

Pure Math 30:

# Exponential and Logarithmic Functions

$$y = \log_2 \left( \frac{A}{B} \right)$$

## Lesson 4

Applications of Exponential / Logarithmic Functions

Pure Math  
30:

**EXPLAINED!**

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# Logarithms Lesson 4

## Part I – Basic Exponential Applications

EXPONENTIAL GROWTH & DECAY: SITUATIONS FOLLOWING THIS TYPE OF CHANGE CAN BE MODELED USING THE FORMULA:  $A = A_0(B)^{\frac{T}{P}}$

A = FUTURE AMOUNT  
 $A_0$  = INITIAL AMOUNT  
 B = TYPE OF GROWTH  
 P = PERIOD FOR GROWTH TO OCCUR  
 T = ELAPSED TIME

### B & P Examples

A bacteria doubles in 4 hours (b = 2, P = 4)  
 A radioactive sample has a half life of 8 years (b = 1/2, P = 8)  
 A population triples every 10 years. (b = 3, P = 10)

**Example 1:** A bacterial culture doubles every 2 hours. If the culture started with 24000 bacteria, how many bacteria will be present in 5 hours?

$$A = A_0(b)^{\frac{t}{P}}$$

$$A = 24000(2)^{\frac{5}{2}}$$

$$A = 135765 \text{ bacteria}$$

**Example 2:** The half life of a radioactive sample is 4 hours. If 60 g of the sample was initially present, how much will remain after 7 hours?

$$A = A_0(b)^{\frac{t}{P}}$$

$$A = 60\left(\frac{1}{2}\right)^{\frac{7}{4}}$$

$$A = 17.84 \text{ g}$$

**Example 3:** The population of a town triples every 6 years. If 4000 people are present in 2006, how many will be in the town in 2016?

$$A = A_0(b)^{\frac{t}{P}}$$

$$A = 4000(3)^{\frac{10}{6}}$$

$$A = 24961 \text{ people}$$

### QUESTIONS:

- 1) A bacterial culture doubles every 40 minutes. If the culture started with 24000 bacteria, how many bacteria are present after 20 minutes?
- 2) The half life of a radioactive sample is 4.5 years. If 46000 g of the sample was initially present, how much will remain after 89 years?
- 3) The population of a town doubles every 13 years. If 2330 people were present in 1990, how many people will be in the town in 2020?

### ANSWERS:

- 1) 33941 bacteria
- 2) 0.05 g
- 3) 11536 people

# Logarithms Lesson 4

## Part II – Solving For Other Variables

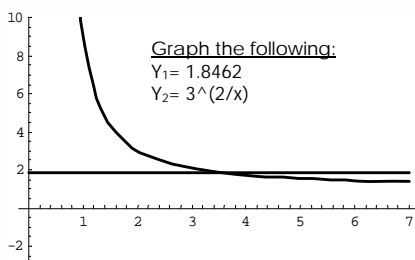
**Example 1:** A bacterial culture triples every  $P$  hours. If the culture started with 13000 bacteria, and there are 24000 after 2 hours, what is the value of  $P$  in hours?

$$A = A_0 (b)^{\frac{t}{P}}$$

$$24000 = 13000(3)^{\frac{2}{P}}$$

$$1.8462 = (3)^{\frac{2}{P}}$$

$$P = 3.58 \text{ hours}$$



AN EASY WAY TO SOLVE FOR VARIABLES IN THESE QUESTIONS IS TO GRAPH AND FIND THE POINT OF INTERSECTION. THE EQUATIONS SHOULD BE SIMPLIFIED AS MUCH AS POSSIBLE BEFORE GRAPHING SO YOU DON'T HAVE TO WORRY ABOUT FINDING HUGE NUMBERS IN YOUR WINDOW SETTINGS.

ALGEBRAIC METHODS INVOLVING LOG RULES ALSO WORK TO SOLVE THESE EQUATIONS. THE METHOD YOU USE IS A MATTER OF PREFERENCE.

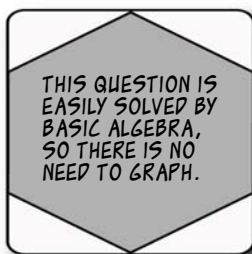
**Example 2:** The half life of a radioactive sample is 6.2 hours. If 2000 g of the sample is present after 7 hours, how much was initially present?

$$A = A_0 (b)^{\frac{t}{P}}$$

$$2000 = A_0 \left(\frac{1}{2}\right)^{\frac{7}{6.2}}$$

$$2000 = A_0 (0.4572)$$

$$A_0 = 4374 \text{ g}$$



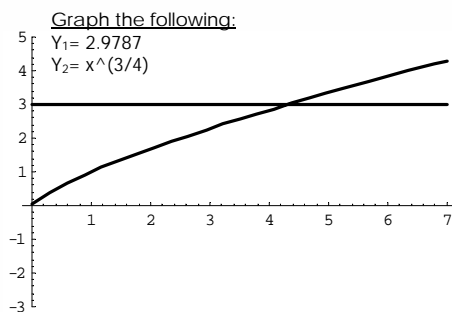
**Example 3:** The population of a town changes by an exponential growth factor  $b$  every 4 years. If 2350 people grows to 7000 in 3 years, what is the value of  $b$ ?

$$A = A_0 (b)^{\frac{t}{P}}$$

$$7000 = 2350(b)^{\frac{3}{4}}$$

$$2.9787 = (b)^{\frac{3}{4}}$$

$$b = 4.29$$



Algebraic solution with logarithms:

**\*YOU CAN ALSO USE LOGARITHM RULES TO OBTAIN THE SOLUTION!**

$$A = A_0 b^{\frac{t}{P}}$$

$$7000 = 2350b^{\frac{3}{4}}$$

$$2.9787 = b^{\frac{3}{4}}$$

$$(2.9787)^{\frac{4}{3}} = \left(b^{\frac{3}{4}}\right)^{\frac{4}{3}}$$

$$b = 4.29$$

**QUESTIONS:**

1) A bacterial culture doubles every  $P$  hours. If the culture started with 5000 bacteria, and there are 40000 after 3 hours, what is the value of  $P$  in hours?

2) The half life of a radioactive sample is 5 years. If 10 g of the sample is present after 12 years, how much was initially present?

3) The population of a town changes by an exponential growth factor  $b$  every 3 years. If a population of 7844 people increases to a population of 8002 in 1 year, what is the value of  $b$ ?

**ANSWERS:**

- 1) 1 hour
- 2) 52.78 g
- 3) 1.062

# Logarithms Lesson 4

## Part III – Ratios of Initial & Final Amounts

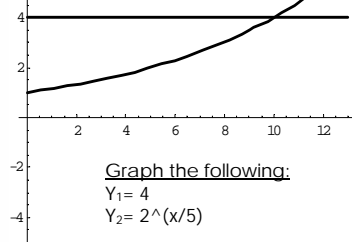
**Example 1:** A bacterial culture doubles every 5 hours. How long will it take for a culture to quadruple?

$$A = A_0(b)^{\frac{t}{P}}$$

$$4A_0 = A_0(2)^{\frac{n}{5}}$$

$$4 = (2)^{\frac{n}{5}}$$

$$n = 10 \text{ hours}$$



IF YOU ARE NOT GIVEN ACTUAL NUMBERS FOR THE INITIAL AND FUTURE VALUES, YOU CAN DO IT ALGEBRAICALLY. KEEP THE INITIAL AMOUNT AS  $A_0$ , AND THEN EXPRESS THE FUTURE VALUE IN TERMS OF  $A_0$ . THIS WILL CANCEL OUT THE  $A_0$ , ALLOWING YOU TO SOLVE THE QUESTION. IN THIS EXAMPLE, THE FUTURE VALUE IS 4 TIMES GREATER THAN THE INITIAL AMOUNT, SO WRITE THE FUTURE VALUE AS  $4A_0$ .

Algebraic solution with logarithms:

$$A = A_0b^{\frac{t}{P}}$$

$$4A_0 = A_0(2)^{\frac{n}{5}}$$

$$4 = (2)^{\frac{n}{5}}$$

$$\log 4 = \log(2)^{\frac{n}{5}}$$

$$\log 4 = \frac{n}{5} \log 2$$

$$\frac{5 \log 4}{\log 2} = n$$

$$n = 10$$

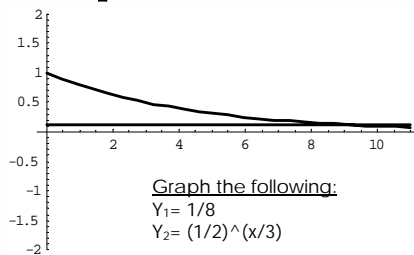
**Example 2:** A radioactive sample has a half life of 3 days. How long will it take for only 1/8 of the sample to remain?

$$A = A_0(b)^{\frac{t}{P}}$$

$$\frac{1}{8}A_0 = A_0\left(\frac{1}{2}\right)^{\frac{n}{3}}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{n}{3}}$$

$$n = 9 \text{ hours}$$



Algebraic solution with logarithms:

$$A = A_0b^{\frac{t}{P}}$$

$$\frac{1}{8}A_0 = A_0\left(\frac{1}{2}\right)^{\frac{n}{3}}$$

$$\frac{1}{8} = \left(\frac{1}{2}\right)^{\frac{n}{3}}$$

$$\log \frac{1}{8} = \log \left(\frac{1}{2}\right)^{\frac{n}{3}}$$

$$\log \frac{1}{8} = \frac{n}{3} \log \left(\frac{1}{2}\right)$$

$$3 \log \left(\frac{1}{8}\right) = n \log \left(\frac{1}{2}\right)$$

$$n = \frac{3 \log \left(\frac{1}{8}\right)}{\log \left(\frac{1}{2}\right)}$$

$$n = 9$$

Cross Multiply

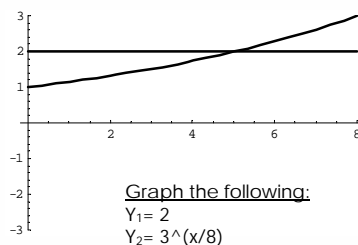
**Example 3:** The population of a town triples every 8 years. How many years will it take for the population to double?

$$A = A_0(b)^{\frac{t}{P}}$$

$$2A_0 = A_0(3)^{\frac{n}{8}}$$

$$2 = (3)^{\frac{n}{8}}$$

$$n = 5.05 \text{ years}$$



Algebraic solution with logarithms:

$$A = A_0b^{\frac{t}{P}}$$

$$2A_0 = A_0(3)^{\frac{n}{8}}$$

$$2 = (3)^{\frac{n}{8}}$$

$$\log 2 = \log(3)^{\frac{n}{8}}$$

$$\log 2 = \frac{n}{8} \log(3)$$

$$\frac{8 \log 2}{\log 3} = n$$

$$n = 5.05$$

Cross multiply and divide

### QUESTIONS:

- 1) A bacterial culture doubles every 3 hours. How long will it take for a culture to triple?
- 2) A radioactive sample has a half life of 2 days. How long will it take for only 1/6 of the sample to remain?
- 3) The population of a town quadruples every 20 years. How many years will it take for the population to double?

### ANSWERS:

- 1) 4.75 hours
- 2) 5.17 g
- 3) 10 years

# Logarithms Lesson 4

## Part IV – Percents of Initial & Final Amounts

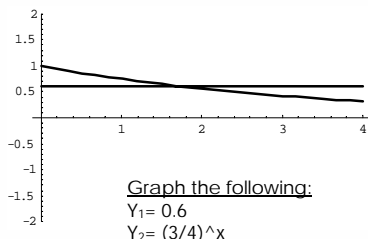
**Example 1:** Light passing through murky water retains  $\frac{3}{4}$  of its intensity for every metre of water. At what depth will the light intensity be 60% of what it is at the surface?

$$A = A_0(b)^{\frac{t}{P}}$$

$$0.6A_0 = A_0\left(\frac{3}{4}\right)^{\frac{n}{1}}$$

$$0.6 = \left(\frac{3}{4}\right)^n$$

$$n = 1.78 \text{ m}$$



THE FINAL AMOUNT IS ALWAYS WHAT YOU HAVE REMAINING.

IN THIS EXAMPLE, THE REMAINING LIGHT INTENSITY IS 60%, SO YOU CAN USE  $0.6A_0$  WITHOUT ANY PROBLEMS.

Algebraic solution with logarithms:

$$A = A_0b^{\frac{t}{P}}$$

$$0.6A_0 = A_0\left(\frac{3}{4}\right)^{\frac{n}{1}}$$

$$0.6 = \left(\frac{3}{4}\right)^n$$

$$\log 0.6 = \log \left(\frac{3}{4}\right)^n$$

$$\log 0.6 = n \log \left(\frac{3}{4}\right)$$

$$n = \frac{\log 0.6}{\log \left(\frac{3}{4}\right)}$$

$$n = 1.78$$

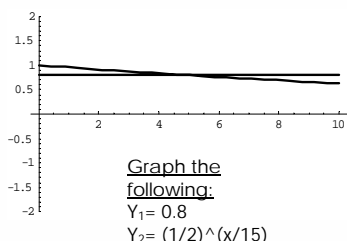
**Example 2:** The population of a town halves every 15 years. In how many years will 20% of the population have fled?

$$A = A_0(b)^{\frac{t}{P}}$$

$$0.8A_0 = A_0\left(\frac{1}{2}\right)^{\frac{n}{15}}$$

$$0.8 = \left(\frac{1}{2}\right)^{\frac{n}{15}}$$

$$n = 4.83 \text{ years}$$



IN THIS QUESTION, THE FUTURE AMOUNT WILL NOT BE  $0.2A_0$ , SINCE THAT IS THE AMOUNT LOST.

THE REMAINING AMOUNT WILL BE 80% OF THE POPULATION, SO USE  $0.8A_0$  ON THE LEFT SIDE.

Algebraic solution with logarithms:

$$A = A_0b^{\frac{t}{P}}$$

$$0.8A_0 = A_0\left(\frac{1}{2}\right)^{\frac{n}{15}}$$

$$0.8 = \left(\frac{1}{2}\right)^{\frac{n}{15}}$$

$$\log 0.8 = \log \left(\frac{1}{2}\right)^{\frac{n}{15}}$$

$$\log 0.8 = \frac{n}{15} \log \left(\frac{1}{2}\right)$$

$$n = \frac{15 \log 0.8}{\log \left(\frac{1}{2}\right)}$$

$$n = 4.83$$

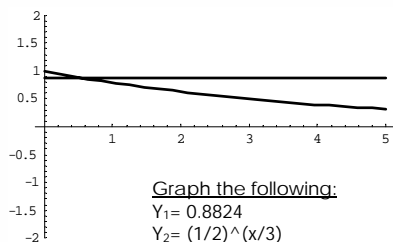
**Example 3:** A radioactive sample has a half life of 3 years, and has an initial mass of 68 g. How long will it take for the sample to lose 8 g?

$$A = A_0(b)^{\frac{t}{P}}$$

$$60 = 68\left(\frac{1}{2}\right)^{\frac{n}{3}}$$

$$0.8824 = \left(\frac{1}{2}\right)^{\frac{n}{3}}$$

$$n = 0.54 \text{ years}$$



THE FINAL

AMOUNT IS SIMPLY

$$68 \text{ g} - 8 \text{ g} = 60 \text{ g.}$$

Algebraic solution with logarithms:

$$A = A_0b^{\frac{t}{P}}$$

$$60 = 68(0.5)^{\frac{n}{3}}$$

$$0.8824 = (0.5)^{\frac{n}{3}}$$

$$\log 0.8824 = \log (0.5)^{\frac{n}{3}}$$

$$\log 0.8824 = \frac{n}{3} \log (0.5)$$

$$n = \frac{3 \log 0.8824}{\log (0.5)}$$

$$n = 0.54$$

### QUESTIONS:

1) Light passing through murky water retains  $\frac{6}{7}$  of its intensity of for every metre of water. At what depth will the light intensity be 68% of what it is at the surface?

2) The population of a town halves every 12 years. In how many years will 27% of the population have fled?

3) A radioactive sample has a half life of 2 years, and has an initial mass of 90 g. How long will it take for the sample to lose 10 g?

### ANSWERS:

- 1) 2.5 m
- 2) 5.45 years
- 3) 0.34 years

# Logarithms Lesson 4

## Part V – Rate Questions

**Example 1:** A crack in a window grows by 4.5 % every hour. If the crack starts at a length of 3 cm, how long will it be in 3 hours?

$$A = A_0 (b)^{\frac{t}{p}}$$

$$A = 3(1.045)^{\frac{3}{1}}$$

$$A = 3.42 \text{ cm}$$

USE 1.045 FOR THE B-VALUE SINCE WE ARE ADDING 4.5% ON TOP OF THE INITIAL VALUE FOR EVERY HOUR THAT PASSES.

### b - Values

IF THE RATE IS AN INCREASING PERCENT, **ADD** IT TO 1.

IF THE RATE IS A DECREASING PERCENT, **SUBTRACT** IT FROM 1.

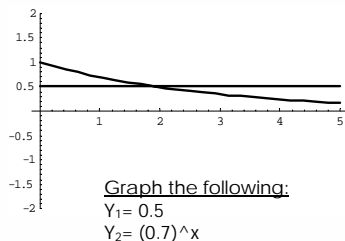
**Example 2:** Light passing through murky water loses 30% of it's intensity for every metre of water. At what depth will the light intensity be half of what it is at the surface?

$$A = A_0 (b)^{\frac{t}{p}}$$

$$\frac{1}{2} A_0 = A_0 (0.7)^{\frac{t}{1}}$$

$$\frac{1}{2} = (0.7)^t$$

$$t = 1.94 \text{ m}$$



USE 0.7 FOR THE B-VALUE, SINCE WE ARE RETAINING 70% OF THE LIGHT FOR EVERY METRE.

Algebraic solution with logarithms:

$$A = A_0 b^{\frac{t}{p}}$$

$$0.5 A_0 = A_0 (0.7)^t$$

$$0.5 = (0.7)^t$$

$$\log 0.5 = \log (0.7)^t$$

$$\log 0.5 = t \log (0.7)$$

$$t = \frac{\log 0.5}{\log 0.7}$$

$$t = 1.94$$

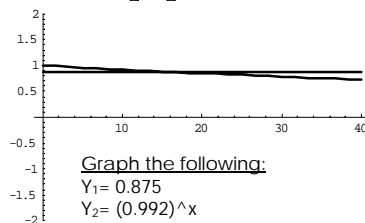
**Example 3:** A town loses 0.8% of its population every year. How many years will it take for a population of 8000 to become 7000?

$$A = A_0 (b)^{\frac{t}{p}}$$

$$7000 = 8000(0.992)^{\frac{t}{1}}$$

$$0.875 = (0.992)^t$$

$$t = 16.62 \text{ years}$$



USE 0.992 FOR THE B-VALUE SINCE WE ARE RETAINING 99.2 % OF THE POPULATION EACH YEAR.

Algebraic solution with logarithms:

$$A = A_0 b^{\frac{t}{p}}$$

$$7000 = 8000(0.992)^{\frac{t}{1}}$$

$$0.875 = (0.992)^t$$

$$\log 0.875 = \log (0.992)^t$$

$$\log 0.875 = t \log (0.992)$$

$$t = \frac{\log 0.875}{\log 0.992}$$

$$t = 16.62$$

### **QUESTIONS:**

- 1) A crack in a window grows by 6.5 % every hour. If the crack starts at a length of 3.7 cm, how long will it be in 4 hours?
- 2) Light passing through murky water loses 26% of it's intensity for every metre of water. At what depth will the light intensity be 1/3 of what it is at the surface?
- 3) A town loses 1.2% of its population every year. How many years will it take for a population of 10000 to drop under 9500?

### **ANSWERS:**

- 1) 4.76 cm
- 2) 3.65 m
- 3) 4.25 years



# Logarithms Lesson 4

## Part VI – Compound Interest

**COMPOUND INTEREST:** THE FORMULA FOR COMPOUND INTEREST IS  $A = A_0(1 + i)^N$

$A$  = FUTURE AMOUNT  
 $A_0$  = INITIAL AMOUNT  
 $i$  = INTEREST RATE PER COMPOUNDING PERIOD. (DIVIDE ANNUAL INTEREST RATE BY PERIOD)  
 $N$  = NUMBER OF COMPOUNDING PERIODS. (MULTIPLY YEARS BY PERIOD)

Common compounding periods are:

Annually: Once per year

Semi-Annually: Twice per year

Quarterly: Four times per year

Monthly: 12 times per year

Daily: 365 times per year.

**Example 1:** A sum of money is invested at 6%, compounded quarterly, for 5 years. What are the values required for  $i$  &  $n$ ?

$$i = 6\% \div 4 = 1.5\% = 0.015$$

$$n = 5 \times 4 = 20$$

**Example 2:** A sum of money is invested at 0.7%, compounded semi-annually, for 3 years. What are the values required for  $i$  &  $n$ ?

$$i = 0.7\% \div 2 = 0.35\% = 0.0035$$

$$n = 3 \times 2 = 6$$

**Example 3:** \$5000 is invested at 7.2% compounded annually for 4 years. What is the amount of money at the end of the 4 years?

$$i = 7.2\% \div 1 = 7.2\% = 0.072$$

$$n = 4 \times 1 = 4$$

$$A = 5000(1 + 0.072)^4 = \$6603.12$$

**Example 4:** \$2300 is invested at 6% compounded monthly for 7 years. How much interest is earned?

$$i = 6\% \div 12 = 0.5\% = 0.005$$

$$n = 7 \times 12 = 84$$

$$A = 2300(1 + 0.005)^{84} = \$3496.85$$

$$\text{Interest earned} = \$3496.85 - \$2300 = \$1196.85$$

# Logarithms Lesson 4

## Part VI – Compound Interest

**Example 5:** \$300 is invested at 10% compounded quarterly. How many years must it stay in the bank to double?

$$r = 10\% \div 4 = 2.5\% = 0.025$$

$$A = 300(1 + 0.025)^n$$

$$600 = 300(1.025)^n$$

$$2 = (1.025)^n$$

$$\log 2 = \log 1.025^n$$

$$\log 2 = n \log 1.025$$

$$n = \frac{\log 2}{\log 1.025} = 28$$

$n = 28$  represents the number of compounding periods.

Since the compounding is done 4 times per year, that means the money has been in the bank for **7 years**.

### QUESTIONS:

1) \$8000 is invested at 5.6% compounded semi-annually for 7 years. What is the amount of money at the end of the 7 years?

2) \$2500 is invested at 9.4% compounded monthly for 1.5 years. How much interest is earned?

3) \$380 is invested at 5% compounded quarterly. How many years must it stay in the bank to triple?

### ANSWERS:

1) \$11775.89

2) \$376.98

3) 22.1 years



# Logarithms Lesson 4

## Part VII – Earthquakes

**EARTHQUAKE INTENSITY: THE FORMULA FOR EARTHQUAKES IS:**  $\frac{I}{I_0} = 10^{M-M_0}$

$\frac{I}{I_0}$  = INTENSITY RATIO OF EARTHQUAKES  
 $M$  = RICHTER MAGNITUDE OF STRONGER EARTHQUAKE  
 $M_0$  = RICHTER MAGNITUDE OF WEAKER EARTHQUAKE

The **Richter Scale** is a logarithmic way of measuring earthquake strength.

**Example 1:** an earthquake of magnitude 6 is ten times stronger than a magnitude 5 earthquake.

**Example 2:** an earthquake of magnitude 7 is 100 times stronger than a magnitude 5 earthquake.

**Example 1:** A small tremor of magnitude 3.4 is followed by a stronger one of magnitude 4.1. How much stronger is the second tremor than the first?

$$\frac{I}{I_0} = 10^{M-M_0}$$

$$\frac{I}{I_0} = 10^{4.1-3.4}$$

$$\frac{I}{I_0} = 5.01$$

The larger earthquake is 5 times stronger than the weaker.

**Example 2:** A weak earthquake has a magnitude of 6.5, and the following day a strong earthquake occurs with double the intensity. What is the magnitude of the stronger earthquake?

$$\frac{I}{I_0} = 10^{M-M_0}$$

$$2 = 10^{M-6.5}$$

$$\log 2 = \log 10^{M-6.5}$$

$$\log 2 = (M - 6.5) \log 10$$

$$\log 2 = (M - 6.5)(1)$$

$$\log 2 = M - 6.5$$

$$M = \log 2 + 6.5$$

$$M = 6.80$$

**Example 3:** A strong earthquake with a magnitude of 6.7 is three times more intense than a weaker earthquake. What is the magnitude of the weaker earthquake?

$$\frac{I}{I_0} = 10^{M-M_0}$$

$$3 = 10^{6.7-M_0}$$

$$\log 3 = \log 10^{6.7-M_0}$$

$$\log 3 = (6.7 - M_0) \log 10$$

$$\log 3 = (6.7 - M_0)(1)$$

$$\log 3 = 6.7 - M_0$$

$$M_0 = 6.7 - \log 3$$

$$M_0 = 6.22$$

### QUESTIONS:

1) A small tremor of magnitude 2.3 is then followed by a stronger one of magnitude 5.3. How much stronger is the second tremor than the first?

2) An earthquake has a magnitude of 6.7, and the following week a stronger earthquake occurs with six times the intensity. What is the magnitude of the stronger earthquake?

3) An earthquake with a magnitude of 5.9 is one thousand times more intense than a weaker earthquake. What is the magnitude of the weaker earthquake?

ANSWERS: 1) 1000, 2) 7.48 3) 2.9

# Logarithms Lesson 4

## Part VIII – Sound Loudness

**SOUND:** THE FORMULA FOR SOUND INTENSITY IS:  $dB - dB_0 = 10 \log \left( \frac{I}{I_0} \right)$

$dB$  = DECIBLE LEVEL OF LOUD SOUND;  $dB_0$  = DECIBLE LEVEL OF SOFT SOUND  
 $\frac{I}{I_0}$  = INTENSITY RATIO OF SOUNDS

**Example 1:** A sound of 80 dB is 45 times louder than a weaker sound.

What is the loudness of the weaker sound?

$$dB - dB_0 = 10 \log \left( \frac{I}{I_0} \right)$$

$$80 - dB_0 = 10 \log (45)$$

$$80 - dB_0 = 16.53$$

$$dB_0 = 80 - 16.53$$

$$dB_0 = 63.47 \text{ dB}$$

**Example 2:** One sound has a loudness of 78 dB, and another sound has a loudness of 100 dB. How much louder is the second sound?

$$dB - dB_0 = 10 \log \left( \frac{I}{I_0} \right)$$

$$100 - 78 = 10 \log \left( \frac{I}{I_0} \right)$$

$$22 = 10 \log \left( \frac{I}{I_0} \right)$$

$$2.2 = \log \left( \frac{I}{I_0} \right) \quad \text{Use the "seven rule"}$$

$$\frac{I}{I_0} = 10^{2.2}$$

$$\frac{I}{I_0} = 158.5$$

The louder sound is 158.5 times louder than the weaker sound.

**Example 3:** If the intensity of one sound is 10 000 times stronger than another sound, what is the difference in decibels between the two sounds?

$$\Delta dB = 10 \log \left( \frac{I}{I_0} \right)$$

$$\Delta dB = 10 \log (10000)$$

$$\Delta dB = 40$$

The louder sound is 40 dB greater than the weaker sound.

### QUESTIONS:

1) A sound of 70 dB is 8900 times louder than weaker sound.  
What is the loudness of the weaker sound?

2) One sound has a loudness of 97 dB, and another sound has a loudness of 110 dB. How many times louder is the second sound?

3) If the intensity of one sound is 500 000 times stronger than another sound, what is the difference in decibels between the two sounds?

### ANSWERS:

- 1) 30.5 dB
- 2) 20
- 3) 57 dB

# Logarithms Lesson 4

## Part IX - Acids

**pH:** THE FORMULA FOR ACID STRENGTH IS:  $pH = -\log[H^+]$

$pH$  = ACID STRENGTH (POWER OF HYDROGEN)  
 $H^+$  = CONCENTRATION OF HYDROGEN ION

**Example 1:** A beaker of acid has a hydrogen concentration of  $3.5 \times 10^{-6}$  mol/L

Calculate the pH of the acid.

$$pH = -\log(H^+)$$

$$pH = -\log(3.5 \times 10^{-6})$$

$$pH = 5.46$$

THE FORMULA FOR FINDING  $[H^+]$ , GIVEN THE pH, IS  $[H^+] = 10^{-pH}$

**Example 2:** If a beaker of acid has a pH of 4.1, Calculate the hydrogen concentration of the acid.

$$[H^+] = 10^{-pH}$$

$$[H^+] = 10^{-4.1}$$

$$[H^+] = 7.94 \times 10^{-5} \text{ mol/L}$$

### DERIVATION OF ABOVE FORMULA:

$$pH = -\log[H^+]$$

TAKE THE NEGATIVE TO THE OTHER SIDE OF THE EQUATION TO ISOLATE THE LOGARITHM

$$-pH = \log_{10}[H^+]$$

NOW USE THE SEVEN RULE

$$10^{-pH} = [H^+]$$

**Example 3:** A beaker of acid has a pH of 4.9, and a second beaker has a pH of 7.6. Determine how many times higher the hydrogen concentration is in the stronger acid as compared to the weaker one.

Step 1: Calculate  $[H^+]$  in the stronger acid

$$[H^+] = 10^{-4.9} = 1.26 \times 10^{-5}$$

Step 2: Calculate  $[H^+]$  in the weaker acid

$$[H^+] = 10^{-7.6} = 2.512 \times 10^{-8}$$

Step 3: Divide the results:  $\frac{\text{strong}}{\text{weak}} = \frac{1.26 \times 10^{-5}}{2.512 \times 10^{-8}} = 501$

### QUESTIONS:

- 1) A beaker of acid has a hydrogen concentration of  $8.9 \times 10^{-3}$  mol/L. Calculate the pH of the acid.
- 2) If a beaker of acid has a pH of 8.7, Calculate the hydrogen concentration of the acid.
- 3) A beaker of acid has a pH of 4.5, and a second beaker has a pH of 1.9. Determine how much higher the hydrogen concentration is in the second beaker as compared to the first beaker.

### ANSWERS:

- 1) 2.05
- 2)  $2 \times 10^{-9}$  mol/L
- 3) 398 times stronger.