



# Pure Math 30

## EXPLAINED!

*Diploma Style  
Practice Exam*

Statistics  
-- ANSWERS --

## **Formulas**

*These are the formulas for Statistics you will be given on your diploma*

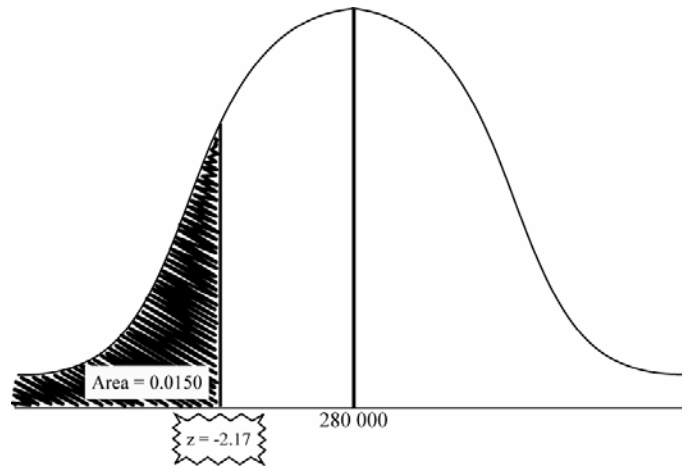
$$P(k) = {}_n C_k p^k (1-p)^{n-k}$$

$$z = \frac{x - \mu}{\sigma}$$

<i>Use this sheet to record your answers</i>
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- |                  |                   |                   |                   |
|------------------|-------------------|-------------------|-------------------|
| <b>1.</b> A      | <b>11.</b> D      | <b>18.</b> D      | <b>27.</b> C      |
| <b>NR 1)</b> 0.6 | <b>NR 2)</b> 0.06 | <b>19.</b> A      | <b>28.</b> C      |
| <b>2.</b> C      | <b>NR 3)</b> 0.75 | <b>20.</b> C      | <b>NR 7)</b> 0.33 |
| <b>3.</b> D      | <b>12.</b> C      | <b>21.</b> A      | <b>29.</b> D      |
| <b>4.</b> D      | <b>13.</b> B      | <b>22.</b> A      | <b>30.</b> B      |
| <b>5.</b> C      | <b>14.</b> A      | <b>23.</b> A      | <b>31.</b> B      |
| <b>6.</b> D      | <b>15.</b> A      | <b>24.</b> C      | <b>32.</b> A      |
| <b>7.</b> C      | <b>16.</b> A      | <b>NR 6)</b> 0.09 | <b>33.</b> C      |
| <b>8.</b> B      | <b>NR 4)</b> 0.01 | <b>25.</b> B      |                   |
| <b>9.</b> A      | <b>17.</b> D      | <b>26.</b> C      |                   |
| <b>10.</b> B     | <b>NR 5)</b> 0.50 |                   |                   |

1. The diagram on the right shows how to draw the normal distribution curve. When the question says only 1.5% of the engines are to be replaced, it means the **area** is 0.0150. Find this area on the z-score table, then line it up with the corresponding z-score of -2.17



At this point, use the formula to calculate the value of x.

$$z = \frac{x - \mu}{\sigma}$$

$$-2.17 = \frac{x - 280000}{23000}$$

$$-49910 = x - 280000$$

$$x = 230090$$

The answer is **A**.

- NR 1.** There are three **S**'s and three **T**'s, leading to six favorable outcomes. There are ten letters altogether. The probability of pulling an **S** or a **T** is  $\frac{6}{10} = 0.60$

The answer is **0.60**

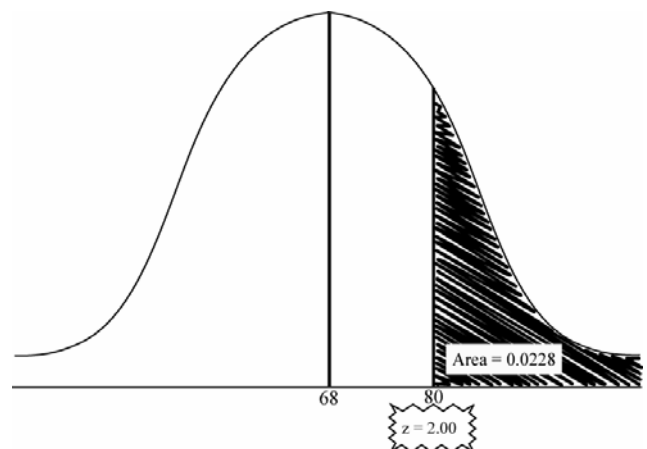
2. Calculate the z-score corresponding to an exam mark of 80.

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{80 - 68}{6}$$

$$z = 2.0$$

From the z-score table, this corresponds to an area of 0.9772. However, we want the area on the right, so  $1 - 0.9772 = 0.0228$



The number of students falling in this region is  $14000 \times 0.0228 = 319$

The answer is **C**.

3. First determine the number of possible arrangements for the PIN number that satisfy the stated restrictions.

$$\underline{1} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{1} = 1000$$

Now determine the total number of arrangements

$$\underline{9} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} = 90000$$

The probability is  $\frac{1000}{90000} = \frac{1}{90}$

The answer is **D**.

4. The question gives the following information ( $x = 57$ ,  $z = -1.4$ ,  $\mu = 68$ ). We want  $\sigma$ .

$$z = \frac{x - \mu}{\sigma}$$

$$-1.4 = \frac{57 - 68}{\sigma}$$

$$-1.4\sigma = 57 - 68$$

$$-1.4\sigma = -11$$

$$\sigma = 7.86$$

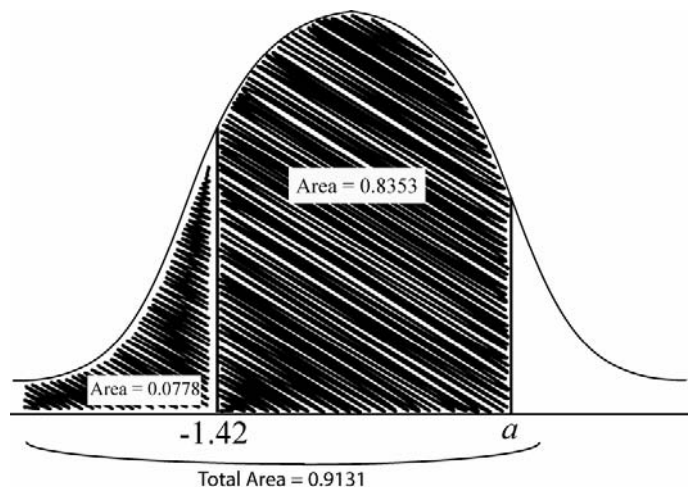
The answer is **D**.

5. First determine the area to the left of  $z = -1.42$

Then add the two areas to find the full area leading up to the unknown  $z$ -score.

Finally, find this area in the  $z$ -score table and it corresponds to  $z = 1.36$

The answer is **C**.



6. The number of ways of selecting two math books **or** two physics books is  ${}_6C_2 + {}_{12}C_2 = 81$

The number of ways to select any two books is  ${}_{18}C_2 = 153$

The probability is  $\frac{81}{153} = \frac{9}{17}$

The answer is **D**.

7.  $z = -1.22 \rightarrow \text{Area} = 0.1112$   
 $z = 0.75 \rightarrow \text{Area} = 0.7734$

The difference in the areas gives the required probability  $0.7734 - 0.1112 = 0.6622$   
 Use the formula:

*Total Number of Students*  $\times$  *Probability* = *Number of Students Between 45 and 65*

Rearranging, we get *Total Number of Students* =  $\frac{\text{Students Between 45 and 65}}{\text{Probability}}$

$$\text{Total Number of Students} = \frac{5000}{0.6622} = 7551$$

The answer is **C**.

8.  $z = -0.63 \rightarrow \text{Area on left side of this } z\text{-score} = 0.2643$   
 The area between  $z = -0.63$  and the mean is  $0.5000 - 0.2643 = 0.2357$

$z = 1.54 \rightarrow \text{Area on left side of this } z\text{-score} = 0.9382$   
 The area to the *right* is  $1 - 0.9382 = 0.0618$



















Total area =  $0.2357 + 0.0618 = 0.2975$

The answer is **B**.

9. First draw out the possible cases:  
 There is only one favorable outcome out of six possible outcomes.

The answer is  $\frac{1}{6}$

This matches with answer **A**.

	A	B	C
Arrangement 1.			
Arrangement 2.			
Arrangement 3.			
Arrangement 4.			
Arrangement 5.			
Arrangement 6.			

**10.** Adding a number to every number in a set will bump the mean up by exactly that number. The answer is  $\mu + n$ .  
The answer is **B**.

**11.** Answers **A** and **B** can be true or false, depending on what the mean & standard deviation are. Answer **C** is false since the z-score is positive.  
The answer is **D**.

**NR2.** The number of ways three unique prizes are won by 3 math teachers, 3 science teachers, 3 English teachers, or 3 social teachers is  
 ${}_7P_3 + {}_{10}P_3 + {}_4P_3 + {}_8P_3 = 1290$  (Add since the cases are independent of each other, and use permutations since the prizes are unique.)

The number of ways three unique prizes are won by anybody is  ${}_{29}P_3 = 21924$

The probability is  $\frac{1290}{21924} = \mathbf{0.06}$

**NR3.** The number of lines with Kevin and Rachel **not** standing together is  $8! - (7! \bullet 2!) = 30240$

The number of lines without restrictions is  $8! = 40320$

The probability is  $\frac{30240}{40320} = \mathbf{0.75}$

**12.** The area given is 0.209, but the area to the left is required to find the z-score.  
 $1 - 0.209 = 0.791$

From this, the z-score is 0.81

Now use the formula to solve for  $\mu$ .

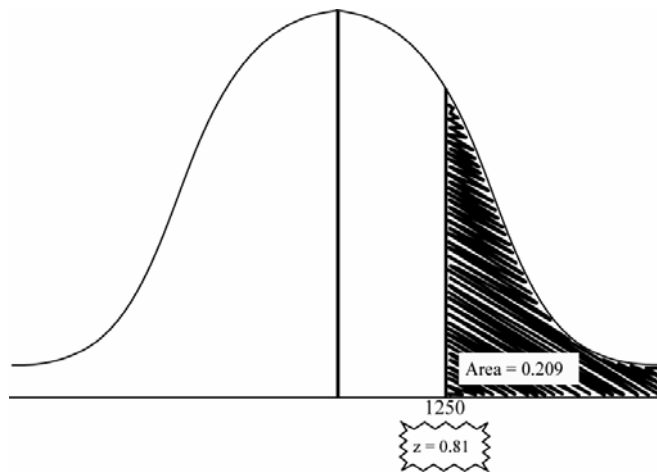
$$z = \frac{x - \mu}{\sigma}$$

$$0.81 = \frac{1250 - \mu}{105}$$

$$85.05 = 1250 - \mu$$

$$\mu = 1165$$

The answer is **C**.



13. Draw in the normal curve as shown. Use the area to find the  $z$ -score. ( $z = 1.81$ )

Now use the formula to calculate  $\sigma$

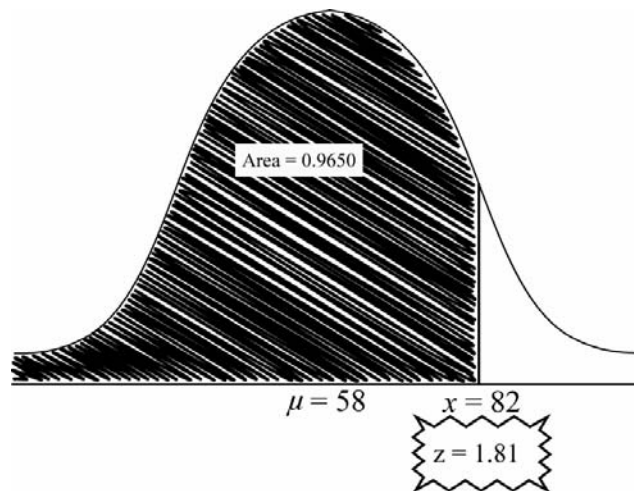
$$z = \frac{x - \mu}{\sigma}$$

$$1.81 = \frac{82 - 58}{\sigma}$$

$$1.81\sigma = 24$$

$$\sigma = 13.26$$

The answer is **B**.



14. The number of ways to select four candies, such that one from each color is obtained, is  ${}_5C_1 \times {}_3C_1 \times {}_7C_1 \times {}_5C_1$ . (Multiply since you are creating one set of items from multiple sets)

The total number of ways to select four candies is  ${}_{20}C_4$

The answer is **A**.

15. The course Jake performed best in can be determined by calculating the  $z$ -score for each course.

Pure Math 30  $\rightarrow z = 1.46$

Physics 30  $\rightarrow z = 0.43$

Chemistry 30  $\rightarrow z = 1.43$

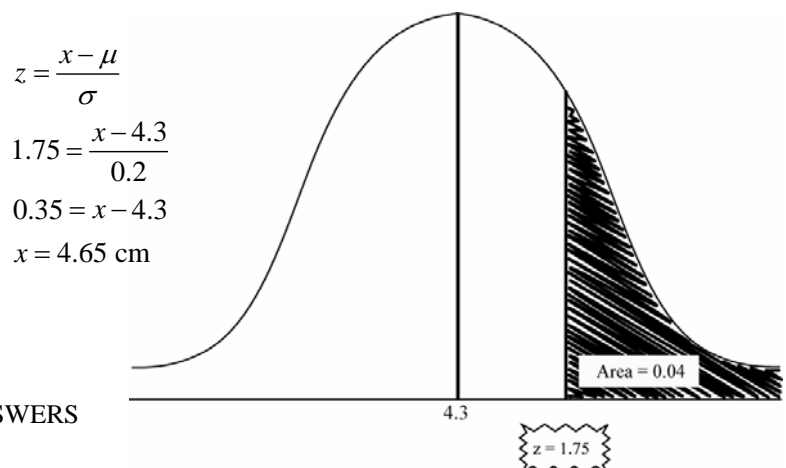
Biology 30  $\rightarrow z = -0.40$

The highest score is for Pure Math 30

The answer is **A**.

16. If 4% of screws have a length too long, the area to the *right* of the required length is 0.04. Area to the left is 0.96, which gives a  $z$ -score of 1.75. Use the formula to calculate  $x$ .

The answer is **A**.



- NR4.** The first two digits must be 7, so there is only 1 possibility for each position.  
 The third digit must NOT be 7, so there are 9 possibilities. (*Recall that exactly two seven's means the first two digits are seven, and the third is not seven*)  
 The remaining digits can be anything from 0 – 9, so there are 10 possibilities.

$$\underline{1} \quad \underline{1} \quad \underline{9} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} = 900\,000$$

There are  $10^8 = 100\,000\,000$  arrangements without restrictions.

The probability is  $\frac{900000}{100000000} = 0.009 = 0.01(\text{Rounded})$

The answer is **A**

- 17.** Calculate the  $z$ -score

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{19 - 17}{1.1}$$

$$z = 1.82$$

The area on the left is 0.9656, and the area on the right is 0.0344

The number of students is  $982 \times 0.0344 = 34$

The answer is **D**.

- NR5.** Calculate the number of even codes, and total possible codes.

Codes ending with an even digit

$$\underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{5} = 500\,000\,000$$

Unrestricted Codes

$$\underline{1} \quad \underline{1} \quad \underline{1} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} \quad \underline{10} = 1\,000\,000\,000$$

The probability is  $\frac{500000000}{1000000000} = 0.50$

The answer is **0.50**

- 18.** The ways the top three prizes can go to men or women is  ${}_{12}P_3 + {}_{14}P_3 = 3504$   
 (*Use permutations since the prizes are different, adding since the groups are independent.*)  
 The number of ways three prizes can be ordered randomly is  ${}_{26}P_3 = 15600$

The probability is  $\frac{3504}{15600} = 0.2246$  The answer is **D**.



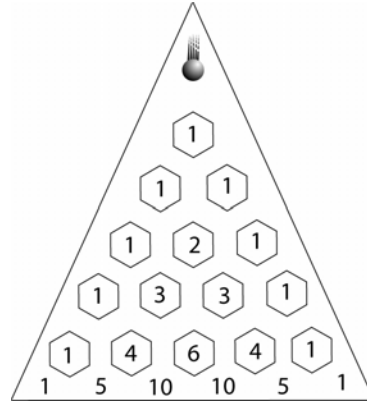
19. The probability of 0.002 is assigned by the manufacturer, so use a binomial distribution.  $\text{binompdf}[10, 0.002, 2] = 0.000177 = 0.0002$   
The answer is **A**.

20. The number of ways the ball does not hit B is 27.

The total number of ways the ball can land is 32.

The probability is  $\frac{27}{32} = 0.84$

The answer is **C**.



21. If there is an equal chance of giving birth to a male or female kitten, the probability of success (which we can arbitrarily call being female) is 0.5  
The number of successes is 2 females.  
Use  $\text{binompdf}[5, 0.5, 2] = 0.31$

The answer is **A**.

(The answer would be the same if we assumed success is male, and we want 3 successes.)

22. Use  $\text{binompdf}[20, 0.32, \{0,1,2,3\}] \rightarrow \text{Enter} \rightarrow \text{sum(ans)} = 0.077$   
The answer is **A**.

23. z-score (12 years) =  $\frac{12-10}{2} = 1.00$  Area = 0.8413

$$\text{z-score (14 years)} = \frac{14-10}{2} = 2.00 \quad \text{Area} = 0.9772$$

Difference in area = 0.1359

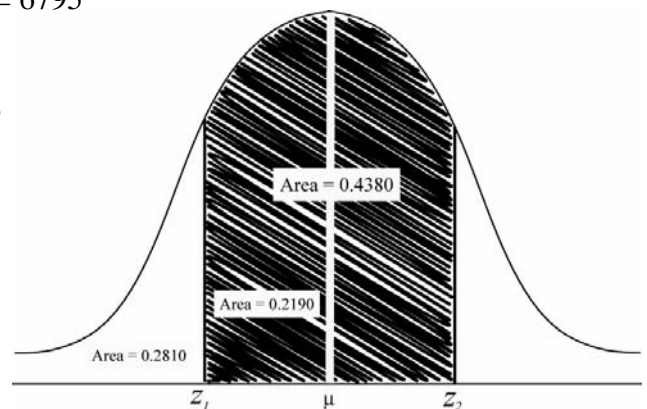
Number of televisions =  $50\,000 \times 0.1359 = 6795$

The answer is **A**.

24. If the area is symmetrical, dividing by two gives the area on the left side of the mean, as shown in the diagram. Once this area is found, go  $0.5 - 0.219 = 0.281$  to get the area to the left of  $z_1$ .

The value of  $z_1$  is -0.58

The answer is **C**.



**NR6.** Use  $\text{binompdf}[12, 0.17, 4] = 0.09$  (Since the probability of 0.17 is the probability of making the shot, the number of successes is how many shots actually scored. If 8 out of the 12 shots missed, then 4 shots scored, so that is the number of successes.)  
The answer is **0.09**.

**25.** Type all these values into  $\text{stat} \rightarrow \text{edit} \rightarrow L_1$ . Then use 1-var statistics.  
The standard deviation is  $\sigma_x = 2.39$   
The answer is **B**.

**26.** Physics exam: (Area = 0.8770  $\rightarrow z = 1.16$ )  
Chemistry exam: (Area = 0.9850  $\rightarrow z = 2.17$ )

<p>Physics exam</p> $z = \frac{x - \mu}{\sigma}$ $1.16 = \frac{x - 58}{9}$ $10.44 = x - 58$ $x = 68.44$	<p>Chemistry exam</p> $z = \frac{x - \mu}{\sigma}$ $2.17 = \frac{x - 54}{12}$ $26.04 = x - 54$ $x = 80.04$
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The difference is 11.60  
The answer is **C**.

**27.** Since each value is going up by the same amount, so does the mean. The standard deviation remains constant since the spread of the data does not change.  
The answer is **C**.

**28.** The number of ways to select a five-person team, such that exactly 4 have a bowling average over 250, is  ${}_8C_4 \times {}_4C_1 = 280$ . The number of ways to select a 5 person team at random is  ${}_{12}C_5 = 792$

The probability is  $\frac{280}{792} = \frac{35}{99}$

The answer is **C**.

**NR7.** In this case, the probability of a bowler having an average over 250 is 0.67, and this can be solved using the binomial distribution.  
 $\text{binompdf}[5, 0.67, 4] = 0.33$   
The answer is **0.33**.

*\*The difference between 28 & 29 is that in 28, the probability changes each time a bowler is selected, so a combination must be used. In 29, the same probability applies to each bowler, so the binomial distribution can be used.*

29. List the possible dollar combinations:

$\$5 + \$5 = \$10$   
 $\$5 + \$10 = \$15$   
 $\$5 + \$20 = \$25$   
 $\$10 + \$10 = \$20$   
 $\$10 + \$20 = \$30$   
 $\$20 + \$20 = \$40$

The probability of pulling out two bills that are at least \$30 is  $\frac{2}{6} = \frac{1}{3}$

The answer is **D**.

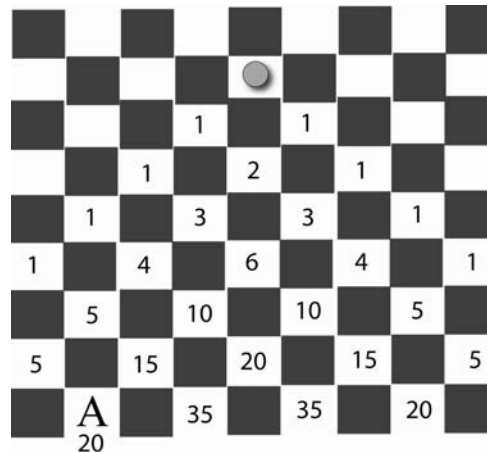
30. The number of paths to point A is 20.

The number of paths in total is 110

The probability is

$$\frac{20}{110} = \frac{2}{11} = 0.18$$

The answer is **B**.



31. The number of ways to spell BUNNY with the letters provided is

$$1 \cdot 1 \cdot 2 \cdot 1 \cdot 1 = 2 \text{ ways}$$

The number of possible arrangements is  $5!$

The probability is  $\frac{2}{5!}$

The answer is **B**. (Note that if you were to divide out repetitions for the possible & total cases, it would cancel out when calculating the probability and you would still get the same answer)

32. The data values all have the same value, so there is no spread. The standard deviation is zero.

The answer is **A**.

33. There is one favorable outcome, that is playing the notes FGABCD

There are  ${}_7P_6$  possible ways to play 6 notes out of 7.

The probability is  $\frac{1}{{}_7P_6}$

The answer is **C**.

### **Written Response 1**

- The number of households with exactly one vehicle is  $700\,000 \times \frac{17}{23} = 517391$   
(2 Marks)
- Use the binomial distribution since the probability of  $\frac{17}{23}$  is the same for every household in the city.  $\text{binompdf}[67, \frac{17}{23}, 55] = 0.09$   
(2 Marks)
- Anna's solution is correct, since the order of the vehicles is not important.  
(2 Marks)

### **Written Response 2**

- Type the first column into  $L_1$ , and the second column into  $L_2$ .  
Then type 1-var-stats  $L_1, L_2 =$   
From here, the mean is 4.57, and the standard deviation is 2.16  
(2 Marks)
- From the table, the favorable cases of rolling a 1, 2, or 3 is the sum of 53  
The total possible cases is 150.  
The probability is  $\frac{53}{150}$   
(2 Marks)
- The binomial distribution may **not** be used for an 8-sided dice since there is more than two possible outcomes, and cannot be represented using success/failure.  
(2 Marks)

### Written Response 3

- Set up a frequency table.

From the table, type the data into  $L_1$  and  $L_2$ , then type 1-var-stats  $L_1$ ,  $L_2 =$  to find the mean and standard deviation.

Grade	Frequency
1	2
2	6
3	8
4	3
5	1
6	0

mean = 2.75, standard deviation = 0.99

**(2 Marks)**

- Use the formula to calculate the z-score for a grade of 5

$$z = \frac{x - \mu}{\sigma}$$

$$z = \frac{5 - 2.75}{0.99}$$

$$z = 2.27$$

This corresponds to an area of 0.9884 on the left, so the area on the right will be 0.0116. Therefore, the number of students receiving a 5 or 6 will be  $325 \times 0.0116 = 3.77 \rightarrow 4$  (rounded)

**(2 Marks)**

- Since the z-scores are the same, we can set up the following:

$$z_1 = z_1$$

$$\frac{x_1 - \mu_1}{\sigma_1} = \frac{x_2 - \mu_2}{\sigma_2}$$

$$\frac{79 - 47}{7} = \frac{x_2 - 59}{6}$$

$$\frac{32}{7} = \frac{x_2 - 59}{6}$$

$$192 = 7x_2 - 413$$

$$605 = 7x_2$$

$$x_2 = 86.4$$

Clarissa's new mark is 86.4%

**(2 Marks)**