

Pure Math 30:

CONICS



LESSON 4

Standard & General Form of Conics

Pure Math
30:

EXPLAINED!

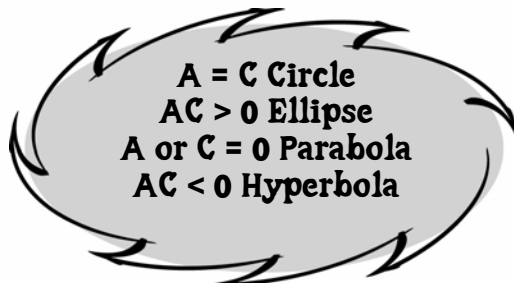
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CONICS LESSON 4

PART I: GENERAL FORM

General Form of a Conic: $Ax^2 + Cy^2 + Dx + Ey + F = 0$

A & C are useful in finding out which conic is produced:



Example 1: What shape is the graph of $3x^2 - 2y^2 + 2x - 3 = 0$

Hyperbola, since $AC = (3)(-2) = -6$, which is negative.

Example 2: What shape is the graph of $y^2 + 2x - y - 5 = 0$

Parabola, since $A = 0$

Questions: Based on the A & C values, determine the type of conic for each of the following:

1) $9x^2 - 4y^2 - 8y + 32 = 0$

2) $-2y^2 - x + 20y - 47 = 0$

3) $y^2 + 6y = 3x - 12$

4) $25x^2 - 9y^2 - 72y - 100x - 269 = 0$

5) $4x^2 + y^2 - 8x + 4y - 9 = 0$

6) $9x^2 - 4y^2 - 40y - 136 = 0$

7) $x^2 + 6x + 8y + 33 = 0$

8) $4x^2 + 4y^2 - 8x + 4y - 9 = 0$

Why is there no B?

The Bxy term involves conic rotation, and is excluded from the Pure Math 30 curriculum.

Answers:

- 1) Hyperbola
- 2) Parabola
- 3) Parabola
- 4) Hyperbola
- 5) Ellipse
- 6) Hyperbola
- 7) Parabola
- 8) Circle

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PART II: STANDARD TO GENERAL FORM

Converting from Standard To General Form: This is done by multiplying both sides of the equation by the common denominator, then simplifying:

Example 1: Convert $y + 3 = \frac{5}{4}(x + 3)^2$ to general form:

Multiply both sides by the common denominator of 4:

$$4[y + 3] = 4\left[\frac{5}{4}(x + 3)^2\right]$$

$$4y + 12 = \cancel{4}\left[\frac{5}{\cancel{4}}(x + 3)^2\right]$$

$$4y + 12 = 5(x + 3)^2$$

$$4y + 12 = 5(x + 3)(x + 3)$$

$$4y + 12 = 5(x^2 + 6x + 9)$$

$$4y + 12 = 5x^2 + 30x + 45$$

$$0 = 5x^2 + 30x - 4y + 33$$

Example 2: Convert $\frac{(x - 10)^2}{10} + \frac{y^2}{25} = 1$ to general form:

Multiply both sides by the common denominator of 50:

$$50\left[\frac{(x - 10)^2}{10} + \frac{y^2}{25}\right] = 50[1]$$

$$5(x - 10)^2 + 2y^2 = 50$$

$$5(x^2 - 20x + 100) + 2y^2 = 50$$

$$5x^2 - 100x + 500 + 2y^2 = 50$$

$$5x^2 + 2y^2 - 100x + 450 = 0$$

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PART II: STANDARD TO GENERAL FORM

Questions: Convert each of the following to general form:

1) $\frac{9x^2}{36} - \frac{4(y+1)^2}{36} = -1$

4) $-2(y-5)^2 = x-3$

2) $\frac{x^2}{4} - \frac{(y+5)^2}{9} = 1$

5) $4(x-1)^2 + (y+2)^2 = 17$

3) $y+3 = -\frac{1}{8}(x+3)^2$

6) $\frac{(x-2)^2}{9} - \frac{(y-4)^2}{25} = 1$

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PART II: STANDARD TO GENERAL FORM

Answers:

1.
$$36 \left[\frac{9x^2}{36} - \frac{4(y+1)^2}{36} \right] = 36[-1]$$
$$9x^2 - 4(y+1)^2 = -36$$
$$9x^2 - 4(y+1)(y+1) = -36$$
$$9x^2 - 4(y^2 + 2y + 1) = -36$$
$$9x^2 - 4y^2 - 8y - 4 = -36$$
$$9x^2 - 4y^2 - 8y + 32 = 0$$

2.
$$36 \left[\frac{x^2}{4} - \frac{(y+5)^2}{9} \right] = 36[1]$$
$$9x^2 - 4(y+5)^2 = 36$$
$$9x^2 - 4(y+5)(y+5) = 36$$
$$9x^2 - 4(y^2 + 10y + 25) = 36$$
$$9x^2 - 4y^2 - 40y - 100 = 36$$
$$9x^2 - 4y^2 - 40y - 136 = 0$$

3.
$$8[y+3] = 8 \left[-\frac{1}{8}(x+3)^2 \right]$$
$$8y + 24 = -(x+3)^2$$
$$8y + 24 = -(x+3)(x+3)$$
$$8y + 24 = -(x^2 + 6x + 9)$$
$$8y + 24 = -x^2 - 6x - 9$$
$$x^2 + 6x + 8y + 33 = 0$$

4.
$$-2(y-5)^2 = x-3$$
$$-2(y-5)(y-5) = x-3$$
$$-2(y^2 - 10y + 25) = x-3$$
$$-2y^2 + 20y - 50 = x-3$$
$$0 = 2y^2 + x - 20y + 47$$

5.
$$4(x-1)^2 + (y+2)^2 = 17$$
$$4(x-1)(x-1) + (y+2)(y+2) = 17$$
$$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = 17$$
$$4x^2 - 8x + 4 + y^2 + 4y + 4 = 17$$
$$4x^2 + y^2 - 8x + 4y + 8 = 17$$
$$4x^2 + y^2 - 8x + 4y - 9 = 0$$

6.
$$225 \left[\frac{(x-2)^2}{9} - \frac{(y-4)^2}{25} \right] = 225[1]$$
$$25(x-2)^2 - 9(y-4)^2 = 225$$
$$25(x-2)(x-2) - 9(y-4)(y-4) = 225$$
$$25(x^2 - 4x + 4) - 9(y^2 - 8y + 16) = 225$$
$$25x^2 - 100x + 100 - 9y^2 + 72y - 144 = 225$$
$$25x^2 - 9y^2 - 100x + 72y - 269 = 0$$

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PART III: GENERAL TO STANDARD FORM

Converting from General To Standard Form: This is done by completing the square:

Example 1: Convert $x^2 + 3y^2 + 10x - 30y + 91 = 0$ to general form.

Step 1: Group terms with x, group terms with y, and take the constant to the other side.

$$x^2 + 10x + 3y^2 - 30y = -91$$

Step 2: Put brackets around the x-terms, and put brackets around the y-terms, leaving a space.

$$(x^2 + 10x \quad) + (3y^2 - 30y \quad) = -91$$

Step 3: Make sure the squared terms have a coefficient of 1. If they don't, factor that number out.

$$(x^2 + 10x \quad) + 3(y^2 - 10y \quad) = -91$$

Step 4: Divide the x-coefficient by 2, square the result, and add inside the bracket. Repeat for y.

$$(x^2 + 10x + 25) + 3(y^2 - 10y + 25) = -91$$

Step 5: Whatever you do to one side, you must do to the other! On the left, you have now added 25 in the x-brackets, and 75 in the y-brackets (multiplying the 3 through the brackets).

Add 25 + 75 to the other side to keep things balanced.

$$(x^2 + 10x + 25) + 3(y^2 - 10y + 25) = -91 + 25 + 75$$

Step 6: Factor the expressions.

$$(x + 5)^2 + 3(y - 5)^2 = 9$$

Step 7: Divide both sides by 9 so the equation equals 1, which is required for standard form:

$$\frac{(x + 5)^2}{9} + \frac{3(y - 5)^2}{9} = \frac{9}{9}$$

$$\frac{(x + 5)^2}{9} + \frac{(y - 5)^2}{3} = 1$$

Example 2: Convert $3x^2 + 12x - 4y - 12 = 0$ to general form.

Step 1: Group terms with x. Since there is no y^2 , take both the 4y & 12 to the other side.

$$3x^2 + 12x = 4y + 12$$

Step 2: Put brackets around the x-terms, leaving a space.

$$(3x^2 + 12x \quad) = 4y + 12$$

Step 3: Make sure the squared terms have a coefficient of 1. If they don't, factor that number out.

$$3(x^2 + 4x \quad) = 4y + 12$$

Step 4: Divide the x-coefficient by 2, square the result and add inside the bracket.

$$3(x^2 + 4x + 4) = 4y + 12 + 12$$

Step 5: Whatever you do to one side, you must do to the other! On the left, you have added 12 (multiplying the 3 into the brackets), so add 12 to the other side too.

$$3(x^2 + 4x + 4) = 4y + 12 + 12$$

Step 6: Factor and simplify the expressions.

$$3(x + 2)^2 = 4y + 24$$

Step 7: Take this to parabola standard form.

$$\frac{3(x + 2)^2}{4} = \frac{4(y + 6)}{4}$$

$$y + 6 = \frac{3}{4}(x + 2)^2$$

If you are ever asked to graph a conic presented in general form, you must first convert it to standard form. Once converted, you can use rules from previous lessons to draw the graph.

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PART III: GENERAL TO STANDARD FORM

Questions: Convert each of the following to standard form.

1) $9x^2 - 4y^2 - 8y + 32 = 0$

5) $4x^2 + y^2 - 8x + 4y - 9 = 0$

2) $-2y^2 - x + 20y - 47 = 0$

6) $9x^2 - 4y^2 - 40y - 136 = 0$

3) $y^2 + 6y = 3x - 12$

7) $x^2 + 6x + 8y + 33 = 0$

4) $25x^2 - 9y^2 - 72y - 100x - 269 = 0$

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PART III: GENERAL TO STANDARD FORM

Answers:

1. $9x^2 - 4y^2 - 8y + 32 = 0$

$$9x^2 - 4(y^2 + 2y) = -32$$

$$9x^2 - 4(y^2 + 2y + 1) = -32 - 4$$

$$9x^2 - 4(y + 1)^2 = -36$$

$$\frac{9x^2}{36} - \frac{4(y + 1)^2}{36} = \frac{-36}{36}$$

$$\frac{x^2}{4} - \frac{(y + 1)^2}{9} = -1$$

2. $-2y^2 - x + 20y - 47 = 0$

$$-2y^2 + 20y = x + 47$$

$$-2(y^2 - 10y) = x + 47$$

$$-2(y^2 - 10y + 25) = x + 47 - 50$$

$$-2(y - 5)^2 = x - 3$$

3. $y^2 + 6y = 3x - 12$

$$(y^2 + 6y + 9) = 3x - 12 + 9$$

$$(y + 3)^2 = 3x - 3$$

$$(y + 3)^2 = 3(x - 1)$$

$$x - 1 = \frac{1}{3}(y + 3)^2$$

4. $25x^2 - 9y^2 - 72y - 100x - 269 = 0$

$$25x^2 - 100x - 9y^2 - 72y = 269$$

$$25(x^2 - 4x) - 9(y^2 + 8y) = 269$$

$$25(x^2 - 4x + 4) - 9(y^2 + 8y + 16) = 269 + 100 - 144$$

$$\frac{25(x - 2)^2}{225} - \frac{9(y + 4)^2}{225} = \frac{225}{225}$$

$$\frac{(x - 2)^2}{9} - \frac{(y + 4)^2}{25} = 1$$

5. $4x^2 + y^2 - 8x + 4y - 9 = 0$

$$4x^2 - 8x + y^2 + 4y - 9 = 0$$

$$4(x^2 - 2x) + (y^2 + 4y) = 9$$

$$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = 9 + 4 + 4$$

$$4(x - 1)^2 + (y + 2)^2 = 17$$

$$\frac{4(x - 1)^2}{17} + \frac{(y + 2)^2}{17} = 1$$

6. $9x^2 - 4y^2 - 40y - 136 = 0$

$$9x^2 - 4(y^2 + 10y) = 136$$

$$9x^2 - 4(y^2 + 10y + 25) = 136 - 100$$

$$\frac{9x^2}{36} - \frac{4(y + 5)^2}{36} = \frac{36}{36}$$

$$\frac{x^2}{4} - \frac{(y + 5)^2}{9} = 1$$

7. $x^2 + 6x + 8y + 33 = 0$

$$(x^2 + 6x) = -8y - 33$$

$$(x^2 + 6x + 9) = -8y - 33 + 9$$

$$(x + 3)^2 = -8y - 24$$

$$(x + 3)^2 = -8(y + 3)$$

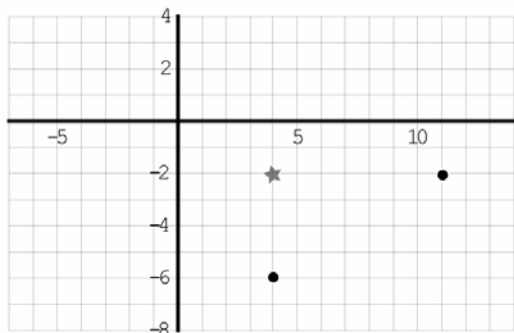
$$y + 3 = -\frac{1}{8}(x + 3)^2$$

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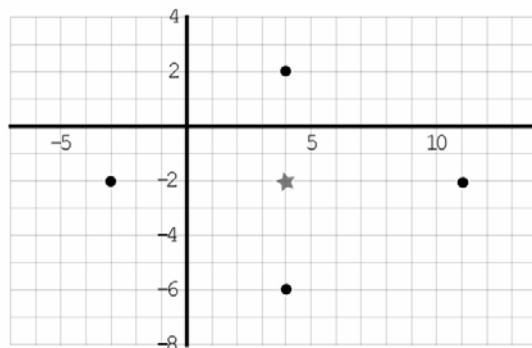
PART IV: DIPLOMA STYLE

Example 1: An ellipse has its centre at (4,-2) and passes through the points (11,-2) & (4, -6). Determine the equation of the ellipse

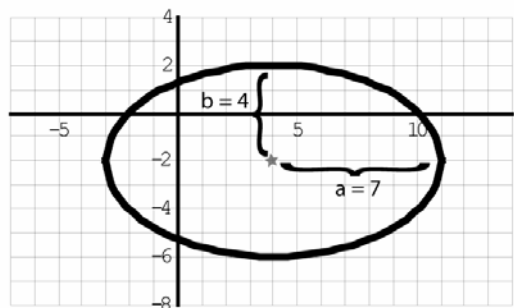
Step 1: Draw in the points given:



Step 2: By symmetry, you can find the following points on the ellipse as well.



Step 3: Read off the a & b values.



Step 4: plug everything into your equation.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-4)^2}{7^2} + \frac{(y-(-2))^2}{4^2} = 1$$

$$\frac{(x-4)^2}{49} + \frac{(y+2)^2}{16} = 1$$

Example 2: A vertical parabola has a vertex at (-3, -2) and passes through the point (-1, 7).

It's a vertical parabola, so the standard form equation is $y - k = a(x - h)^2$.

All you need to do is plug in the numbers and solve for the a-value.

$$y - k = a(x - h)^2$$

$$7 - (-2) = a(-1 - (-3))^2$$

$$9 = 4a$$

$$a = \frac{9}{4}$$

$$\text{The equation is: } y + 2 = \frac{9}{4}(x + 3)^2$$

Example 3: Given the ellipse $\frac{(x-2)^2}{9} + \frac{(y+4)^2}{16} = 1$, determine the new

equation after a translation 3 units up and 7 units right.

First state your original centre point (2,-4)

Then apply the transformation to this point: (2+7, -4+3)

The new centre is (9,-1)

Since there are no other transformations, the new equation is: $\frac{(x-9)^2}{9} + \frac{(y+1)^2}{16} = 1$

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PART IV: DIPLOMA STYLE

Example 4: The general form of a particular ellipse is $2x^2 + y^2 - 2x + 3y - 9 = 0$.

If this conic is translated 2 units left and 1 unit down, determine the new general equation.

Replace x with $(x + 2)$ to represent 2 units left, and replace y with $(y + 1)$ to represent 1 unit down.

$$2(x + 2)^2 + (y + 1)^2 - 2(x + 2) + 3(y + 1) - 9 = 0$$

$$2(x^2 + 4x + 4) + (y^2 + 2y + 1) - 2(x + 2) + 3(y + 1) - 9 = 0$$

$$2x^2 + 8x + 8 + y^2 + 2y + 1 - 2x - 4 + 3y + 3 - 9 = 0$$

$$2x^2 + y^2 + 6x + 5y - 1 = 0$$

Example 5: Determine the y – intercepts of $x^2 + y^2 - x + 5y - 6 = 0$

The y – intercepts of $x^2 + y^2 - x + 5y - 6 = 0$ can be found by substituting zero for x , then solving for y .

$$x^2 + y^2 - x + 5y - 6 = 0$$

$$y^2 + 5y - 6 = 0$$

$$(y + 6)(y - 1) = 0$$

$$y = -6, 1$$

Remember that y -intercepts are actually a coordinate with $x = 0$, so write the y -intercepts as $(0, -6)$ and $(0, 1)$

Example 6: The circle $x^2 + y^2 = 1$ is translated h units left and k units down.

Determine the x -intercepts

Replace x with $(x + h)$ and y with $(y - k)$ to account for the transformations.

$$x^2 + y^2 = 1$$

$$(x + h)^2 + (y - k)^2 = 1$$

To find the x -intercepts, let $y = 0$.

$$(x + h)^2 + (0 - k)^2 = 1$$

$$(x + h)^2 + (-k)^2 = 1$$

$$(x + h)^2 + k^2 = 1$$

$$(x + h)^2 = 1 - k^2$$

$$x + h = \pm\sqrt{1 - k^2}$$

$$x = \pm\sqrt{1 - k^2} - h$$

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PART IV: DIPLOMA STYLE

Questions:

- 1) An ellipse has a centre at $(-4, 6)$ and passes through $(-4, 9)$ and $(2, 6)$. Determine the standard form equation.
- 2) An ellipse has a centre at $(3, 2)$ and passes through $(5, 2)$ and $(3, -3)$. Determine the standard form equation.
- 3) A horizontal parabola has a vertex at $(3, 2)$ and passes through $(-13, 4)$.
- 4) A vertical parabola has a vertex at $(-3, 5)$ and passes through $(-1, -4)$.

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PART IV: DIPLOMA STYLE

5) Given the ellipse $\frac{(x-3)^2}{4} + \frac{(y+1)^2}{9} = 1$, determine the new equation after a translation 2 units right and 2 units down.

6) Given the ellipse $\frac{(x+5)^2}{25} + \frac{y^2}{4} = 1$, determine the new equation after a translation 3 units left and 4 units down.

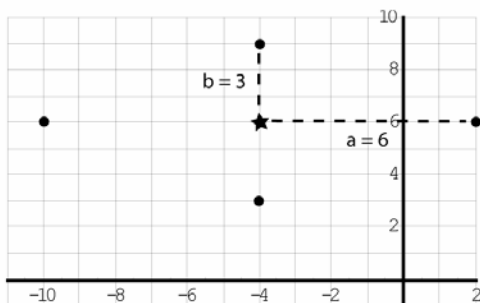
7) The general form of a particular ellipse is $x^2 + y^2 - x + 2y - 4 = 0$. If this conic is translated 1 unit right and 3 units up, determine the new general equation.

8) Determine the x - intercepts of $x^2 + y^2 - x + 5y - 6 = 0$

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PART IV: DIPLOMA STYLE

1) $\frac{(x+4)^2}{36} + \frac{(y-6)^2}{9} = 1$



2) $\frac{(x-3)^2}{4} + \frac{(y-2)^2}{25} = 1$



3)

$$\begin{aligned} x - h &= a(y - k)^2 \\ -13 - 3 &= a(4 - 2)^2 \\ -16 &= 4a \\ -4 &= a \\ \text{The equation is:} \\ x - 3 &= -4(y - 2)^2 \end{aligned}$$

4)

$$\begin{aligned} y - k &= a(x - h)^2 \\ -4 - 5 &= a(-1 + 3)^2 \\ -9 &= 4a \\ \frac{9}{4} &= a \\ \text{The equation is:} \\ y - 5 &= \frac{9}{4}(x + 3)^2 \end{aligned}$$

5) First state your original centre point (3, -1)
Then apply the transformation to this point:
(3+2, -1-2)
The new centre is (5, -3)

The new equation is $\frac{(x-5)^2}{4} + \frac{(y+3)^2}{9} = 1$

7)

$$\begin{aligned} x^2 + y^2 - x + 2y - 4 &= 0 \\ (x-1)^2 + (y-3)^2 - (x-1) + 2(y-3) - 4 &= 0 \\ x^2 - 2x + 1 + y^2 - 6y + 9 - x + 1 + 2y - 6 - 4 &= 0 \\ x^2 + y^2 - 3x - 4y + 1 &= 0 \end{aligned}$$

6) First state your original centre point (-5, 0)
Then apply the transformation to this point:
(-5-3, 0-4)
The new centre is (-8, -4)

The new equation is $\frac{(x+8)^2}{25} + \frac{(y+4)^2}{4} = 1$

8) The x -intercepts of $x^2 + y^2 - x + 5y - 6 = 0$ can be found by substituting zero for y then solving for x .

$$\begin{aligned} x^2 + y^2 - x + 5y - 6 &= 0 \\ x^2 + (0)^2 - x + 5(0) - 6 &= 0 \\ x^2 - x - 6 &= 0 \\ (x-3)(x+2) &= 0 \\ x &= 3, -2 \end{aligned}$$

The x -intercepts are (-2, 0) and (3, 0)