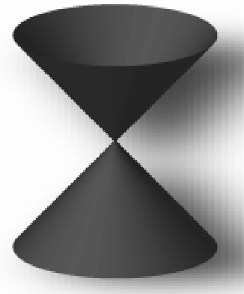
Pre - Calculus Math 40S:

CONICS



LESSON 1

The Double - Napped Cone

Pre - Calculus Math 40S EXPLAINED!

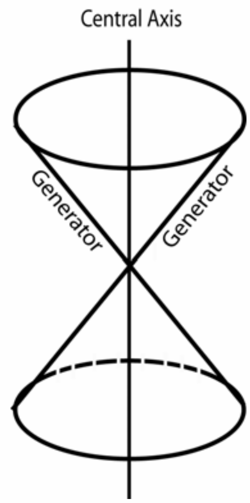
By Barry Mabillard

Conics Lesson 1

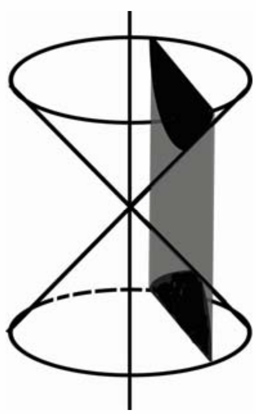
Part I – The Double – Napped Cone

<u>Conic Sections:</u> There are 4 main conic sections: *circle, ellipse, parabola, and hyperbola*. It is possible to create each of these shapes by passing a plane through a three dimensional double napped cone.

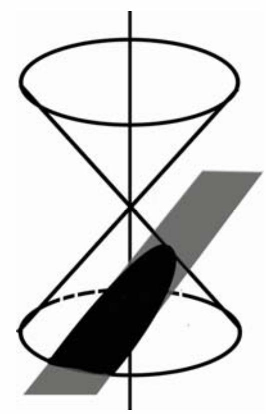
The shape to the right is a double napped cone. The vertical line down the middle is called the central axis, and the diagonal sides are called generators. The point at the centre is called the vertex. In theory, the double napped cone extends forever up & down.



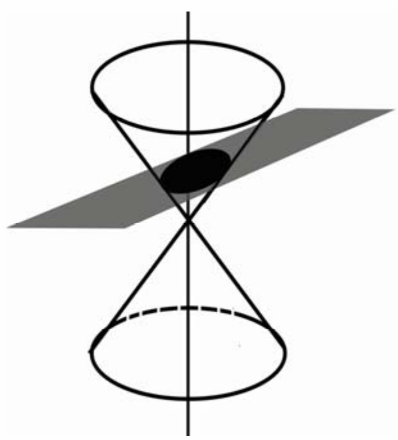
A ***hyperbola*** is produced when the plane passes through both nappes, between the central axis and generator.



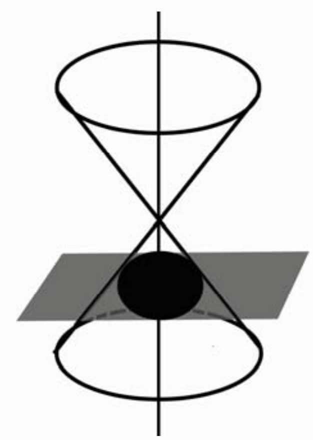
A ***parabola*** is produced when the plane passes through one nappe parallel to the generator.



An ***ellipse*** is produced when the plane passes through one nappe only, between the generator and perpendicular.



A ***circle*** is produced when the plane passes through one nappe only, perpendicular to the central axis.

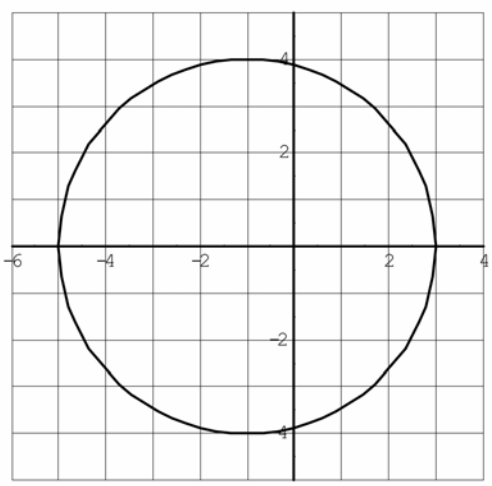


Conics Lesson 1

Part II - Circles

Circles: **The standard form of a circle is given by the equation $(x-h)^2+(y-k)^2=r^2$, where (h, k) is the centre of the circle and r is the radius.**

Example 1: Given the following graph, write the equation.



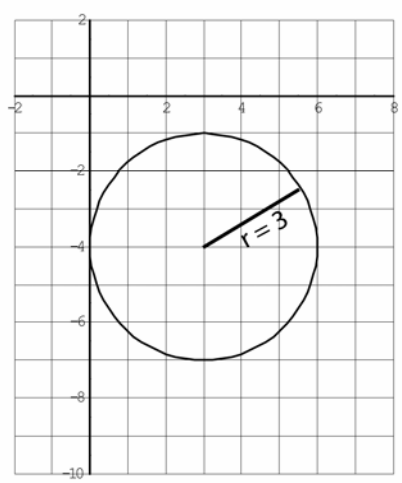
The first thing you should do when given a circle is write down the coordinates of the centre.
In this case, the centre is at (-1,0).
Next, determine the radius, which is 4 units.
Finally, plug the h, k, and r values into the standard form equation and you'll have the equation of the graph!

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-(-1))^2+(y-0)^2=4^2$$
$$(x+1)^2+y^2=16$$

Example 2: Sketch the graph of $(x-3)^2+(y+4)^2=9$ and state the domain and range.

To draw the graph of a circle from a standard form equation, first draw a dot at the centre of the circle. The radius can be found by taking the square root of the number on the right side. *(Remember, you're given r^2 and you just want r.)*

$$(x-3)^2+(y+4)^2=9$$



Quick Tip: An easy way to read off the centre is to use values for x and y that make each bracket go to zero.

$(x-3)$ becomes zero when $x = 3$
$(y+4)$ becomes zero when $y = -4$
So, the centre is at $(3,-4)$

When writing the domain & range for an enclosed shape, we use "*in-between notation*"

Domain: Leftmost Value $\leq x \leq$ Rightmost Value
Range: Bottom Value $\leq y \leq$ Top Value

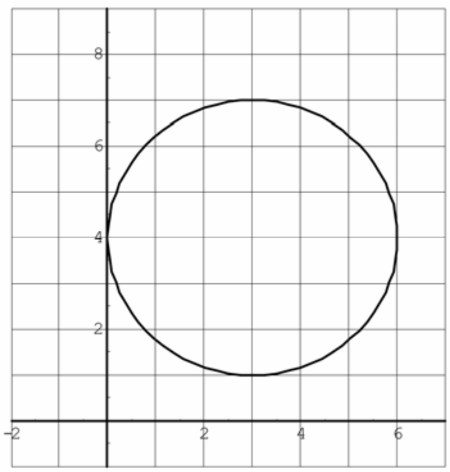
For the circle in this question:
Domain: $0 \leq x \leq 6$ (Read as "the domain is between zero and six")
Range: $-7 \leq y \leq -1$ (Read as "the range is between negative seven and negative one")

Conics Lesson 1

Part II - Circles

Questions: For each of the following graphs, write the equation, then state domain & range:

1.



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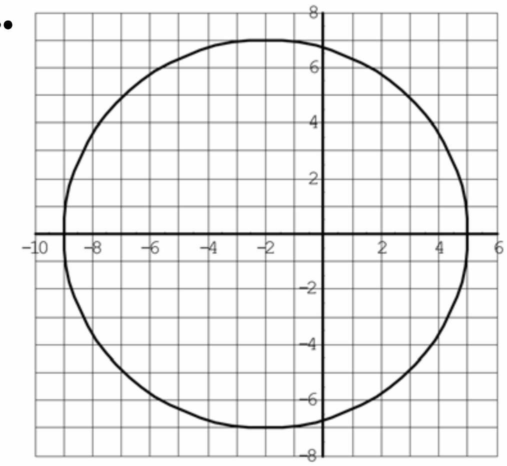

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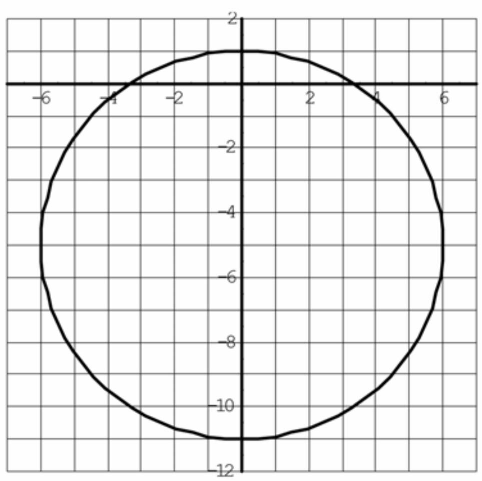

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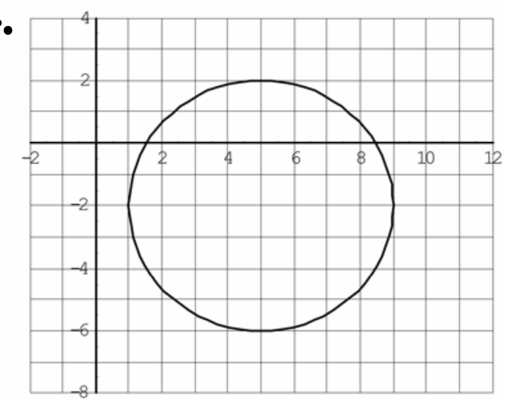
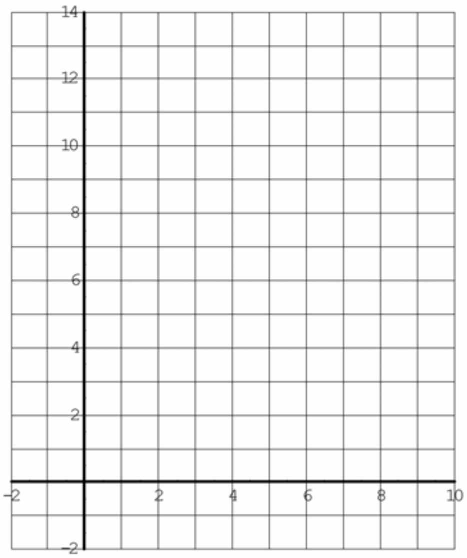


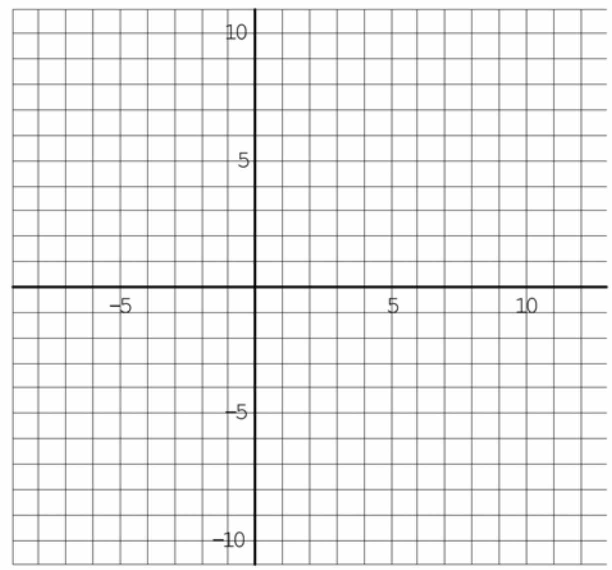
Part II – Circles

Questions: **For each of the following equations, draw the graph and state domain & range:**

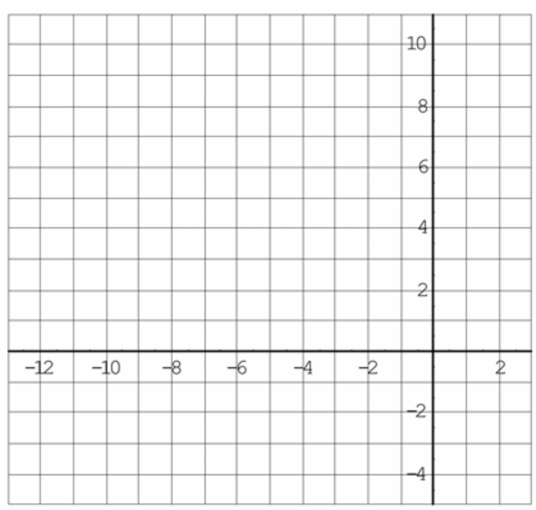
5. $(x-4)^2+(y-6)^2=16$



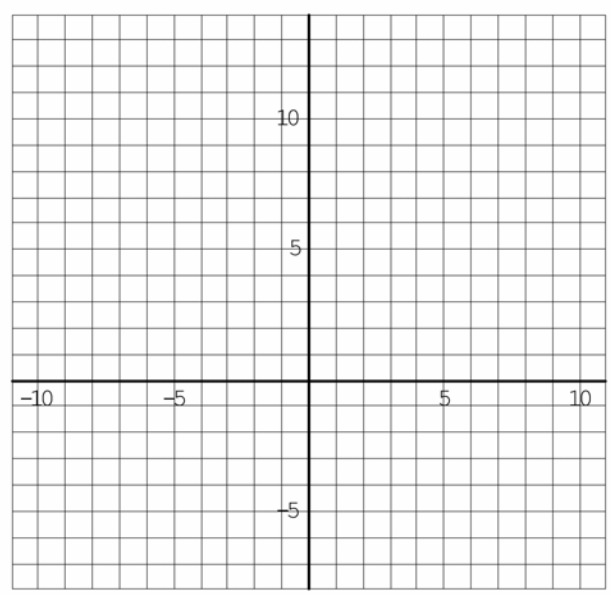
6. $(x-2)^2+y^2=64$



7. $(x+5)^2+(y-3)^2=49$



8. $x^2+(y-3)^2=100$



Answers:

1. $(x-3)^2+(y-4)^2=9$

Domain: $0 \le x \le 6$
Range: $1 \le y \le 7$

2. $(x+2)^2+y^2=49$

Domain: $-9 \le x \le 5$
Range: $-7 \le y \le 7$

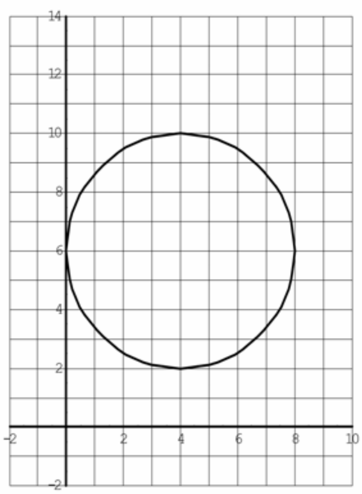
3. $x^2+(y+5)^2=36$

Domain: $-6 \le x \le 6$
Range: $-11 \le y \le 1$

4. $(x-5)^2+(y+2)^2=16$

Domain: $1 \le x \le 9$
Range: $-6 \le y \le 2$

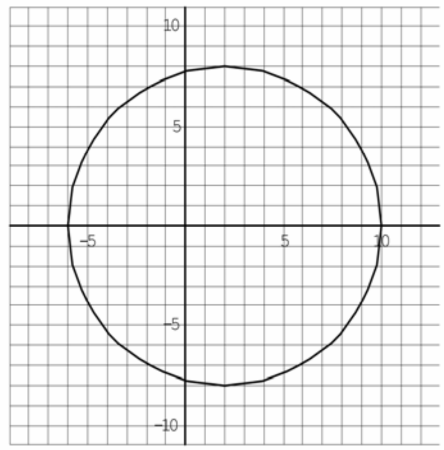
5.



Domain:
$0 \le x \le 8$

Range:
$2 \le y \le 10$

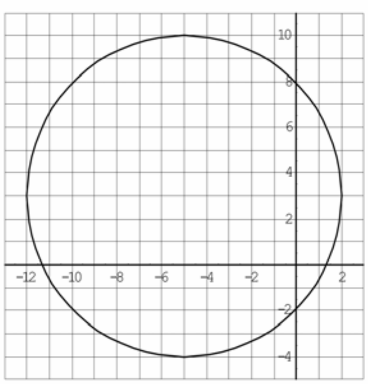
6.



Domain:
$-6 \le x \le 10$

Range:
$-8 \le y \le 8$

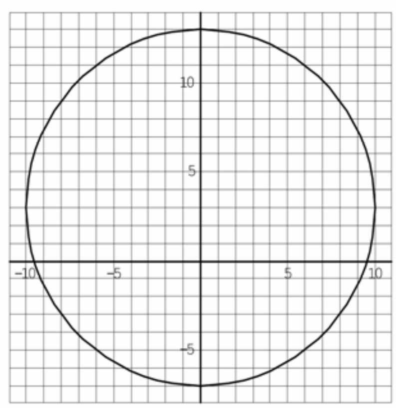
7.



Domain:
$-12 \le x \le 2$

Range:
$-4 \le y \le 10$

8.



Domain:
$-10 \le x \le 10$

Range:
$-7 \le y \le 13$

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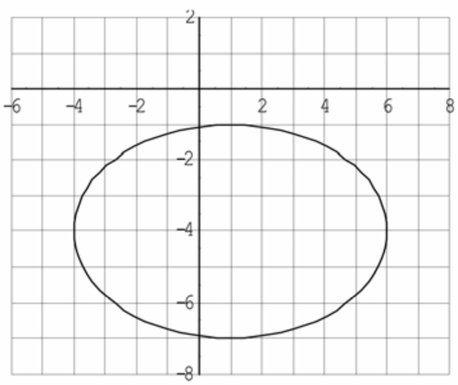
Part III- Ellipses

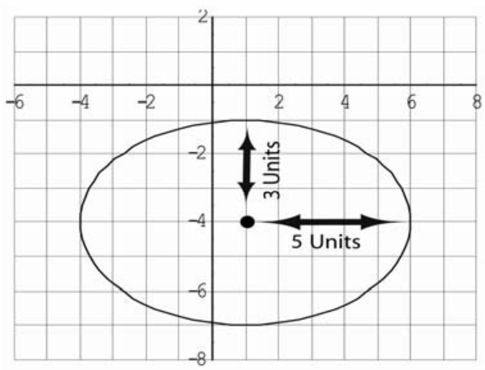
Ellipses: **The equation of an ellipse is given by** $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1,$

(h, k) **is the centre of the ellipse.**
"*a*" represents the horizontal distance from the centre to the edge of the ellipse.
"*b*" represents the vertical distance from the centre to the edge of the ellipse.

Example 1: Given the following graph, find the equation of the ellipse:



First identify the centre of the ellipse, which in this case is (1,-4). To find the a-value, count horizontally from the centre to the right edge and you will get 5. To find the b-value, count vertically from the centre to the upper edge, and you will get 3.

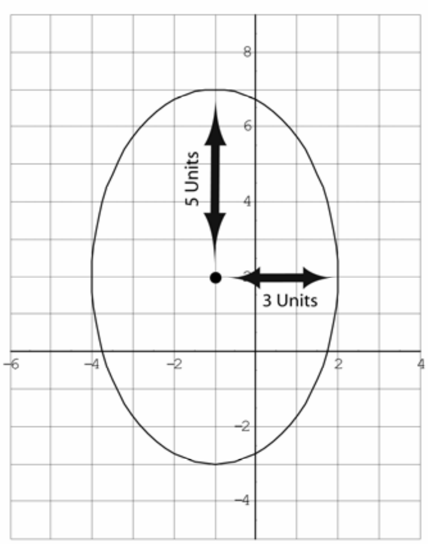


When you put the a & b values into the equation, remember to square them!

$$\frac{(x-1)^2}{25}+\frac{(y+4)^2}{9}=1$$

Example 2: Sketch the graph of $\frac{(x+1)^2}{9}+\frac{(y-2)^2}{25}=1$

Place a point at the centre of the ellipse (-1, 2).
The a-value is $\sqrt{9}=3$
The b-value is $\sqrt{25}=5$



Quick Tip: What happens when both a and b are the same number? This will give you a circle. When writing the equation of an ellipse that is really a circle, you should use a circle equation instead.

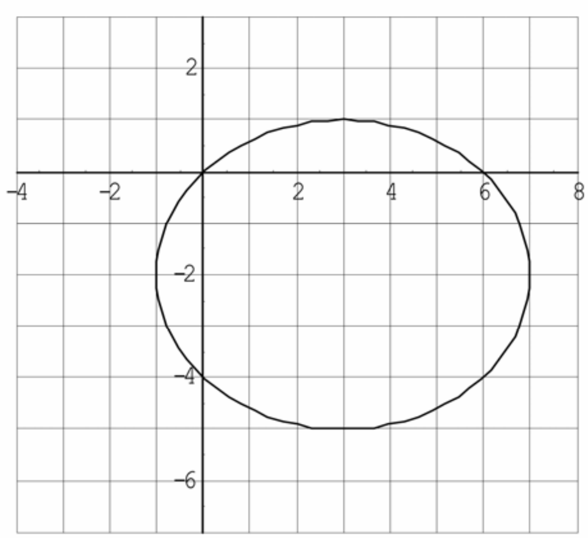
Don t write $\frac{x^2}{9}+\frac{y^2}{9}=1$

Write $x^2+y^2=9$

When a^2 is bigger *(the number under x)* the ellipse is horizontal.
When b^2 is bigger, *(the number under y)*, the ellipse is vertical

Questions: Given the following graphs, write the equation.

1.



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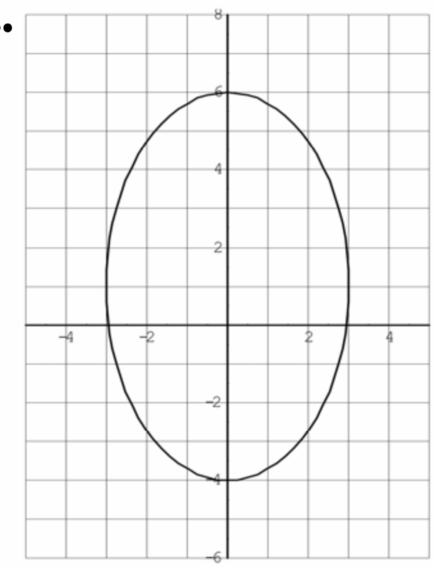

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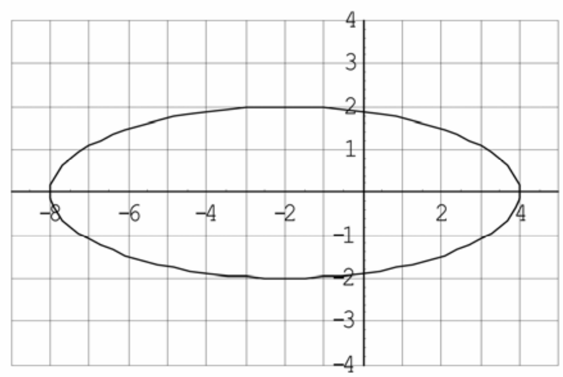

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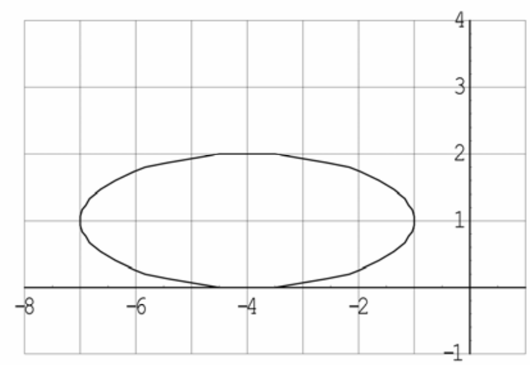

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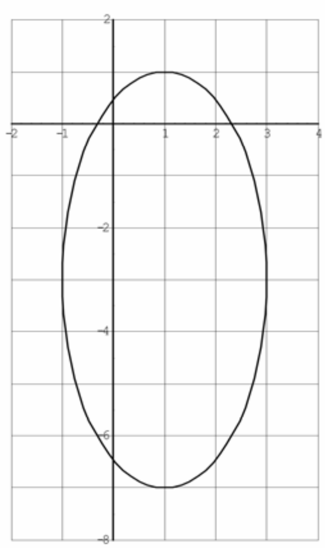

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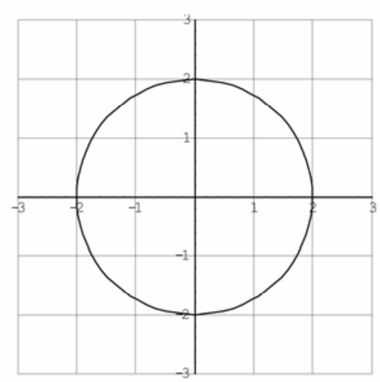

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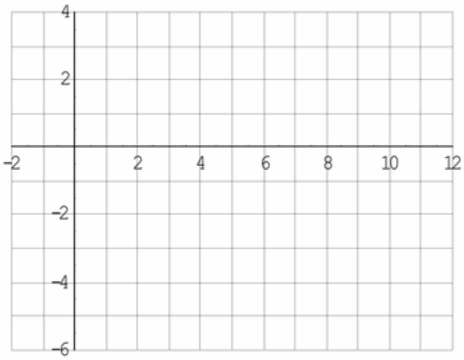
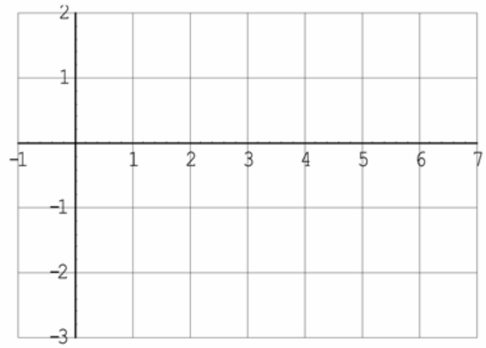
Part III - Ellipses

Questions: Given the following equations, sketch the graph.

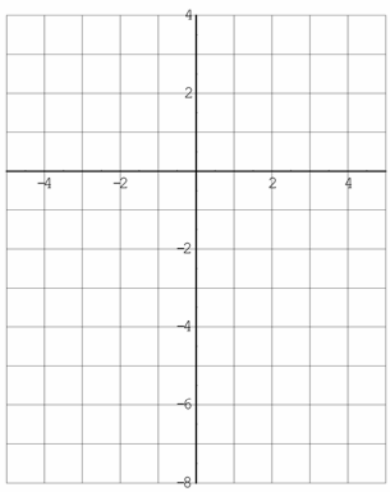
7. $\frac{(x-5)^2}{9}+\frac{(y+1)^2}{16}=1$



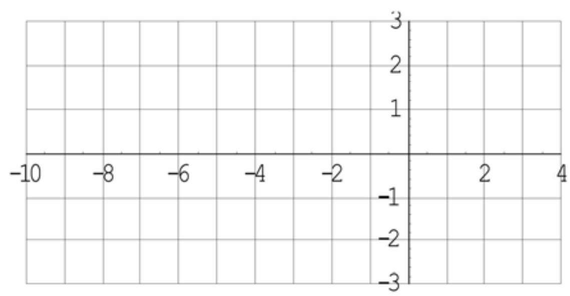
8. $\frac{(x-3)^2}{4}+(y+1)^2=1$



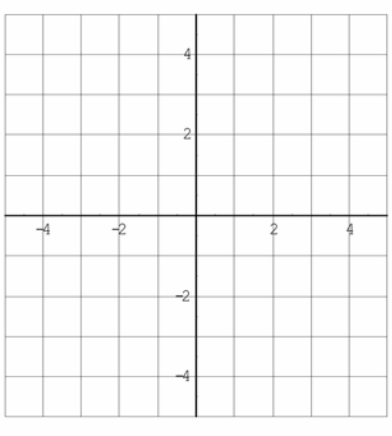
9. $\frac{x^2}{16}+\frac{(y+2)^2}{25}=1$



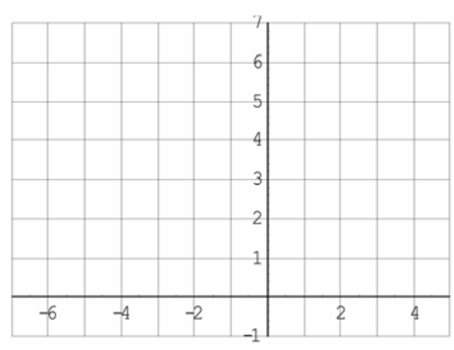
10. $\frac{(x+3)^2}{36}+\frac{y^2}{4}=1$



11. $\frac{x^2}{16}+\frac{y^2}{16}=1$



12. $\frac{(x+1)^2}{25}+\frac{(y-3)^2}{9}=1$



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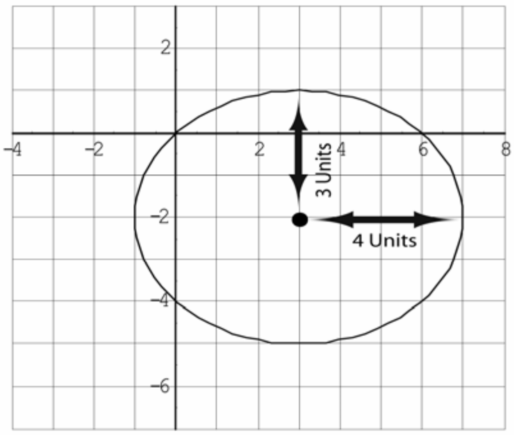
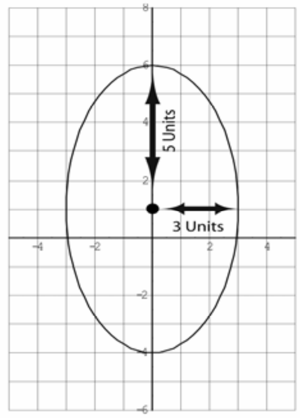
Part III- Ellipses

Answers:

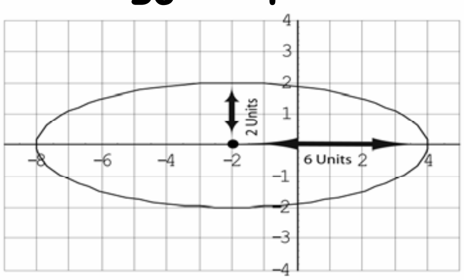
1. $\frac{(x-3)^2}{16}+\frac{(y+2)^2}{9}=1$



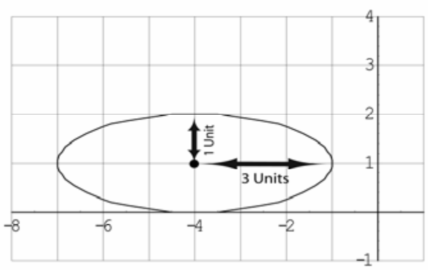
2. $\frac{x^2}{9}+\frac{(y-1)^2}{25}=1$



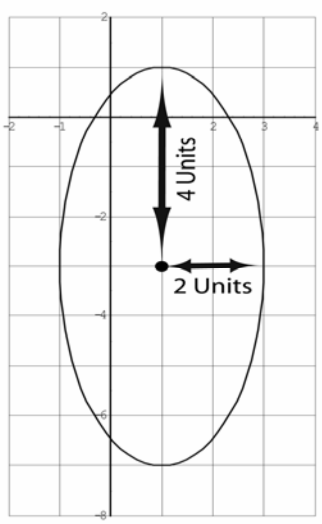
3. $\frac{(x+2)^2}{36}+\frac{y^2}{4}=1$



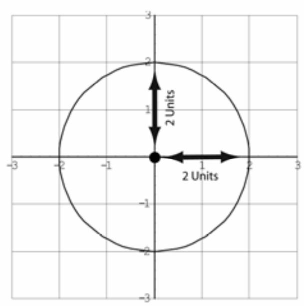
4. $\frac{(x+4)^2}{9}+(y-1)^2=1$



5. $\frac{(x-1)^2}{4}+\frac{(y+3)^2}{16}=1$

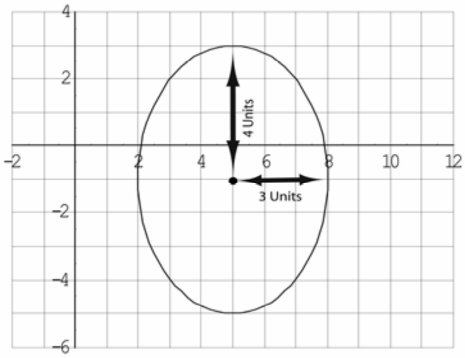


6. $\frac{x^2}{4}+\frac{y^2}{4}=1 \rightarrow x^2+y^2=4$



Answers:

7.



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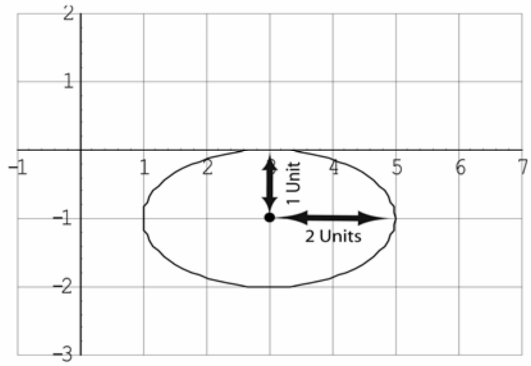

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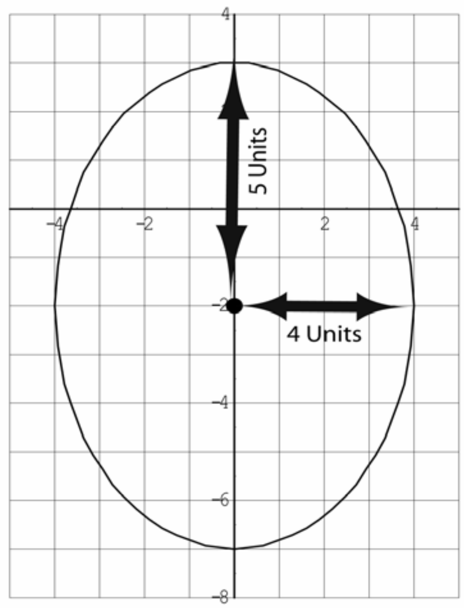

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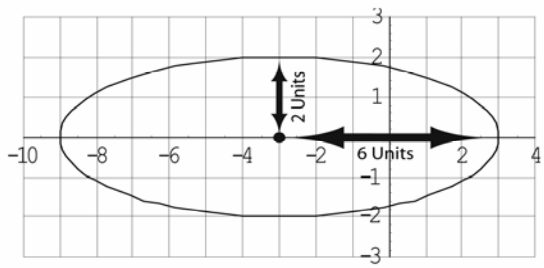

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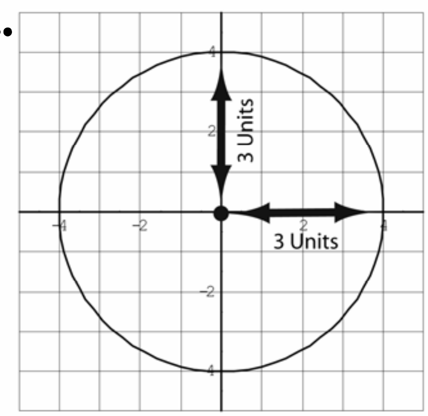

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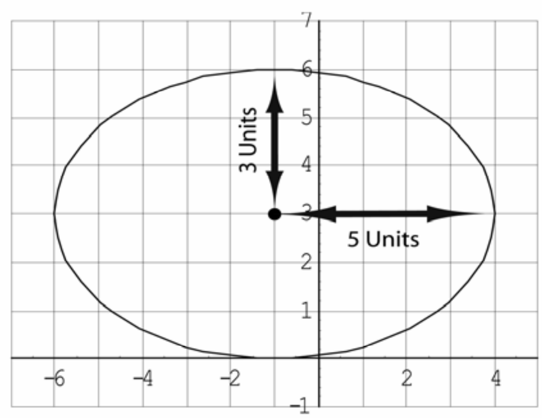


Parabolas: **There are two different standard form equations for parabolas.**

Vertical parabolas are given by: $y - k = a(x - h)^2$. "a" is the vertical stretch factor

(Vertical parabolas that open down have a negative sign with the a-value, those opening up have a positive sign.)

Horizontal parabolas are given by: $x - h = a(y - k)^2$. "a" is the horizontal stretch factor.

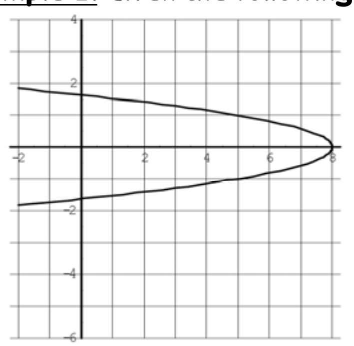
(Horizontal parabolas that open left have a negative sign with the a-value, those opening right have a positive sign.)

(h, k) is the *vertex* of the parabola.

Try to remember the following rules when it comes to standard form parabolas:

If you have an x^2, but no y^2 → vertical parabola.
If you have a y^2, but no x^2 → horizontal parabola.

Example 1: **Given the following graph, write the equation.**



First note the coordinates of the vertex: (8,0). This gives you h & k

To obtain the a-value, find another point on the parabola.
By inspection, the point (5, 1) lies on on the graph.
This can now be plugged in for x & y.

Take the values above and insert them into the standard form of a **horizontal** parabola:

$x - h = a(y - k)^2$

$5 - 8 = a(1 - 0)^2$

$-3 = a$

To obtain the final equation, plug in numbers for a, h, & k, leaving x & y as variables.

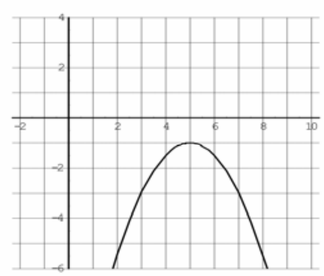
$$x - 8 = -3y^2$$

Example 2: **Sketch the graph of $y + 1 = -\frac{1}{2}(x - 5)^2$**

The vertex is located at the point (5,-1), and it's a upside down vertical parabola.

When given a parabola equation, it may be graphed in your calculator by isolating y:

$y = -\frac{1}{2}(x - 5)^2 - 1$



Example 3: **Sketch the graph of $y + 4 = \frac{1}{4}(x + 2)^2$**

The vertex is located at the point (-2,-4), and it's a right-side up vertical parabola.

This time, graph the parabola using x & y intercepts instead of the calculator. *(The x & y intercept method is being used in this example to illustrate an alternative to using your graphing calculator.)*

x – intercepts:
Set y = 0, then solve for x.

$y + 4 = \frac{1}{4}(x + 2)^2$

$0 + 4 = \frac{1}{4}(x + 2)^2$

$4 = \frac{1}{4}(x + 2)^2$

$16 = (x + 2)^2$

$\pm 4 = x + 2$

$x = -6, 2$

y – intercept:
Set x = 0, then solve for y.

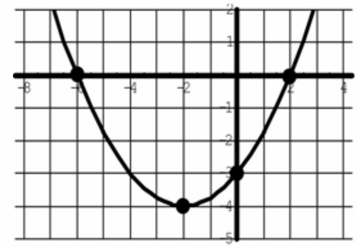
$y + 4 = \frac{1}{4}(x + 2)^2$

$y + 4 = \frac{1}{4}(0 + 2)^2$

$y + 4 = \frac{1}{4}(4)$

$y + 4 = 1$

$y = -3$

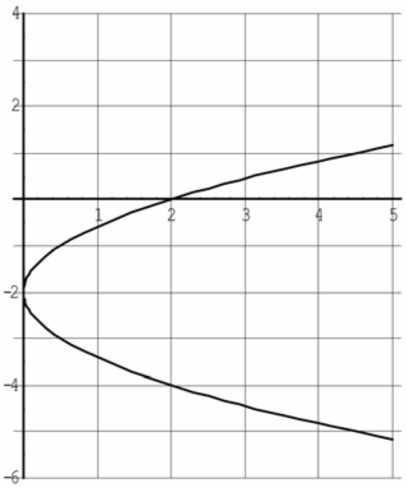


Conics Lesson 1

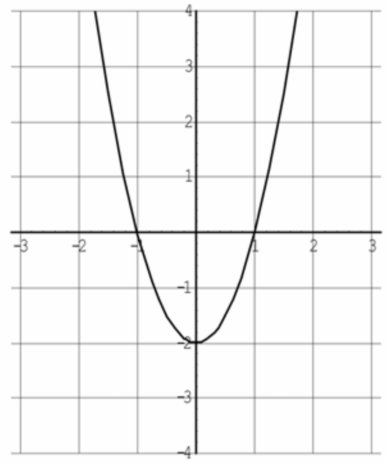
Part IV - Parabolas

Questions: Given the following graphs, write the equation.

1.



2.



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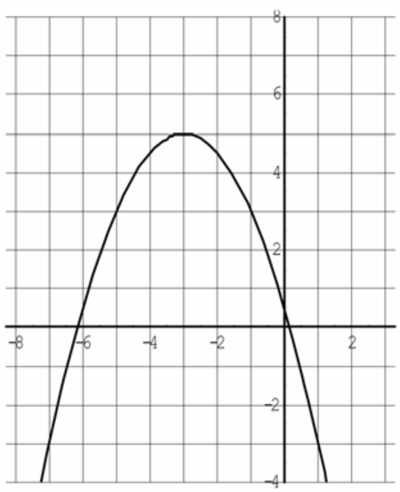

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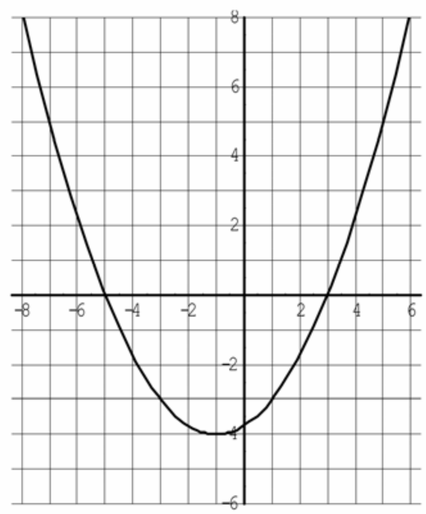

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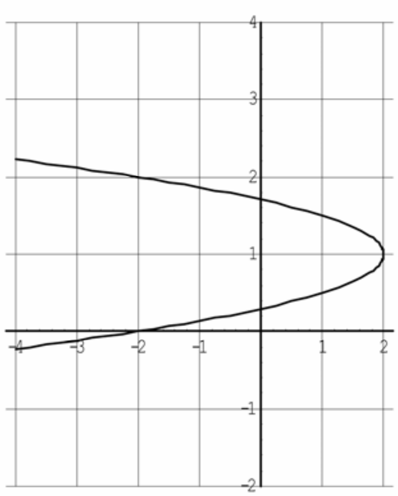

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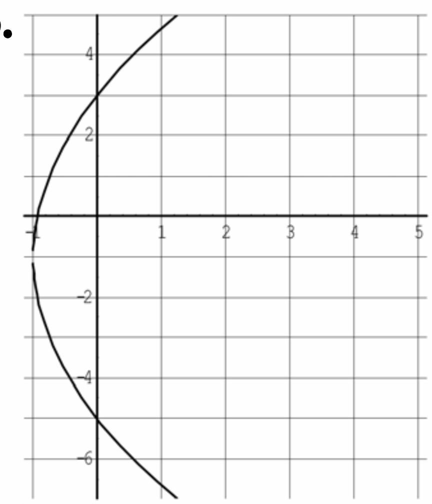
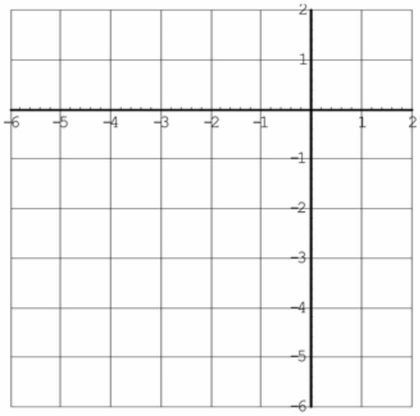
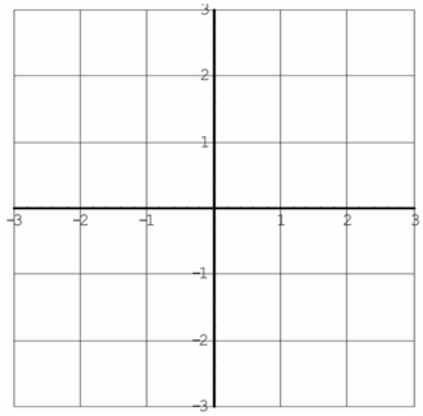


Questions: Isolate y and then sketch the graph:

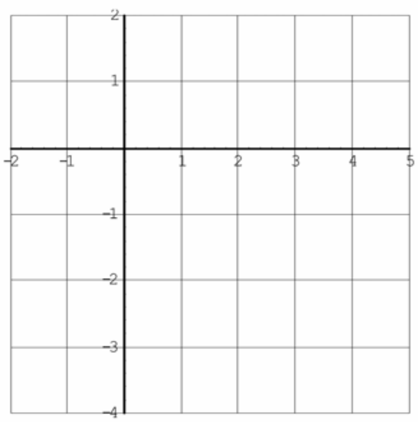
7. $x = -(y+2)^2$



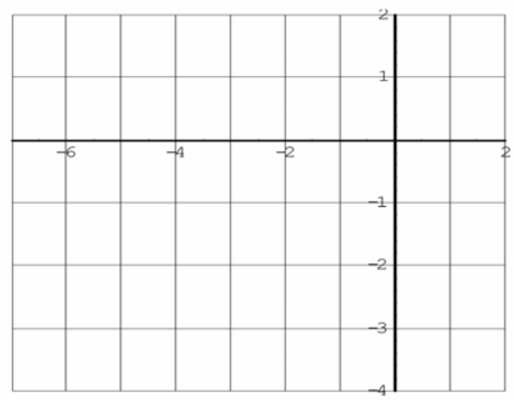
8. $y + 2 = 3x^2$



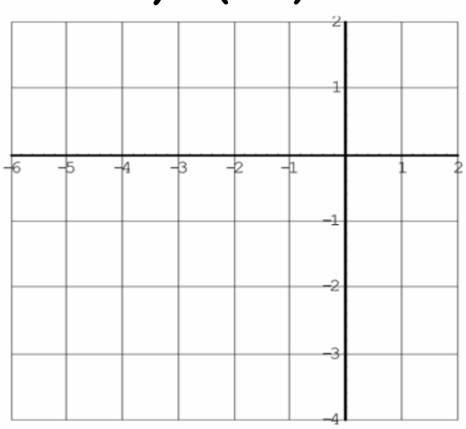
9. $x - 2 = \frac{1}{2}(y+1)^2$



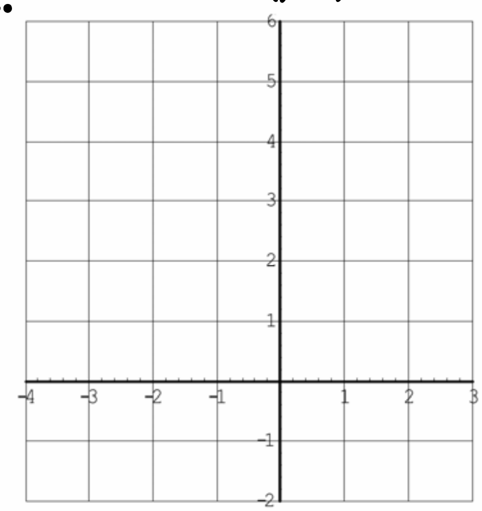
10. $y - 1 = -\frac{1}{2}(x+3)^2$



11. $y = -(x+2)^2$



12. $x + 3 = 2(y-3)^2$

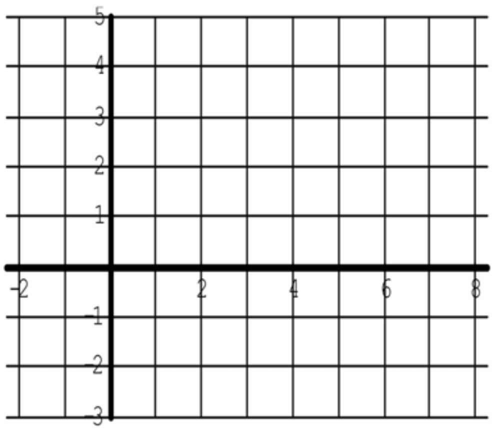


Conics Lesson 1

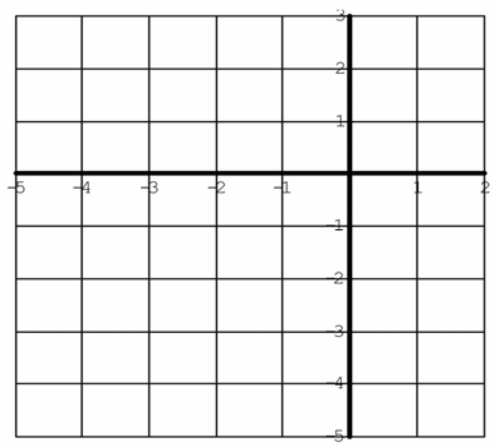
Part IV - Parabolas

Questions: Using x & y intercepts, graph the following parabolas

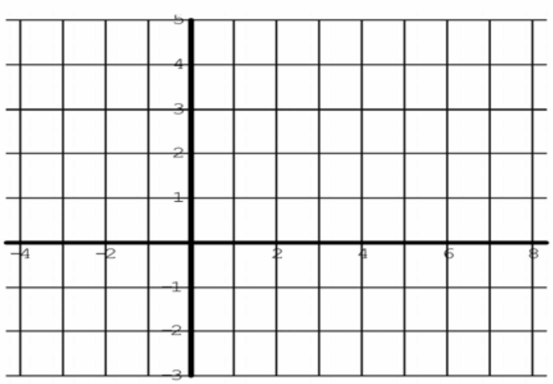
13. $y+2=\frac{1}{2}(x-3)^2$



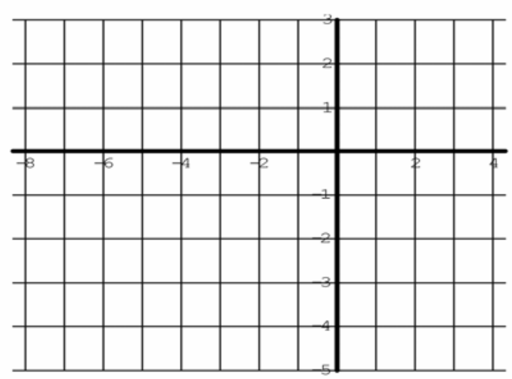
14. $x+4=(y+1)^2$



15. $x+4=(y-1)^2$



16. $y+4=\frac{1}{4}(x+2)^2$



Conics Lesson 1

Part IV - Parabolas

Answers:

1. The vertex is at (0, -2)
A point is (2, 0)

$$x - h = a(y - k)^2$$
$$2 - 0 = a(0 - (-2))^2$$
$$2 = 4a$$
$$a = \frac{1}{2}$$
$$\mathbf{x = \frac{1}{2}(y + 2)^2}$$

2. The vertex is at (0, -2)
A point is (1, 0)

$$y - k = a(x - h)^2$$
$$0 - (-2) = a(1 - 0)^2$$
$$2 = a$$
$$\mathbf{y + 2 = 2x^2}$$

3. The vertex is at (-3, 5)
A point is (1, -3)

$$y - k = a(x - h)^2$$
$$-3 - 5 = a(1 - (-3))^2$$
$$-8 = a(4)^2$$
$$\frac{-8}{16} = a$$
$$a = -\frac{1}{2}$$
$$\mathbf{y - 5 = -\frac{1}{2}(x + 3)^2}$$

4. The vertex is at (-1, -4)
A point is (3, 0)

$$y - k = a(x - h)^2$$
$$0 - (-4) = a(3 - (-1))^2$$
$$4 = a(4)^2$$
$$\frac{4}{16} = a$$
$$a = \frac{1}{4}$$
$$\mathbf{y + 4 = \frac{1}{4}(x + 1)^2}$$

5. The vertex is at (2, 1)
A point is (-2, 0)

$$x - h = a(y - k)^2$$
$$-2 - 2 = a(0 - 1)^2$$
$$-4 = a$$
$$\mathbf{x - 2 = -4(y - 1)^2}$$

6. The vertex is at (-1, -1)
A point is (0, 3)

$$x - h = a(y - k)^2$$
$$0 - (-1) = a(3 - (-1))^2$$
$$1 = a(4)^2$$
$$a = \frac{1}{16}$$
$$\mathbf{x + 1 = \frac{1}{16}(y + 1)^2}$$

Conics Lesson 1

Part IV - Parabolas

Answers:

7. $y = \pm\sqrt{-x} - 2$

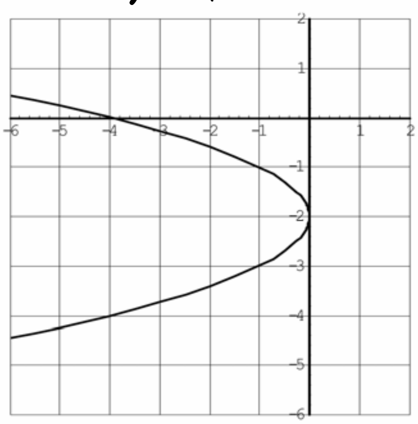
$x = -(y+2)^2$

$-x = (y+2)^2$

$\sqrt{-x} = \sqrt{(y+2)^2}$

$\pm\sqrt{-x} = y+2$

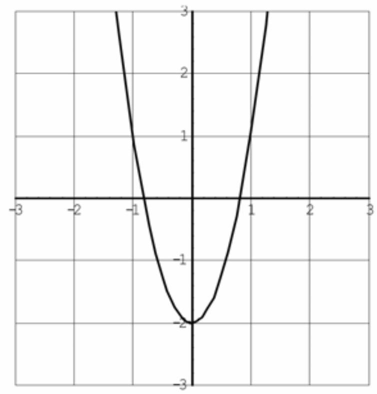
$y = \pm\sqrt{-x} - 2$



8. $y = 3x^2 - 2$

$y + 2 = 3x^2$

$y = 3x^2 - 2$



9. $y = \pm\sqrt{2(x-2)} - 1$

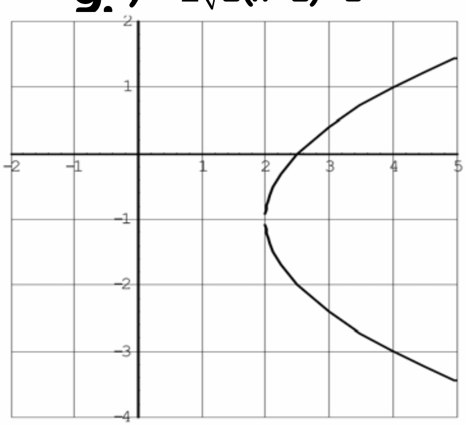
$x - 2 = \frac{1}{2}(y+1)^2$

$2(x-2) = (y+1)^2$

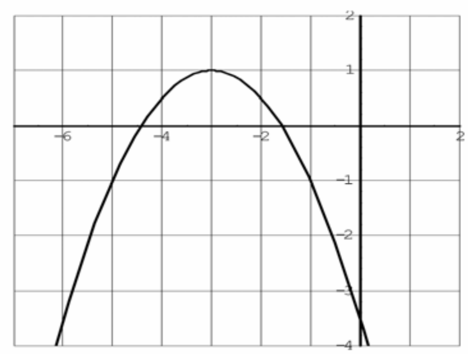
$\sqrt{2(x-2)} = \sqrt{(y+1)^2}$

$\pm\sqrt{2(x-2)} = y+1$

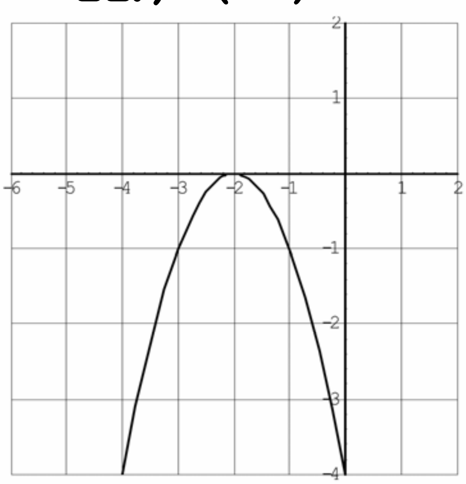
$y = \pm\sqrt{2(x-2)} - 1$



10. $y = -\frac{1}{2}(x+3)^2 + 1$



11. $y = -(x+2)^2$



12. $y = \pm\sqrt{\left(\frac{x+3}{2}\right)} + 3$

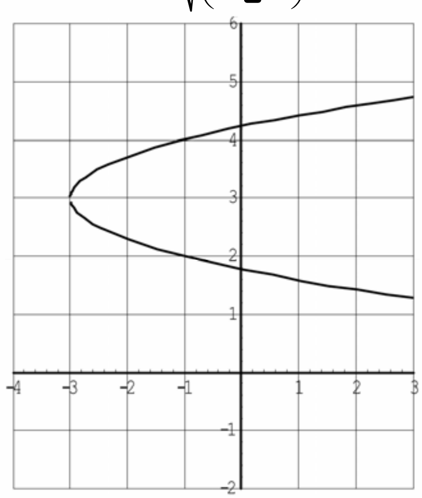
$x + 3 = 2(y-3)^2$

$\frac{x+3}{2} = (y-3)^2$

$\sqrt{\frac{x+3}{2}} = \sqrt{(y-3)^2}$

$\pm\sqrt{\frac{x+3}{2}} = y - 3$

$y = \pm\sqrt{\frac{x+3}{2}} + 3$



Conics Lesson 1

Part IV – Parabolas

Answers:

13.

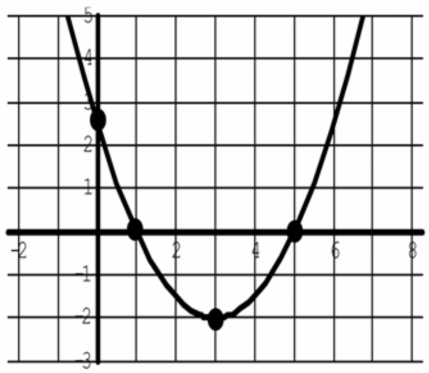
Vertex
(3, -2)

x-intercepts

$$y+2=\frac{1}{2}(x-3)^2$$
$$0+2=\frac{1}{2}(x-3)^2$$
$$4=(x-3)^2$$
$$\pm 2=x-3$$
$$x=1,5$$

y-intercept

$$y+2=\frac{1}{2}(x-3)^2$$
$$y+2=\frac{1}{2}(0-3)^2$$
$$y+2=\frac{9}{2}$$
$$y=\frac{5}{2}=2.5$$



14.

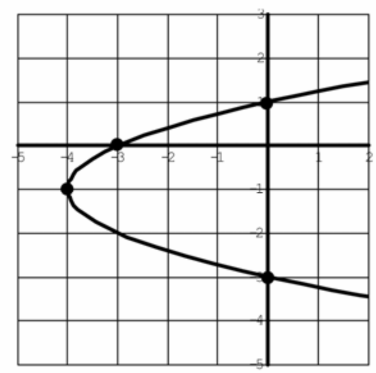
Vertex
(-4, -1)

x-intercept:

$$x+4=(y+1)^2$$
$$x+4=(0+1)^2$$
$$x+4=1$$
$$x=-3$$

y-intercepts

$$x+4=(y+1)^2$$
$$0+4=(y+1)^2$$
$$\pm 2=y+1$$
$$y=-3,1$$



15.

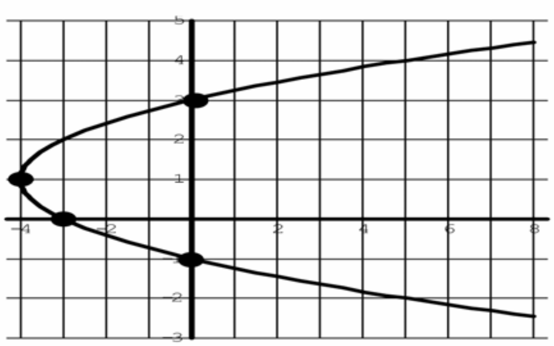
Vertex
(-4, 1)

x-intercept

$$x+4=(y-1)^2$$
$$x+4=(0-1)^2$$
$$x+4=1$$
$$x=-3$$

y-intercepts

$$x+4=(y-1)^2$$
$$0+4=(y-1)^2$$
$$\pm 2=y-1$$
$$y=-1,3$$



16.

Vertex
(-2, -4)

x-intercepts

$$y+4=\frac{1}{4}(x+2)^2$$
$$0+4=\frac{1}{4}(x+2)^2$$
$$16=(x+2)^2$$
$$\pm 4=x+2$$
$$x=-6,2$$

y-intercept

$$y+4=\frac{1}{4}(x+2)^2$$
$$y+4=\frac{1}{4}(0+2)^2$$
$$y+4=\frac{1}{4}(4)$$
$$y+4=1$$
$$y=-3$$