



AP[®] Calculus AB 2006 Scoring Guidelines Form B

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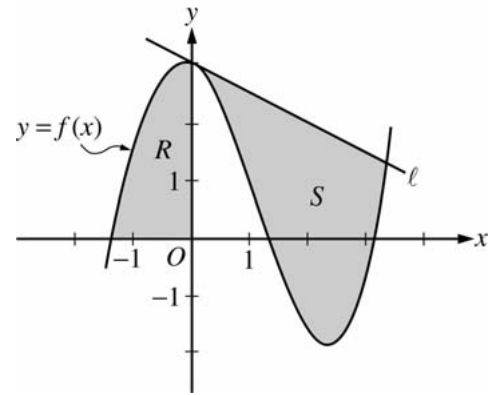
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Question 1

Let f be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f , and let S be the shaded region bounded by the graph of f and line ℓ , the line tangent to the graph of f at $x = 0$, as shown above.



- (a) Find the area of R .
 (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.
 (c) Write, but do not evaluate, an integral expression that can be used to find the area of S .

For $x < 0$, $f(x) = 0$ when $x = -1.37312$.
 Let $P = -1.37312$.

(a) Area of $R = \int_P^0 f(x) dx = 2.903$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Volume = $\pi \int_P^0 ((f(x) + 2)^2 - 4) dx = 59.361$

4 : $\begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) The equation of the tangent line ℓ is $y = 3 - \frac{1}{2}x$.

The graph of f and line ℓ intersect at $A = 3.38987$.

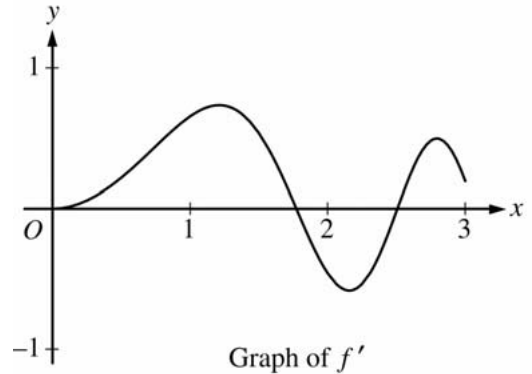
Area of $S = \int_0^A \left(\left(3 - \frac{1}{2}x \right) - f(x) \right) dx$

3 : $\begin{cases} 1 : \text{tangent line} \\ 1 : \text{integrand} \\ 1 : \text{limits} \end{cases}$

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Question 2

Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f' , the first derivative of f , given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown above.



- (a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.
- (b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.
- (c) Write an equation for the line tangent to the graph of f at $x = 2$.

(a) On the interval $1.7 < x < 1.9$, f' is decreasing and thus f is concave down on this interval.

(b) $f'(x) = 0$ when $x = 0, \sqrt{\pi}, \sqrt{2\pi}, \sqrt{3\pi}, \dots$
On $[0, 3]$ f' changes from positive to negative only at $\sqrt{\pi}$. The absolute maximum must occur at $x = \sqrt{\pi}$ or at an endpoint.

$$f(0) = 5$$

$$f(\sqrt{\pi}) = f(0) + \int_0^{\sqrt{\pi}} f'(x) dx = 5.67911$$

$$f(3) = f(0) + \int_0^3 f'(x) dx = 5.57893$$

This shows that f has an absolute maximum at $x = \sqrt{\pi}$.

(c) $f(2) = f(0) + \int_0^2 f'(x) dx = 5.62342$

$$f'(2) = e^{-0.5} \sin(4) = -0.45902$$

$$y - 5.623 = (-0.459)(x - 2)$$

2 : { 1 : answer
1 : reason

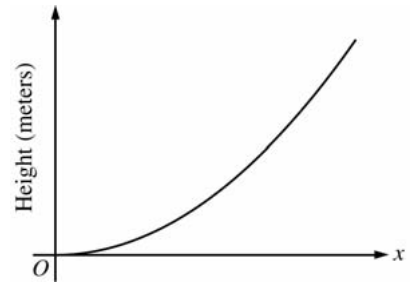
3 : { 1 : identifies $\sqrt{\pi}$ and 3 as candidates
- or -
indicates that the graph of f increases, decreases, then increases
1 : justifies $f(\sqrt{\pi}) > f(3)$
1 : answer

4 : { 2 : $f(2)$ expression
1 : integral
1 : including $f(0)$ term
1 : $f'(2)$
1 : equation

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Question 3

The figure above is the graph of a function of x , which models the height of a skateboard ramp. The function meets the following requirements.



- (i) At $x = 0$, the value of the function is 0, and the slope of the graph of the function is 0.
 - (ii) At $x = 4$, the value of the function is 1, and the slope of the graph of the function is 1.
 - (iii) Between $x = 0$ and $x = 4$, the function is increasing.
- (a) Let $f(x) = ax^2$, where a is a nonzero constant. Show that it is not possible to find a value for a so that f meets requirement (ii) above.
- (b) Let $g(x) = cx^3 - \frac{x^2}{16}$, where c is a nonzero constant. Find the value of c so that g meets requirement (ii) above. Show the work that leads to your answer.
- (c) Using the function g and your value of c from part (b), show that g does not meet requirement (iii) above.
- (d) Let $h(x) = \frac{x^n}{k}$, where k is a nonzero constant and n is a positive integer. Find the values of k and n so that h meets requirement (ii) above. Show that h also meets requirements (i) and (iii) above.

(a) $f(4) = 1$ implies that $a = \frac{1}{16}$ and $f'(4) = 2a(4) = 1$
implies that $a = \frac{1}{8}$. Thus, f cannot satisfy (ii).

2 : $\left\{ \begin{array}{l} 1 : a = \frac{1}{16} \text{ or } a = \frac{1}{8} \\ 1 : \text{shows } a \text{ does not work} \end{array} \right.$

(b) $g(4) = 64c - 1 = 1$ implies that $c = \frac{1}{32}$.
When $c = \frac{1}{32}$, $g'(4) = 3c(4)^2 - \frac{2(4)}{16} = 3\left(\frac{1}{32}\right)(16) - \frac{1}{2} = 1$

1 : value of c

(c) $g'(x) = \frac{3}{32}x^2 - \frac{x}{8} = \frac{1}{32}x(3x - 4)$
 $g'(x) < 0$ for $0 < x < \frac{4}{3}$, so g does not satisfy (iii).

2 : $\left\{ \begin{array}{l} 1 : g'(x) \\ 1 : \text{explanation} \end{array} \right.$

(d) $h(4) = \frac{4^n}{k} = 1$ implies that $4^n = k$.
 $h'(4) = \frac{n4^{n-1}}{k} = \frac{n4^{n-1}}{4^n} = \frac{n}{4} = 1$ gives $n = 4$ and $k = 4^4 = 256$.

4 : $\left\{ \begin{array}{l} 1 : \frac{4^n}{k} = 1 \\ 1 : \frac{n4^{n-1}}{k} = 1 \\ 1 : \text{values for } k \text{ and } n \\ 1 : \text{verifications} \end{array} \right.$

$$h(x) = \frac{x^4}{256} \Rightarrow h(0) = 0.$$

$$h'(x) = \frac{4x^3}{256} \Rightarrow h'(0) = 0 \text{ and } h'(x) > 0 \text{ for } 0 < x < 4.$$

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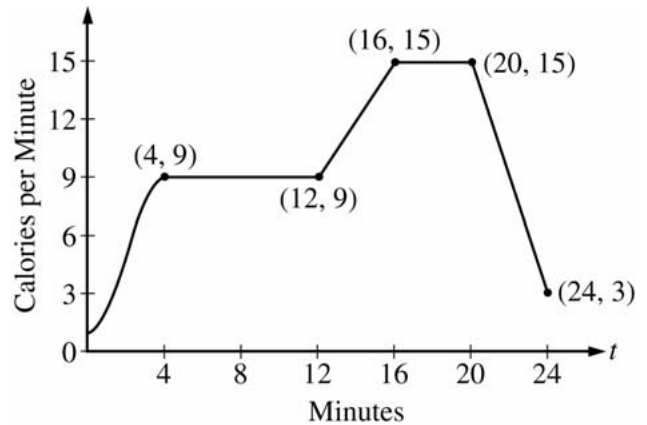
Question 4

The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function

f . In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for

$0 \leq t \leq 4$ and f is piecewise linear for $4 \leq t \leq 24$.

- (a) Find $f'(22)$. Indicate units of measure.
- (b) For the time interval $0 \leq t \leq 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.
- (c) Find the total number of calories burned over the time interval $6 \leq t \leq 18$ minutes.
- (d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \leq t \leq 18$.



(a) $f'(22) = \frac{15 - 3}{20 - 24} = -3$ calories/min/min

(b) f is increasing on $[0, 4]$ and on $[12, 16]$.

On $(12, 16)$, $f'(t) = \frac{15 - 9}{16 - 12} = \frac{3}{2}$ since f has constant slope on this interval.

On $(0, 4)$, $f'(t) = -\frac{3}{4}t^2 + 3t$ and

$f''(t) = -\frac{3}{2}t + 3 = 0$ when $t = 2$. This is where f' has a maximum on $[0, 4]$ since $f'' > 0$ on $(0, 2)$ and $f'' < 0$ on $(2, 4)$.

On $[0, 24]$, f is increasing at its greatest rate when $t = 2$ because $f'(2) = 3 > \frac{3}{2}$.

(c) $\int_6^{18} f(t) dt = 6(9) + \frac{1}{2}(4)(9 + 15) + 2(15)$
 $= 132$ calories

(d) We want $\frac{1}{12} \int_6^{18} (f(t) + c) dt = 15$.

This means $132 + 12c = 15(12)$. So, $c = 4$.

OR

Currently, the average is $\frac{132}{12} = 11$ calories/min.

Adding c to $f(t)$ will shift the average by c .

So $c = 4$ to get an average of 15 calories/min.

1 : $f'(22)$ and units

4 : $\left\{ \begin{array}{l} 1 : f' \text{ on } (0, 4) \\ 1 : \text{shows } f' \text{ has a max at } t = 2 \text{ on } (0, 4) \\ 1 : \text{shows for } 12 < t < 16, f'(t) < f'(2) \\ 1 : \text{answer} \end{array} \right.$

2 : $\left\{ \begin{array}{l} 1 : \text{method} \\ 1 : \text{answer} \end{array} \right.$

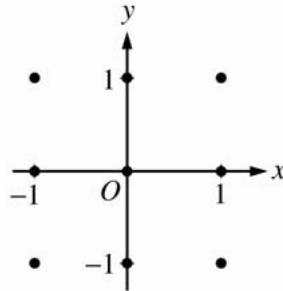
2 : $\left\{ \begin{array}{l} 1 : \text{setup} \\ 1 : \text{value of } c \end{array} \right.$

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Question 5

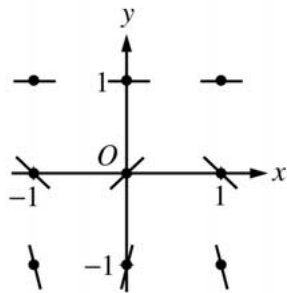
Consider the differential equation $\frac{dy}{dx} = (y - 1)^2 \cos(\pi x)$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
 (Note: Use the axes provided in the exam booklet.)



- (b) There is a horizontal line with equation $y = c$ that satisfies this differential equation. Find the value of c .
 (c) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(1) = 0$.

(a)



- (b) The line $y = 1$ satisfies the differential equation, so $c = 1$.

(c)
$$\frac{1}{(y - 1)^2} dy = \cos(\pi x) dx$$

$$-(y - 1)^{-1} = \frac{1}{\pi} \sin(\pi x) + C$$

$$\frac{1}{1 - y} = \frac{1}{\pi} \sin(\pi x) + C$$

$$1 = \frac{1}{\pi} \sin(\pi) + C = C$$

$$\frac{1}{1 - y} = \frac{1}{\pi} \sin(\pi x) + 1$$

$$\frac{\pi}{1 - y} = \sin(\pi x) + \pi$$

$$y = 1 - \frac{\pi}{\sin(\pi x) + \pi} \text{ for } -\infty < x < \infty$$

2 : $\begin{cases} 1 : \text{zero slopes} \\ 1 : \text{all other slopes} \end{cases}$

1 : $c = 1$

6 : $\begin{cases} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{answer} \end{cases}$

Note: max 3/6 [1-2-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

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Question 6

t (sec)	0	15	25	30	35	50	60
$v(t)$ (ft/sec)	-20	-30	-20	-14	-10	0	10
$a(t)$ (ft/sec ²)	1	5	2	1	2	4	2

A car travels on a straight track. During the time interval $0 \leq t \leq 60$ seconds, the car's velocity v , measured in feet per second, and acceleration a , measured in feet per second per second, are continuous functions. The table above shows selected values of these functions.

- (a) Using appropriate units, explain the meaning of $\int_{30}^{60} |v(t)| dt$ in terms of the car's motion. Approximate $\int_{30}^{60} |v(t)| dt$ using a trapezoidal approximation with the three subintervals determined by the table.
- (b) Using appropriate units, explain the meaning of $\int_0^{30} a(t) dt$ in terms of the car's motion. Find the exact value of $\int_0^{30} a(t) dt$.
- (c) For $0 < t < 60$, must there be a time t when $v(t) = -5$? Justify your answer.
- (d) For $0 < t < 60$, must there be a time t when $a(t) = 0$? Justify your answer.

- (a) $\int_{30}^{60} |v(t)| dt$ is the distance in feet that the car travels from $t = 30$ sec to $t = 60$ sec.

Trapezoidal approximation for $\int_{30}^{60} |v(t)| dt$:

$$A = \frac{1}{2}(14 + 10)5 + \frac{1}{2}(10)(15) + \frac{1}{2}(10)(10) = 185 \text{ ft}$$

- (b) $\int_0^{30} a(t) dt$ is the car's change in velocity in ft/sec from $t = 0$ sec to $t = 30$ sec.

$$\begin{aligned} \int_0^{30} a(t) dt &= \int_0^{30} v'(t) dt = v(30) - v(0) \\ &= -14 - (-20) = 6 \text{ ft/sec} \end{aligned}$$

- (c) Yes. Since $v(35) = -10 < -5 < 0 = v(50)$, the IVT guarantees a t in $(35, 50)$ so that $v(t) = -5$.

- (d) Yes. Since $v(0) = v(25)$, the MVT guarantees a t in $(0, 25)$ so that $a(t) = v'(t) = 0$.

Units of ft in (a) and ft/sec in (b)

2 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{value} \end{cases}$

2 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{value} \end{cases}$

2 : $\begin{cases} 1 : v(35) < -5 < v(50) \\ 1 : \text{Yes; refers to IVT or hypotheses} \end{cases}$

2 : $\begin{cases} 1 : v(0) = v(25) \\ 1 : \text{Yes; refers to MVT or hypotheses} \end{cases}$

1 : units in (a) and (b)