

Pure Math 30:

# CONICS



## LESSON 3

Hyperbolas

Pure Math  
30:

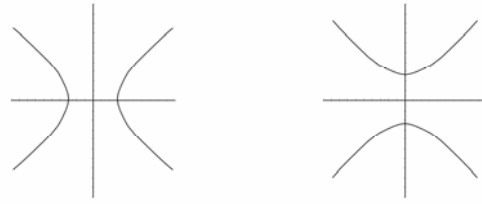
**EXPLAINED!**

By  
Barry  
Mabillard

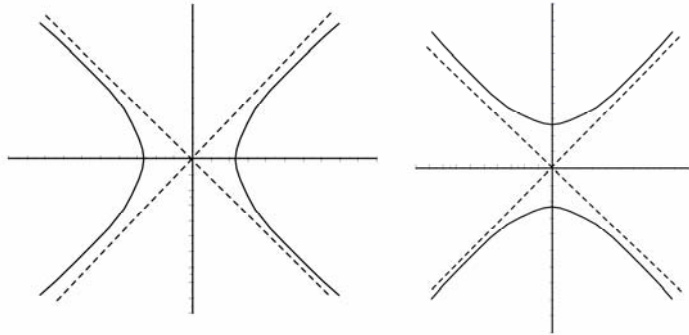
# CONICS LESSON 3

## PART I - ELEMENTS OF HYPERBOLAS

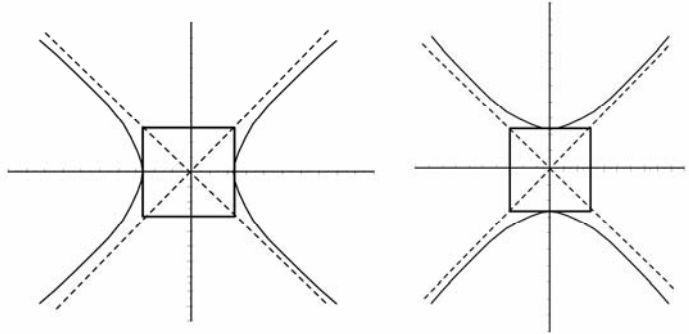
**Hyperbolas:** The shapes you see to the right are called hyperbolas. In this course, you will study horizontal and vertical hyperbolas.



Hyperbolas have *slant asymptotes*. These are shown by the dashed lines. They always intersect at the centre of the hyperbola. The tips of the hyperbola arms are called *vertices*.



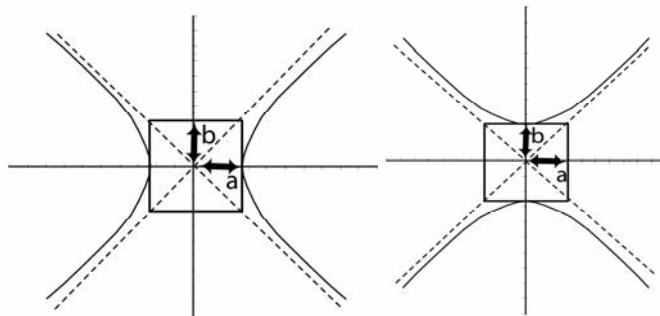
To properly read the hyperbola graph, draw in a reference box. Make sure the box touches both vertices, and the corners will always intersect the asymptotes.



**The following is true for both orientations:**

The a-value is the horizontal distance from the centre to the edge of the box.

The b-value is the vertical distance from the centre to the edge of the box.

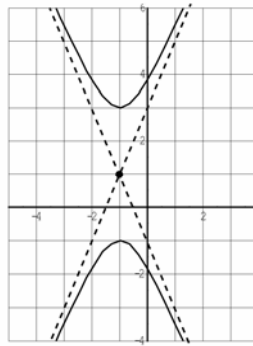


# CONICS LESSON 3

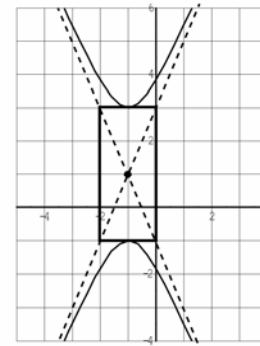
## PART I - ELEMENTS OF HYPERBOLAS

**Example 1:** Given the following graph, find:

- Coordinates of the centre
- Coordinates of the vertices
- The  $a$  and  $b$  values
- The domain and range.
- Distance between vertices



The first thing you should do is draw in the reference box.

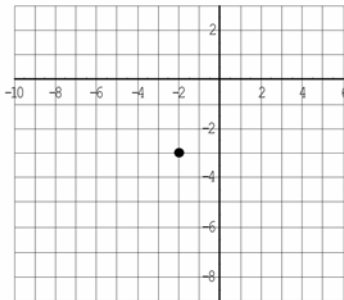


- From the graph, we can read off the coordinates of the centre. They are  $(-1, 1)$
- The upper vertex is located at  $(-1, 3)$ . The lower vertex is located at  $(-1, -1)$
- The  $a$ -value of the box is 1 (horizontal distance from centre to edge).  
The  $b$ -value of the box is 2 (vertical distance from centre to edge.)
- The domain is  $x \in \mathbb{R}$ , and the range is:  $\{y \leq -1, y \geq 3\}$ .
- The vertical distance between vertices is 4.

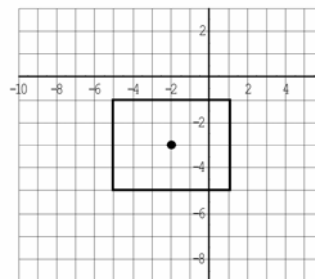
The distance between vertices is always double the distance from a vertex to the centre.

**Example 2:** If the centre is  $(-2, -3)$ ,  $a = 3$ , and  $b = 2$ , sketch the graph of the horizontal hyperbola.

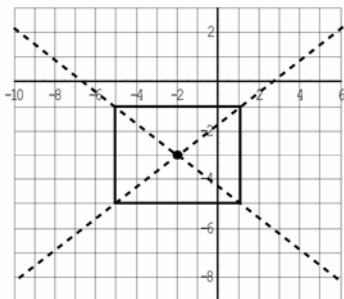
**Step 1:** Draw a dot for your centre point.



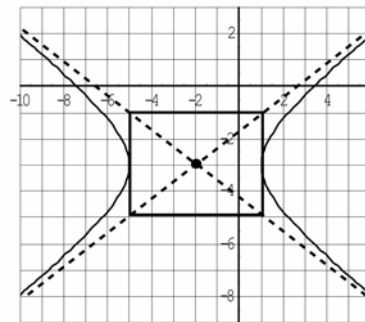
**Step 2:** We know  $a = 3$  and  $b = 2$ . Draw in the box with these values.



**Step 3:** Draw in your slant asymptotes. (Remember that they always pass through the corners.)



**Step 4:** Complete the hyperbola. (The vertices will touch the box)



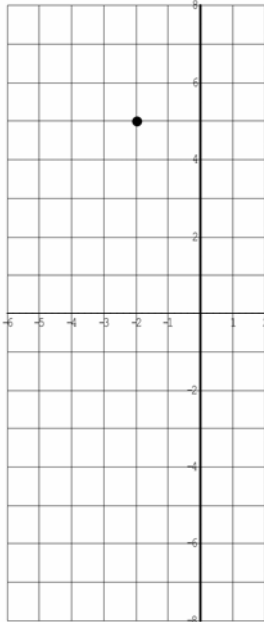
# CONICS LESSON 3

## PART I - ELEMENTS OF HYPERBOLAS

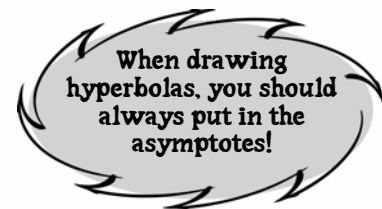
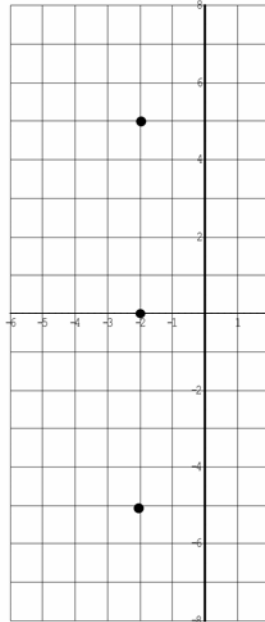
**Example 3:** A vertical hyperbola has an upper vertex at  $(-2, 5)$ ,  $a = 3$ , and  $b = 5$ .

**Sketch the graph.**

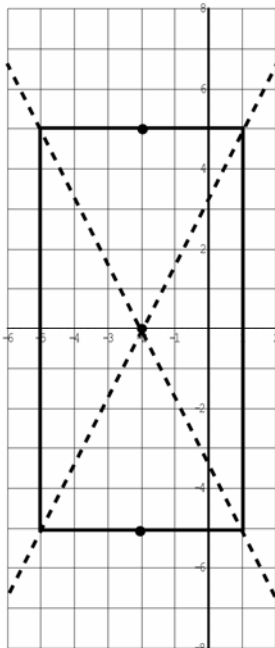
**Step 1:** Draw a point at the vertex given.



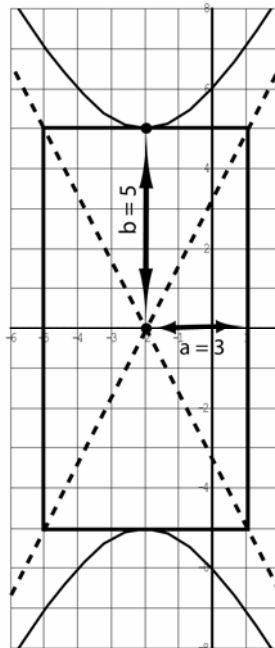
**Step 2:** We know that  $a = 3$  and  $b = 5$ . Since  $b$  is the vertical, we know it is 5 units down from the vertex to the centre, then another 5 units down to the lower vertex. The lower vertex is at  $(-2, -5)$



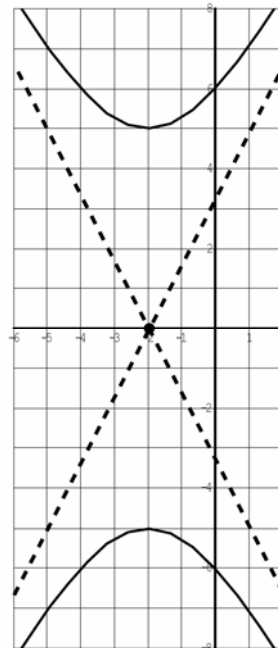
**Step 3:** Draw in the reference box and the asymptotes.



**Step 4:** Check to make sure the information is all correct.



**Step 5 (Optional):** Erase the excess information so you're left with just the graph.



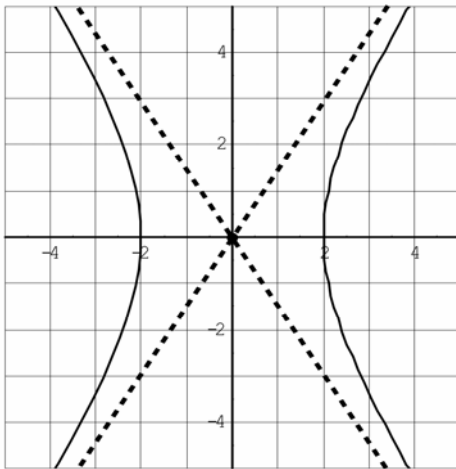
# CONICS LESSON 3

## PART I - ELEMENTS OF HYPERBOLAS

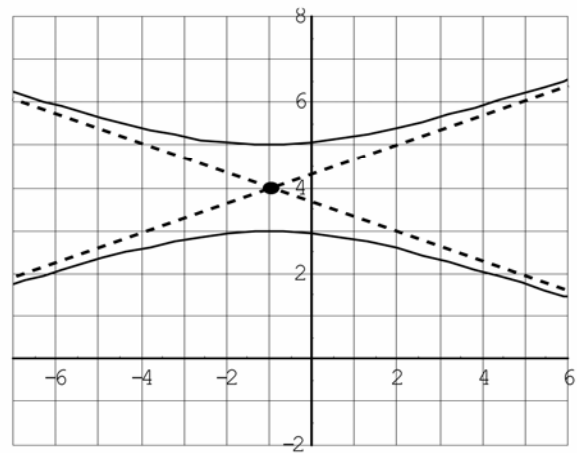
**Questions:** For each of the following hyperbolas, state:

- a) **Coordinates of the centre**
- b) **Coordinates of the vertices**
- c) **The  $a$  and  $b$  values**
- d) **The domain and range.**
- e) **Distance between vertices**

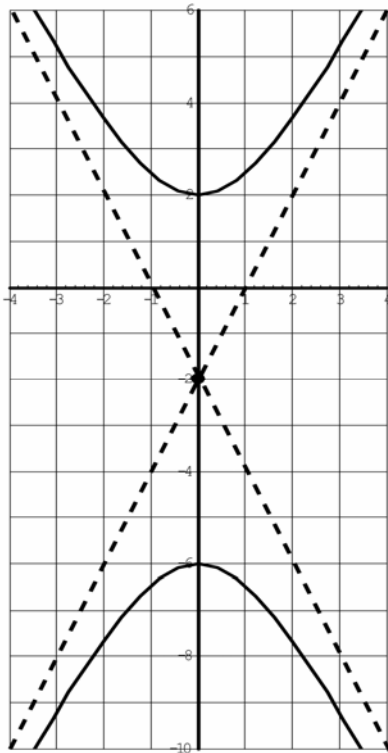
**1.**



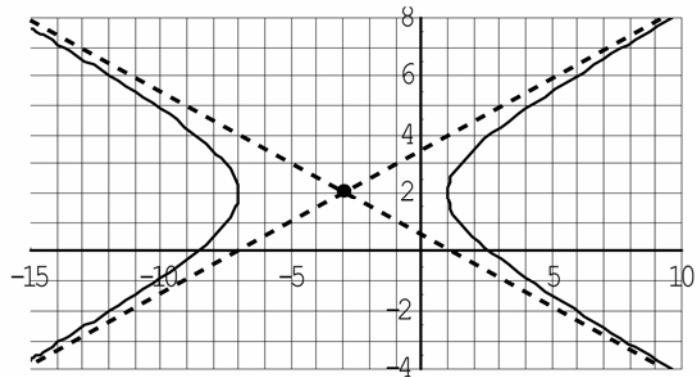
**2.**



**3.**



**4.**



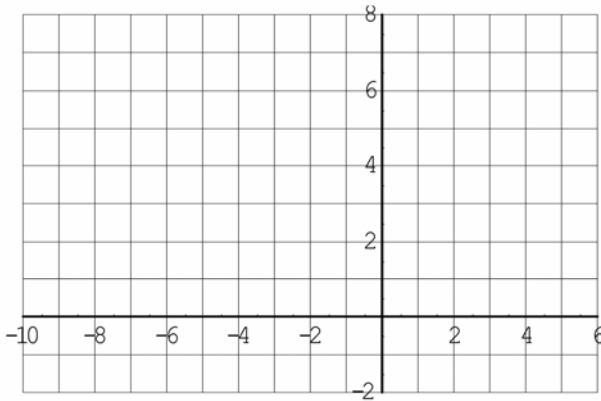
**PURE MATH 30: EXPLAINED!**

# CONICS LESSON 3

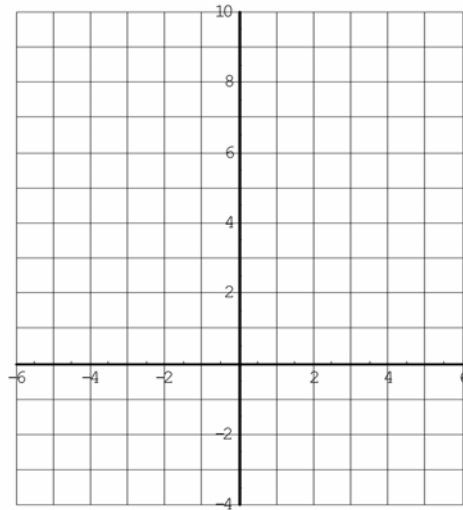
## PART I - ELEMENTS OF HYPERBOLAS

**Questions:** For each of the following, sketch the hyperbola:

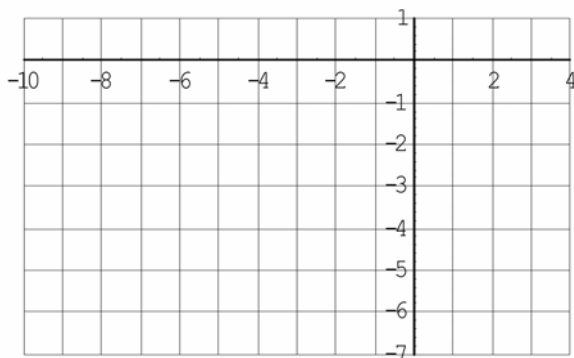
5) A horizontal hyperbola has a centre at  $(-2, 4)$ , with  $a = 2$ , and  $b = 1$ .



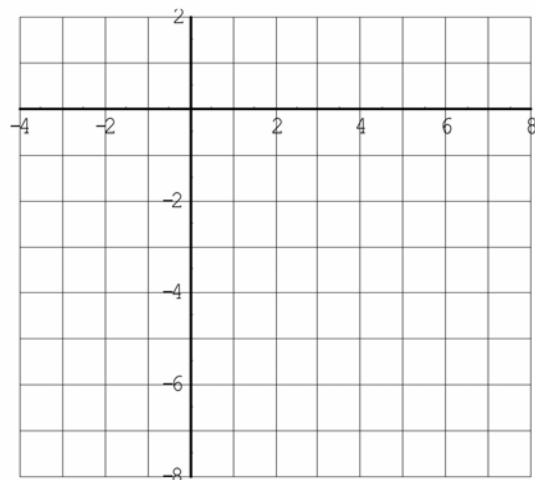
6) A vertical hyperbola has a centre at  $(0, 3)$ , with  $a = 3$ , and  $b = 4$ .



7) A horizontal hyperbola has a centre at  $(-3, -3)$ , with  $a = 5$ , and  $b = 2$ .



8) A vertical hyperbola has a centre at  $(2, -3)$ , with  $a = 1$ , and  $b = 1$ .

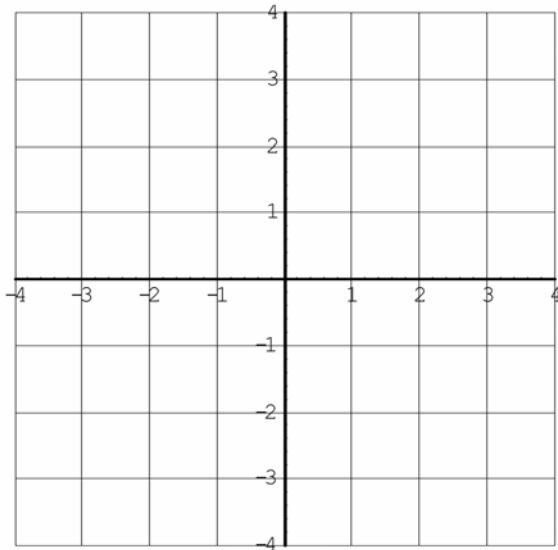


# CONICS LESSON 3

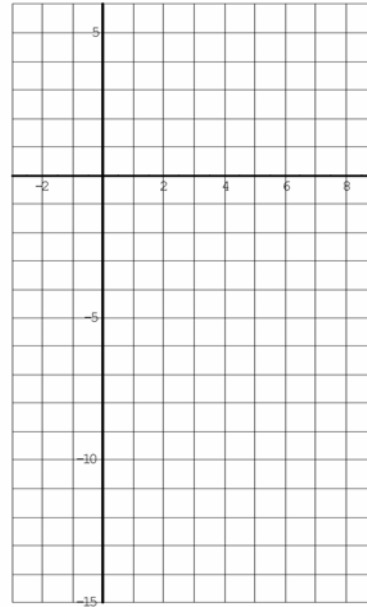
## PART I - ELEMENTS OF HYPERBOLAS

**Questions:** For each of the following, sketch the hyperbola:

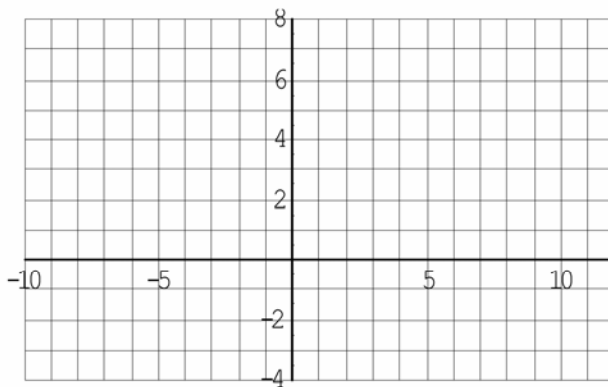
9) A horizontal hyperbola has a left vertex at  $(-2, 0)$ , with  $a = 2$ , and  $b = 1$ .



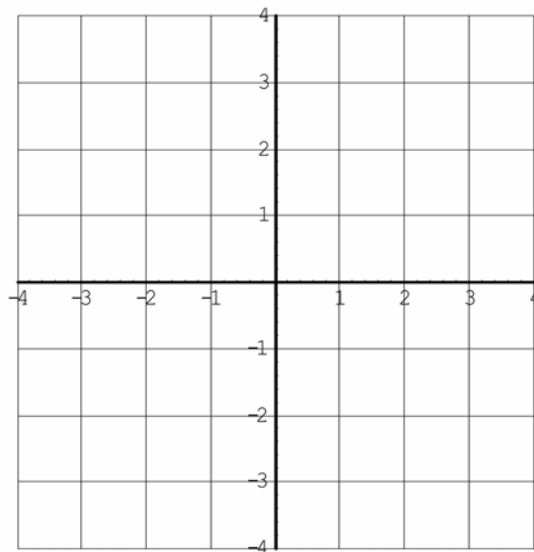
10) A vertical hyperbola has an upper vertex at  $(3, 0)$ , with  $a = 2$ , and  $b = 5$ .



11) A horizontal hyperbola has a left vertex at  $(-4, 2)$ , with  $a = 5$ , and  $b = 3$ .



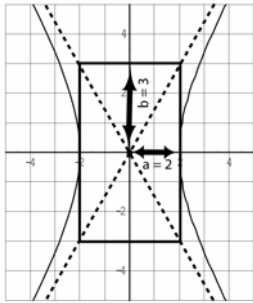
12) A vertical hyperbola has a lower vertex at  $(0, -2)$ , with  $a = 1$ , and  $b = 2$ .



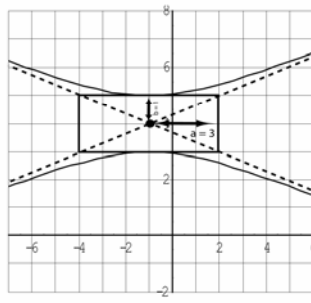
# CONICS LESSON 3

## PART I - ELEMENTS OF HYPERBOLAS

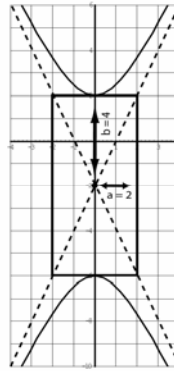
### Answers:



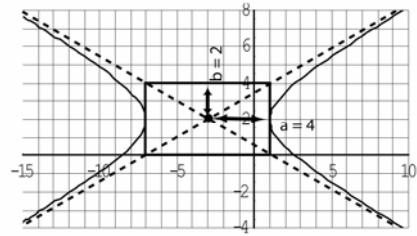
1. a) (0,0)  
b) (-2,0) & (2,0)  
c)  $a = 2$ ,  $b = 3$   
d) Domain:  $\{x \leq -2, x \geq 2\}$   
Range:  $y \in \mathbb{R}$   
e) 4 units.



2. a) (-1,4)  
b) (-1,5) & (-1,3)  
c)  $a = 3$ ,  $b = 1$   
d) Domain:  $x \in \mathbb{R}$   
Range:  $\{y \leq 3, y \geq 5\}$   
e) 2 units.

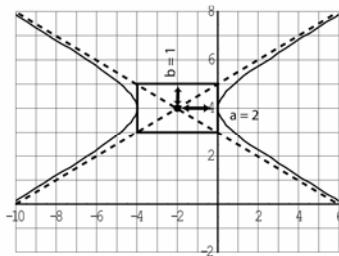


3. a) (0,-2)  
b) (0,2) & (0,-6)  
c)  $a = 2$ ,  $b = 4$   
d) Domain:  $x \in \mathbb{R}$   
Range:  $\{y \leq -6, y \geq 2\}$   
e) 8 units.

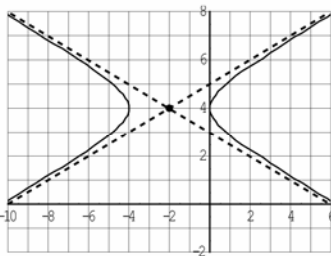


4. a) (-3,2)  
b) (-7,2) & (1,2)  
c)  $a = 4$ ,  $b = 2$   
d) Domain:  $\{x \leq -7, x \geq 1\}$   
Range:  $y \in \mathbb{R}$   
e) 8 units.

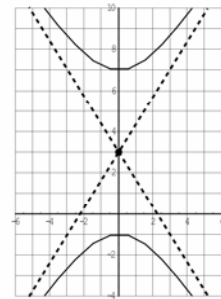
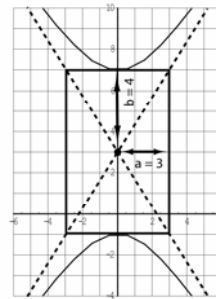
5. First draw in the box ← necessary information



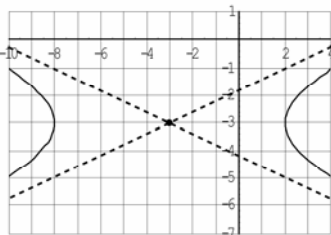
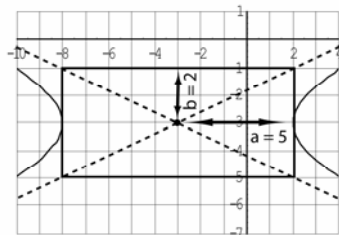
Then erase it so you're left with the hyperbola.



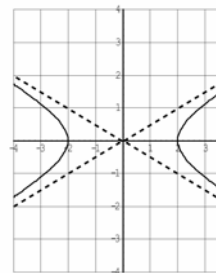
6.



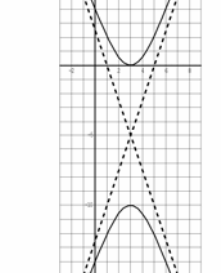
7.



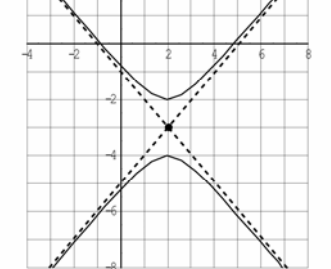
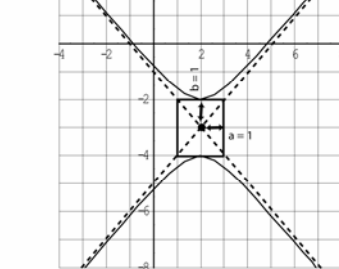
9.



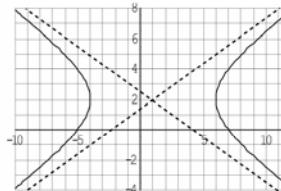
10.



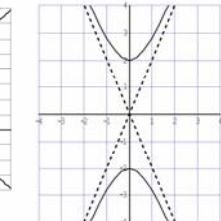
8.



11.



12.



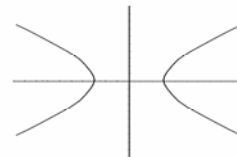


# CONICS LESSON 3

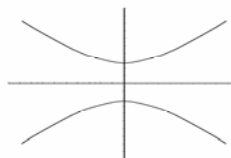
## PART II - HYPERBOLA EQUATIONS

**Equations of Hyperbolas:** The following are standard form equations for hyperbolas:

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$  represents a horizontal hyperbola.



$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = -1$  represents a vertical hyperbola.



(h, k) are the coordinates of the centre.

a & b are the reference box values.

Note that in these equations, a & b are squared!

- When placing these values into an equation, you must square them.
- When reading a & b off the equation, you will need to square root!

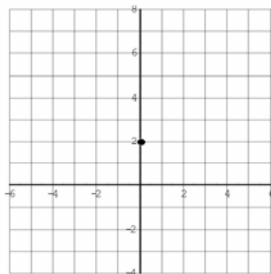
**Example 1: Sketch the graph of  $\frac{x^2}{16} - \frac{(y-2)^2}{9} = -1$**

This equation has -1 on the right side, so it's a vertical hyperbola.

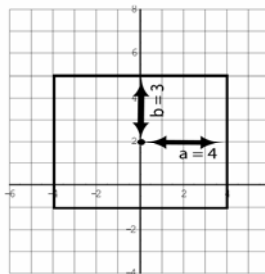
- Centre = (0, 2)
- a-value = 4 \*Remember to square root!
- b-value = 3 \*Remember to square root!

The line between vertices is often called the *transverse axis*.

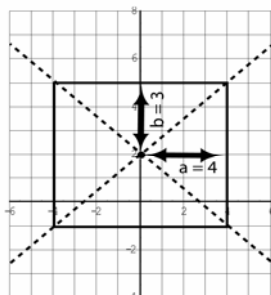
**Step 1:** Draw a point at the centre.



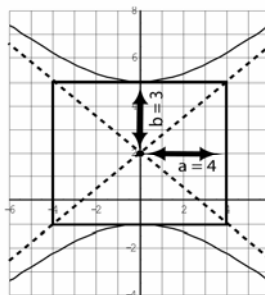
**Step 2:** Draw in the reference box.



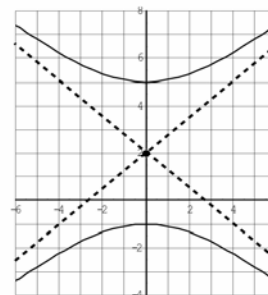
**Step 3:** Draw in the asymptotes.



**Step 4:** Finish the graph.



**Step 5:** Erase the excess information so you just have the graph. \*Optional



# CONICS LESSON 3

## PART II - HYPERBOLA EQUATIONS

**Example 2:** A horizontal hyperbola has a centre at  $(-2, -3)$ , with  $a = 5$  and  $b = 3$ . Find the equation.

Since the centre is given, you know  $h = -2$  and  $k = -3$ .

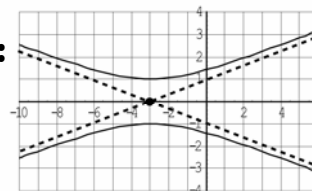
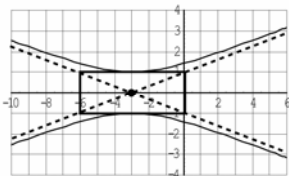
You are given  $a = 5$  and  $b = 3$

Put these values into your standard form and you're done:

$$\frac{(x + 2)^2}{25} - \frac{(y + 3)^2}{9} = 1$$

**Example 3:** Given the following graph, write the equation:

**Step 1:** Draw the reference box in.



**Step 2:** State everything you know: ( $h = -3$ ,  $k = 0$ ,  $a = 3$ ,  $b = 1$ )

**Step 3:** Fill in the equation:  $\frac{(x + 3)^2}{9} - y^2 = 1$

**Example 4:** Sketch the graph of  $\frac{9(x - 3)^2}{4} - \frac{4(y + 1)^2}{16} = 1$

In a standard form equation, you are not allowed to have any numbers in the numerator outside the brackets. You can fix this by "swinging" the top number down below the bottom number as follows:

$$\frac{9(x - 3)^2}{4} - \frac{4(y + 1)^2}{16} = 1$$

$$\frac{(x - 3)^2}{\frac{4}{9}} - \frac{(y + 1)^2}{\frac{16}{4}} = 1$$

$$\frac{(x - 3)^2}{\frac{4}{9}} - \frac{(y + 1)^2}{4} = 1$$

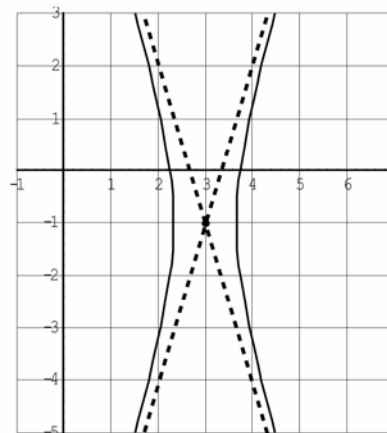
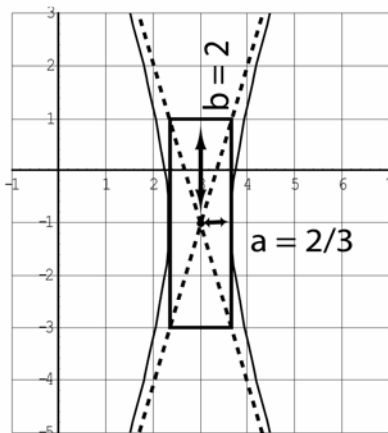
Now state what you know:  
 $h = 3$ ,  $k = -1$

$$a^2 = \frac{4}{9} \quad \text{so,} \quad a = \sqrt{\frac{4}{9}} = \frac{2}{3}$$

$$b^2 = 4, \quad \text{so,} \quad b = \sqrt{4} = 2$$



Erase the box to see a clean graph.



### Remember These Rules!

Hyperbolas will only have a - separating terms

Ellipses will only have a + separating terms.

Hyperbolas can equal either +1 (horizontal) or -1 (vertical).

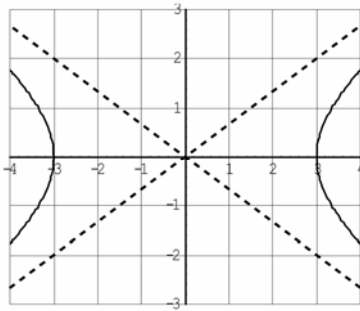
Ellipses only equal +1

# CONICS LESSON 3

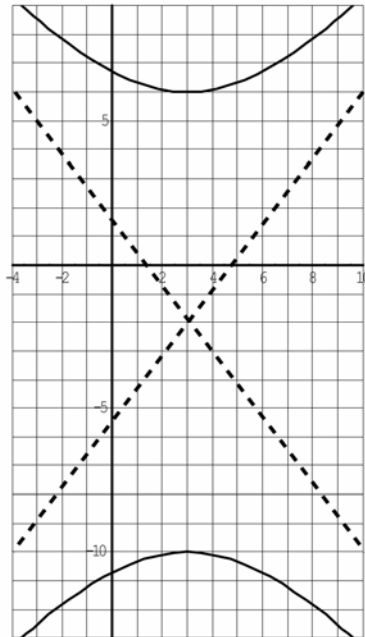
## PART II - HYPERBOLA EQUATIONS

**Questions:** For each of the following graphs, write the equation:

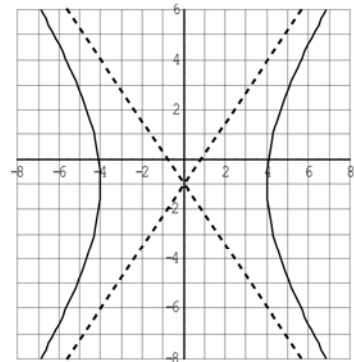
1.



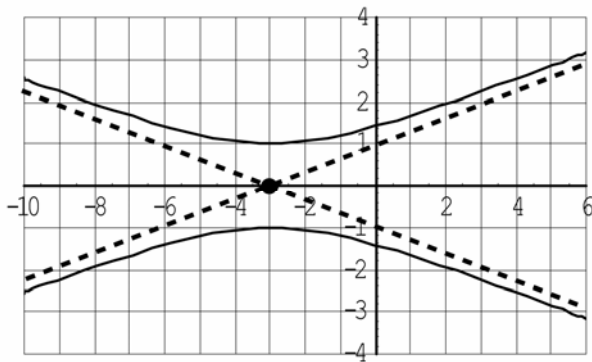
2.



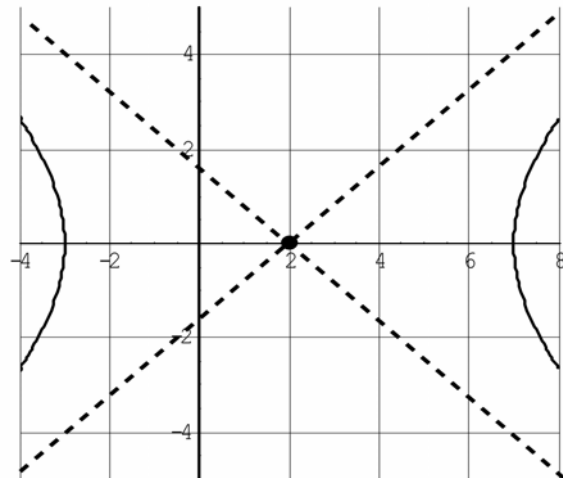
3.



4.



5.

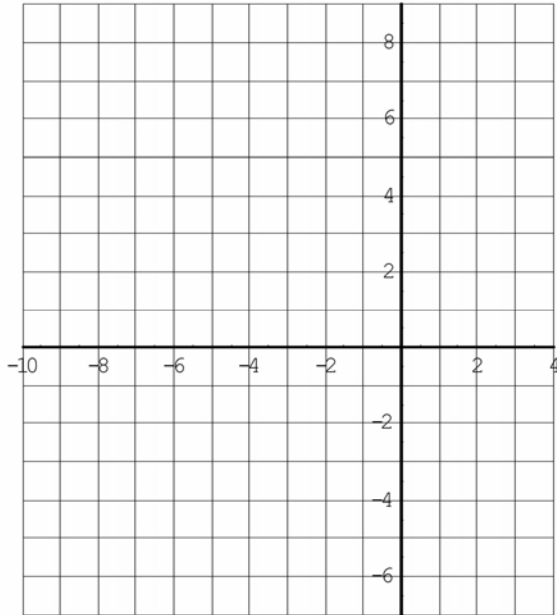


# CONICS LESSON 3

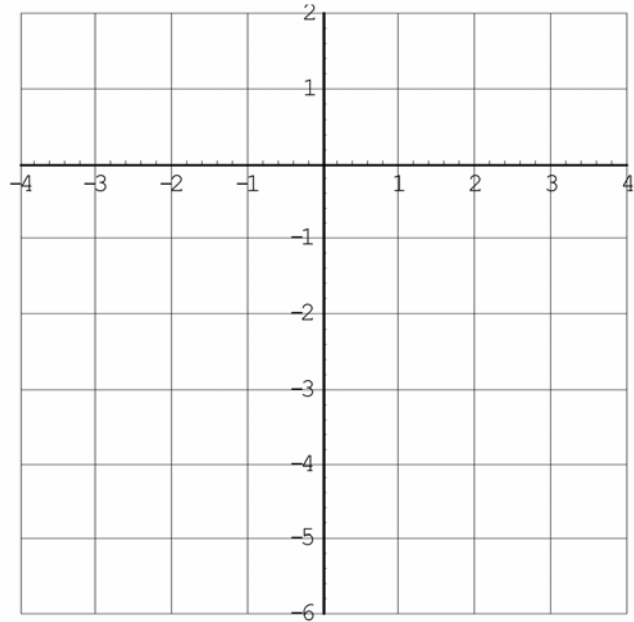
## PART II - HYPERBOLA EQUATIONS

**Questions:** For each of the following equations, sketch the graph:

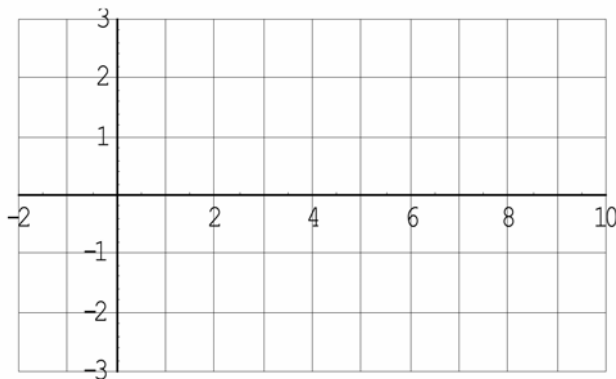
6.  $\frac{(x+3)^2}{36} - \frac{(y-1)^2}{25} = -1$



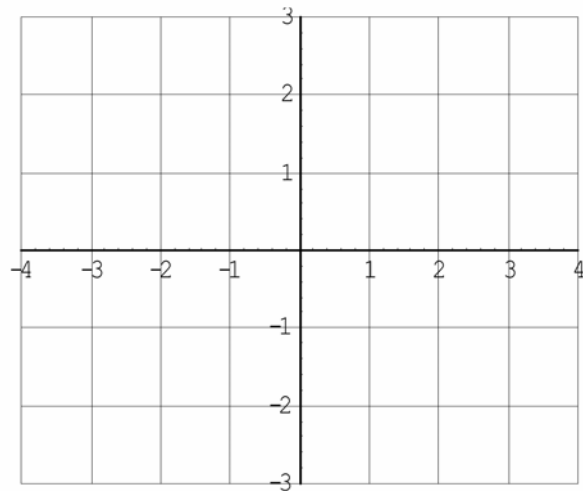
7.  $\frac{x^2}{9} - \frac{(y+2)^2}{16} = 1$



8.  $\frac{(x-4)^2}{4} - y^2 = -1$



9.  $\frac{4x^2}{9} - 4y^2 = 1$



# CONICS LESSON 3

## PART II - HYPERBOLA EQUATIONS

### **Answers:**

1.  $\frac{x^2}{9} - \frac{y^2}{4} = 1$

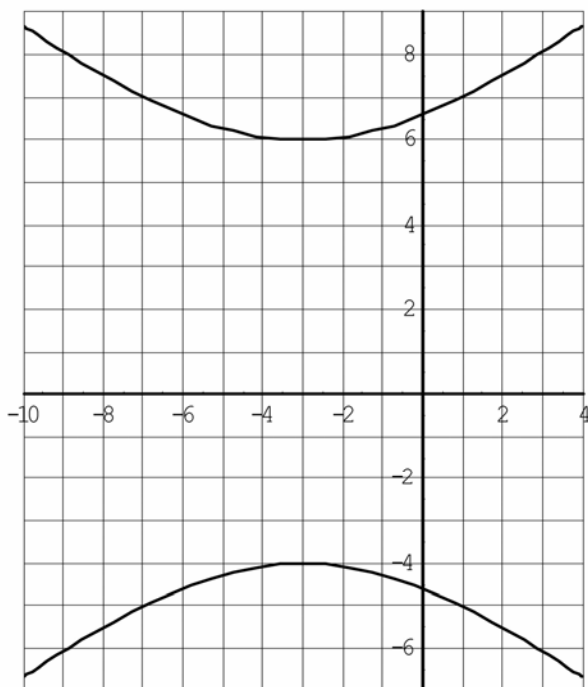
4.  $\frac{(x+3)^2}{9} - y^2 = -1$

2.  $\frac{(x-3)^2}{49} - \frac{(y+2)^2}{64} = -1$

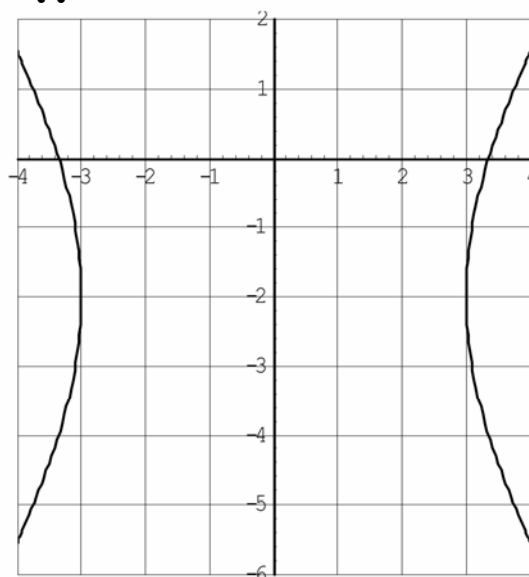
5.  $\frac{(x-2)^2}{25} - \frac{y^2}{16} = 1$

3.  $\frac{x^2}{16} - \frac{(y+1)^2}{25} = 1$

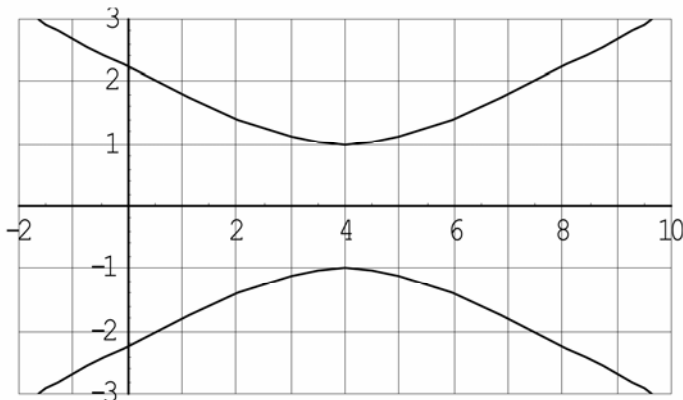
6.



7.



8.



9.

