



AP[®] Calculus AB
2007 Free-Response Questions
Form B

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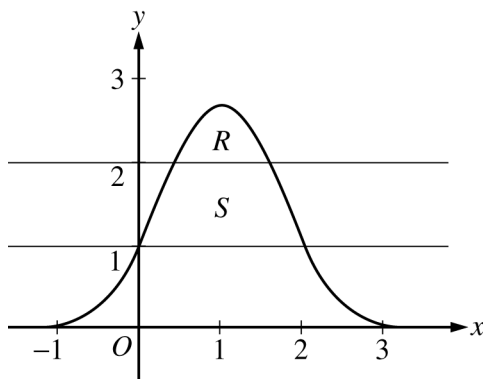
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2007 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

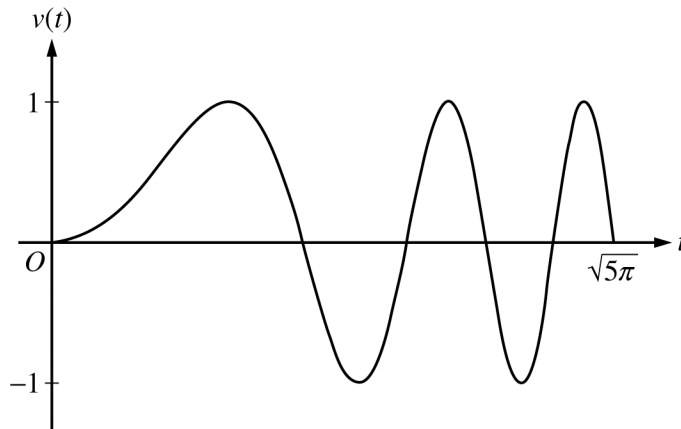
A graphing calculator is required for some problems or parts of problems.



1. Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.
- (a) Find the area of R .
- (b) Find the area of S .
- (c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1$.
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WRITE ALL WORK IN THE EXAM BOOKLET.

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2. A particle moves along the x -axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.
- Find the acceleration of the particle at time $t = 3$.
 - Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.
 - Find the position of the particle at time $t = 3$.
 - For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.
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3. The wind chill is the temperature, in degrees Fahrenheit ($^{\circ}\text{F}$), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v , in miles per hour (mph). If the air temperature is 32°F , then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \leq v \leq 60$.
- (a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.
- (b) Find the average rate of change of W over the interval $5 \leq v \leq 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \leq v \leq 60$.
- (c) Over the time interval $0 \leq t \leq 4$ hours, the air temperature is a constant 32°F . At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.
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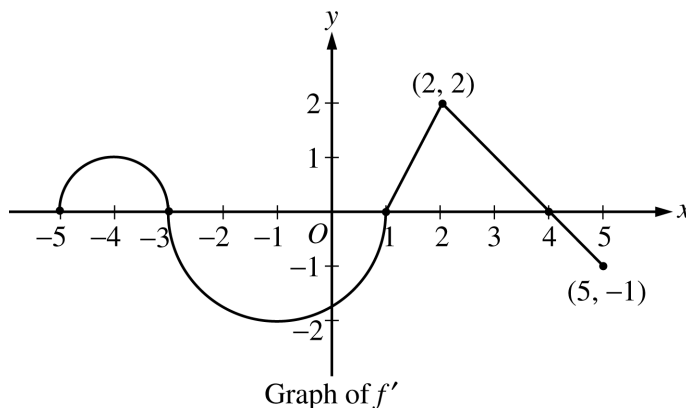
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END OF PART A OF SECTION II

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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. Let f be a function defined on the closed interval $-5 \leq x \leq 5$ with $f(1) = 3$. The graph of f' , the derivative of f , consists of two semicircles and two line segments, as shown above.
- For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
 - For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
 - Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
 - Find the absolute minimum value of $f(x)$ over the closed interval $-5 \leq x \leq 5$. Explain your reasoning.

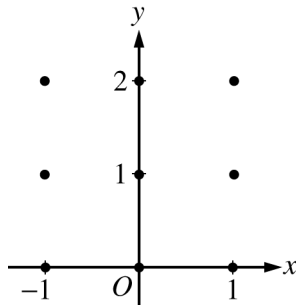
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5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Describe the region in the xy -plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the values of the constants m and b , for which $y = mx + b$ is a solution to the differential equation.

6. Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

(a) Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

(b) Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

(c) Show that if $f''(x) = 0$ for all x , then the graph of g does not have a point of inflection.

(d) Let $h(x) = f(x) - x$. Explain why there must be a value r for $2 < r < 5$ such that $h(r) = 0$.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM