

## 삼각함수 공식 정리

### 1. Trigonometric Functions of Special Angles (특수각일때의 함수값)

Angle	$\frac{\pi}{6} = 30^\circ$	$\frac{\pi}{4} = 45^\circ$	$\frac{\pi}{3} = 60^\circ$
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

### 2. Functions in terms of Angles in the First Quadrant (일사분면각으로 표현한 함수)

$n$  이 정수일때,

	$-\alpha$	$\frac{\pi}{2} \pm \alpha$	$\pi \pm \alpha$	$\frac{3\pi}{2} \pm \alpha$	$2n\pi \pm \alpha$
sin	$-\sin \alpha$	$\cos \alpha$	$\mp \sin \alpha$	$-\cos \alpha$	$\pm \sin \alpha$
cos	$\cos \alpha$	$\mp \sin \alpha$	$-\cos \alpha$	$\pm \sin \alpha$	$+\cos \alpha$
tan	$-\tan \alpha$	$\mp \cot \alpha$	$\pm \tan \alpha$	$\mp \tan \alpha$	$\pm \cot \alpha$

### 3. Fundamental Identities (기본 항등식)

#### (1) Reciprocal relations (상호관계)

$$\sin \alpha = \frac{1}{\csc \alpha},$$

$$\cos \alpha = \frac{1}{\sec \alpha},$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{1}{\cot \alpha}$$

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(2) Pythagorean theorem (피타고라스 정리)

$$\sin^2 z + \cos^2 z = 1$$

$$\sec^2 z - \tan^2 z = 1$$

$$\csc^2 z - \cot^2 z = 1$$

(3) Product relations (곱의 관계)

$$\sin \alpha = \tan \alpha \cos \alpha,$$

$$\sin \alpha = \tan \alpha \cos \alpha$$

$$\tan \alpha = \sin \alpha \sec \alpha,$$

$$\cot \alpha = \cos \alpha \csc \alpha$$

$$\sec \alpha = \csc \alpha \tan \alpha,$$

$$\csc \alpha = \sec \alpha \cot \alpha$$

(4) Quotient relations (지수관계)

$$\sin \alpha = \frac{\tan \alpha}{\sec \alpha},$$

$$\cos \alpha = \frac{\cot \alpha}{\csc \alpha},$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$$

4. One Circular Function in terms of Another (다른 삼각함수로 표현한 삼각함수)

	$\sin x$	$\cos x$	$\tan x$
$\sin x =$	$\sin x$	$\sqrt{1 - \cos^2 x}$	$\frac{\tan x}{\sqrt{1 + \tan^2 x}}$
$\cos x =$	$\sqrt{1 - \sin^2 x}$	$\cos x$	$\frac{1}{\sqrt{1 + \tan^2 x}}$
$\tan x =$	$\frac{\sin x}{\sqrt{1 - \sin^2 x}}$	$\frac{\sqrt{1 - \cos^2 x}}{\cos x}$	$\tan x$

5. Circulation Functions in terms of Exponentials (지수함수로 형태로 나타낸 삼각함수)

$$\cos z = \frac{e^{iz} + e^{-iz}}{2},$$

$$e^{iz} = \cos z + i \sin z$$



$$\sin z = \frac{e^{iz} - e^{-iz}}{2i},$$

$$e^{-iz} = \cos z - i \sin z$$

$$\tan z = \frac{\sin z}{\cos z} = \frac{e^{iz} - e^{-iz}}{i(e^{iz} + e^{-iz})}$$

### 6. Angle Sum and Difference Relationships (각도 합과 차의 관계)

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$$

### 7. Multiple-angle Formulae (배각공식)

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha}$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha = \cos^2 \alpha - \sin^2 \alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\sin 3\alpha = -4 \sin^3 \alpha + 3 \sin \alpha$$

$$\cos 3\alpha = 4 \cos^3 \alpha - 3 \cos \alpha$$

$$\tan 3\alpha = \frac{-\tan^3 \alpha + 3 \tan \alpha}{-3 \tan^2 \alpha + 1}$$

### 8. Half-Angle Formulae (반각공식)

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}} \quad \left( \frac{\alpha}{2} \text{ 이 일사분면이나 사사분면에 있으면 양수, 아니면 음수} \right)$$

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}} \quad \left( \frac{\alpha}{2} \text{ 이 일사분면이나 이사분면에 있으면 양수, 아니면 음수} \right)$$

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$$\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 + \cos \alpha} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

( $\frac{\alpha}{2}$  이 일사분면이나 삼사분면에 있으면 양수, 아니면 음수)

### 9. Powers of Circular Functions (삼각함수의 지수)

$$\sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha),$$

$$\cos^2 \alpha = \frac{1}{2}(1 + \cos 2\alpha)$$

$$\sin^3 \alpha = \frac{1}{4}(-\sin 3\alpha + 3\sin \alpha),$$

$$\cos^3 \alpha = \frac{1}{4}(\cos 3\alpha + 3\cos \alpha)$$

$$\tan^2 \alpha = \frac{1 - \cos 2\alpha}{1 + \cos 2\alpha},$$

### 10. Products of Sine and Cosine (곱의 공식)

$$\cos \alpha \cos \beta = \frac{1}{2} \cos(\alpha - \beta) + \frac{1}{2} \cos(\alpha + \beta)$$

$$\sin \alpha \sin \beta = \frac{1}{2} \cos(\alpha - \beta) - \frac{1}{2} \cos(\alpha + \beta)$$

$$\sin \alpha \cos \beta = \frac{1}{2} \sin(\alpha - \beta) + \frac{1}{2} \sin(\alpha + \beta)$$

### 11. Sums of Circular Functions (합의 공식)

$$\sin \alpha \pm \sin \beta = 2 \sin \frac{\alpha \pm \beta}{2} \cos \frac{\alpha \mp \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

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