Definition I_n, J_n : Homogeneous Polynomial of degree n.

① $I_n(\vec{q}) = I_n(q_0, q_1, q_2, \cdots)$

$$= \sum_{F \in F_n} q_0^{t_0(F)} q_1^{t_1(F)} q_2^{t_2(F)} \cdots$$

where $t_i(F) = \#$ of vertices v s.t. $inv_F(v) = i$.

especially, $t_0(F) = lead(F)$

② $J_n(\vec{q}) = J_n(q_0, q_1, q_2, \cdots)$

$$= \sum_{P \in PF_n} q_0^{s_0(P)} q_1^{s_1(P)} q_2^{s_2(P)} \cdots$$

where $s_i(P) = \#$ of cars c s.t. $push_P(c) = i$.

especially, $s_0(P) = lucky(P)$

Theorem. $I_n(\vec{q}) = J_n(\vec{q})$

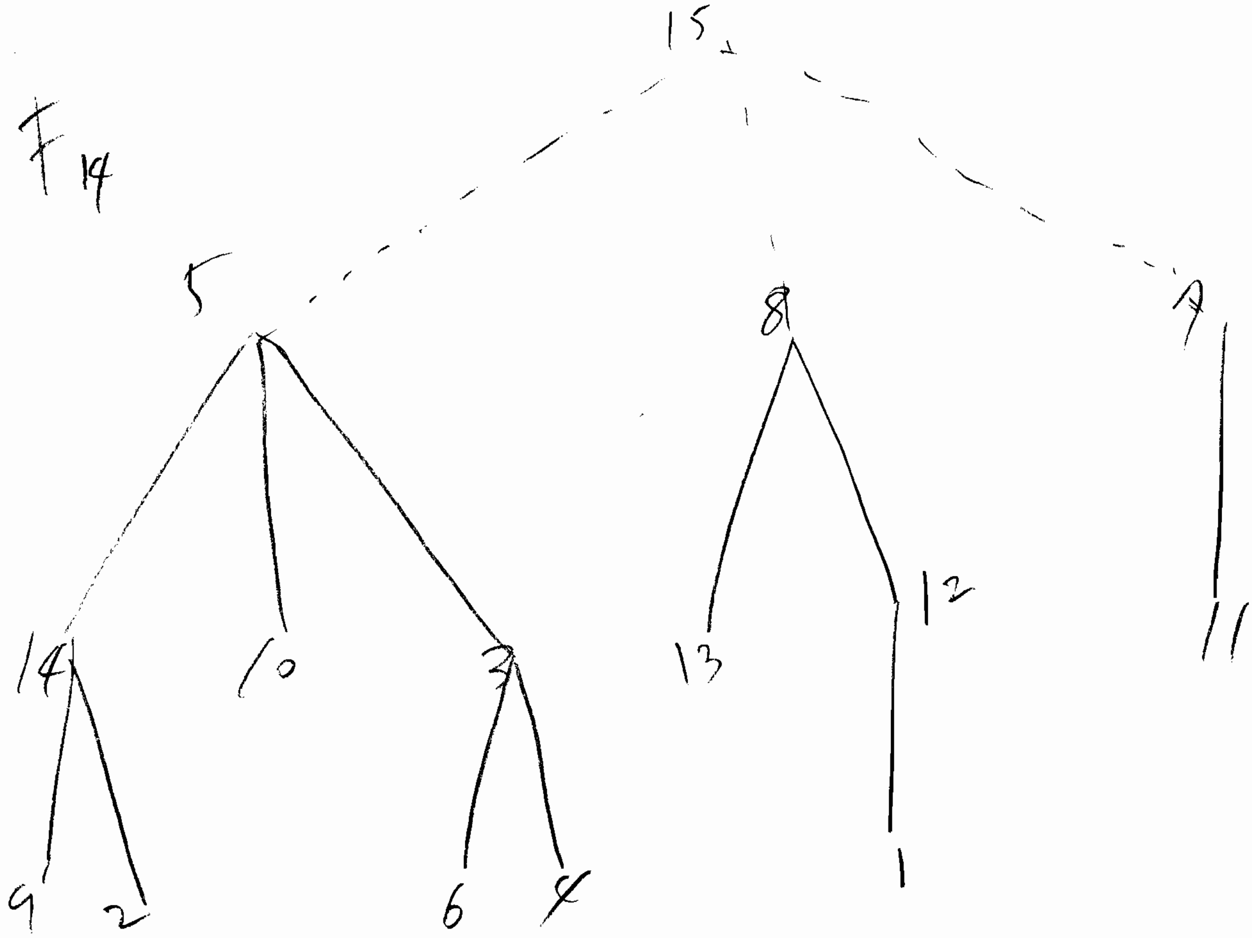
especially, $I_n(t, q, q^2, q^3, \cdots) = J_n(t, q, q^2, \cdots)$

i.e. $$\sum_{F \in F_n} t^{lead(F)} q^{inv(F)} = \sum_{P \in PF_n} t^{lucky(P)} q^{\binom{n}{2} - \sum_i P_i}$$

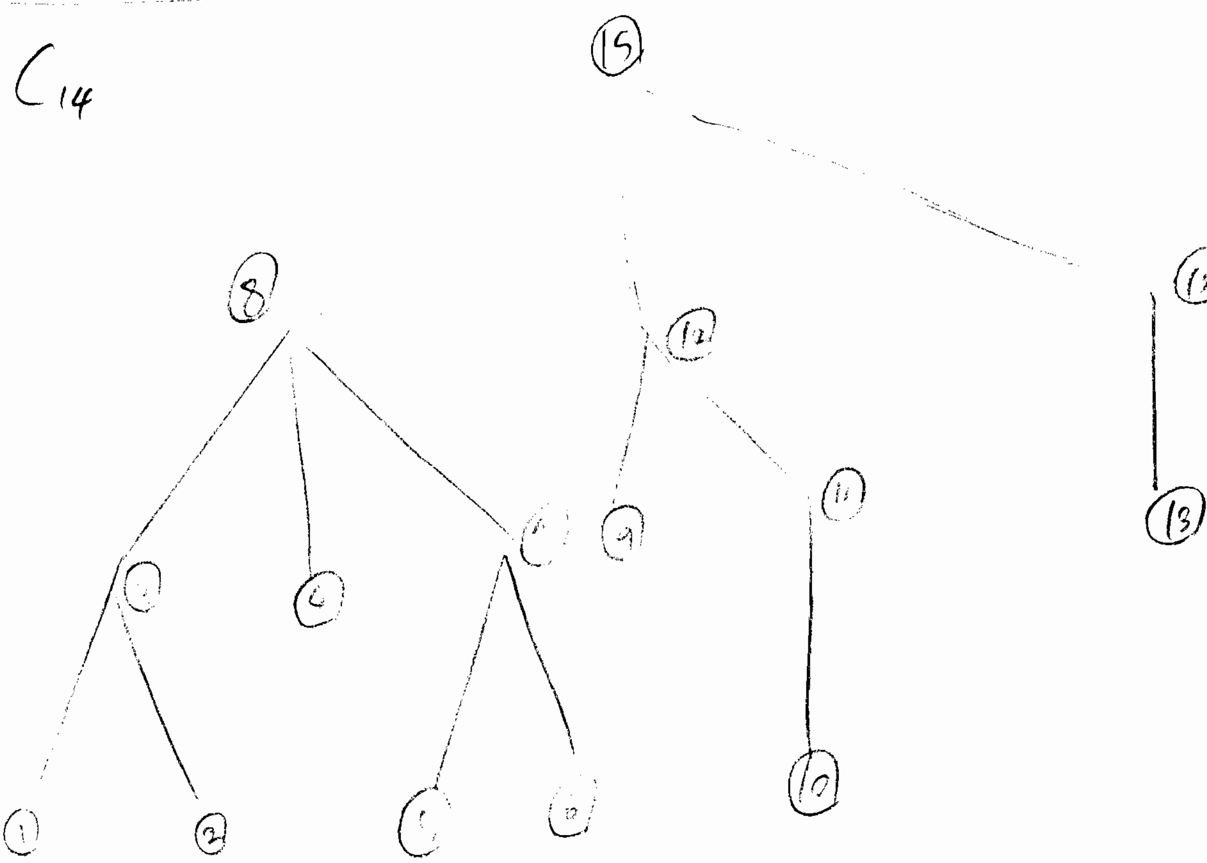
Note) $inv(F) = \sum_i i\, t_i(F)$

$$\binom{n}{2} = \sum_i P_i + \sum_i i\, s_i(P)$$

F_{14}

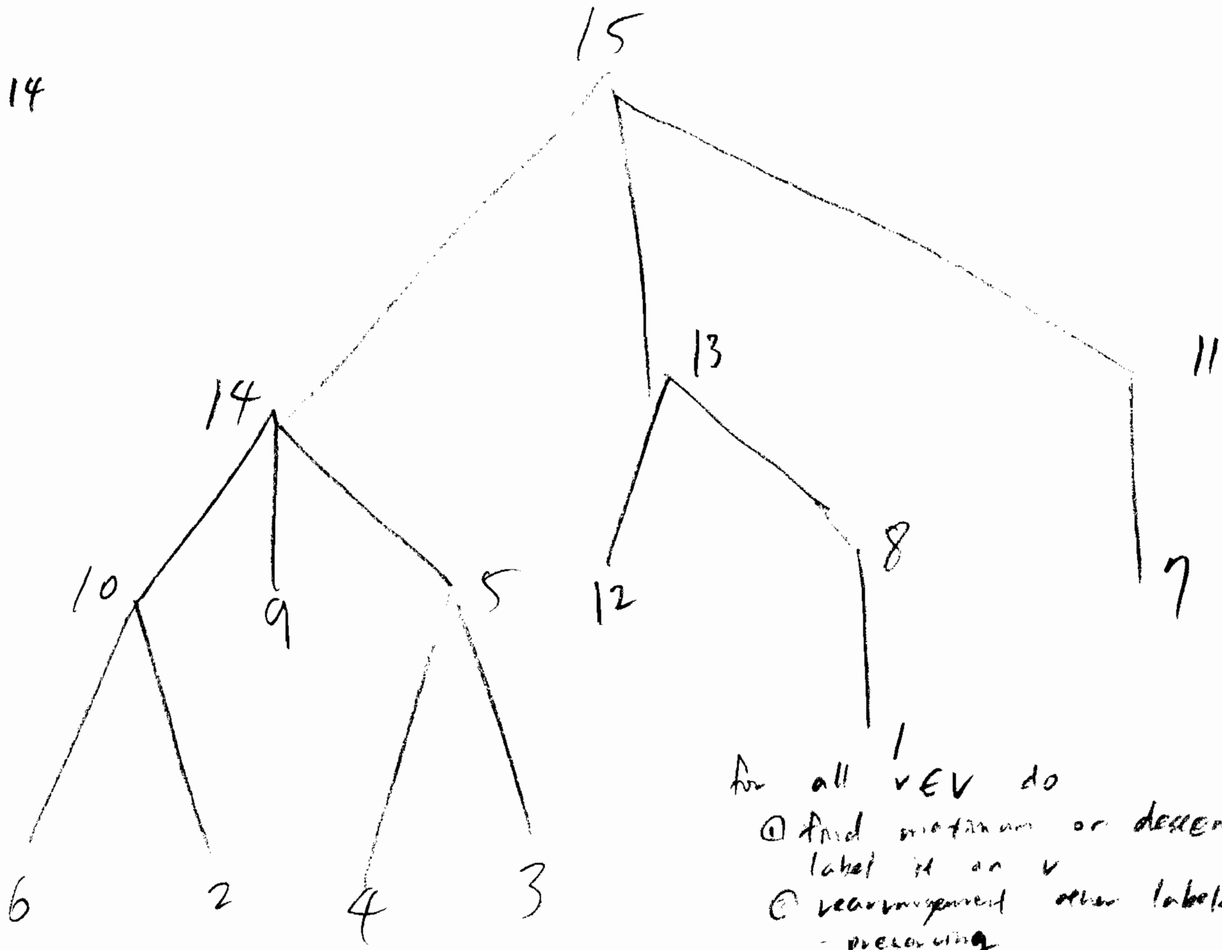


C_{14}



indexed by post-order.

D_{14}



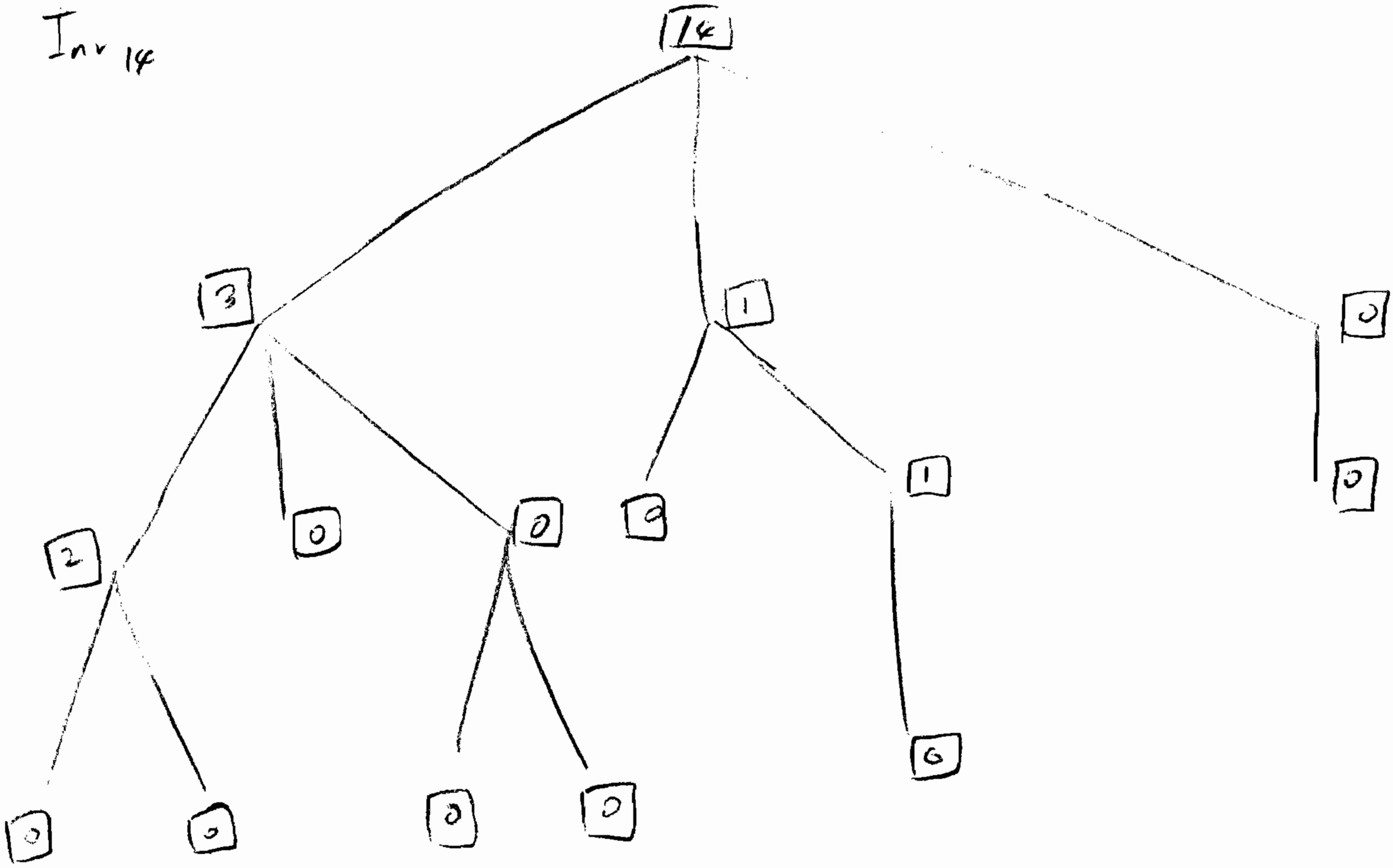
for all $v \in V$ do

① find minimum on descendants label it on v

② rearrangement other labels by order-preserving

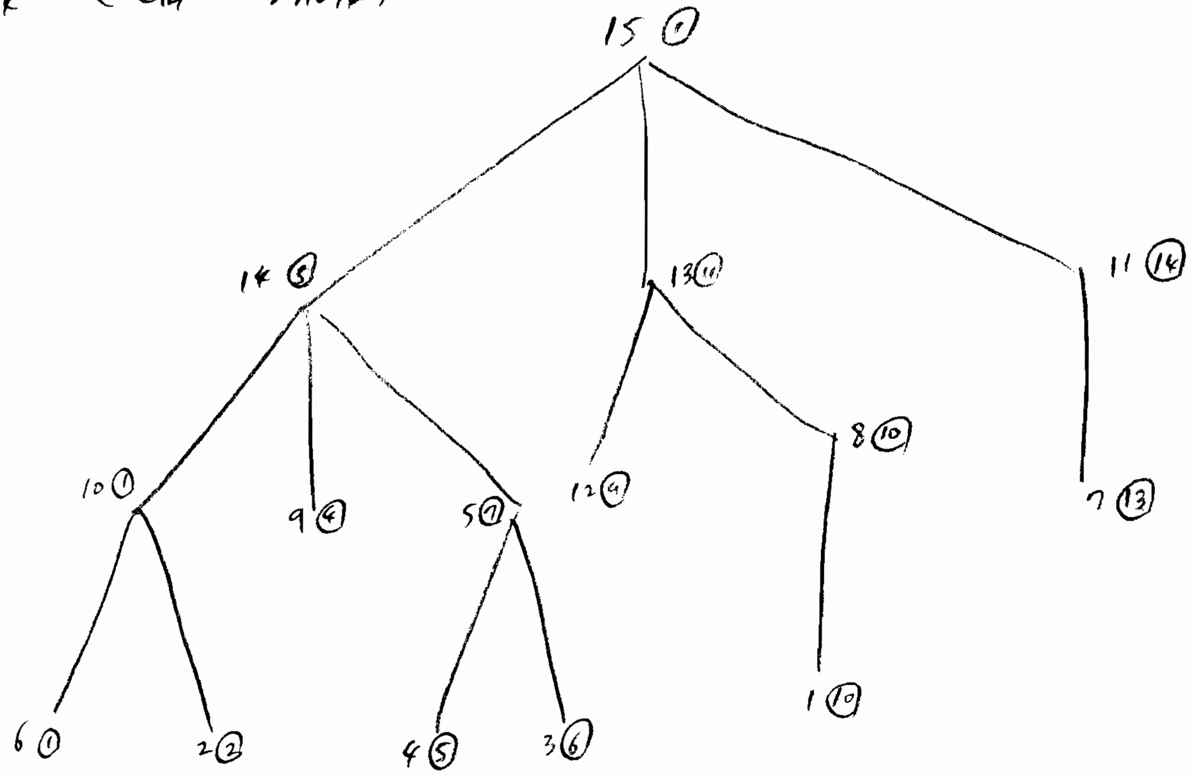
end do

Inv_{14}



# of inversion on descendent

$D_{14} \times (C_{14} - Inv_{14})$



PF_{14}

Car #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hope #	10	2	6	5	7	1	13	10	4	1	14	9	11	5	1

After parking cars.

Parking #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Car #	6	2	10	9	4	3	5	14	12	1	8	13	7	11	15
pushed #	0	0	2	0	0	0	0	3	0	0	1	1	0	0	14

edges between car c and the leftest car on its right which is larger than c.

# of trees in forest with vertices n = tree(F)

= degree of root in tree with edges n

= # of right-to left maximum after parking

= # of cars such that there are no empty right parking zone after parking it = fortune(P)

$$P_n(a,b,c) = c\prod_{i=1}^{n-1}(ia+(n-i)b+c)$$

$$\Rightarrow \sum_{F\in F_n} t^{\mathrm{lead}(F)} c^{\mathrm{tree}(F)} = \sum_{P\in PF_n} t^{\mathrm{lucky}(P)} c^{\mathrm{fortune}(P)} = P_n(1,t,ct)$$

$$I_n(\vec{q}:c) = I_n(q_0,q_1,\cdots:c) = \sum_{F\in F_n} q_0^{t_0(F)} q_1^{t_1(F)} \cdots c^{\mathrm{conn}(F)}$$

$$J_n(\vec{q}:c) = J_n(q_0,q_1,\cdots\ c) = \sum_{P\in PF_n} q_0^{s_0(P)} q_1^{s_1(P)} \cdots c^{\mathrm{fortune}(P)}$$

$$\Rightarrow I_n(\vec{q}:c) = J_n(\vec{q}:c)$$

especially, $I_n(t,q,q^2,\cdots,c) = J_n(t,q,q^2,\cdots;c)$

i.e. $$\sum_{F\in F_n} t^{\mathrm{lead}(F)} q^{\mathrm{inv}(F)} c^{\mathrm{conn}(F)} = \sum_{P\in PF_n} t^{\mathrm{lucky}(P)} q^{\binom{n}{2}-\sum p_i} c^{\mathrm{fortune}(P)}$$

Note) $I_n(t,1,1,\cdots:c) = J_n(t,1,1,\cdots:c) = P_n(1,t,ct)$