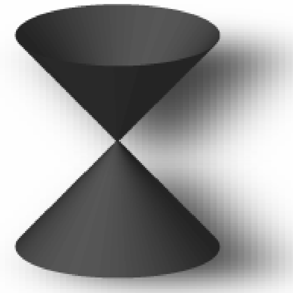
Pre - Calculus Math 40S:

Conics



Lesson 3

Standard & General Form of Conics

Pre - Calculus Math 40S

EXPLAINED!

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Part I: General Form

General Form of a Conic: $Ax^2 + Cy^2 + Dx + Ey + F = 0$

A & C are useful in finding out which conic is produced:

A = C Circle
AC > 0 Ellipse
A or C = 0 Parabola
AC < 0 Hyperbola

Example 1: What shape is the graph of $3x^2 - 2y^2 + 2x - 3 = 0$
Hyperbola, since AC = (3)(-2) = -6, which is negative.

Example 2: What shape is the graph of $y^2 + 2x - y - 5 = 0$
Parabola, since A = 0

Questions: **Based on the A & C values, determine the type of conic for each of the following:**

1) $9x^2 - 4y^2 - 8y + 32 = 0$

2) $-2y^2 - x + 20y - 47 = 0$

3) $y^2 + 6y = 3x - 12$

4) $25x^2 - 9y^2 - 72y - 100x - 269 = 0$

5) $4x^2 + y^2 - 8x + 4y - 9 = 0$

6) $9x^2 - 4y^2 - 40y - 136 = 0$

7) $x^2 + 6x + 8y + 33 = 0$

8) $4x^2 + 4y^2 - 8x + 4y - 9 = 0$

Why is there no B?

The Bxy term involves conic rotation, and is excluded from the Math 40S curriculum.

Answers:
1) Hyperbola
2) Parabola
3) Parabola
4) Hyperbola
5) Ellipse
6) Hyperbola
7) Parabola
8) Circle

Converting from Standard To General Form: This is done by multiplying both sides of the equation by the common denominator, then simplifying:

Example 1: Convert $y+3=\frac{5}{4}(x+3)^2$ to general form:

Multiply both sides by the common denominator of 4:

$$4\left[y+3\right]=4\left[\frac{5}{4}(x+3)^2\right]$$

$$4y+12=\cancel{4}\left[\frac{5}{\cancel{4}}(x+3)^2\right]$$

$$4y+12=5(x+3)^2$$

$$4y+12=5(x+3)(x+3)$$

$$4y+12=5\left(x^2+6x+9\right)$$

$$4y+12=5x^2+30x+45$$

$$0=5x^2+30x-4y+33$$

Example 2: Convert $\frac{(x-10)^2}{10}+\frac{y^2}{25}=1$ to general form:

Multiply both sides by the common denominator of 50:

$$50\left[\frac{(x-10)^2}{10}+\frac{y^2}{25}\right]=50\left[1\right]$$

$$5(x-10)^2+2y^2=50$$

$$5(x^2-20x+100)+2y^2=50$$

$$5x^2-100x+500+2y^2=50$$

$$5x^2+2y^2-100x+450=0$$

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Part II: Standard to General Form

Questions: **Convert each of the following to general form:**

1) $\frac{9x^2}{36} - \frac{4(y+1)^2}{36} = -1$

2) $\frac{x^2}{4} - \frac{(y+5)^2}{9} = 1$

3) $y + 3 = -\frac{1}{8}(x+3)^2$

4) $-2(y-5)^2 = x - 3$

5) $4(x-1)^2 + (y+2)^2 = 17$

6) $\frac{(x-2)^2}{9} - \frac{(y-4)^2}{25} = 1$

Answers:

1.

$$36\left[\frac{9x^2}{36}-\frac{4(y+1)^2}{36}\right]=36[-1]$$
$$9x^2-4(y+1)^2=-36$$
$$9x^2-4(y+1)(y+1)=-36$$
$$9x^2-4(y^2+2y+1)=-36$$
$$9x^2-4y^2-8y-4=-36$$
$$9x^2-4y^2-8y+32=0$$

2.

$$36\left[\frac{x^2}{4}-\frac{(y+5)^2}{9}\right]=36[1]$$
$$9x^2-4(y+5)^2=36$$
$$9x^2-4(y+5)(y+5)=36$$
$$9x^2-4(y^2+10y+25)=36$$
$$9x^2-4y^2-40y-100=36$$
$$9x^2-4y^2-40y-136=0$$

3.

$$8[y+3]=8\left[-\frac{1}{8}(x+3)^2\right]$$
$$8y+24=-(x+3)^2$$
$$8y+24=-(x+3)(x+3)$$
$$8y+24=-(x^2+6x+9)$$
$$8y+24=-x^2-6x-9$$
$$x^2+6x+8y+33=0$$

4.

$$-2(y-5)^2=x-3$$
$$-2(y-5)(y-5)=x-3$$
$$-2(y^2-10y+25)=x-3$$
$$-2y^2+20y-50=x-3$$
$$0=2y^2+x-20y+47$$

5.

$$4(x-1)^2+(y+2)^2=17$$
$$4(x-1)(x-1)+(y+2)(y+2)=17$$
$$4(x^2-2x+1)+(y^2+4y+4)=17$$
$$4x^2-8x+4+y^2+4y+4=17$$
$$4x^2+y^2-8x+4y+8=17$$
$$4x^2+y^2-8x+4y-9=0$$

6.

$$225\left[\frac{(x-2)^2}{9}-\frac{(y-4)^2}{25}\right]=225[1]$$
$$25(x-2)^2-9(y-4)^2=225$$
$$25(x-2)(x-2)-9(y-4)(y-4)=225$$
$$25(x^2-4x+4)-9(y^2-8y+16)=225$$
$$25x^2-100x+100-9y^2+72y-144=225$$
$$25x^2-9y^2-100x+72y-269=0$$

Converting from General To Standard Form: This is done by completing the square:

Example 1: Convert $x^2 + 3y^2 + 10x - 30y + 91 = 0$ to general form.

Step 1: Group terms with x, group terms with y, and take the constant to the other side.

$x^2 + 10x + 3y^2 - 30y = -91$

Step 2: Put brackets around the x-terms, and put brackets around the y-terms, leaving a space.

$(x^2 + 10x \quad) + (3y^2 - 30y \quad) = -91$

Step 3: Make sure the squared terms have a coefficient of 1. If they don't, factor that number out.

$(x^2 + 10x \quad) + 3(y^2 - 10y \quad) = -91$

Step 4: Divide the x-coefficient by 2, square the result, and add inside the bracket. Repeat for y.

$(x^2 + 10x + 25) + 3(y^2 - 10y + 25) = -91$

Step 5: Whatever you do to one side, you must do to the other! On the left, you have now added 25 in the x-brackets, and 75 in the y-brackets (multiplying the 3 through the brackets). Add 25 + 75 to the other side to keep things balanced.

$(x^2 + 10x + 25) + 3(y^2 - 10y + 25) = -91 + 25 + 75$

Step 6: Factor the expressions.

$(x + 5)^2 + 3(y - 5)^2 = 9$

Step 7: Divide both sides by 9 so the equation equals 1, which is required for standard form:

$$\frac{(x+5)^2}{9} + \frac{3(y-5)^2}{9} = \frac{9}{9}$$

$$\frac{(x+5)^2}{9} + \frac{(y-5)^2}{3} = 1$$

Example 2: Convert $3x^2 + 12x - 4y - 12 = 0$ to general form.

Step 1: Group terms with x. Since there is no y^2, take both the 4y & 12 to the other side.

$3x^2 + 12x = 4y + 12$

Step 2: Put brackets around the x-terms, leaving a space.

$(3x^2 + 12x \quad) = 4y + 12$

Step 3: Make sure the squared terms have a coefficient of 1. If they don't, factor that number out.

$3(x^2 + 4x \quad) = 4y + 12$

Step 4: Divide the x-coefficient by 2, square the result and add inside the bracket.

$3(x^2 + 4x + 4) = 4y + 12 + 12$

Step 5: Whatever you do to one side, you must do to the other! On the left, you have added 12 (multiplying the 3 into the brackets), so add 12 to the other side too.

$3(x^2 + 4x + 4) = 4y + 12 + 12$

Step 6: Factor and simplify the expressions.

$3(x + 2)^2 = 4y + 24$

Step 7: Take this to parabola standard form.

$$\frac{3(x+2)^2}{4} = \frac{4(y+6)}{4}$$

$$y + 6 = \frac{3}{4}(x+2)^2$$

> **If you are ever asked to graph a conic presented in general form, you must first convert it to standard form. Once converted, you can use rules from previous lessons to draw the graph.**

Conics Lesson 3

Part III: General to Standard Form

Questions: **Convert each of the following to standard form.**

1) $9x^2 - 4y^2 - 8y + 32 = 0$

2) $-2y^2 - x + 20y - 47 = 0$

3) $y^2 + 6y = 3x - 12$

4) $25x^2 - 9y^2 - 72y - 100x - 269 = 0$

5) $4x^2 + y^2 - 8x + 4y - 9 = 0$

6) $9x^2 - 4y^2 - 40y - 136 = 0$

7) $x^2 + 6x + 8y + 33 = 0$

Answers:

1.
$$9x^2 - 4y^2 - 8y + 32 = 0$$
$$9x^2 - 4(y^2 + 2y) = -32$$
$$9x^2 - 4(y^2 + 2y + 1) = -32 - 4$$
$$9x^2 - 4(y + 1)^2 = -36$$
$$\frac{9x^2}{36} - \frac{4(y+1)^2}{36} = \frac{-36}{36}$$
$$\frac{x^2}{4} - \frac{(y+1)^2}{9} = -1$$

2.
$$-2y^2 - x + 20y - 47 = 0$$
$$-2y^2 + 20y = x + 47$$
$$-2(y^2 - 10y) = x + 47$$
$$-2(y^2 - 10y + 25) = x + 47 - 50$$
$$-2(y - 5)^2 = x - 3$$

3.
$$y^2 + 6y = 3x - 12$$
$$(y^2 + 6y + 9) = 3x - 12 + 9$$
$$(y + 3)^2 = 3x - 3$$
$$(y + 3)^2 = 3(x - 1)$$
$$x - 1 = \frac{1}{3}(y + 3)^2$$

4.
$$25x^2 - 9y^2 - 72y - 100x - 269 = 0$$
$$25x^2 - 100x - 9y^2 - 72y = 269$$
$$25(x^2 - 4x) - 9(y^2 + 8y) = 269$$
$$25(x^2 - 4x + 4) - 9(y^2 + 8y + 16) = 269 + 100 - 144$$
$$\frac{25(x-2)^2}{225} - \frac{9(y+4)^2}{225} = \frac{225}{225}$$
$$\frac{(x-2)^2}{9} - \frac{(y+4)^2}{25} = 1$$

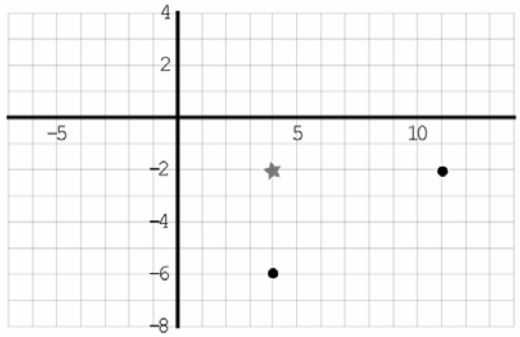
5.
$$4x^2 + y^2 - 8x + 4y - 9 = 0$$
$$4x^2 - 8x + y^2 + 4y - 9 = 0$$
$$4(x^2 - 2x) + (y^2 + 4y) = 9$$
$$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = 9 + 4 + 4$$
$$4(x - 1)^2 + (y + 2)^2 = 17$$
$$\frac{4(x-1)^2}{17} + \frac{(y+2)^2}{17} = 1$$

6.
$$9x^2 - 4y^2 - 40y - 136 = 0$$
$$9x^2 - 4(y^2 + 10y) = 136$$
$$9x^2 - 4(y^2 + 10y + 25) = 136 - 100$$
$$\frac{9x^2}{36} - \frac{4(y+5)^2}{36} = \frac{36}{36}$$
$$\frac{x^2}{4} - \frac{(y+5)^2}{9} = 1$$

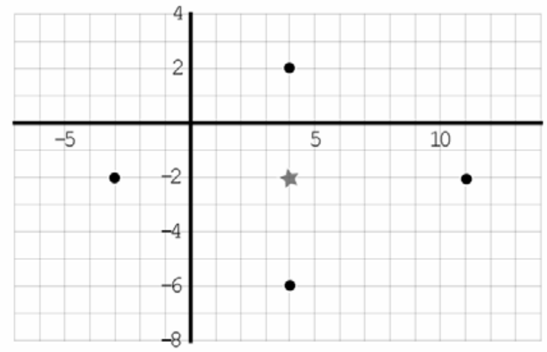
7.
$$x^2 + 6x + 8y + 33 = 0$$
$$(x^2 + 6x) = -8y - 33$$
$$(x^2 + 6x + 9) = -8y - 33 + 9$$
$$(x + 3)^2 = -8y - 24$$
$$(x + 3)^2 = -8(y + 3)$$
$$y + 3 = -\frac{1}{8}(x + 3)^2$$

Example 1: An ellipse has its centre at (4,-2) and passes through the points (11,-2) & (4, -6). Determine the equation of the ellipse

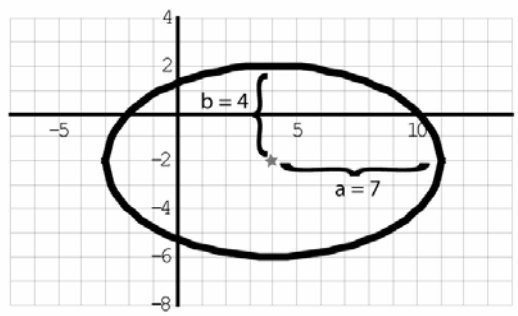
Step 1: Draw in the points given:



Step 2: By symmetry, you can find the following points on the ellipse as well.



Step 3: Read off the a & b values.



Step 4: plug everything into your equation.

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$$\frac{(x-4)^2}{7^2}+\frac{(y-(-2))^2}{4^2}=1$$

$$\frac{(x-4)^2}{49}+\frac{(y+2)^2}{16}=1$$

Example 2: A vertical parabola has a vertex at (-3, -2) and passes through the point (-1, 7).

It's a vertical parabola, so the standard form equation is $y-k=a(x-h)^2$.
All you need to do is plug in the numbers and solve for the a-value.

$y-k=a(x-h)^2$

$7-(-2)=a(-1-(-3))^2$

$9=4a$

$a=\frac{9}{4}$

The equation is: $y+2=\frac{9}{4}(x+3)^2$

Example 3: Given the ellipse $\frac{(x-2)^2}{9}+\frac{(y+4)^2}{16}=1$, determine the new equation after a translation 3 units up and 7 units right.

First state your original centre point (2,-4)
Then apply the transformation to this point: (2+7, -4+3)
The new centre is (9,-1)

Since there are no other transformations, the new equation is: $\frac{(x-9)^2}{9}+\frac{(y+1)^2}{16}=1$

<u>Example 4:</u> The general form of a particular ellipse is $2x^2+y^2-2x+3y-9=0$. If this conic is translated 2 units left and 1 unit down, determine the new general equation.

Replace x with $(x+2)$ to represent 2 units left, and replace y with $(y+1)$ to represent 1 unit down.

$$2(x+2)^2+(y+1)^2-2(x+2)+3(y+1)-9=0$$
$$2\left(x^2+4x+4\right)+\left(y^2+2y+1\right)-2(x+2)+3(y+1)-9=0$$
$$2x^2+8x+8+y^2+2y+1-2x-4+3y+3-9=0$$
$$2x^2+y^2+6x+5y-1=0$$

<u>Example 5:</u> Determine the y – intercepts of $x^2+y^2-x+5y-6=0$

The $y-$ intercepts of $x^2+y^2-x+5y-6=0$ can be found by substituting zero for x, then solving for y.

$$x^2+y^2-x+5y-6=0$$
$$y^2+5y-6=0$$
$$(y+6)(y-1)=0$$
$$y=-6,1$$

Remember that y-intercepts are actually a coordinate with x = 0, so write the y-intercepts as (0, -6) and (0,1)

<u>Example 6:</u> The circle $x^2+y^2=1$ is translated *h* units left and *k* units down. Determine the *x*-intercepts

Replace x with $(x+h)$ and y with $(y-k)$ to account for the transformations.

$$x^2+y^2=1$$
$$(x+h)^2+(y-k)^2=1$$

To find the x-intercepts, let $y=0$.

$$(x+h)^2+(0-k)^2=1$$
$$(x+h)^2+(-k)^2=1$$
$$(x+h)^2+k^2=1$$
$$(x+h)^2=1-k^2$$
$$x+h=\pm\sqrt{1-k^2}$$
$$x=\pm\sqrt{1-k^2}-h$$

Conics Lesson 3

Part IV: Diploma Style

Questions:

1) An ellipse has a centre at (-4, 6) and passes through (-4, 9) and (2, 6). Determine the standard form equation.

2) An ellipse has a centre at (3, 2) and passes through (5, 2) and (3, -3). Determine the standard form equation.

3) A horizontal parabola has a vertex at (3, 2) and passes through (-13, 4).

4) A vertical parabola has a vertex at (-3, 5) and passes through (-1, -4).

5) Given the ellipse $\frac{(x-3)^2}{4}+\frac{(y+1)^2}{9}=1$, determine the new equation after a translation 2 units right and 2 units down.

6) Given the ellipse $\frac{(x+5)^2}{25}+\frac{y^2}{4}=1$, determine the new equation after a translation 3 units left and 4 units down.

7) The general form of a particular ellipse is $x^2+y^2-x+2y-4=0$.
If this conic is translated 1 unit right and 3 units up, determine the new general equation.

8) Determine the $x-$intercepts of $x^2+y^2-x+5y-6=0$

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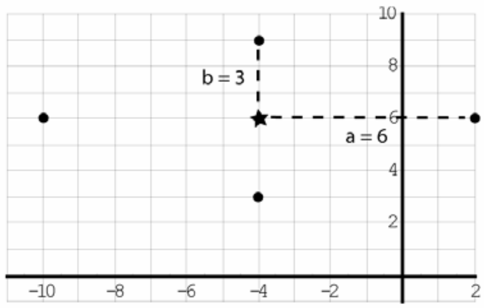
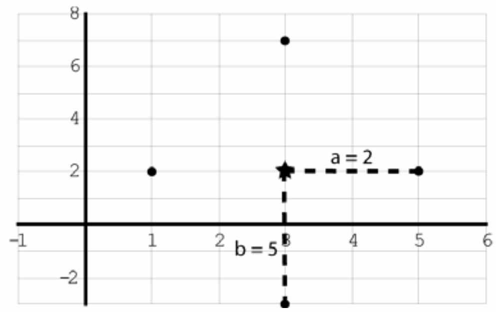
PART IV: DIPLOMA STYLE

1) $\frac{(x+4)^2}{36}+\frac{(y-6)^2}{9}=1$



2) $\frac{(x-3)^2}{4}+\frac{(y-2)^2}{25}=1$



3)

$x - h = a(y-k)^2$

$-13-3 = a(4-2)^2$

$-16 = 4a$

$-4 = a$

The equation is :

$x-3 = -4(y-2)^2$

4)

$y-k = a(x-h)^2$

$-4-5 = a(-1+3)^2$

$-9 = 4a$

$-\frac{9}{4} = a$

The equation is :

$y-5 = -\frac{9}{4}(x+3)^2$

5) First state your original centre point (3,-1)
Then apply the transformation to this point:
(3+2, -1-2)
The new centre is (5,-3)

The new equation is $\frac{(x-5)^2}{4}+\frac{(y+3)^2}{9}=1$

6) First state your original centre point (-5, 0)
Then apply the transformation to this point:
(-5-3, 0-4)
The new centre is (-8,-4)

The new equation is $\frac{(x+8)^2}{25}+\frac{(y+4)^2}{4}=1$

7)

$x^2+y^2-x+2y-4=0$

$(x-1)^2+(y-3)^2-(x-1)+2(y-3)-4=0$

$x^2-2x+1+y^2-6y+9-x+1+2y-6-4=0$

$x^2+y^2-3x-4y+1=0$

8) The $x-$ intercepts of $x^2+y^2-x+5y-6=0$ can be found by substituting zero for y then solving for x.

$x^2+y^2-x+5y-6=0$

$x^2+(0)^2-x+5(0)-6=0$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3,-2$

The x-intercepts are (-2, 0) and (3,0)