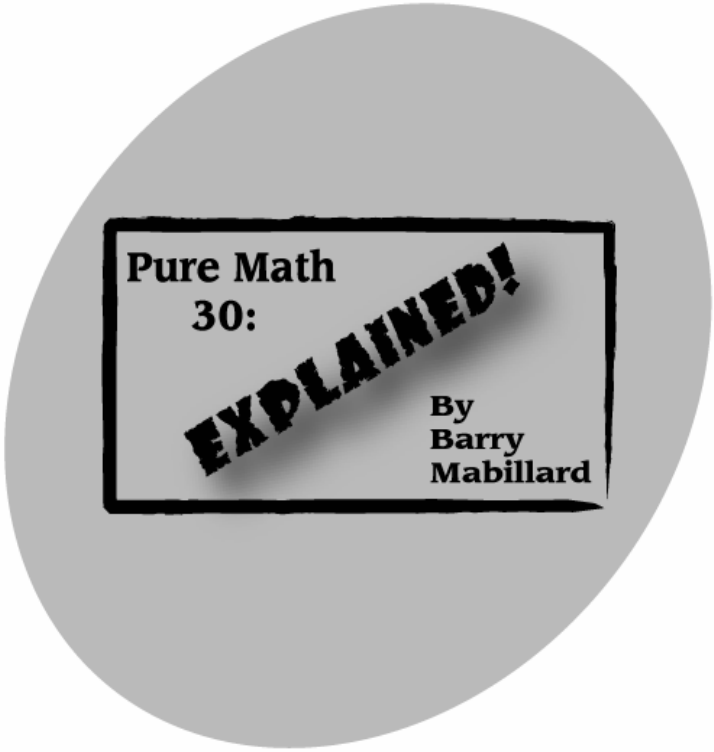
Pure Math 30:

Exponential and Logarithmic Functions

$$y = \log_2\left(\frac{A}{B}\right)$$

LESSON 1

Graphing Exponential / Logarithmic Functions

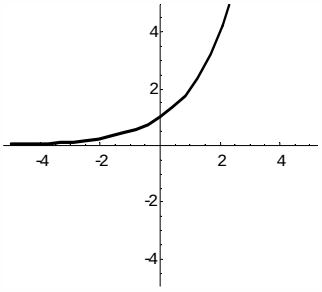


LOGARITHMS LESSON 1
PART I: EXPONENTIAL FUNCTIONS

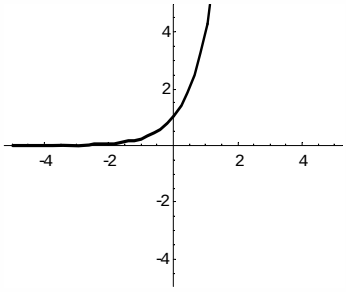
EXPONENTIAL FUNCTIONS: THESE ARE FUNCTIONS WHERE THE VARIABLE IS AN EXPONENT.

The first type of exponential graph occurs when the base has a value greater than 1:

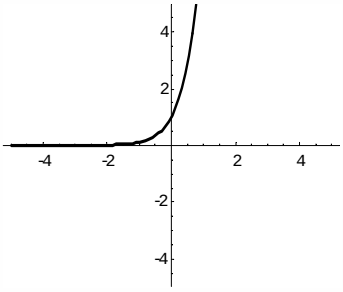
$y = 2^x$



$y = 4^x$



$y = 8^x$



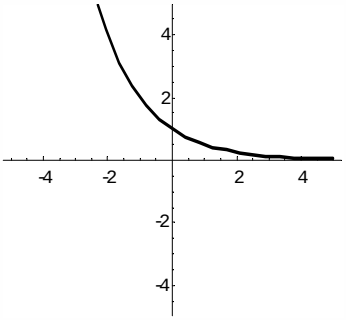
$y = b^x$

$b > 1$

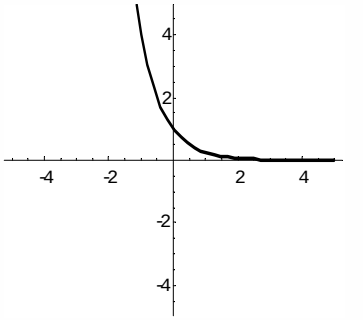
When the base is greater than 1, the graph starts low and gets higher from left to right.

The second type occurs when the base has a value between 0 and 1.

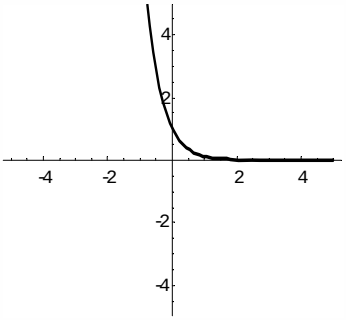
$y = \left(\frac{1}{2}\right)^x$



$y = \left(\frac{1}{4}\right)^x$



$y = \left(\frac{1}{8}\right)^x$



$y = b^x$

$0 < b < 1$

When the base is between 0 and 1, the graph starts high and gets lower from left to right.

- *Notice that the point (0,1) is common to all untransformed exponential graphs. This is because anything raised to the power of zero is one! You can use this feature as an "anchor point" when drawing these graphs.*
- *All exponential graphs have a **horizontal asymptote**. In the above graphs, the asymptote occurs along the x-axis. (Equation: y = 0)*
- *The domain of untransformed exponential graphs is $x \in R$ since the graph goes left & right forever. The range is $y > 0$. (The $\geq$ symbol is NOT used, due to the presence of the asymptote.)*
- *Remember all of the above rules are based on untransformed exponential graphs. Once transformations are involved, these points & lines will move.*

LOGARITHMS LESSON 1
PART II: LOGARITHMIC FUNCTIONS

LOGARITHMIC FUNCTIONS: A LOGARITHMIC FUNCTION IS THE INVERSE OF AN EXPONENTIAL FUNCTION.

$$y = \log_b x \leftarrow \text{Variable}$$

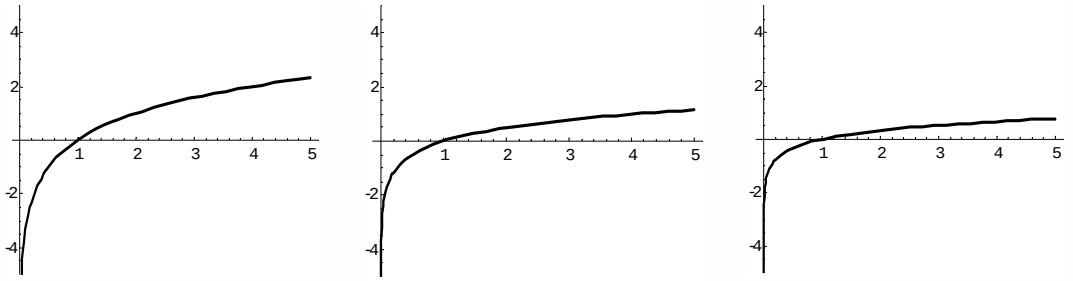
b ↑ **Base**

To draw log graphs in your TI-83, you must type in log(variable) / log(base).

Example: To graph $\log_2 x$, you would type in log(x) ÷ log(2)

There are two basic types of log graphs you will need to memorize:
The first type occurs when the base of the logarithm is bigger than 1.

$y = \log_2 x$ $y = \log_4 x$ $y = \log_8 x$



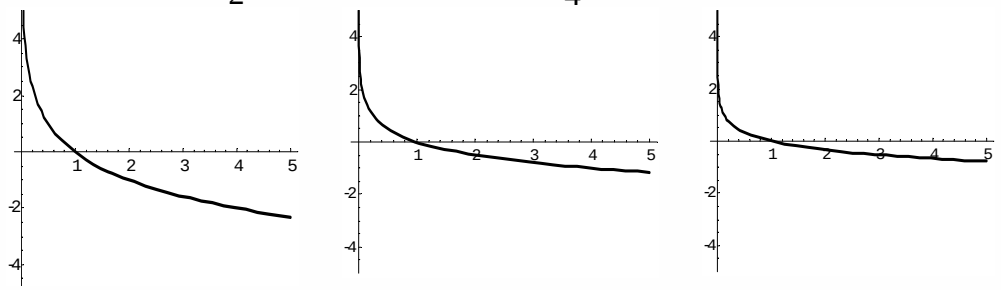
$y = \log_b x$

$b > 1$

When the base is greater than 1, the graph starts low and gets higher from left to right.

The second type is when the base of the logarithm is between 0 and 1.

$y = \log_{\frac{1}{2}} x$ $y = \log_{\frac{1}{4}} x$ $y = \log_{\frac{1}{8}} x$



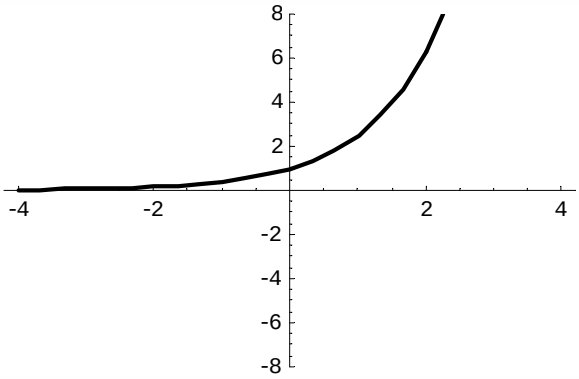
$y = \log_b x$

$0 < b < 1$

When the base is between 0 & 1, the graph starts high and gets lower from left to right.

- *Notice that the point (1,0) is common to all untransformed log graphs. This occurs because a log graph is the inverse of an exponential graph. So, if exponential graphs have the point (0,1), it follows that log graphs should pass through (1,0)*
- *log graphs have a vertical asymptote. In the above graphs, the asymptote occurs along the y-axis. (Equation: x = 0)*
- *The domain of untransformed log graphs is* $x > 0$ *since the graph is always to the right of the vertical asymptote.*
- *The range is* $y \in R$ *since the graph goes up & down forever.*
- *If you ever have negative numbers, 0, or 1 as a base, no graph exists since the logarithm is undefined.*
- *Remember that transformations will change the above values.*

Example 1: Given $y = \left(\frac{5}{2}\right)^x$, answer the following:



a) Draw the graph

Graph in your calculator as (5 ÷ 2) ^ x
Use the window settings:
x: [-4 , 4 , 1]
y: [-8 , 8 , 2]

b) What is the domain & range

The domain is $x \in \mathbb{R}$
The range is $y > 0$

c) What is the equation of the asymptote?

The asymptote is the x-axis, so the equation is y = 0

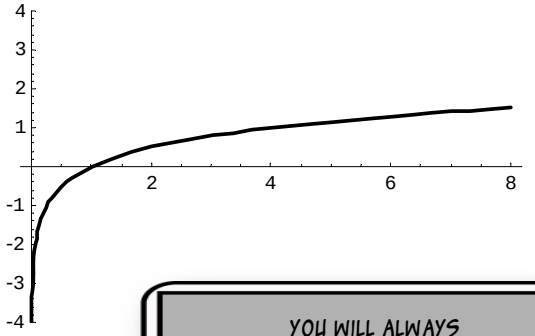
d) What are the x & y intercepts?

There is no x-intercept due to the asymptote..
Find the y-intercept by using 2nd → Trace → Value → x = 0 in your TI-83. Answer = (0, 1)

e) What is the value of the graph when x = 2?

You could plug x = 2 into the equation and solve, but an easier way is to use the TI-83.
Go 2nd → Trace → Value → x = 2. This will give you the resulting y-value automatically.
Answer = 6.25

Example 2: Given $y = \log_4 x$, answer the following:



a) Draw the graph

Graph in your calculator as log(x) ÷ log(4)
Use the window settings:
x: [0 , 8 , 1]
y: [-4 , 4 , 1]

b) What is the domain & range

The domain is $x > 0$ due to the vertical asymptote at the y-axis.
The range is $y \in \mathbb{R}$

c) What is the equation of the asymptote?

The asymptote is the y-axis, so x = 0

d) What are the x & y intercepts?

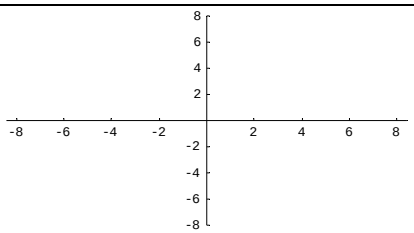
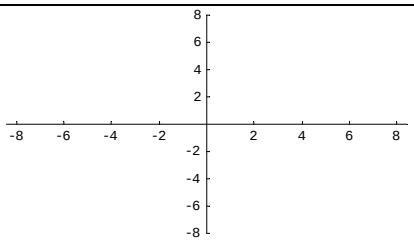
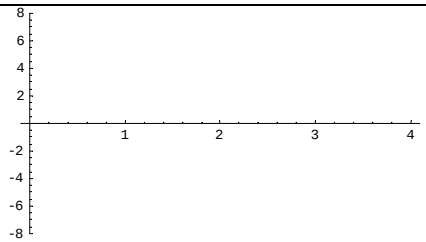
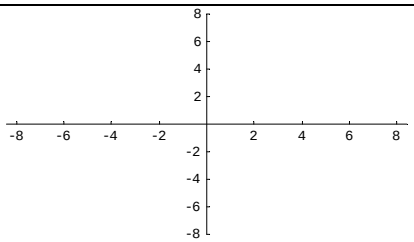
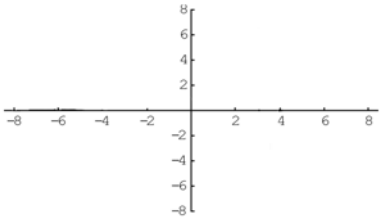
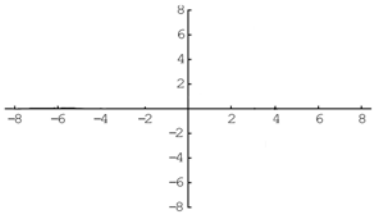
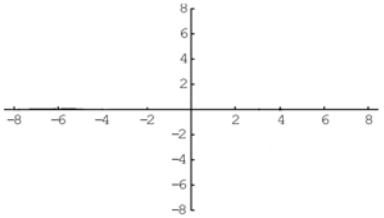
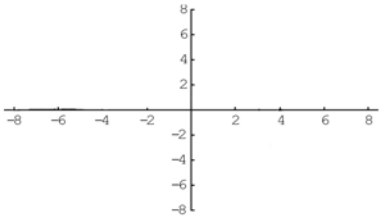
The x – intercept can be found by going 2nd → Trace → Zero in your TI-83. Answer = (1, 0)
There is no y-intercept due to the vertical asymptote at the y-axis.

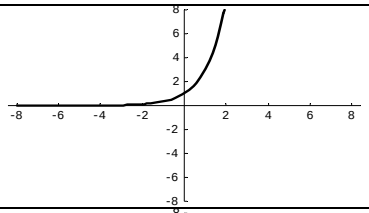
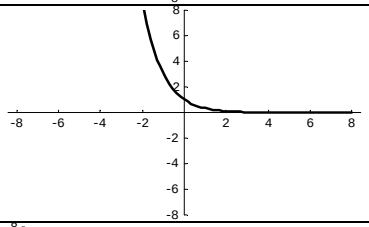
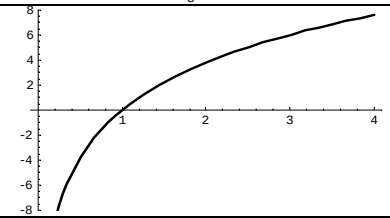
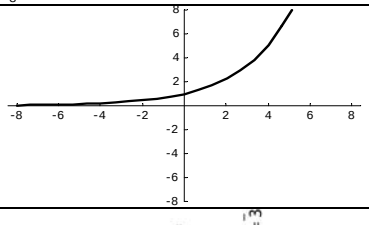
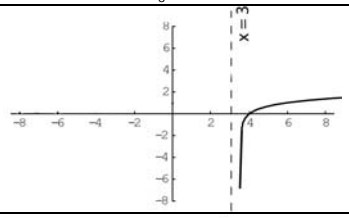
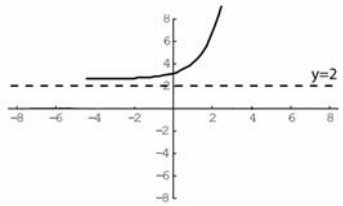
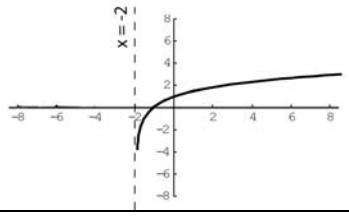
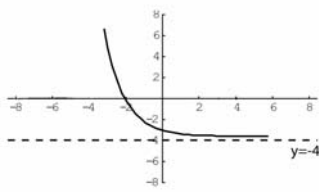
e) *What is the value of the graph when x = 2?*
Go 2nd → Trace → Value → x = 2
Answer = 0.5

YOU WILL ALWAYS HAVE TO TYPE LOGARITHMS INTO YOUR CALCULATOR AS A FRACTION... WITH ONE EXCEPTION:

A LOGARITHM WITHOUT A BASE, SUCH AS Y = LOG X, CAN BE TYPED IN AS IS AND YOU'LL GET THE PROPER GRAPH.

LOGARITHMS WITHOUT BASES ARE CALLED COMMON LOGARITHMS. THEY ACTUALLY HAVE A BASE OF 10, IT'S JUST NOT WRITTEN IN. THE LOG BUTTON ON YOUR CALCULATOR IS A COMMON LOGARITHM.

Function	Graph	Domain	Range	Equation of Asymptote	x-intercept	y-intercept	y-value when x = 2
$y = 3^x$							
$y = \left(\frac{1}{3}\right)^x$							
$y = \log_{1.2} x$							
$y = \left(\frac{3}{2}\right)^x$							
$y = \log(x-3)$							
$y = 2^x + 2$							
$y = \log(x+2)$							
$y = \left(\frac{1}{2}\right)^x - 4$							

Function	Graph	Domain	Range	Equation of Asymptote	x-intercept	y-intercept	y-value when x = 2
$y = 3^x$		*x ε R*	*y > 0*	*y = 0*	*None*	*1*	*9*
$y = \left(\frac{1}{3}\right)^x$		*x ε R*	*y > 0*	*y = 0*	*None*	*1*	$\frac{1}{9}$
$y = \log_{1.2} x$		*x > 0*	*y ε R*	*x = 0*	*1*	*None*	*3.80*
$y = \left(\frac{3}{2}\right)^x$ Notice how the base is greater than 1.		*x ε R*	*y > 0*	*y = 0*	*None*	*1*	*2.25*
$y = \log(x-3)$		*x > 3*	*y ε R*	*x = 3*	*4*	*None*	*Undefined*
$y = 2^x + 2$		*x ε R*	*y > 2*	*y = 2*	*None*	*3*	*6*
$y = \log(x+2)$		*x > -2*	*y ε R*	*x = -2*	*-1*	*0.30*	*0.60*
$y = \left(\frac{1}{2}\right)^x - 4$		*x ε R*	*y > -4*	*y = -4*	*-2*	*-3*	*-3.75*

LOGARITHMS LESSON 1
PART III: EXPONENTIAL REGRESSION

EXPONENTIAL REGRESSION: USING THE TI-83, IT IS POSSIBLE TO FIND AN EXPONENTIAL EQUATION FROM A LIST OF DATA.

Example 1: Given the following data, determine the exponential regression equation.

x	y
0	2
1	2.13
2	2.28
3	2.49
4	2.74
5	3.13
6	3.78

Step 1: *Type: Stat → Edit to bring up the list function of your calculator.*

Step 2: *Fill in the x-values for L1 and the y-values for L2. (If there is data in the first list, put the cursor at the very top over L1 and type clear → enter to empty the entire column.)*

Step 3: *Type 2nd → Quit to return to the main screen.*

Step 4: *Type Stat → Calc → ExpReg (Just hit the zero button to bring it up quickly) → Enter*

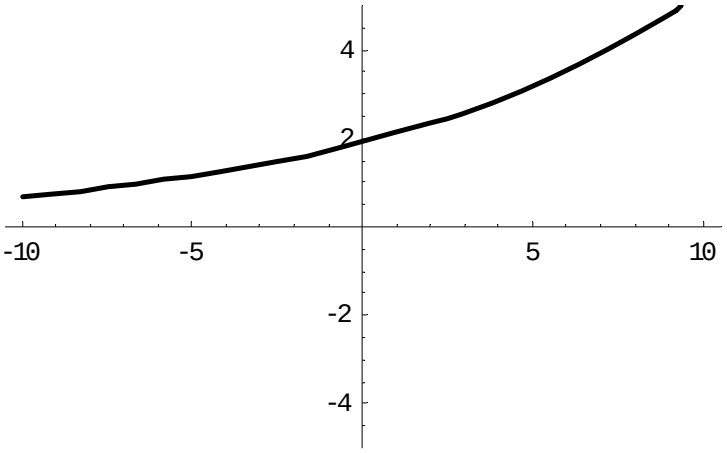
The screen that comes up next tells you the equation is of the form $y = a(b)^x$*, and the numerical values for a & b are given below.*

The exponential regression equation is $y = 1.9065(1.1077)^x$

Step 5: *If you draw the graph, manually type this equation in as you would any other graph.*

**Alternatively, you could copy the equation by doing the following:*

Type:
Y= → Vars → Statistics → EQ → RegEQ



Example 2: Given the following table:

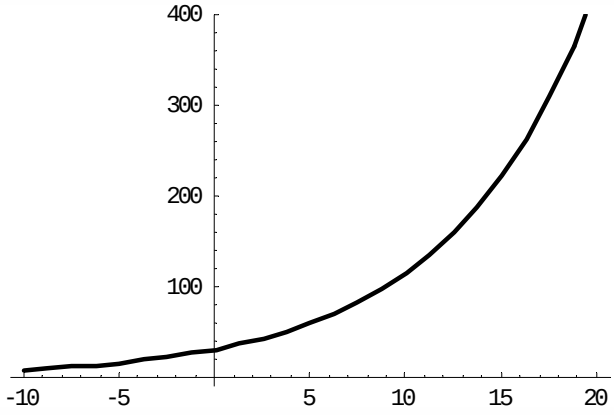
x	y
2	45
5	56
7	74
10	103
11	132
16	246
17	301

a) Determine the exponential regression equation.

Using the steps from the previous example, the regression equation is:

$y = 31.0840(1.1383)^x$

b) Draw the graph.



c) Find y when x = 9

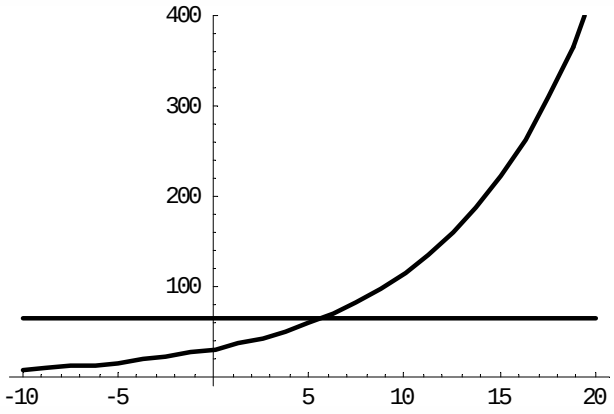
Now that you have the graph in your TI-83, type:

2nd → Trace → Value → x = 9

Answer: y = 99.7449

d) Find x when y = 65

In order to solve this, draw in a second graph, the line y = 65.



Now find the point of intersection of these two graphs, and the x-value will be the solution.

Answer: x = 5.6944

LOGARITHMS LESSON 1

PART III: EXPONENTIAL REGRESSION

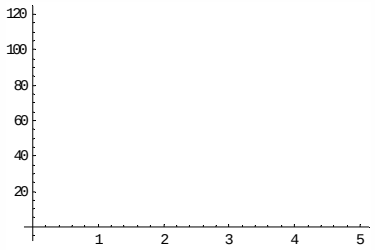
QUESTIONS: DETERMINE THE EXPONENTIAL REGRESSION EQUATION FOR EACH OF THE FOLLOWING SETS OF DATA. GRAPH EACH RESULT.

1.

x	y
0	54
1	60
2	69
3	84
4	98
5	113

a) Determine the regression equation.

b) Draw the graph.



c) Find y when x = 9 (Nearest Hundredth)

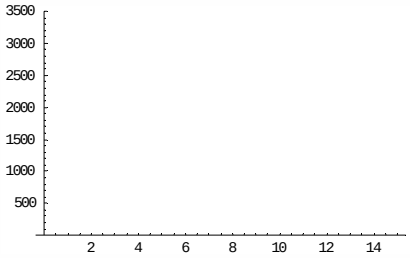
d) Find x when y = 633 (Nearest Hundredth)

2.

x	y
0	934
3	990
6	1120
9	1439
12	2138
15	3244

a) Determine the regression equation.

b) Draw the graph.



c) Find y when x = 32 (Nearest Hundredth)

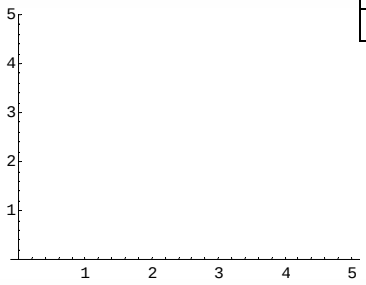
d) Find x when y = 10000 (Nearest Hundredth)

3.

x	y
0	0.05
1	0.1
2	0.4
3	0.8
4	1.6
5	3.2

a) Determine the regression equation.

b) Draw the graph.



c) Find y when x = 10 (Nearest Hundredth)

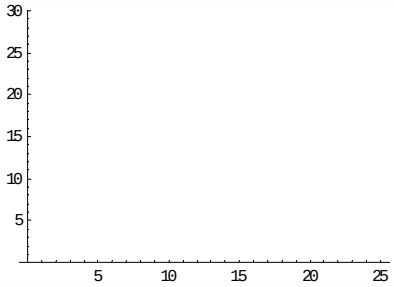
d) Find x when y = 0.08 (Nearest Hundredth)

4.

x	y
0	30.9
5	14.7
10	7.5
15	4.2
20	2.3
25	1

a) Determine the regression equation.

b) Draw the graph.



c) Find y when x = 3 (Nearest Hundredth)

d) Find x when y = 7 (Nearest Hundredth)

LOGARITHMS LESSON 1

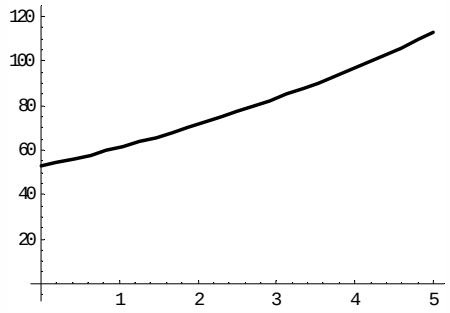
PART III: EXPONENTIAL REGRESSION

ANSWERS:

1.

a) $52.4854(1.1655)^x$

b)



c) Find y when x = 9

y = 208.30

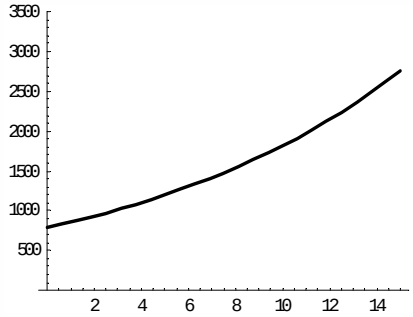
d) Find x when y = 633

16.26 (By graphing)

2.

a) $787.9764(1.0873)^x$

b)



c) Find y when x = 32

11464.94

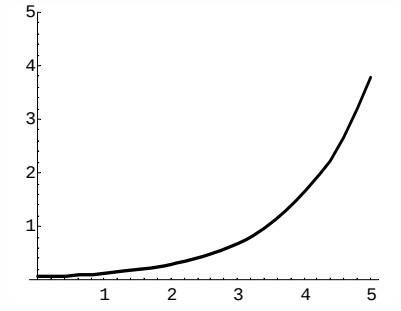
d) Find x when y = 10000

30.37

3.

a) $0.0534(2.3433)^x$

b)



c) Find y when x = 10

266.69

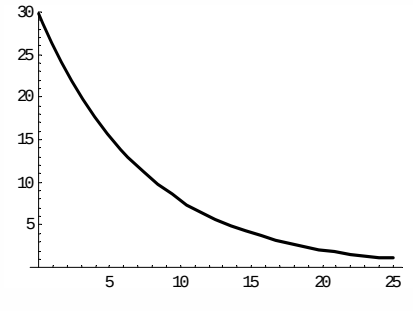
d) Find x when y = 0.08

0.47 (By graphing)

4.

a) $29.8969(0.8753)^x$

b)



c) Find y when x = 3

20.05

d) Find x when y = 7

10.91 (By graphing)