- B. The range of  $y = \cos x$  is  $-1 \leq y \leq 1$
- C.  $\cos^2 x - 2 = 0$  cannot be factored
- D.  $\cos^2 x$  must be replaced with  $\sin^2 x - 1$  before factoring

Use the following information to answer the next question.

A student graphs the following function in a graphing calculator.

$$f(x) = 8 - 3\sin^2 x$$

$x$  is measured in radians, and the student wishes to analyze the graph for  $-2\pi \leq x \leq 2\pi$

**Written Response – 10%**

1.

- Explain how the student would have to type the above equation into their graphing calculator in order to obtain the correct graph. Indicate appropriate window settings.
- The student now wishes to solve the equation  $6.2 = f(x)$ . State the general solution to this equation in radian decimal form, to the nearest hundredth.
- The graph of  $f(x)$  can be expressed in the form  $g(x) = a \cos b[x - c] + d$ . Write the equation for  $g(x)$
- Algebraically solve the equation  $7 + \sin^2 x = 8 - 3\sin^2 x$ . Show all steps required in obtaining the answer.



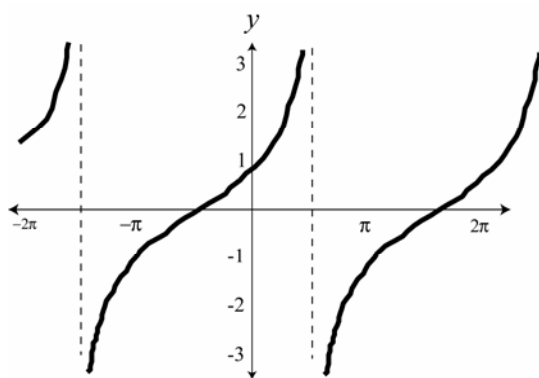
Written Response – 10%

2.

- Verify the identity  $\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$  for  $x = \frac{\pi}{6}$

Use the following additional information to answer the next part of the question.

The graphs of  $y_1 = \frac{\cos x}{1 - \sin x}$  and  $y_2 = \frac{1 + \sin x}{\cos x}$  are shown below.



- The graphs of  $y_1 = \frac{\cos x}{1 - \sin x}$  and  $y_2 = \frac{1 + \sin x}{\cos x}$  are **not** identical. Explain the difference between the graphs of  $y_1$  and  $y_2$ .

- Algebraically prove the identity  $\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}$

- Algebraically show that  $\frac{\cos x}{1 - \sin x} + \frac{1 + \sin x}{\cos x} = \frac{2 \cos x}{1 - \sin x}$

**Written Response – 10%**

**3.**

- Prove the identity  $\frac{1 + \cos 2x}{\sin 2x} = \cot x$

- Prove the identity  $(\sin x + \cos x)^2 = 1 + \sin 2x$

- Prove the identity  $\sin 2x = 2 \sin x \cos x$

- Solve algebraically:  $2 \sin x \cos x = \cos x$

- Solve algebraically:  $\frac{\sin x}{2} = \frac{\sin x}{3}$

- Solve algebraically:  $\frac{\csc x}{5} + \frac{\csc x}{3} = \frac{16}{15}$