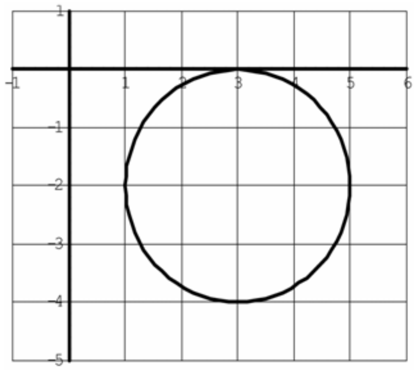
Pre - Calculus Mathematics 40S

Standards Test Practice Exam - ANSWERS

Conics

1. Sketch the graph and write the equation of a circle with a centre at (3, -2) and is tangent to the x – axis.



The phrase "tangent to the x-axis" means the circle touches the x-axis.

Start with

$$(x - h)^2 + (y - k)^2 = r^2$$

The centre is (3, -2), and the radius is 2. Plug these into the standard form equation.

$$(x - 3)^2 + (y - (-2))^2 = 2^2$$

$$\boldsymbol{(x - 3)^2 + (y + 2)^2 = 4}$$

2. Identify the conic represented by the equation $x^2 - 6x = y + 3$

Since the y^2 term is missing, the conic is a ***parabola****.*

3. State the coordinates of the vertex in the relation $y^2 = 2x + 4$

$$y^2 = 2x + 4$$

$$y^2 = 2(x + 2)$$

The vertex is located at (-2, 0)

4. Change the following to standard form, then sketch: $4y^2 + 40y - 4x^2 + 32x + 20 = 0$

$$4y^2 + 40y - 4x^2 + 32x + 20 = 0$$

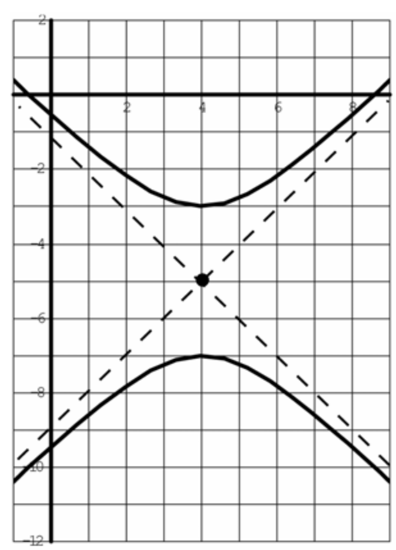
$$4(y^2 + 10y \qquad) - 4(x^2 - 8x \qquad) = -20$$

$$4(y^2 + 10y + 25) - 4(x^2 - 8x + 16) = -20 + 100 - 64$$

$$4(y + 5)^2 - 4(x - 4)^2 = 16$$

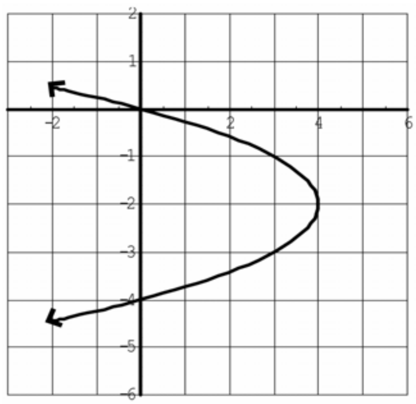
$$\frac{4(y + 5)^2}{16} - \frac{4(x - 4)^2}{16} = \frac{16}{16}$$

$$\boldsymbol{\frac{(y + 5)^2}{4} - \frac{(x - 4)^2}{4} = 1}$$



This is a vertical hyperbola since y comes first
Centre = (4, -5), a = 2, b = 2

5. a) Given the parabola below, find the equation if the vertex is at (4, -2)



The vertex is located at (4, -2). These are the h & k values.

A point on the graph is located at (0, 0) This is a value you can use for x & y.

Plug everything in and solve for a.

Standard form for horizontal parabolas $\rightarrow x - h = a(y - k)^2$

$$0 - 4 = a(0 - (-2))^2$$
$$0 - 4 = a(0 + 2)^2$$
$$-4 = a(4)$$
$$a = -1$$

The equation is: $\boldsymbol{x - 4 = -(y + 2)^2}$

b) Verify the intercepts algebraically

<u>x – intercept:</u>

Set y = 0, then solve for x.

$$x - 4 = -(0 + 2)^2$$
$$x - 4 = -4$$
$$\boldsymbol{x = 0}$$

<u>y – intercepts:</u>

Set x = 0, then solve for y.

$$0 - 4 = -(y + 2)^2$$
$$-4 = -(y + 2)^2$$
$$4 = (y + 2)^2$$
$$\sqrt{4} = \sqrt{(y + 2)^2}$$
$$\pm 2 = y + 2$$

First evaluate for + 2

$$+2 = y + 2$$
$$\boldsymbol{y = 0}$$

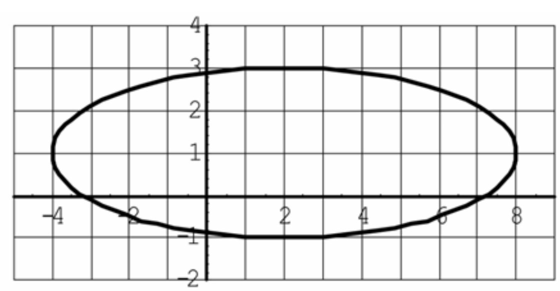
Then evaluate for - 2

$$-2 = y + 2$$
$$\boldsymbol{y = -4}$$

6. A conic is represented by $x^2 - 4x + 9y^2 - 18y = 23$

a) Sketch the conic

$$x^2 - 4x + 9y^2 - 18y = 23$$
$$\left(x^2 - 4x \qquad\right) + 9\left(y^2 - 2y \qquad\right) = 23$$
$$\left(x^2 - 4x + 4\right) + 9\left(y^2 - 2y + 1\right) = 23 + 4 + 9$$
$$(x - 2)^2 + 9(y - 1)^2 = 36$$
$$\frac{(x - 2)^2}{36} + \frac{9(y - 1)^2}{36} = \frac{36}{36}$$
$$\boldsymbol{\frac{(x - 2)^2}{36} + \frac{(y - 1)^2}{4} = 1}$$



Centre = (2, 1)
a = 6, b = 2

b) State the domain & range

Domain is $\{\boldsymbol{x \mid -4 \le x \le 8}\}$

Range is $\{\boldsymbol{y \mid -1 \le y \le 3}\}$

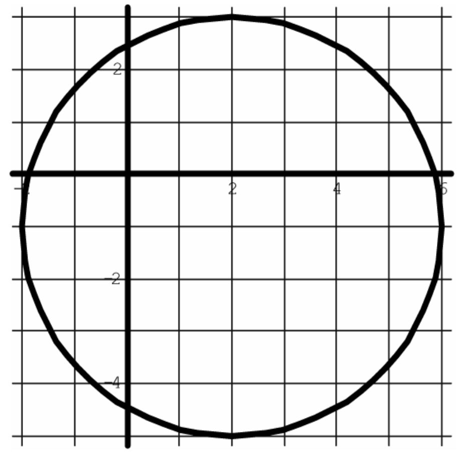
7. Identify the conic with the equation $3x^2 - y^2 - 7x + 2 = 0$

Since the product of A & C is negative, the conic is a ***hyperbola****.*

8. The equation of a circle is given by the equation $2x^2 + 2y^2 - 8x + 4y - 22 = 0$

a) Sketch the conic

$$2x^2 + 2y^2 - 8x + 4y - 22 = 0$$
$$2x^2 - 8x + 2y^2 + 4y = 22$$
$$2\left(x^2 - 4x \quad\right) + 2\left(y^2 + 2y \quad\right) = 22$$
$$2\left(x^2 - 4x + 4\right) + 2\left(y^2 + 2y + 1\right) = 22 + 8 + 2$$
$$2(x - 2)^2 + 2(y + 1)^2 = 32$$
$$\frac{2(x - 2)^2}{2} + \frac{2(y + 1)^2}{2} = \frac{32}{2}$$
$$\boldsymbol{(x - 2)^2 + (y + 1)^2 = 16}$$

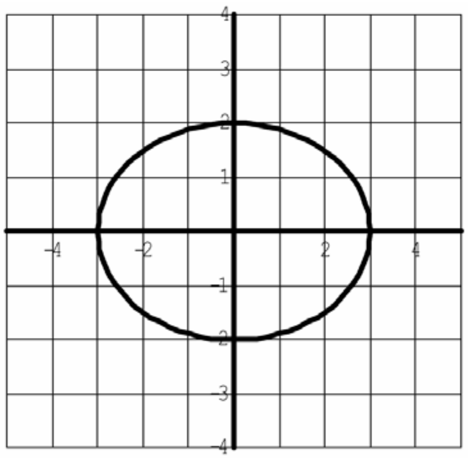


b) State the radius

The form is $(x - h)^2 + (y - k)^2 = r^2$.

Taking the square root of the right side gives ***4 units****.*

9. Determine the equation of the ellipse shown below



The a – value is 3, and the b – value is 2.
The centre is (0, 0)

Plug into the equation

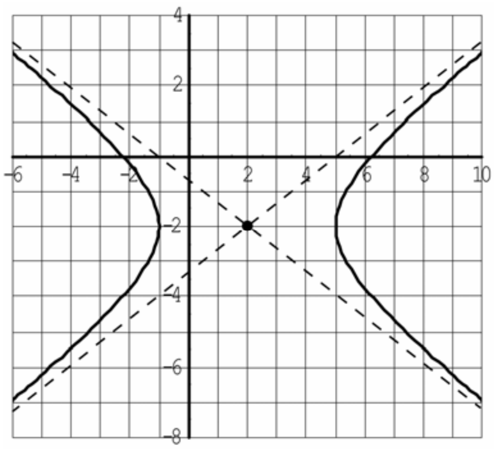
$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$
$$\frac{(x - 0)^2}{3^2} + \frac{(y - 0)^2}{2^2} = 1$$
$$\boldsymbol{\frac{x^2}{9} + \frac{y^2}{4} = 1}$$

10. The equation of a conic is $\frac{(x-2)^2}{9}-\frac{(y+2)^2}{4}=1$

a) Identify this conic section

*The minus between the terms indicates this is a **hyperbola***

b) Sketch a clearly labeled graph



Centre = (2, -2)
a = 3, b = 2

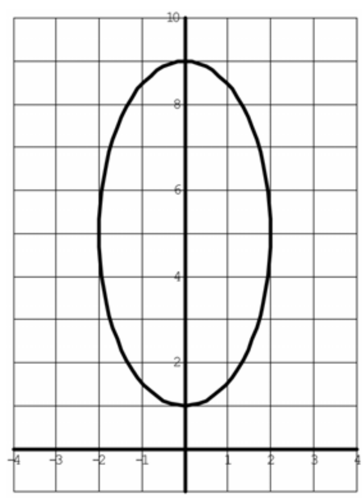
c) State the domain & range

Domain is $\{\boldsymbol{x \mid x \le -1,\ x \ge 5}\}$

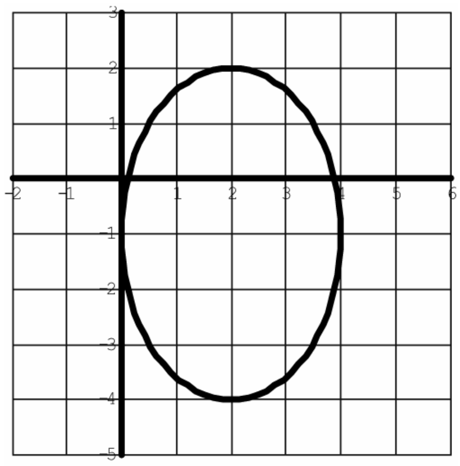
Range is $\{\boldsymbol{y \mid y \in R}\}$

11. Find the coordinates of the vertices and sketch the equation: $9x^2 + 4y^2 + 40y + 64 = 0$

$$9x^2 + 4y^2 + 40y + 64 = 0$$
$$9x^2 + 4(y^2 + 10y) = -64$$
$$9x^2 + 4(y^2 + 10y + 25) = -64 + 100$$
$$\frac{9x^2}{36} + \frac{4(y+5)^2}{36} = \frac{36}{36}$$
$$\boldsymbol{\frac{x^2}{4} + \frac{(y+5)^2}{9} = 1}$$



12. Find the equation of the ellipse sketched below:



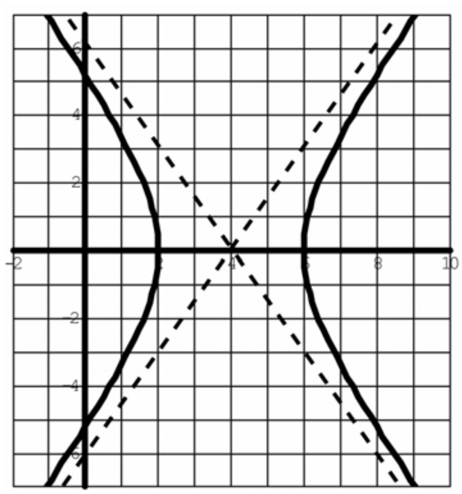
Start with: $\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$

Centre = (2, -1) ; a = 2, b = 3

$$\frac{(x - 2)^2}{(2)^2} + \frac{(y + 1)^2}{(3)^2} = 1$$

$$\boldsymbol{\frac{(x - 2)^2}{4} + \frac{(y + 1)^2}{9} = 1}$$

13. Sketch the equation and state the domain and range for $\frac{(x-4)^2}{4} - \frac{y^2}{9} = 1$

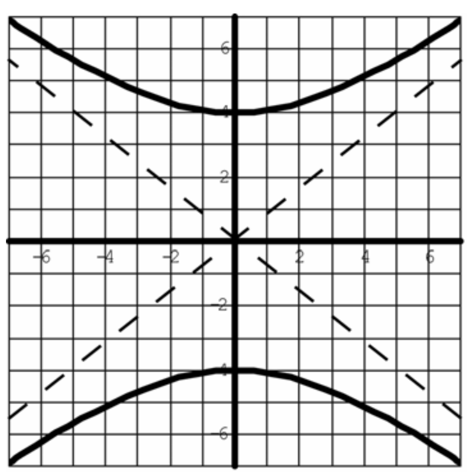


Centre = (4, 0) ; a = 2, b = 3

Domain is $\{\boldsymbol{x \mid x \le 2,\ x \ge 6}\}$

Range is $\{\boldsymbol{y \mid y \in R}\}$

14. Sketch the equation and state the axis of symmetry: $\frac{y^2}{16} - \frac{x^2}{25} = 1$



This is a vertical hyperbola since y comes first.

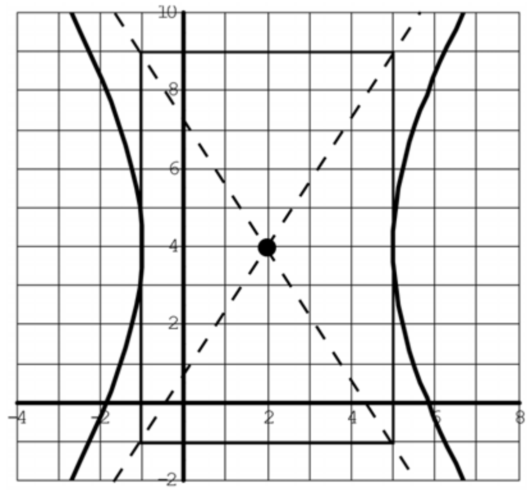
Since the centre is at the origin, the ***<u>x-axis</u>*** *is the axis of symmetry.*

15. The equation of a conic is $25x^2 - 9y^2 - 100x + 72y - 269 = 0$

a) Write the equation in standard form

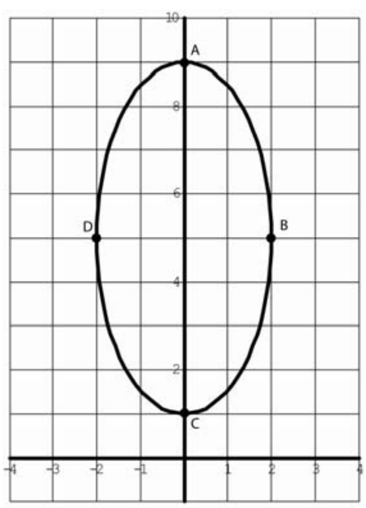
$$25x^2 - 9y^2 - 100x + 72y - 269 = 0$$
$$25x^2 - 100x - 9y^2 + 72y = 269$$
$$25\left(x^2 - 4x \qquad\right) - 9\left(y^2 - 8y \qquad\right) = 269$$
$$25\left(x^2 - 4x + 4\right) - 9\left(y^2 - 8y + 16\right) = 269 + 100 - 144$$
$$25\left(x - 2\right)^2 - 9\left(y - 4\right)^2 = 225$$
$$\frac{25\left(x - 2\right)^2}{225} - \frac{9\left(y - 4\right)^2}{225} = \frac{225}{225}$$
$$\boldsymbol{\frac{\left(x - 2\right)^2}{9} - \frac{\left(y - 4\right)^2}{25} = 1}$$

b) Sketch the graph



Centre = (2, 4)
a = 3, b = 5

16. An ellipse has the following vertices: A(0, 9), B(2, 5), C(0, 1), and D(-2, 5)
Draw the ellipse and determine the equation.



The a – value is 2, and the b – value is 4.
The centre is (0, 5)

Plug into the equation

$$\frac{\left(x - h\right)^2}{a^2} + \frac{\left(y - k\right)^2}{b^2} = 1$$
$$\frac{\left(x - 0\right)^2}{2^2} + \frac{\left(y - 5\right)^2}{4^2} = 1$$
$$\boldsymbol{\frac{x^2}{4} + \frac{\left(y - 5\right)^2}{16} = 1}$$

17. For the conic $2y^2 - 2x - 4y - 6 = 0$

a) Find the intercepts of the conic section

<u>*x* – intercept:</u>

Set y = 0, then solve for x.

$2(0)^2 - 2x - 4(0) - 6 = 0$

$-2x = 6$

$\boldsymbol{x = -3}$

<u>*y* – intercepts:</u>

Set x = 0, then solve for y.

$2y^2 - 2(0) - 4y - 6 = 0$

$2y^2 - 4y - 6 = 0$

$2(y^2 - 2y - 3) = 0$

$y^2 - 2y - 3 = 0$

$(y-3)(y+1) = 0$

$\boldsymbol{y = -1, 3}$

b) Sketch a clearly labeled graph

$2y^2 - 2x - 4y - 6 = 0$

$2y^2 - 4y = 2x + 6$

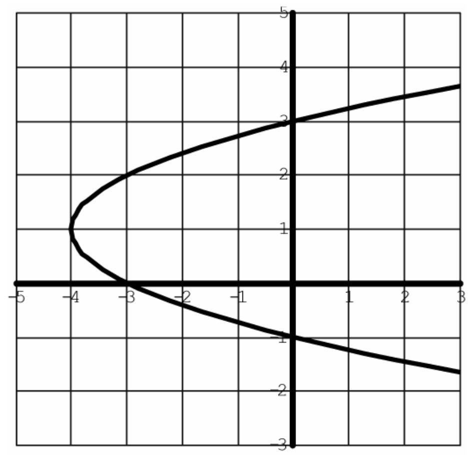
$2(y^2 - 2y) = 2x + 6$

$2(y^2 - 2y + 1) = 2x + 6 + 2$

$2(y-1)^2 = 2x + 8$

$2(y-1)^2 = 2(x+4)$

$\boldsymbol{(y-1)^2 = (x+4)}$

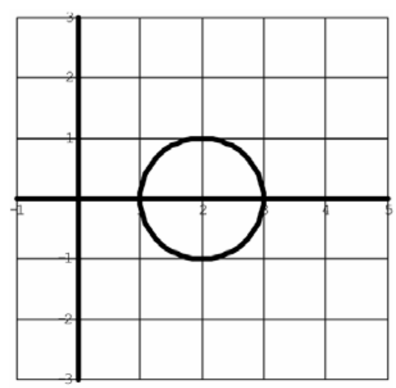


18. The conic $x^2 + y^2 = 1$ is translated 2 units to the right

a) Write the equation of the new conic and sketch it

A translation 2 units right can be accomplished by replacing x with x – 2.

$\boldsymbol{(x-2)^2 + y^2 = 1}$



b) State the domain of the new conic

$\boldsymbol{\{x \mid 1 \le x \le 3\}}$

c) State the range of the new conic

$\boldsymbol{\{y \mid -1 \le y \le 1\}}$