

Pure Math 30:

CONICS



LESSON 1

The Double - Napped Cone

Pure Math
30:

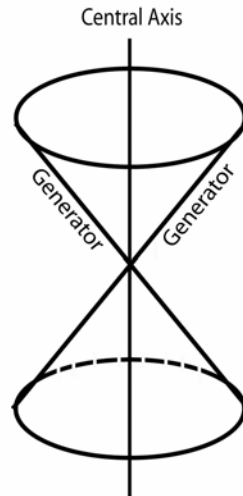
EXPLAINED!

By
Barry
Mabillard

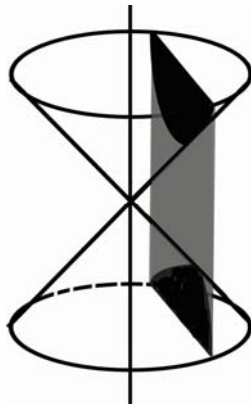
CONICS LESSON 1 - THE DOUBLE-NAPPED CONE

Conic Sections: There are 4 main conic sections: *circle*, *ellipse*, *parabola*, and *hyperbola*. It is possible to create each of these shapes by passing a plane through a three dimensional double napped cone.

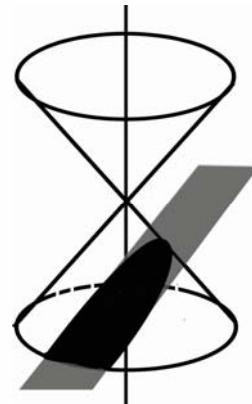
The shape to the right is a double napped cone. The vertical line down the middle is called the central axis, and the diagonal sides are called generators. The point at the centre is called the vertex. In theory, the double napped cone extends forever up & down.



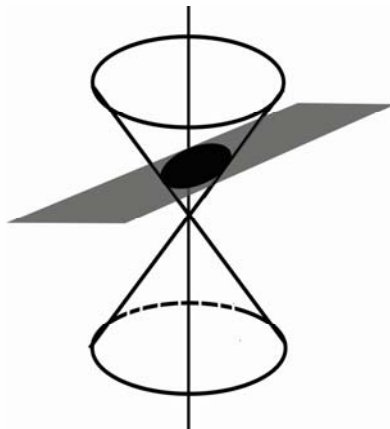
A *hyperbola* is produced when the plane passes through both nappes, between the central axis and generator.



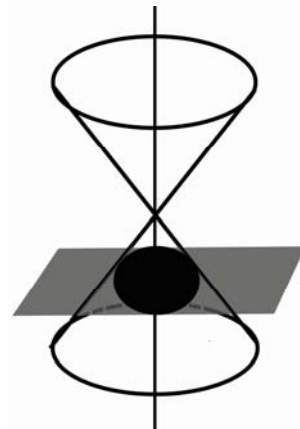
A *parabola* is produced when the plane passes through one nappe parallel to the generator.



An *ellipse* is produced when the plane passes through one nappe only, between the generator and perpendicular.



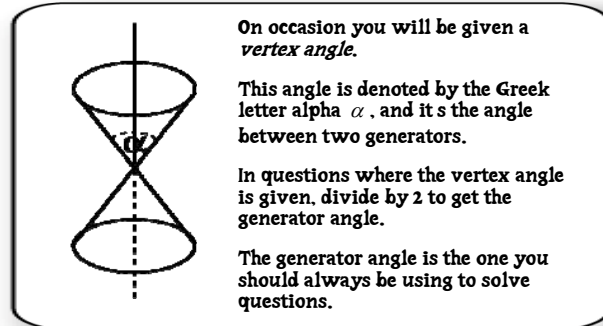
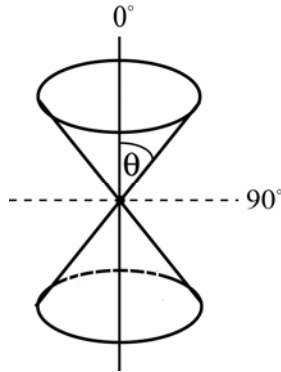
A *circle* is produced when the plane passes through one nappe only, perpendicular to the central axis.



CONICS LESSON 1 - THE DOUBLE-NAPPED CONE

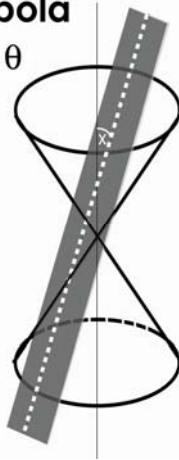
Conic Section Angles: The central axis (the vertical line) is defined as 0° , and angles are measured clockwise from this point. The angle between the central axis and generator is θ .

The line perpendicular to the central axis is 90°



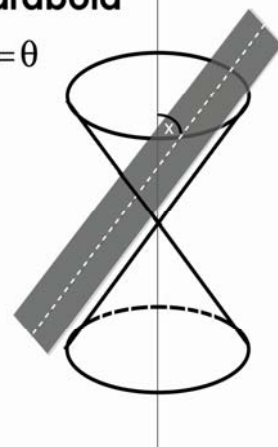
A hyperbola is produced when the plane passes between the central axis and generator

Hyperbola
 $0^\circ \leq x < \theta$



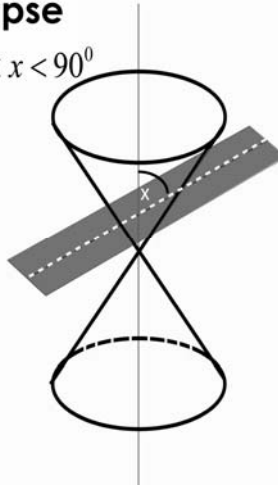
A parabola is produced when the plane passes through at exactly the same angle as the generator.

Parabola
 $x = \theta$



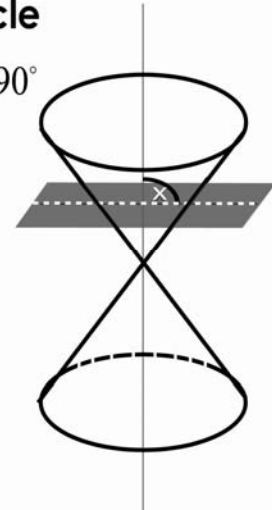
An ellipse is produced when the plane passes between the generator and perpendicular

Ellipse
 $\theta < x < 90^\circ$



A circle is produced when the plane passes through perpendicular to the central axis.

Circle
 $x = 90^\circ$

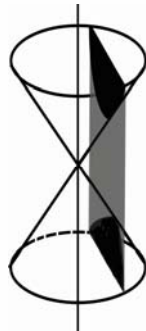


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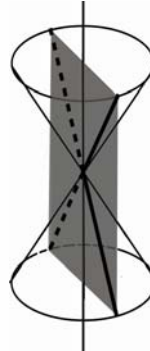
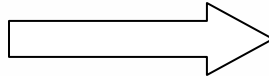
CONICS LESSON 1 - THE DOUBLE-NAPPED CONE

Degenerate Conic Sections: The basic four conic sections (circle, ellipse, parabola, hyperbola) can only be produced when the plane does **NOT** pass through the vertex. If it does pass through the vertex, the following degenerate conics will be produced:

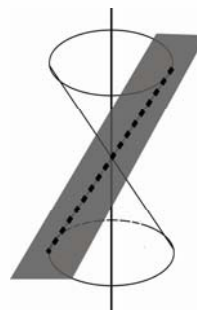
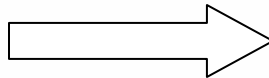
A hyperbola will degenerate into two intersecting lines.



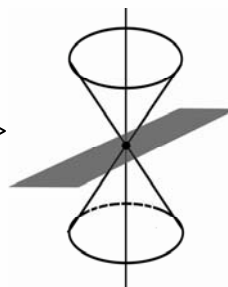
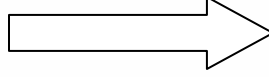
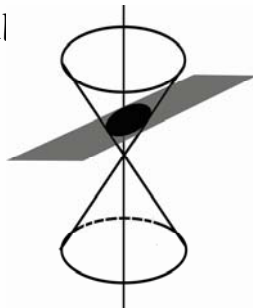
Imagine sliding the plane to the left and try to visualize the result on the graph shape.



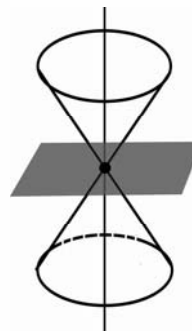
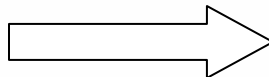
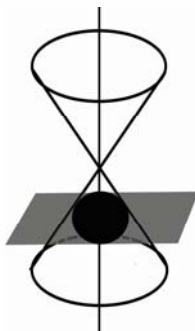
A parabola will degenerate into a single line.



An ellipse will degenerate into a point.



A circle will degenerate into a point.

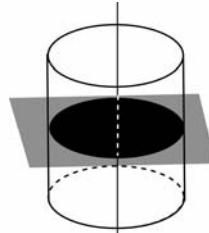


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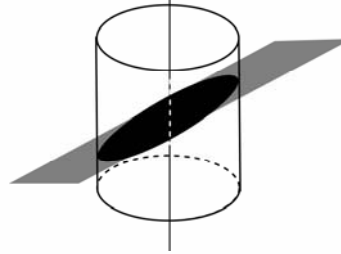
CONICS LESSON 1 - THE DOUBLE-NAPPED CONE

Cylinder Conics: The following conics may be obtained by passing a plane through a cylinder:

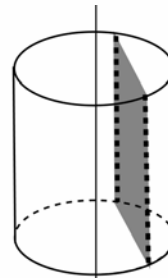
A circle is produced when the plane passes perpendicular to the central axis.



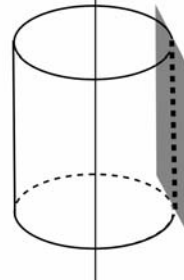
An ellipse is produced when the plane passes at an angle to the central axis.



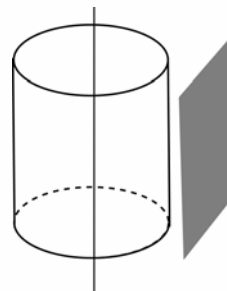
Two parallel lines are produced when the plane is parallel to the central axis and cuts through twice.



One line is formed when the plane passes parallel to the central axis and touches the edge of the cylinder.



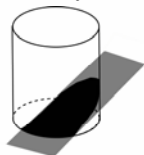
No graph is produced when the plane is parallel to the central axis and misses the cylinder.



CONICS LESSON 1 - THE DOUBLE-NAPPED CONE

Questions:

- 1) What are the four non-degenerate conics?
- 2) What are the five degenerate conics?
- 3) How do you turn a non-degenerate conic on the double-napped cone & cylinder into a degenerate conic?
- 4) What conic is produced in each of the following cases involving a double napped cone?
 - a) Generator angle of 33° , plane passes through at 56°
 - b) Generator angle of 45° , plane passes through at 2°
 - c) Generator angle of 53° , plane passes through at 53°
 - d) Generator angle of 27° , plane passes through at 90°
 - e) Vertex angle of 100° , plane passes through at 50°
 - f) Vertex angle of 88° , plane passes through at 33°
 - g) Vertex angle of 110° , plane passes through at 56°
 - h) Vertex angle of 60° , plane passes through at 90°
- 5) What conic(s) are produced in each of the following cases involving a double napped cone, and what causes the different shapes?
 - a) The plane is parallel to the central axis.
 - b) The plane is parallel to the generator.
 - c) The plane is perpendicular to the central axis.
 - d) The plane passes through the vertex.
 - e) The plane is parallel to the central axis. (Cylinder)
- 6) Suppose you want to create a parabola using a cylinder & a plane. What restriction would you have to make?



Answers:

- 1) Circle, Ellipse, Parabola, Hyperbola
- 2) A point, a single line, two intersecting lines, two parallel lines, and no graph.
- 3) For a double napped-cone, pass the plane through the vertex to turn the conic degenerate. For a cylinder, make the plane parallel to the central axis.
- 4) a) Ellipse, b) Hyperbola, c) Parabola, d) Circle
e) Parabola, f) Hyperbola, g) Ellipse, h) Circle
- 5) a) Hyperbola if it doesn't pass through the vertex, two intersecting lines if the plane passes through the vertex.
b) Parabola if the plane does not pass through the vertex, a single line if the plane does pass through the vertex.
c) Circle if the plane does not pass through the vertex, a point if the plane does pass through the vertex.
d) A point, a single line, or two intersecting lines
e) Two parallel lines if the plane passes within the cylinder, a single line if the plane is just touching the cylinder, and no graph if the plane is outside the cylinder.
- 6) Normally the double-napped cone & cylinder extend vertically forever, so this shape would technically turn into an ellipse. However, if you give the cylinder an actual bottom, the parabola can exist.