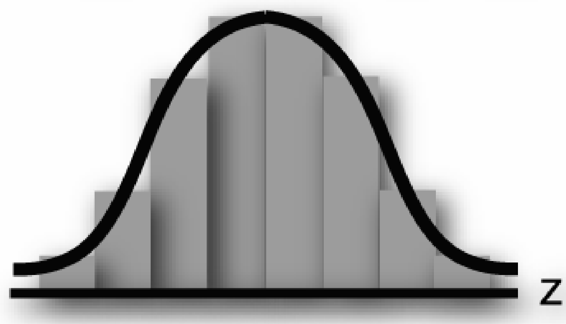
Pure Math 30:

STATISTICS



Statistics Lesson 2

The Binomial Distribution

Pure Math 30: EXPLAINED!

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Statistics Lesson 2
Part 1: Binomial Experiments

Binomial Experiments: A binomial experiment is one in which there are only two possible outcomes: success or failure. Also, the probability must remain constant throughout the experiment.

Example 1: Is flipping a coin a binomial experiment?
A coin being tossed will result in only two outcomes, heads or tails. The probability will remain 0.5 throughout the experiment since the probability of a head is ½, and the probability of a tail is ½ . Because of this, it is a binomial experiment.

Example 2: 4 marbles are in a box. If two marbles are drawn out successively without replacement, is this a binomial experiment?
This is ***not*** a binomial experiment since the probability for the second marble has changed. (We've changed the situation from pulling one marble out of four to pulling one marble out of three.)

Questions:

For each of the following, determine if it is a binomial experiment or not.

1) A coin is flipped 20 times. What is the probability of getting 8 tails?

2) 60% of students from a particular high school take a physics course. What is the probability that out of 15 students from this school, 5 have taken a physics course?

3) The probability of pulling a diamond out of a standard deck of 52 cards is 13/52. What is the probability of pulling 2 diamonds out consecutively, without replacement?

4) The probability of pulling a diamond out of a standard deck of 52 cards is 13/52. What is the probability of pulling 2 diamonds out consecutively, with replacement?

5) A student randomly guesses on seven multiple choice questions. If each question has four choices, find the probability the student gets 3 questions correct.

6) In a group of 10 laser printers, only 7 work. If a sample of 3 printers is taken, what is the probability that exactly 2 work?

7) A company knows that at any time in its production process, 95% of its laser printers work. What is the probability, to the nearest hundredth, that in a sample of 4 printers, exactly 3 work?

Answers:

1) Yes. There are only two outcomes (heads or tails), and the probability is the same for each trial.
2) Yes. There are only two outcomes (they take physics or they don't), and the probability is the same for each trial.
3) No. The probability for each trial will change as the number of cards is reduced. Use combination probability instead.
4) Yes. There are only two outcomes (diamond or not a diamond), and the probability is the same for each trial.
5) Yes. There are only two outcomes (correct or incorrect), and the probability is the same for each trial.
6) No. The probability for each trial will change as the number of printers is reduced. Use combination probability instead.
7) Yes. The 95% chance of working is the same for each printer, This probability is assigned by the manufacturer, and is independent of our sample.

Statistics Lesson 2
Part 2: Binomial Distributions

<u>Binomial Distribution:</u> The following formula can be used to find the probability of success in a binomial experiment.

$$P(k) = {}_nC_k\, p^k (1-p)^{n-k}$$

n = Total number of trials.
p = Probability of success
k = Number of successes
P(k) = Probability of getting *k* successes.

<u>**Example 1:**</u> A coin is flipped 4 times. Determine the probability of obtaining no heads, one head, two heads, three heads, and four heads. Draw a probability histogram displaying your results.

The number of trials (*n*) is 4

The probability (*p*) of obtaining a head (which we will define as a success) is 0.5

We must do the calculation five times, once for each different value of *k*.

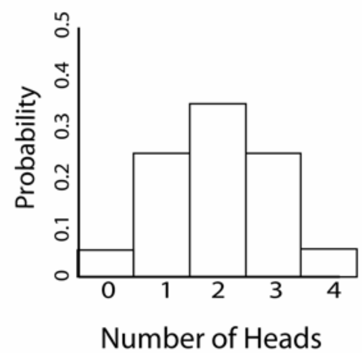
Zero heads *(k=0)* $P(0) = {}_4C_0(0.5)^0(1-0.5)^{4-0} = 0.0625$

One Head *(k=1)* $P(1) = {}_4C_1(0.5)^1(1-0.5)^{4-1} = 0.25$

Two Heads *(k=2)* $P(2) = {}_4C_2(0.5)^2(1-0.5)^{4-2} = 0.375$

Three Heads *(k=3)* $P(3) = {}_4C_3(0.5)^3(1-0.5)^{4-3} = 0.25$

Four Heads *(k=4)* $P(4) = {}_4C_4(0.5)^4(1-0.5)^{4-4} = 0.0625$



<u>***There is a way to do this calculation in your calculator very quickly!**</u>

Use the command binompdf (n, p, k) to evaluate a binomial probability in your calculator.
Find this by typing: 2nd → Distr → Binompdf. (scroll down to find this command, or type 0)

If you want to use this command to find only one case (e.g. *k* = 2) type in:
binompdf(4 , 0.5 , 2)

If you want to use this command to find all the cases, (k = 0 to 5) type in:
binompdf(4 , 0.5 , {0 , 1, 2, 3, 4}) *(Make sure you use the proper brackets.)*
The answers for all five cases will be displayed, separated by commas. Scroll right to see them all.

If you want to find the sum of all the probabilities, you can type in:
2nd → list → math → sum→Enter→2nd → ans
Notice that the sum adds up to 1, since the above probabilities account for all possible cases!

Statistics Lesson 2
Part 2: Binomial Distributions

Example 2: A particular brand of CD player has a 20% chance of having a defect when it leaves the factory. If a store sells 7 of these CD players, what is the probability at most 2 have a defect?

The number of trials is 7
The probability of having a defect (*which in this questions counts as a success*) is 0.2

At most 2 players having a defect means we want to add up the probabilities of zero having a defect, one having a defect, and two having a defect.

Evaluate using **binompdf(7, 0.2, {0, 1, 2})**, then **sum(ans)**

Answer = 0.852

Example 3: 35% of university students regularly take the bus to school. If 13 students are randomly sampled, what is the probability at least 4 take the bus?

The number of trials is 13
The probability of taking the bus (which in this questions counts as a success) is 0.35

If we want the probability of at least 4 taking the bus, we could add the cases where k = 4, 5, 6, … 13.
binompdf(13, 0.35, {4,5,6,7,8,9,10,11,12,13}), then **sum(ans) = 0.7217**

Alternatively, you could subtract all the probabilities you ***don't*** want from 1.

Evaluate the unwanted cases using **binompdf(13, 0.35, {0, 1, 2, 3})**, then **sum(ans) = 0.2783**

Probability that 0 to 3 students take the bus = 0.2783

The probability at least 4 take the bus is 1 – 0.2783 = 0.7217

Example 4: 3% of candies in a mix bag are peppermint. What is the probability that in a sample of 17 candies, at least 1 is peppermint?

The probability that no candy is peppermint is: binompdf(17, 0.03, 0) = 0.5958
The probability that at least one candy is peppermint is: 1 – 0.5958 = **0.4042**

Part 2: Binomial Distributions

Questions:

1) The probability that a high school student will graduate is 0.81. Out of 9 students, determine the probability that:

a) three students graduate.

b) 2 students do not graduate.

c) at least 7 students graduate.

d) at most 1 student graduates.

e) at least one student graduates.

2) The probability of a dart player getting a Bullseye is 0.04. Determine the probability that in five throws she will:

a) Score 1 Bullseye.

b) Score at most 1 Bullseye

c) Score at least 3 Bullseyes

3) What is the probability of drawing 2 red cards consecutively (*without replacement*) from a standard deck of 52 cards?

Answers:

1)
a) binompdf(9,0.81,3) = 0.0021
b) binompdf(9,0.81,7) = 0.2973
c) binompdf(9,0.81,{7,8,9}) → sum(ans) = 0.7642
d) binompdf(9,0.81,{0,1}) → sum(ans) = 0.00001270
e) binompdf(9,0.81,{1,2,3,4,5,6,7,8,9}) → sum(ans) = 0.9999996773

2)
a) binompdf(5,0.04,1) = 0.1699
b) binompdf(5,0.04,{0,1}) → sum(ans) = 0.9852
c) binompdf(5,0.04,{3,4,5}) → sum(ans) = 0.0006022

3) Since the cards are not replaced, this does not follow a binomial distribution. Use combination probability instead:

$$\frac{{}_{26}C_2}{{}_{52}C_2} = 0.2451$$