

Pure Math 30:

CONICS



LESSON 2

Circles, Ellipses, and Parabolas

Pure Math
30:

EXPLAINED!

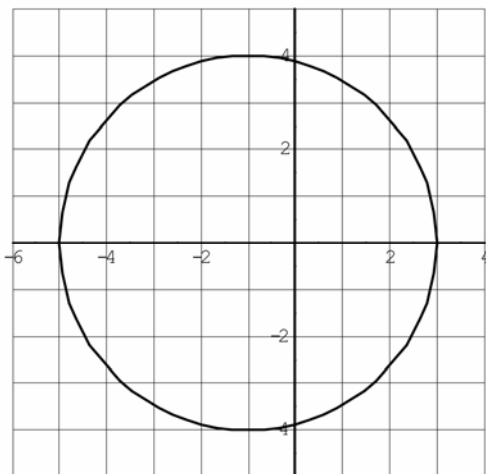
By
Barry
Mabillard

CONICS LESSON 2

PART I - CIRCLES

Circles: The standard form of a circle is given by the equation $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the centre of the circle and r is the radius.

Example 1: Given the following graph, write the equation.



The first thing you should do when given a circle is write down the coordinates of the centre. In this case, the centre is at $(-1, 0)$. Next, determine the radius, which is 4 units. Finally, plug the h , k , and r values into the standard form equation and you'll have the equation of the graph!

$$(x - h)^2 + (y - k)^2 = r^2$$

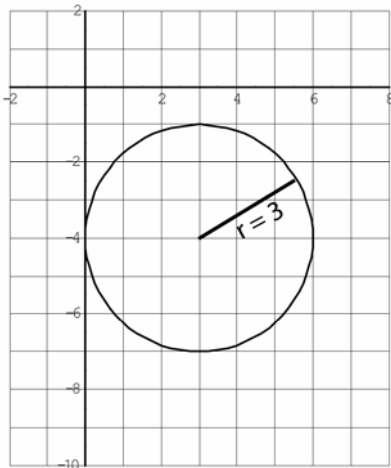
$$(x - (-1))^2 + (y - 0)^2 = 4^2$$

$$(x + 1)^2 + y^2 = 16$$

Example 2: Sketch the graph of $(x - 3)^2 + (y + 4)^2 = 9$ and state the domain and range.

To draw the graph of a circle from a standard form equation, first draw a dot at the centre of the circle. The radius can be found by taking the square root of the number on the right side. (Remember, you're given r^2 and you just want r .)

$$(x - 3)^2 + (y + 4)^2 = 9$$



Quick Tip: An easy way to read off the centre is to use values for x and y that make each bracket go to zero.

$(x - 3)$ becomes zero when $x = 3$
 $(y + 4)$ becomes zero when $y = -4$

So, the centre is at $(3, -4)$

When writing the domain & range for an enclosed shape, we use "in-between notation"

Domain: Leftmost Value $\leq x \leq$ Rightmost Value

Range: Bottom Value $\leq y \leq$ Top Value

For the circle in this question:

Domain: $0 \leq x \leq 6$ (Read as "the domain is between zero and six")

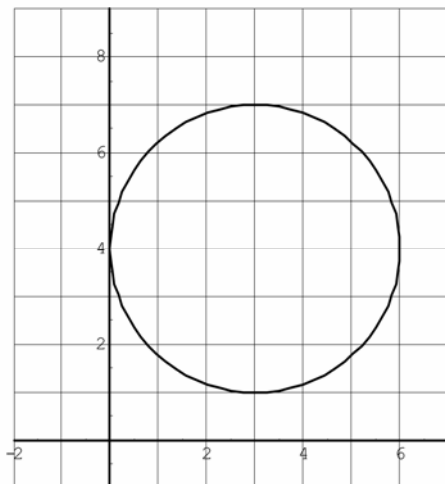
Range: $-7 \leq y \leq -1$ (Read as "the range is between negative seven and negative one")

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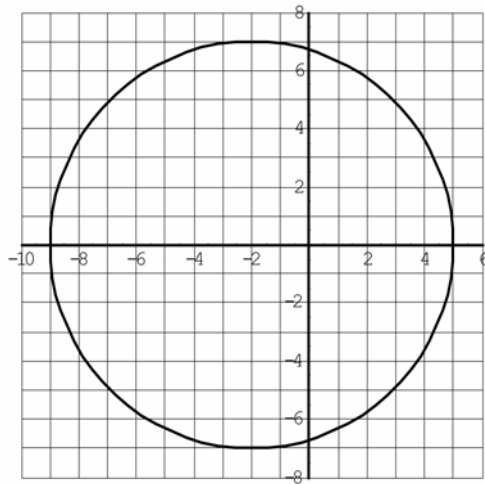
PART I - CIRCLES

Questions: For each of the following graphs, write the equation, then state domain & range:

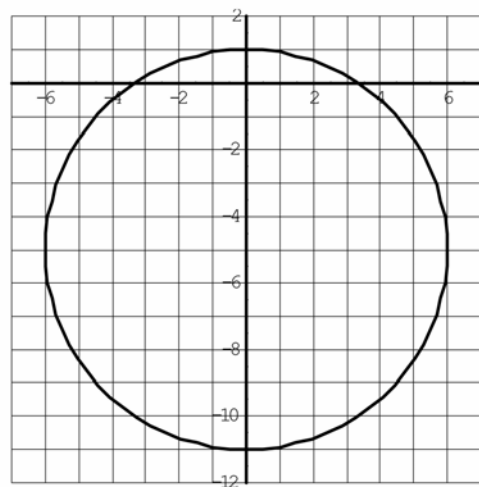
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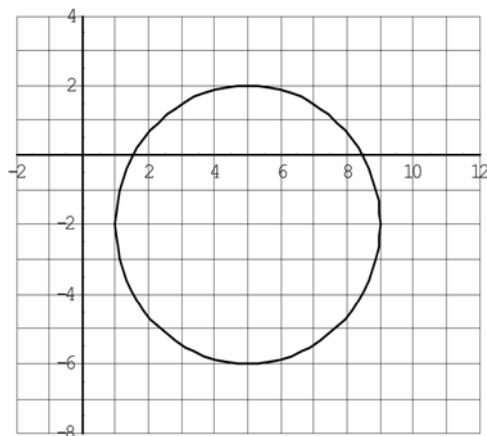
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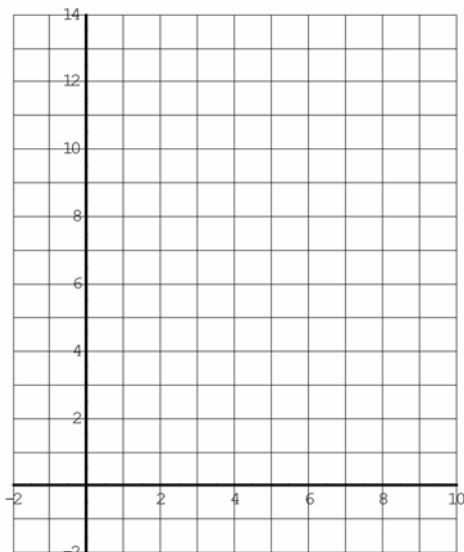


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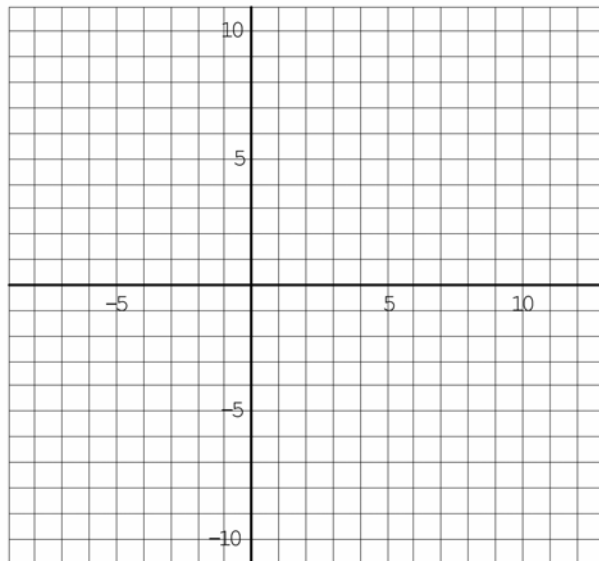
PART I - CIRCLES

Questions: For each of the following equations, draw the graph and state domain & range:

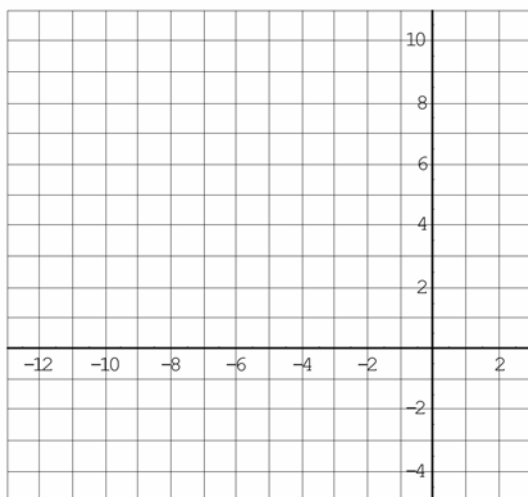
5. $(x - 4)^2 + (y - 6)^2 = 16$



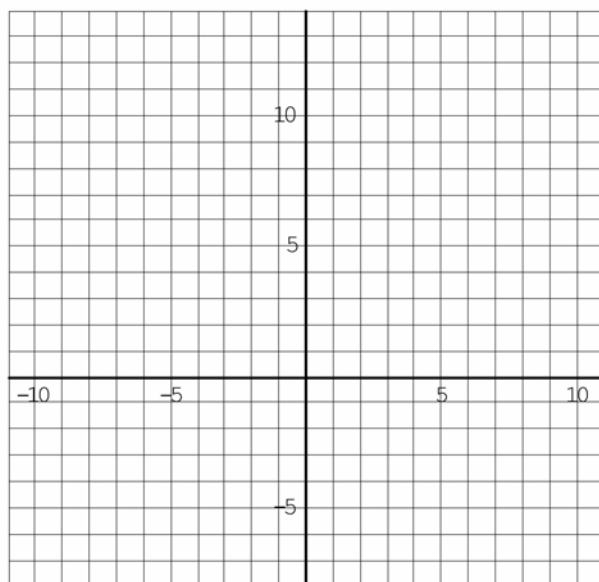
6. $(x - 2)^2 + y^2 = 64$



7. $(x + 5)^2 + (y - 3)^2 = 49$



8. $x^2 + (y - 3)^2 = 100$



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PART I - CIRCLES

Answers:

1. $(x - 3)^2 + (y - 4)^2 = 9$

Domain: $0 \leq x \leq 6$

Range: $1 \leq y \leq 7$

2. $(x + 2)^2 + y^2 = 49$

Domain: $-9 \leq x \leq 5$

Range: $-7 \leq y \leq 7$

3. $x^2 + (y + 5)^2 = 36$

Domain: $-6 \leq x \leq 6$

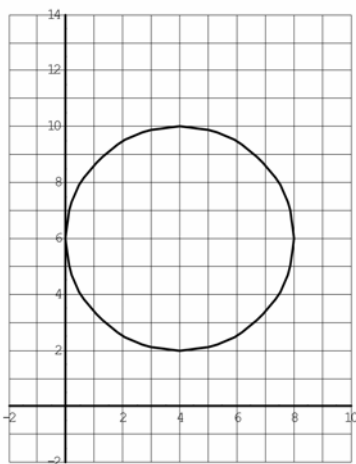
Range: $-11 \leq y \leq 1$

4. $(x - 5)^2 + (y + 2)^2 = 16$

Domain: $1 \leq x \leq 9$

Range: $-6 \leq y \leq 2$

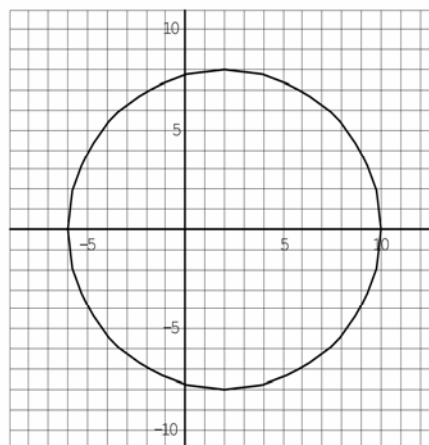
5.



Domain:
 $0 \leq x \leq 8$

Range:
 $2 \leq y \leq 10$

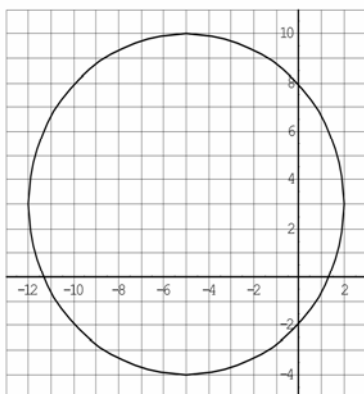
6.



Domain:
 $-6 \leq x \leq 10$

Range:
 $-8 \leq y \leq 8$

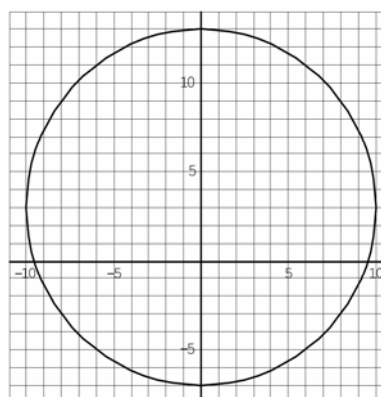
7.



Domain:
 $-12 \leq x \leq 2$

Range:
 $-4 \leq y \leq 10$

8.



Domain:
 $-10 \leq x \leq 10$

Range:
 $-7 \leq y \leq 13$

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PART II- ELLIPSES

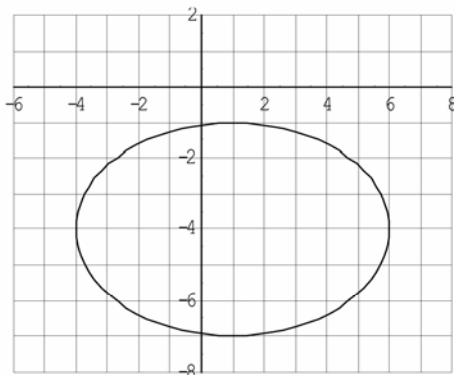
Ellipses: The equation of an ellipse is given by $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$,

(h, k) is the centre of the ellipse.

“ a ” represents the horizontal distance from the centre to the edge of the ellipse.

“ b ” represents the vertical distance from the centre to the edge of the ellipse.

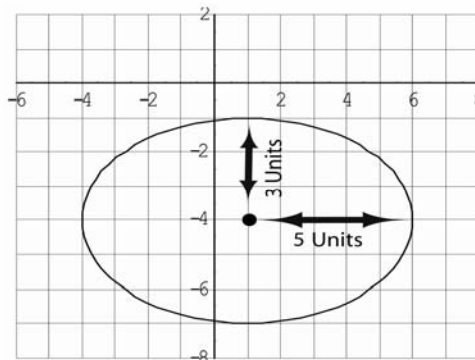
Example 1: Given the following graph, find the equation of the ellipse:



First identify the centre of the ellipse, which in this case is $(1, -4)$.

To find the a -value, count horizontally from the centre to the right edge and you will get 5.

To find the b -value, count vertically from the centre to the upper edge, and you will get 3.



When you put the a & b values into the equation, remember to square them!

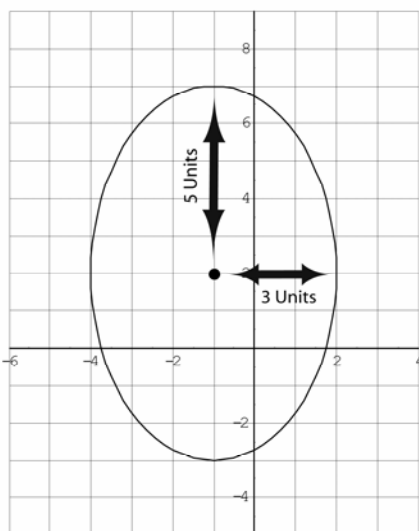
$$\frac{(x-1)^2}{25} + \frac{(y+4)^2}{9} = 1$$

Example 2: Sketch the graph of $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{25} = 1$

Place a point at the centre of the ellipse $(-1, 2)$.

The a -value is $\sqrt{9} = 3$

The b -value is $\sqrt{25} = 5$



Quick Tip: What happens when both a and b are the same number? This will give you a circle. When writing the equation of an ellipse that is really a circle, you should use a circle equation instead.

Don't write $\frac{x^2}{9} + \frac{y^2}{9} = 1$

Write $x^2 + y^2 = 9$

When a^2 is bigger
(the number under x)
the ellipse is horizontal.

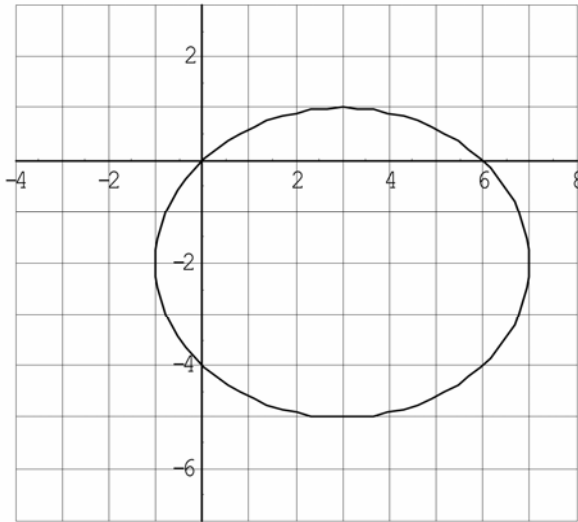
When b^2 is bigger.
(the number under y).
the ellipse is vertical

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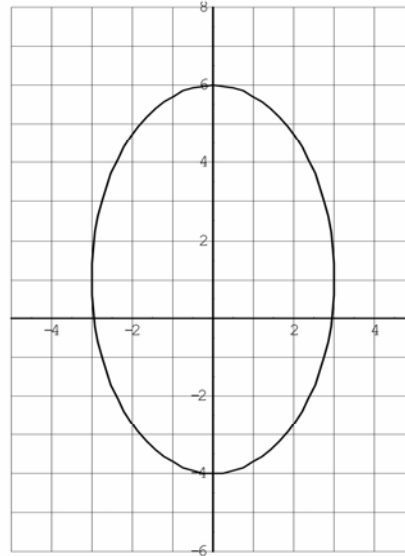
PART II - ELLIPSES

Questions: Given the following graphs, write the equation.

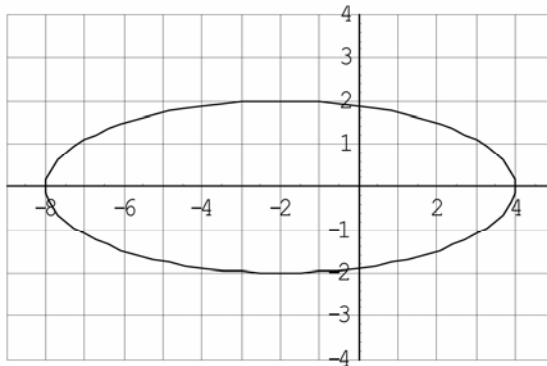
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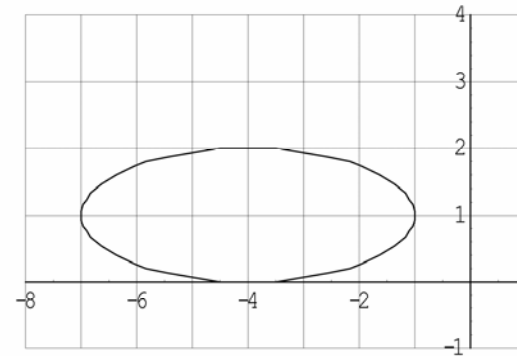
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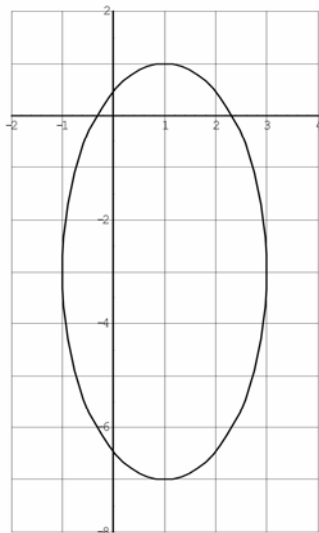
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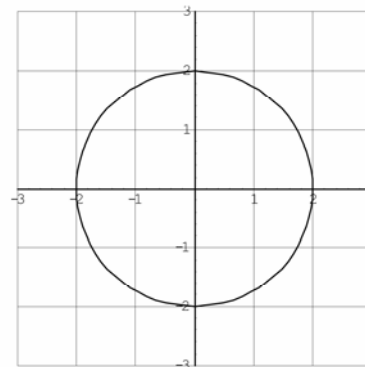
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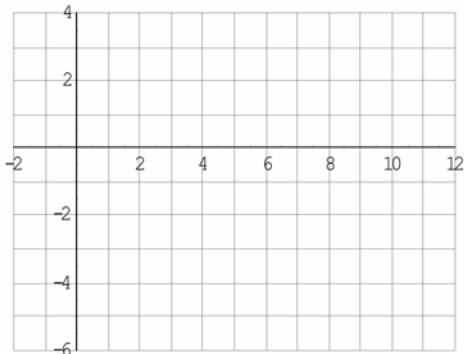


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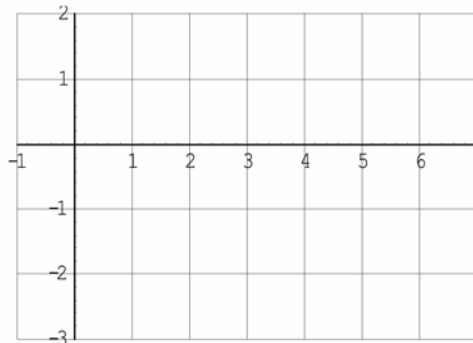
PART II- ELLIPSES

Questions: Given the following equations, sketch the graph.

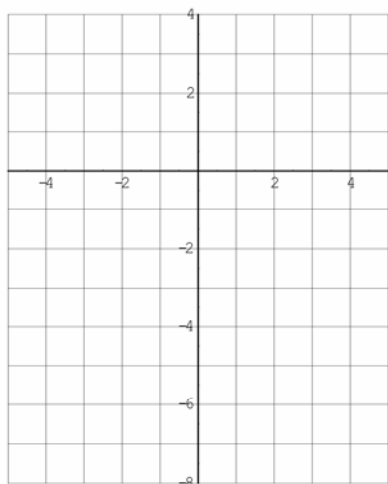
7. $\frac{(x-5)^2}{9} + \frac{(y+1)^2}{16} = 1$



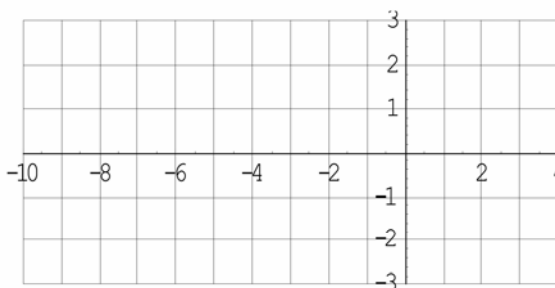
8. $\frac{(x-3)^2}{4} + (y+1)^2 = 1$



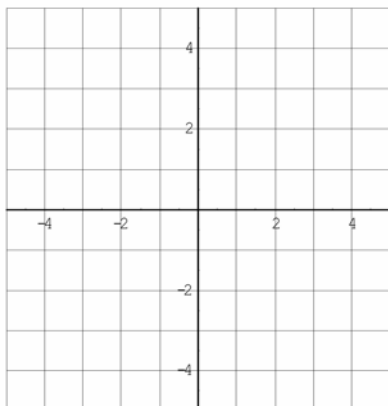
9. $\frac{x^2}{16} + \frac{(y+2)^2}{25} = 1$



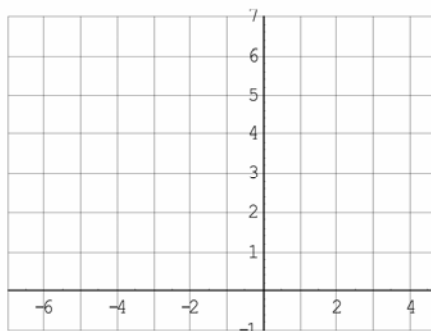
10. $\frac{(x+3)^2}{36} + \frac{y^2}{4} = 1$



11. $\frac{x^2}{16} + \frac{y^2}{16} = 1$



12. $\frac{(x+1)^2}{25} + \frac{(y-3)^2}{9} = 1$

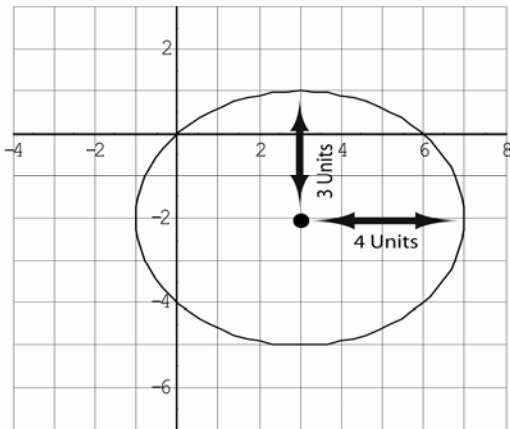


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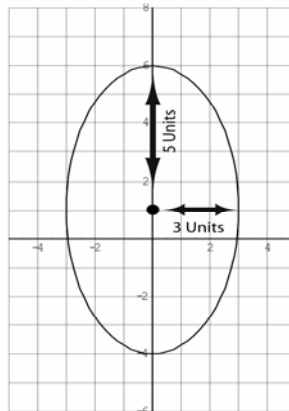
PART II- ELLIPSES

Answers:

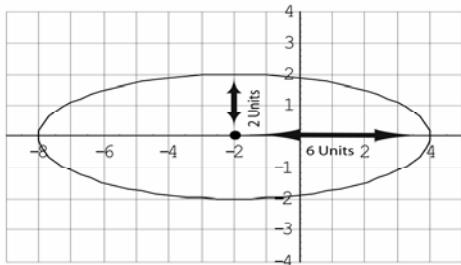
1. $\frac{(x-3)^2}{16} + \frac{(y+2)^2}{9} = 1$



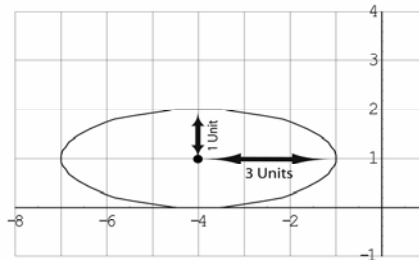
2. $\frac{x^2}{9} + \frac{(y-1)^2}{25} = 1$



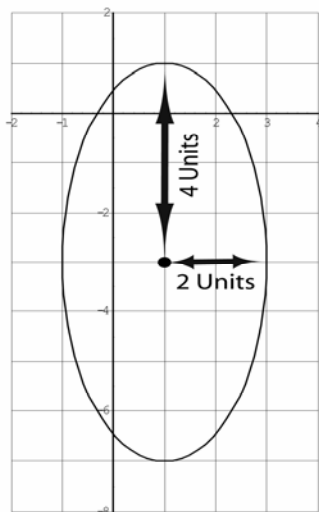
3. $\frac{(x+2)^2}{36} + \frac{y^2}{4} = 1$



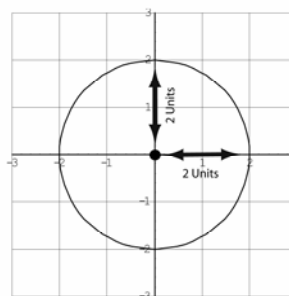
4. $\frac{(x+4)^2}{9} + (y-1)^2 = 1$



5. $\frac{(x-1)^2}{4} + \frac{(y+3)^2}{16} = 1$



6. $\frac{x^2}{4} + \frac{y^2}{4} = 1 \rightarrow x^2 + y^2 = 4$

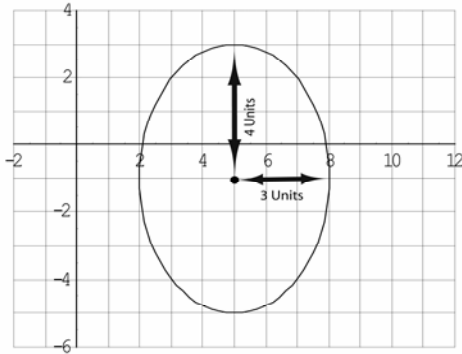


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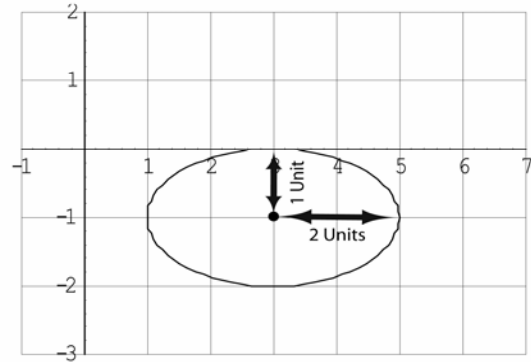
PART II- ELLIPSES

Answers:

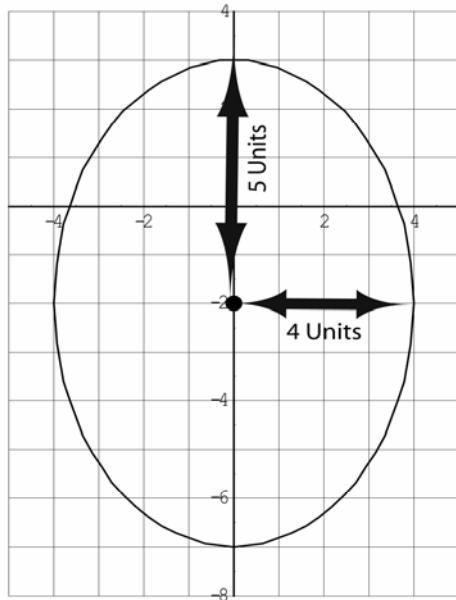
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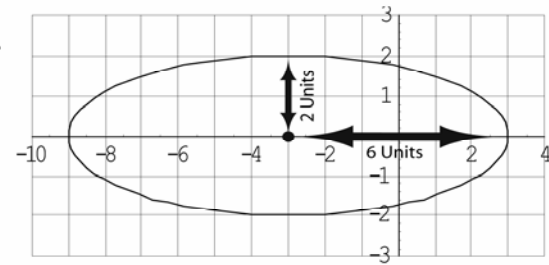
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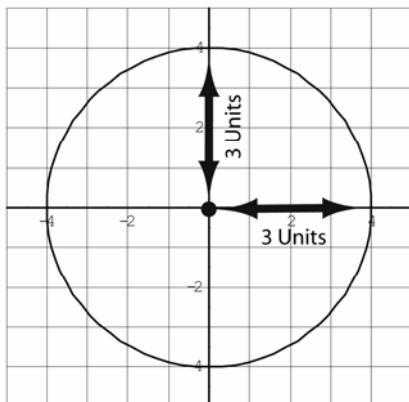
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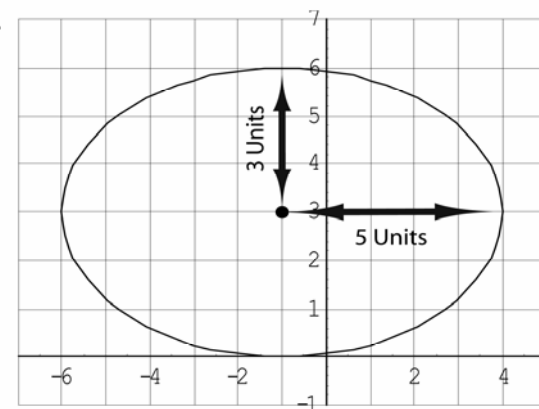
10.



11.



12.



CONICS LESSON 2

PART III - PARABOLAS

Parabolas: There are two different standard form equations for parabolas.

Vertical parabolas are given by: $y - k = a(x - h)^2$. "a" is the vertical stretch factor

(Vertical parabolas that open down have a negative sign with the a-value, those opening up have a positive sign.)

Horizontal parabolas are given by: $x - h = a(y - k)^2$. "a" is the horizontal stretch factor.

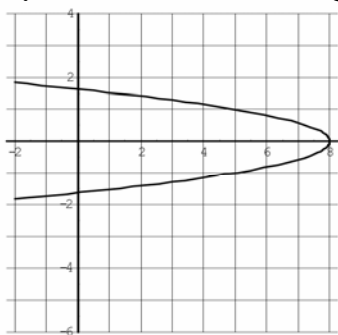
(Horizontal parabolas that open left have a negative sign with the a-value, those opening right have a positive sign.)

(h, k) is the **vertex** of the parabola.

Try to remember the following rules when it comes to standard form parabolas:

If you have an x^2 , but no $y^2 \rightarrow$ vertical parabola.
If you have a y^2 , but no $x^2 \rightarrow$ horizontal parabola.

Example 1: Given the following graph, write the equation.



First note the coordinates of the vertex: (8,0). This gives you h & k

To obtain the a-value, find another point on the parabola.

By inspection, the point (5, 1) lies on the graph.

This can now be plugged in for x & y.

Take the values above and insert them into the standard form of a **horizontal** parabola:

$$x - h = a(y - k)^2$$

$$5 - 8 = a(1 - 0)^2$$

$$-3 = a$$

To obtain the final equation, plug in numbers for a, h, & k, leaving x & y as variables.

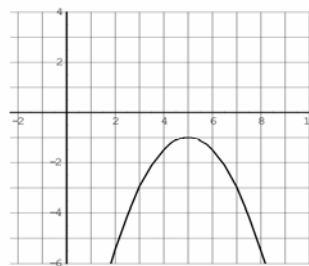
$$x - 8 = -3y^2$$

Example 2: Sketch the graph of $y + 1 = -\frac{1}{2}(x - 5)^2$

The vertex is located at the point (5,-1), and it's a upside down vertical parabola.

When given a parabola equation, it may be graphed in your calculator by isolating y:

$$y = -\frac{1}{2}(x - 5)^2 - 1$$



Example 3: Sketch the graph of $y + 4 = \frac{1}{4}(x + 2)^2$

The vertex is located at the point (-2,-4), and it's a right-side up vertical parabola.

This time, graph the parabola using x & y intercepts instead of the calculator. (The x & y intercept method is being used in this example to illustrate an alternative to using your graphing calculator.)

x - intercepts:

Set y = 0, then solve for x.

$$y + 4 = \frac{1}{4}(x + 2)^2$$

$$0 + 4 = \frac{1}{4}(x + 2)^2$$

$$4 = \frac{1}{4}(x + 2)^2$$

$$16 = (x + 2)^2$$

$$\pm 4 = x + 2$$

$$x = -6, 2$$

y - intercept:

Set x = 0, then solve for y.

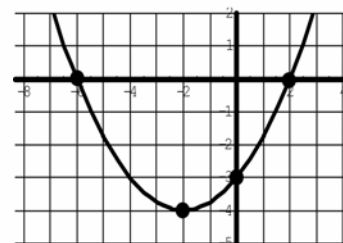
$$y + 4 = \frac{1}{4}(x + 2)^2$$

$$y + 4 = \frac{1}{4}(0 + 2)^2$$

$$y + 4 = \frac{1}{4}(4)$$

$$y + 4 = 1$$

$$y = -3$$

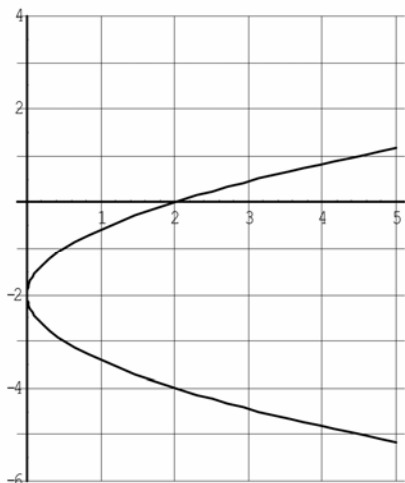


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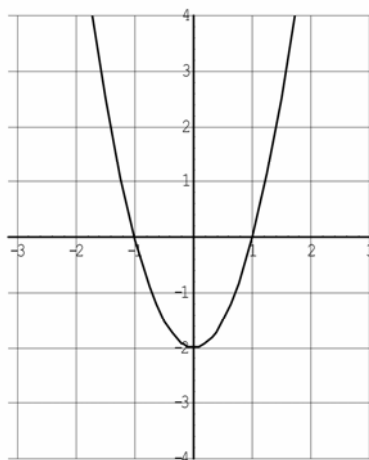
PART III - PARABOLAS

Questions: Given the following graphs, write the equation.

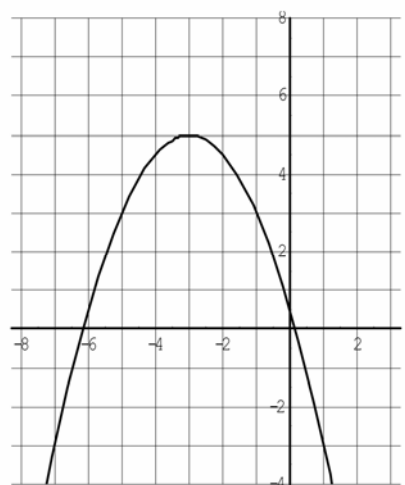
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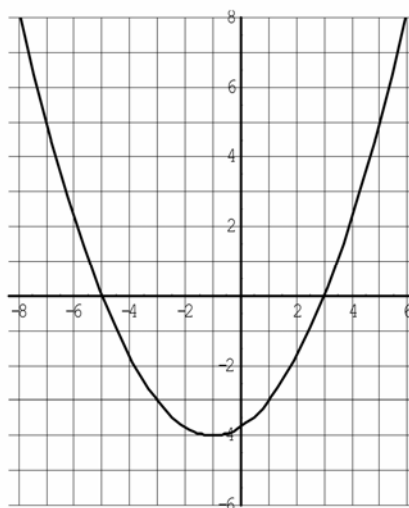
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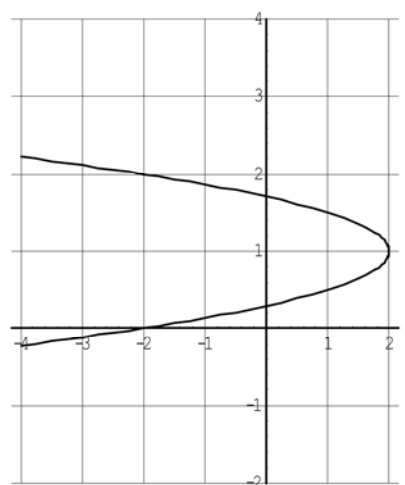
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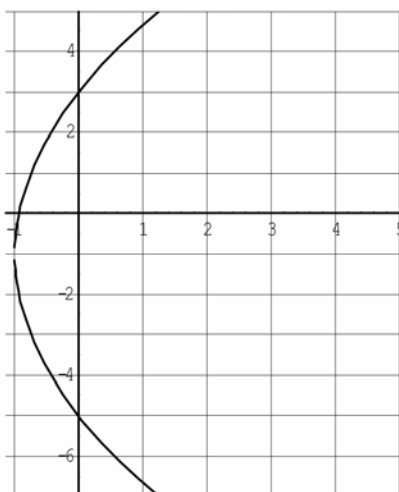
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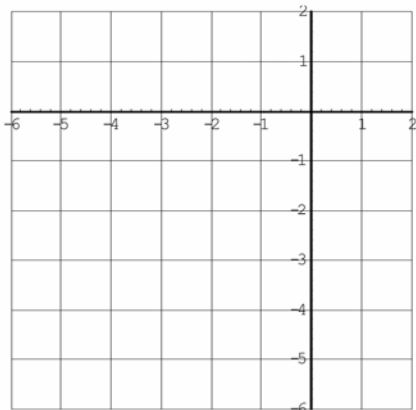


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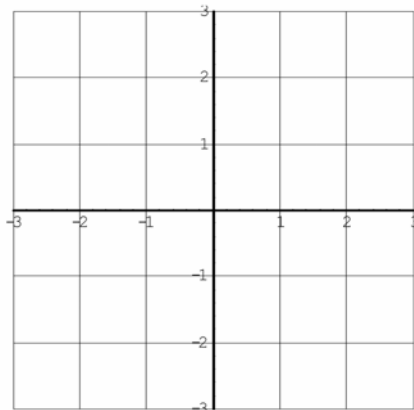
PART III - PARABOLAS

Questions: Isolate y and then sketch the graph:

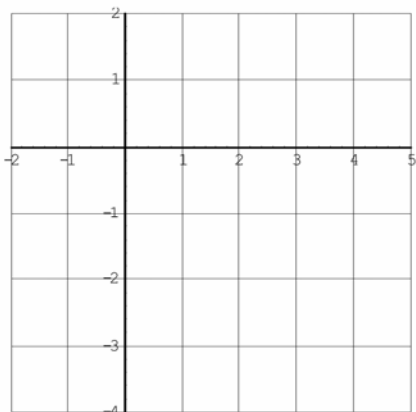
7. $x = -(y + 2)^2$



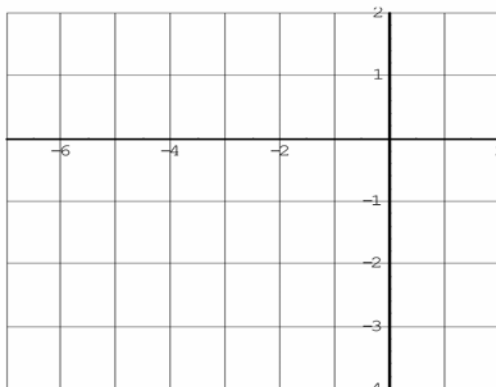
8. $y + 2 = 3x^2$



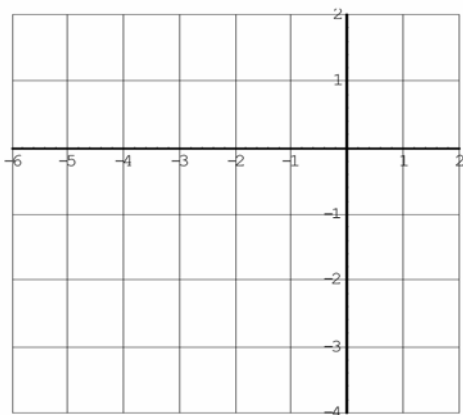
9. $x - 2 = \frac{1}{2}(y + 1)^2$



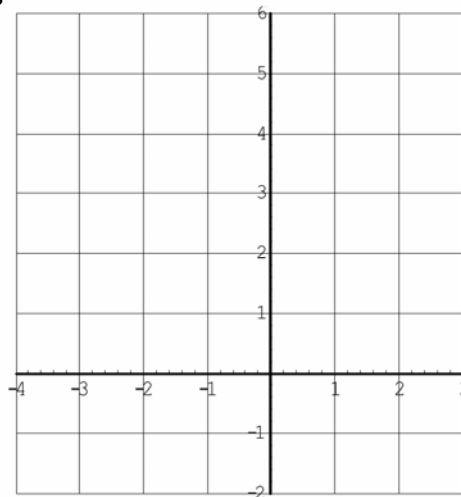
10. $y - 1 = -\frac{1}{2}(x + 3)^2$



11. $y = -(x + 2)^2$



12. $x + 3 = 2(y - 3)^2$



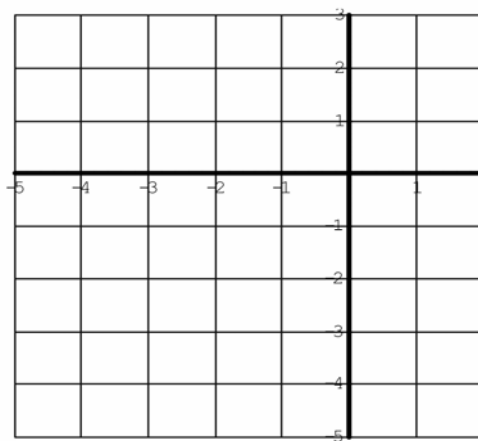
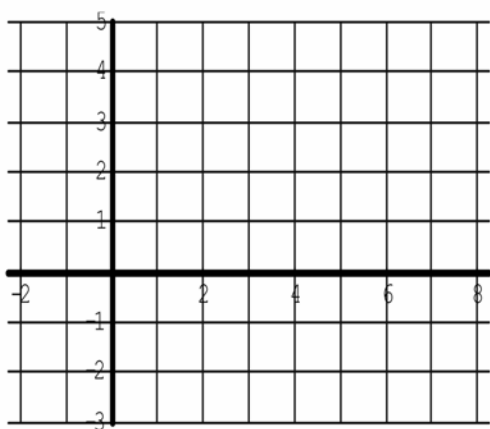
CONICS LESSON 2

PART III - PARABOLAS

Questions: Using x & y intercepts, graph the following parabolas

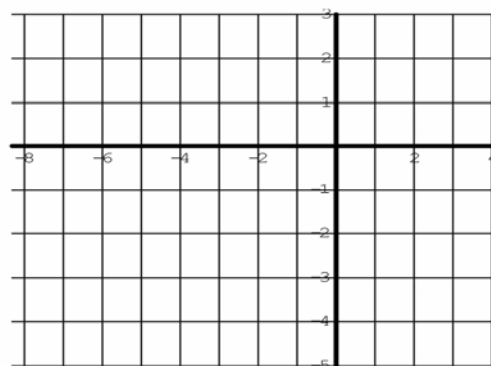
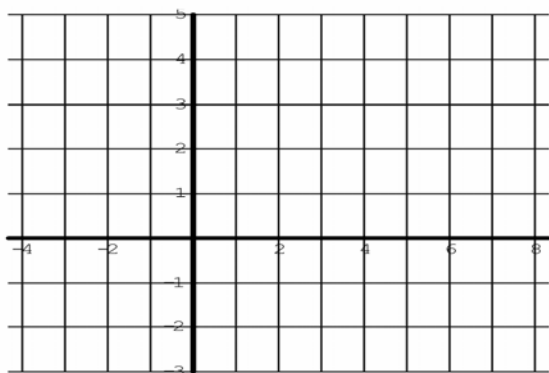
13. $y + 2 = \frac{1}{2}(x - 3)^2$

14. $x + 4 = (y + 1)^2$



15. $x + 4 = (y - 1)^2$

16. $y + 4 = \frac{1}{4}(x + 2)^2$



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PART III - PARABOLAS

Answers:

1. The vertex is at (0, -2)

A point is (2, 0)

$$x - h = a(y - k)^2$$

$$2 - 0 = a(0 - (-2))^2$$

$$2 = 4a$$

$$a = \frac{1}{2}$$

$$x = \frac{1}{2}(y + 2)^2$$

2. The vertex is at (0, -2)

A point is (1, 0)

$$y - k = a(x - h)^2$$

$$0 - (-2) = a(1 - 0)^2$$

$$2 = a$$

$$y + 2 = 2x^2$$

3. The vertex is at (-3, 5)

A point is (1, -3)

$$y - k = a(x - h)^2$$

$$-3 - 5 = a(1 - (-3))^2$$

$$-8 = a(4)^2$$

$$\frac{-8}{16} = a$$

$$a = -\frac{1}{2}$$

$$y - 5 = -\frac{1}{2}(x + 3)^2$$

4. The vertex is at (-1, -4)

A point is (3, 0)

$$y - k = a(x - h)^2$$

$$0 - (-4) = a(3 - (-1))^2$$

$$4 = a(4)^2$$

$$\frac{4}{16} = a$$

$$a = \frac{1}{4}$$

$$y + 4 = \frac{1}{4}(x + 1)^2$$

5. The vertex is at (2, 1)

A point is (-2, 0)

$$x - h = a(y - k)^2$$

$$-2 - 2 = a(0 - 1)^2$$

$$-4 = a$$

$$x - 2 = -4(y - 1)^2$$

6. The vertex is at (-1, -1)

A point is (0, 3)

$$x - h = a(y - k)^2$$

$$0 - (-1) = a(3 - (-1))^2$$

$$1 = a(4)^2$$

$$a = \frac{1}{16}$$

$$x + 1 = \frac{1}{16}(y + 1)^2$$

CONICS LESSON 2

PART III - PARABOLAS

Answers:

7. $y = \pm\sqrt{-x-2}$

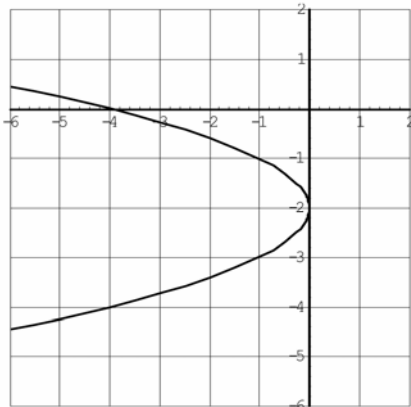
$$x = -(y+2)^2$$

$$-x = (y+2)^2$$

$$\sqrt{-x} = \sqrt{(y+2)^2}$$

$$\pm\sqrt{-x} = y+2$$

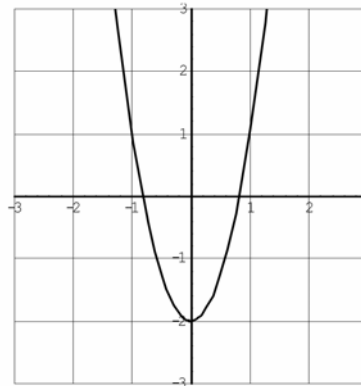
$$y = \pm\sqrt{-x-2}$$



8. $y = 3x^2 - 2$

$$y+2 = 3x^2$$

$$y = 3x^2 - 2$$



9. $y = \pm\sqrt{2(x-2)} - 1$

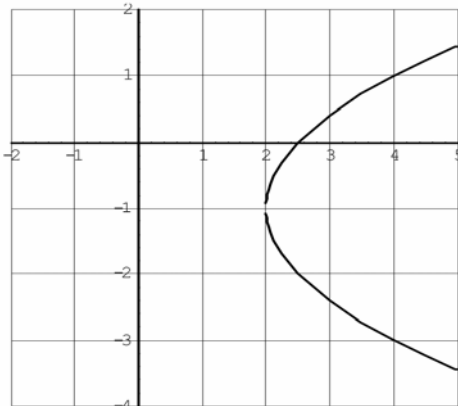
$$x-2 = \frac{1}{2}(y+1)^2$$

$$2(x-2) = (y+1)^2$$

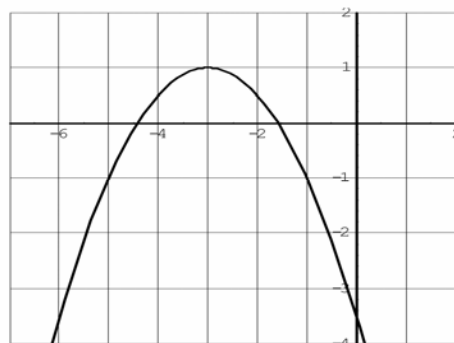
$$\sqrt{2(x-2)} = \sqrt{(y+1)^2}$$

$$\pm\sqrt{2(x-2)} = y+1$$

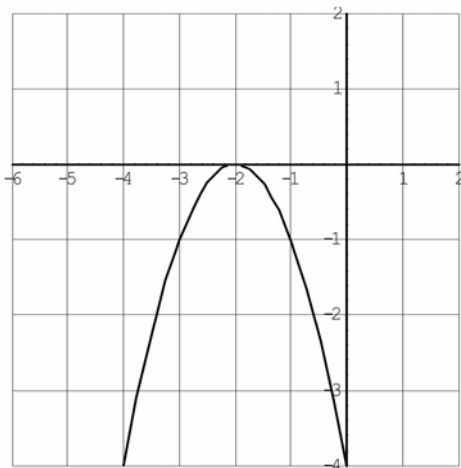
$$y = \pm\sqrt{2(x-2)} - 1$$



10. $y = -\frac{1}{2}(x+3)^2 + 1$



11. $y = -(x+2)^2$



12. $y = \pm\sqrt{\left(\frac{x+3}{2}\right)} + 3$

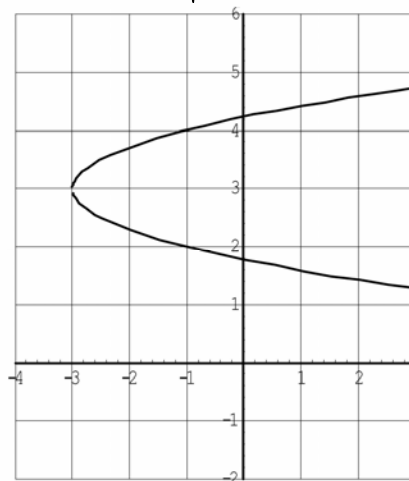
$$x+3 = 2(y-3)^2$$

$$\frac{x+3}{2} = (y-3)^2$$

$$\sqrt{\frac{x+3}{2}} = \sqrt{(y-3)^2}$$

$$\pm\sqrt{\frac{x+3}{2}} = y-3$$

$$y = \pm\sqrt{\frac{x+3}{2}} + 3$$



CONICS LESSON 2

PART III - PARABOLAS

Answers:

13.

x-intercepts

$$y + 2 = \frac{1}{2}(x - 3)^2$$

Vertex

(3, -2)

$$0 + 2 = \frac{1}{2}(x - 3)^2$$

$$4 = (x - 3)^2$$

$$\pm 2 = x - 3$$

$$x = 1, 5$$

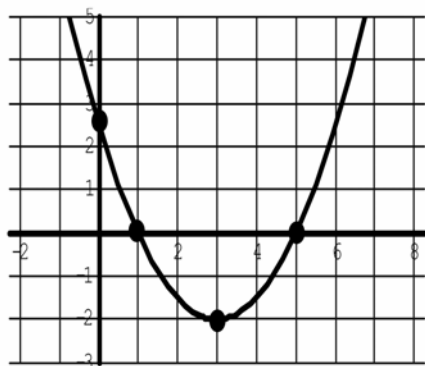
y-intercept

$$y + 2 = \frac{1}{2}(x - 3)^2$$

$$y + 2 = \frac{1}{2}(0 - 3)^2$$

$$y + 2 = \frac{9}{2}$$

$$y = \frac{5}{2} = 2.5$$



14.

Vertex

(-4, -1)

x-intercept:

$$x + 4 = (y + 1)^2$$

$$x + 4 = (0 + 1)^2$$

$$x + 4 = 1$$

$$x = -3$$

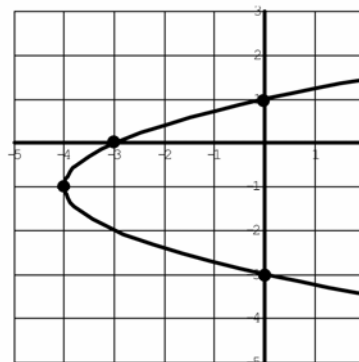
y-intercepts

$$x + 4 = (y + 1)^2$$

$$0 + 4 = (y + 1)^2$$

$$\pm 2 = y + 1$$

$$y = -3, 1$$



Vertex

(-2, -4)

15.

Vertex

(-4, 1)

x-intercept

$$x + 4 = (y - 1)^2$$

$$x + 4 = (0 - 1)^2$$

$$x + 4 = 1$$

$$x = -3$$

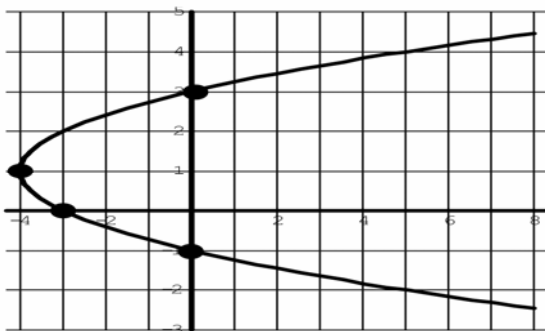
y-intercepts

$$x + 4 = (y - 1)^2$$

$$0 + 4 = (y - 1)^2$$

$$\pm 2 = y - 1$$

$$y = -1, 3$$



16.

x-intercepts

$$y + 4 = \frac{1}{4}(x + 2)^2$$

$$0 + 4 = \frac{1}{4}(x + 2)^2$$

$$16 = (x + 2)^2$$

$$\pm 4 = x + 2$$

$$x = -6, 2$$

y-intercept

$$y + 4 = \frac{1}{4}(x + 2)^2$$

$$y + 4 = \frac{1}{4}(0 + 2)^2$$

$$y + 4 = \frac{1}{4}(4)$$

$$y + 4 = 1$$

$$y = -3$$

