



# Pure Math 30

## EXPLAINED!

*Diploma Style  
Practice Exam*

**Exponential and  
Logarithmic Functions  
– QUESTIONS –**

## **Logarithms Diploma Style Practice Exam**

*These are the formulas for logarithms you will be given on your diploma*

$$A = P(1+i)^n$$

$$\log_a \left( \frac{M}{N} \right) = \log_a M - \log_a N$$

$$\log_a (MN) = \log_a M + \log_a N$$

$$\log_b c = \frac{\log_a c}{\log_a b}$$

<i>Use this sheet to record your answers</i>
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- |       |       |       |     |
|-------|-------|-------|-----|
| 1.    | 10.   | 19.   | 27. |
| 2.    | 11.   | 20.   | 28. |
| 3.    | 12.   | NR 5. | 29. |
| 4.    | NR 3. | NR 6. | 30. |
| 5.    | 13.   | 21.   | 31. |
| 6.    | 14.   | 22.   | 32. |
| NR 1. | NR 4. | 23.   | 33. |
| 7.    | 15.   | 24.   |     |
| 8.    | 16.   | NR 7. |     |
| NR 2. | 17.   | 25.   |     |
| 9.    | 18.   | 26.   |     |

## **Logarithms Diploma Style Practice Exam**

1. The graph of  $f(x) = b^x$  and the graph of  $g(x) = \left(\frac{1}{b}\right)^x$ , where  $b > 0$ , are reflections of each other about the line
- A.  $y = x$   
B.  $y = b$   
C.  $x = 0$   
D.  $y = 0$

*Use the following information to answer the next question.*

**Equation I**       $y = \frac{\log x}{\log 3}$

**Equation II**       $y = x^3$

**Equation III**       $y = x - 6$

**Equation IV**       $y = (x - 6)^3$

2. A solution to the equation  $\log_3 x = x - 6$  could be approximated using technology by graphing equations
- A. I and III  
B. I and IV  
C. II and III  
D. II and IV
- 

3. The expression  $\log_{\frac{1}{5}}\left(\frac{1}{x}\right)$  is equivalent to
- A.  $\log_5\left(\frac{1}{x}\right)$   
B.  $\log_{\frac{1}{x}} 5$   
C.  $\log(5x)$   
D.  $\log_5 x$

Use the following information to answer the next question.

The power rating of a particular dynamic electronic circuit is given by the equation

$$P = 1 - w^{-0.246t}$$

where  $P$  is the power rating,  $t$  is amount of time since the circuit is switched on, and  $w$  is a constant.

4. After the circuit has been operational for 43 seconds, the power rating is 0.83. The value of  $w$ , to the nearest hundredth, is
- A. 0.09  
B. 0.25  
C. 1.18  
D. 10.58
- 
5. The expression  $\log_x(y^3z) - \log_x(yz^2)$  is equivalent to
- A.  $\log_x(y^2z^3)$   
B.  $\log_x\left(\frac{y^2}{z}\right)$   
C.  $3\log_x y + \log_x z - \log_x y + 2\log_x z$   
D. 1
6. The value of  $b$  in the equation  $7 = (3 + b)^4$  is equivalent to
- A.  $\frac{\log 7}{3^4}$   
B.  $\frac{\log_4 7}{\log_4 3}$   
C.  $7^4 - 3$   
D.  $\sqrt[4]{7} - 3$

### Numerical Response

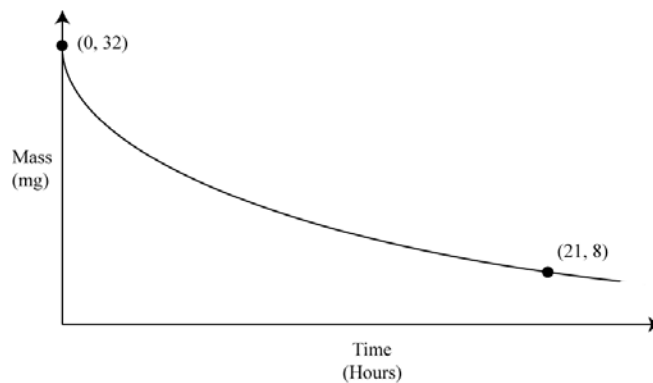
1.

If  $xy = 8$ , then to the nearest tenth, the value of  $5\log_2 x + 5\log_2 y$  is \_\_\_\_\_.

7. The solution to the equation  $2^{3x} = 5^{-x-1}$ , correct to the nearest hundredth, is
- A. -0.081
  - B. -0.436
  - C. 0.413
  - D. 1.455

*Use the following information to answer the next question.*

The mass of a radioactive sample is represented in the graph below. The initial mass of 32 mg decays to 8 mg after 21 hours.

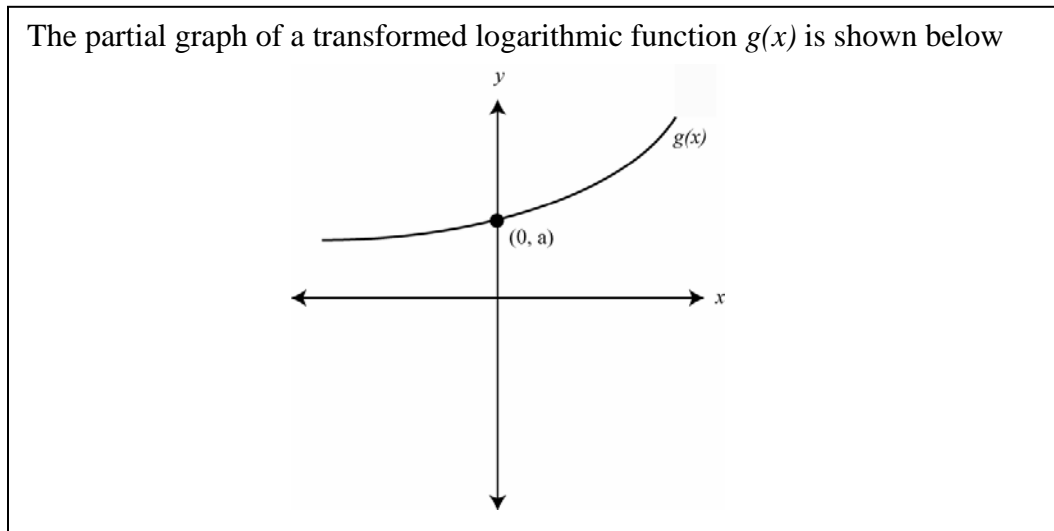


8. The half-life of the radioactive sample, in minutes, is
- A. 60
  - B. 420
  - C. 630
  - D. 1260

### Numerical Response

2. Given  $a^{5x} = (\log_c c^a)^{3x+8}$ , the value of  $x$ , to the nearest hundredth is \_\_\_\_\_.

Use the following information to answer the next question.



9. If the graph of  $g(x)$  is transformed to a new graph  $h(x)$ , and the point  $(0, a)$  becomes  $(a, 0)$ , then a possible transformation is
- A.  $h(x) = g^{-1}(x)$
  - B.  $h(x) = ag(x)$
  - C.  $h(x) = g(x) + a$
  - D.  $h(x) = g^{-1}(x) + a$
- 
10. A skilled player at the video game “Dot – Gobbler” has an average high score of 50000 points. For every day the player is on vacation, she can expect to lose 2.7% of her gaming ability. An equation that may be used to predict the average score  $S$  of the player after  $d$  days is
- A.  $S = 50000(2.7)^{\frac{1}{d}}$
  - B.  $S = 50000(0.027)^d$
  - C.  $S = 50000(0.973)^{\frac{1}{d}}$
  - D.  $S = 50000(0.973)^d$

Use the following information to answer the next two questions.

The decibel level of a sound may be calculated using the formula

$$L = 10 \log(10^{12} \cdot I)$$

where  $L$  is the loudness of the sound ( $dB$ ) and  $I$  is the intensity of the sound.

11. An equation that can be used to solve for the value of  $I$  is

- A.  $I = \frac{L}{120 \times \log 10}$   
B.  $I = \log\left(\frac{L}{10}\right) \times 10^{12}$   
C.  $I = 10^{\frac{L-120}{10}}$   
D.  $I = \frac{L}{10^{13}}$

12. The loudness of a jet engine is 150  $dB$ . The magnitude of the sound intensity is

- A. 1.25  
B.  $1.18 \times 10^{12}$   
C. 1000  
D.  $1.5 \times 10^{-11}$

### Numerical Response

3. The expression  $\log_b\left(\frac{1}{b^{-100}}\right)$  is equivalent to a numerical value of \_\_\_\_\_.

13. The inverse of  $f(x) = 3^x + 4$  is

- A.  $f^{-1}(x) = 3^{-x} + 4$
- B.  $f^{-1}(x) = \log_3(x - 4)$
- C.  $f^{-1}(x) = 4 - 3^x$
- D.  $f^{-1}(x) = \log_{x-4} 3$

14. The graph of  $x = (b)^{-y}$ , where  $b < 0$ , is the same as the graph of

- A.  $y = \log_x b$  reflected in the line  $y = x$
- B.  $y = \log_b x$  reflected in the line  $y = x$
- C.  $y = \log_b x$  reflected in the line  $y = 0$
- D.  $y = b^{-x}$

*Use the following information to answer the next question.*

Newton's Law of cooling can be represented by the equation

$$T(t) = T_0 e^{-kt}$$

where  $T(t)$  is the final temperature in degrees Celsius,  $T_0$  is the initial temperature in degrees Celsius,  $t$  is the elapsed time in minutes, and both  $e$  &  $k$  are constants.

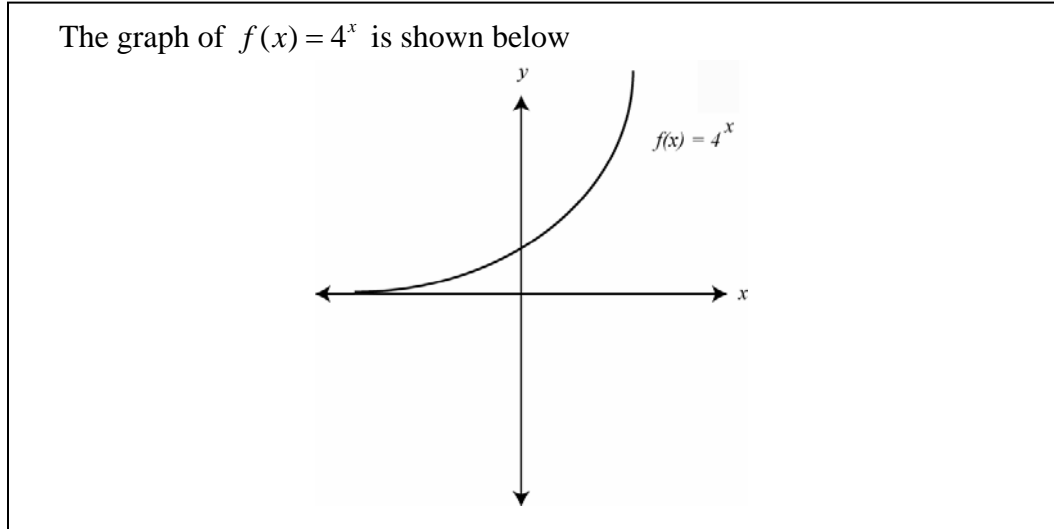
$$\begin{aligned} e &= 2.718 \\ k &= 0.043 \end{aligned}$$

### Numerical Response

4. The length of time, in minutes, required for a cup of coffee to cool from  $82^\circ\text{C}$  to  $65^\circ\text{C}$  is \_\_\_\_\_.



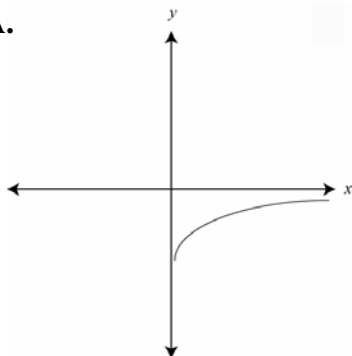
Use the following information to answer the next four questions.



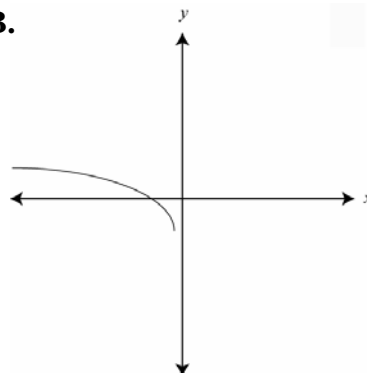
15. The graph of  $f(x) = 4^x$  and the graph of  $g(x) = \log_4 x$  are symmetrical with respect to the line
- A.  $x = 0$
  - B.  $y = 0$
  - C.  $y = x$
  - D.  $y = -x$
16. If the graph of  $g(x) = \log_4 x$  undergoes the transformation  $y = g(3x - 12) + 2$ , the new domain of the graph is
- A.  $x > 2$
  - B.  $x > 3$
  - C.  $x > 4$
  - D.  $x > 12$
17. A student wishes to solve the equation  $4^x = 8$ . An **incorrect** procedure to determine the solution is
- A. Take the logarithm of both sides, use the power rule of logarithms, then isolate the variable by dividing both sides of the equation by  $\log 4$ .
  - B. Graph  $y_1 = 4^x$  and  $y_2 = 8$  in a graphing calculator, find the point of intersection, then state the y-value of this point as the solution.
  - C. Draw  $y_1 = 4^x$  and  $y_2 = 8$  carefully on graph paper, then approximate the coordinates of the point of intersection.
  - D. Find a common base for each side of the equation, then simplify and solve.

18. The equation  $4^{-2y} = x$  is represented by graph

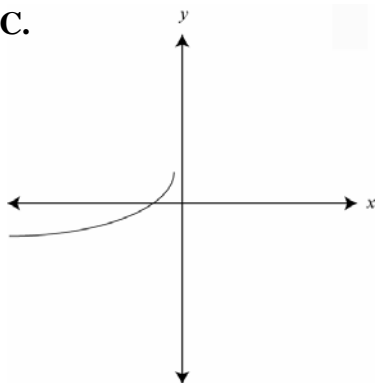
A.



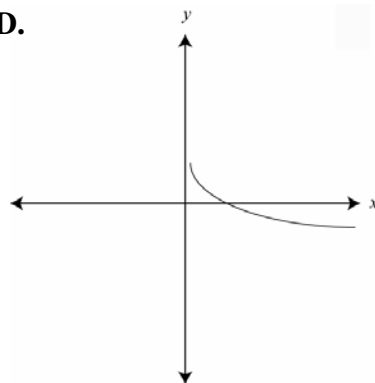
B.



C.



D.



19. The equation  $f(x) = 7a^{2+x} - b$ , has an  $x$  – intercept equivalent to

A.  $x = \frac{\log b - \log 7}{\log a} - 2$

B.  $x = 7a^2 - b$

C.  $x = \frac{y+b}{7a^2}$

D.  $x = 0$

20. A student solves for  $a$  in the equation  $\log_{27}(81a) = b$ . The student determines  $a$  is equivalent to the expression

- A.  $a = \frac{\log 81}{b \log 27}$
- B.  $a = 3^{3b-4}$
- C.  $a = 27^b - 81$
- D.  $a = 3$

*Use the following information to answer the next three questions.*

The profit of a small business is given below

Year	2001	2002	2003	2004	2005
Profit	\$28 000	\$32 000	\$40 000	\$49 000	\$60 000

#### Numerical Response

5. If the owner of the business uses an exponential regression to predict the profits in the year 2010, the expected profit (in thousands) is \_\_\_\_\_.

#### Numerical Response

6. The business will achieve a profit of \$800000 in the year \_\_\_\_\_.

21. A rival business has their profit increase modeled by the function  $P(t) = 13500(1.24)^t$ . The profits of this business will overtake the profits of the first business, for the first time, in the year
- A. 2022
  - B. 2025
  - C. 2029
  - D. 2030

22. Given the equation  $a^{\frac{5}{4}} = 2b$ , an expression for  $a$  is

- A.  $2b^{\frac{4}{5}}$
- B.  $(2b)^{\frac{4}{5}}$
- C.  $2b^{-\frac{4}{5}}$
- D.  $\frac{1}{(2b)^{\frac{4}{5}}}$

*Use the following information to answer the next question.*

A student analyzes the following graph:

$$f(x) = \log_x(6 - x)$$

23. The domain of this graph is
- A.  $x < 0$
  - B.  $x < 6$
  - C.  $0 < x < 6, x \neq 1$
  - D.  $1 < x < 6$
24. The  $x$ -intercept of the graph  $y = b \log_c ax$  is
- A.  $a$
  - B.  $\left(\frac{1}{a}\right)$
  - C.  $x = 0$
  - D.  $2b$

### Numerical Response

7. Given the equation  $2 \log x + 3 \log x = 8$ , a student determines the value of  $x$  to be \_\_\_\_\_.

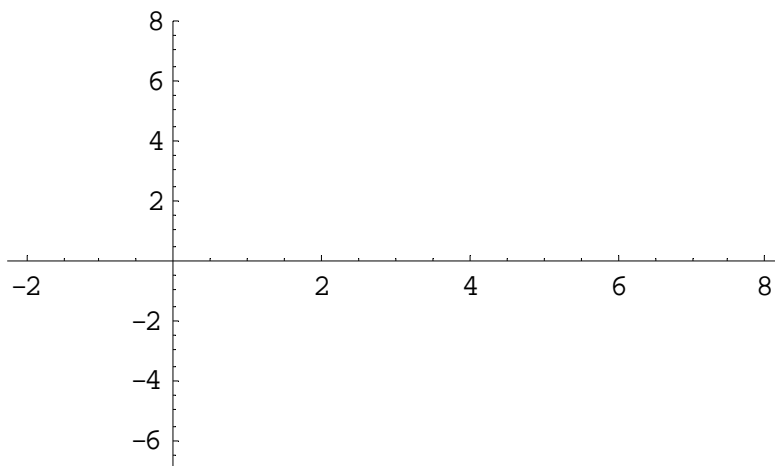
25. One third of  $3^{234}$  is
- A.  $3^{78}$
  - B.  $1^{234}$
  - C.  $3^{233}$
  - D.  $1 \times 10^{234}$
26. Given the equation  $\log_a x + y = \log_a z$ , an expression for  $y$  is
- A.  $y = \log_a \left( \frac{x}{z} \right)$
  - B.  $y = \log_a \left( \frac{z}{x} \right)$
  - C.  $y = \log_a (z - x)$
  - D.  $y = z - x$
27. The price of a vintage video game with the box and instructions doubles every 20 years. If the video game initially cost \$60.00, the value of the game in 33 years will be
- A. 159.59
  - B. 163.28
  - C. 188.30
  - D. 200.00
28. The population of a city can be determined using the equation  $P = 100000(1.03)^t$  where  $P$  is the future population, and  $t$  is the time in years. An equation representing  $t$  as a function of  $P$  is
- A.  $t = \frac{P}{103000}$
  - B.  $t = \log P - 5 - \log 1.03$
  - C.  $t = \frac{\log P}{5 \log 1.03}$
  - D.  $t = \frac{\log P - 5}{\log 1.03}$

29. The value of  $x$  in the equation  $\log(2-x) + \log(2+x) = \log 3$  is
- A.  $x = -1$
  - B.  $x = 1$
  - C.  $x = \pm 1$
  - D. No Solution
30. If  $\log_6 x = 120$ , then  $\log_6 \left( \frac{1}{36} x \right)$  is
- A. 0.52
  - B. 3.33
  - C. 84
  - D. 118
31. An expression equivalent to  $(a^{\log_b c})(a^{\log_b c})$  is
- A.  $a^2 + a^{\log_b c}$
  - B.  $(a^2)^{\log_b c}$
  - C.  $2a^{\log_b c}$
  - D.  $2^{\log_c a^b}$
32. The graph of  $y = b^x$ , where  $b < 1$ , undergoes the transformation  $y + 3 = f(x - 2)$ . The range of the transformed graph is
- A.  $y < -3$
  - B.  $y > -3$
  - C.  $y \leq -3$
  - D.  $y \geq -3$
33. The solution of  $\log(x+2) + \log(x-1) = 1$  is
- A. -4
  - B. 3
  - C. -4, 3
  - D. No Solution

**Written Response – 10%**

**1.**

- Draw the graph of  $f(x) = \log(x+2)$  in the space provided below



- Complete the following chart to describe the graph above

<b>Domain</b>	
<b>Range</b>	
<b>Equation of Asymptote</b>	
<b>x-intercept</b>	
<b>y-intercept</b>	
<b>y-value when <math>x = 2</math></b>	

- Describe the transformations applied to the graph of  $y = \log x$  in order to obtain the graph of  $f(x) = \log(x+2)$
- The domain of the general expression  $f(x) = a \log(bx+c)+d$  is

*Use the following information to answer the next question.*

A student is asked to solve the equation  $27 \bullet 81^{x-2} = 243^{-2x}$  using different techniques learned in Pure Math 30.

**Written Response – 10%**

2.

- Explain how to solve the equation graphically. Indicate appropriate window settings and state the solution.
- **Algebraically** show how to solve this equation using a common base.
- **Algebraically** show how to solve this equation by taking the logarithm of both sides and solving for  $x$ .
- The student is now asked to solve the equation  $\log_3 x = 4$ . Explain how this can be done with a graphing calculator.



*Use the following information to answer the next question.*

A useful equation for solving application questions is

$$A = A_0 (b)^{\frac{t}{P}}$$

where  $A$  is the future amount,  $A_0$  is the initial amount,  $b$  is the rate of growth or decay,  $P$  is the period, and  $t$  is the elapsed time.

**Written Response – 10%**

**3.**

- **Algebraically** solve for  $P$
- A particular bacteria doubles every  $P$  hours. If a bacterial culture starts with 60000 bacteria and has 93000 bacteria after 3 hours, determine the doubling period.
- The population of a town triples every 8 years. Determine the number of years it will take for the population to double.

- Light passing through dirty water retains only  $\frac{3}{4}$  of its intensity for every metre of water. Determine the depth at which the light will have 64% of its surface intensity.
- A particular painting goes down in value by 4.3% each year due to improper storage. Determine the number of years it will take for the value of the painting be half the initial worth.

***You have now completed the examination. Please check over your answers carefully before self-marking. Good luck on your real exam!***