The 13th Algebraic Combinatorics Seminar

Organized by M.Hirasaka and J.Koolen

10th of June in 2006

Date

10th of June in 2006

Place

Audio Visual Room C32-313,
Department of Mathematics in Pusan National University

Program

11:00–11:50, Seunghyun Seo,
A Generalized Enumeration of Labeled Trees and Reverse Prüfer Algorithm

14:00–14:50, Heesung Shin,
Forests and Parking Functions

15:00–15:50, Soon-Yi Kang,
Generalizations of two fundamental product identities and their applications

16:10–17:00, Hiroshi Hirai,
Combinatorics of Stellar Subdivisions of Simplex

17:10–18:00, Dong-Chan Kim,
Construction of linear MDS-type codes on $\mathbb{P}$-metric space

18:00–18:30, Open to All

19:00–21:00, Banquet (free of charge)

Available Devices for Presentation

One white board with a small black board

Seunghyun Seo (Seoul National University)
Title: A Generalized Enumeration of Labeled Trees and Reverse Prüfer Algorithm
Abstract: A *leader* of a tree T on $[n]$ is a vertex which has no smaller descendants in T. In 2005, Gessel and Seo showed

$$\sum_{T\in\mathcal{T}_n} u^{(\#\text{ of leaders in } T)}\cdot c^{(\text{degree of 1 in } T)} = uP_{n-1}(1,u,cu),$$

which is a generalization of Cayley formula, where $\mathcal{T}_n$ is the set of trees on $[n]$ and P_n is given by

$$P_n(a,b,c) = c\prod_{i=1}^{n-1}(\,ia + (n-i)b + c\,).$$

In this talk, using a variation of Prufer code which is called a *reverse Prüfer code*, we give a simple bijective proof of Gessel and Seo's formula.
This is joint work with Heesung Shin.

Heesung Shin (KAIST)
Title: Forests and Parking Functions
Abstract: First of all, we define *trees* and *forests*. A *leader* and an *inversion* are defined for trees and forests. Also, we describe the *Reverse Prüfer Algorithm* which makes each tree correspond to a integer sequence. This sequence is called *RP-code* of a tree. This Algorithm can give the order of vertices naturally.
In parallel, we define *parking functions*. For a given parking function, cars are parked into parking space by the *Parking Algorithm*. A *lucky* and a *jump* are defined for parking functions through this algorithm.
By using two algorithms, we construct the bijection between forests and parking functions. This preserves each statistic of them, that is, a statistic of leaders (resp. inversions) in trees is equal to one of luckys (resp. jumps) in parking functions. This reveals that two combinatorial objects, forests and parking functions, have the same structure.

Soon-Yi Kang (KIAS)
Title: Generalizations of two fundamental product identities and their applications
Abstract: Using some standard transformations in the theory of basic hypergeometric series, we first derive a generalization of a reciprocity theorem for a certain q-series found in Ramanujan's lost notebook and

utilize it to derive four variable generalizations of Jacobi's triple and quintuple product identities. Then we present some applications of the generalized product identities, including new representations for generating functions for sums of six squares and those for overpartitions.

Hiroshi Hirai (Kyoto University)
Title: Combinatorics of Stellar Subdivisions of Simplex
Abstract: Stellar subdivision is one of most fundamental operations of polyhedral complex. In this talk, we discuss combinatorics of sequences of stellar subdivisions of a simplex. For a simplex Δ_0 and a pre-specified set $\mathcal{A}$ of its faces, consider the set of orderings of the faces $\mathcal{A}$ which refine the inclusion order of the faces. We can define the equivalence relation on the orderings such that two orderings of $\mathcal{A}$ are *equivalent* if successive stellar subdivisions with respect to this two orderings produce the same triangulation. Then the set of the orderings of $\mathcal{A}$ is partitioned into the equivalence classes. Our interest is the structure of the equivalence classes. Then we have the following results. First, there exists the set of posets $\mathcal{H}(\mathcal{A})$ on $\mathcal{A}$ such that the collection of orderings

$$\{\{\text{linear extensions of } \mathcal{P}\} \mid \mathcal{P} \in \mathcal{H}(\mathcal{A})\} \tag{1}$$

coincides with the equivalence classes. Second, this set of posets $\mathcal{H}(\mathcal{A})$ has a certain nice geometric property: $\mathcal{H}(\mathcal{A})$ can be naturally regarded as a polyhedral fan coarsening the braid arrangement. In a certain sence, $\mathcal{H}(\mathcal{A})$ is an analogue of the secondary fan of regular triangulations of a point set. Furthermore, we give a characterization of a poset of $\mathcal{H}(\mathcal{A})$. Using this characterization, we show that a certain class of convex geometries naturally arises as members of $\mathcal{H}(\mathcal{A})$.

Dong-Chan Kim (Sogang University)
Title: Construction of linear MDS-type codes on $\mathbb{P}$-metric space
Abstract: R. A. Brualdi introduced a poset code with $\mathbb{P}$-metric. In this paper, we construct the linear maximum distance separable(MDS)-type poset code C over a chain $\mathbb{P}$ with prescribed parameters n, k, q, having the interesting $\mathbb{P}$-weight distribution. This work improves the upper bound of a minimum distance of a linear $[n, k, q]$-codes.