Pure Math 30

EXPLAINED!

Diploma Style Practice Exam

Exponential and Logarithmic Functions

-- ANSWERS --

Logarithms Diploma Style Practice Exam

Answers

1. **C**	10. **D**	19. **A**	27. **C**
2. **A**	11. **C**	20. **B**	28. **D**
3. **D**	12. **C**	NR 5. **158**	29. **C**
4. **C**	NR 3. **100**	NR 6. **2019**	30. **D**
5. **B**	13. **B**	21. **B**	31. **B**
6. **D**	14. **C**	22. **B**	32. **B**
NR 1. **15**	NR 4. **5.40**	23. **C**	33. **B**
7. **B**	15. **C**	24. **B**	
8. **C**	16. **C**	NR 7. **39.8**	
NR 2. **4.00**	17. **B**	25. **C**	
9. **A**	18. **D**	26. **B**	

1) Graphical Solution: The question tells you that $b > 0$, so you could graph $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$ to see what each case would look like. These graphs are symmetrical with respect to the y – axis ($x = 0$).

Algebraic solution: $\left(\frac{1}{b}\right)^x = \left(b^{-1}\right)^x = b^{-x}$. Compared to the original of b^x **,** it's reflected in the y-axis.

The answer is **C**

2) To graph $y = \log_3 x$, change of base is required since the calculator only accepts base 10 logs.

In your calculator, you would use $y_1 = \frac{\log x}{\log 3}$ and $y_2 = x - 6$. The answer is **A.**

3)

$$\log_{\frac{1}{5}}\left(\frac{1}{x}\right)$$

$= \dfrac{\log\left(\frac{1}{x}\right)}{\log\left(\frac{1}{5}\right)}$ Change of Base

$= \dfrac{\text{log1-logx}}{\text{log1-log5}}$ Division Law

$= \dfrac{\text{-logx}}{\text{-log5}}$ Since log1=0

$= \dfrac{\text{logx}}{\text{log5}}$ Cancel out the negatives

$= \log_5 x$ Change of Base in reverse

The answer is **D**.

4)

$P = 1 - w^{-0.246t}$

$0.83 = 1 - w^{-0.246(43)}$ Plug in P = 0.83 and t = 43

$0.83 = 1 - w^{-10.578}$ Subtract 1 on both sides

$-0.17 = -w^{-10.578}$ Cancel out the negatives

$0.17 = w^{-10.578}$

$\left(0.17\right)^{\frac{1}{-10.578}} = \left(w^{-10.578}\right)^{\frac{1}{-10.578}}$ Isolate w by raising each side to the reciprocal exponent

$w = 1.18$

The answer is **C.**

(You can also solve this equation by graphing $y_1 = 0.83$ *and* $y_2 = 1 - w^{-0.246(43)}$*, then find the x-value of the point of intersection.)*

5)

$\log_x\left(y^3z\right) - \log_x\left(yz^2\right)$

$= \log_x \dfrac{y^3z}{yz^2}$

$= \log_x \dfrac{y^2}{z}$

The answer is **B.**

6)

$7 = (3+b)^4$

$[7]^{\frac{1}{4}} = \left[(3+b)^4\right]^{\frac{1}{4}}$ Raise to reciprocal exponents

$\sqrt[4]{7} = 3 + b$ Convert fractional exponent to a radical

$b = \sqrt[4]{7} - 3$

The answer is **D**.

NR #1)

$5\log_2 x + 5\log_2 y$

$= 5\left(\log_2 x + \log_2 y\right)$ Factor out the 5

$= 5\log_2 xy$ Multiplication Law

$= 5\log_2 8$ We know xy=8

$= 5(3)$ Evaluate $\log_2 8$ using change of base

$= 15$

The answer is **15**.

7) Graph $y_1 = 2^{3x}$ and $y_2 = 5^{-x-1}$ in your calculator and find the *x*-value of the point of intersection. Remember to keep your exponent in brackets! The answer is **B**.

8) The initial amount A_0 is 32. The final amount A is 8. The length of time t is 21 hours. The growth b is ½. We want to solve for P.

$$A = A_0 (b)^{\frac{t}{P}}$$

$$8 = 32\left(\frac{1}{2}\right)^{\frac{21}{P}}$$

$$\frac{8}{32} = \left(\frac{1}{2}\right)^{\frac{21}{P}}$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{21}{P}}$$

$$\left(\frac{1}{2}\right)^{2} = \left(\frac{1}{2}\right)^{\frac{21}{P}}$$

$$2 = \frac{21}{P}$$

$$2P = 21$$

$$P = 10.5$$

The half life is 10.5 hours, which is 630 minutes. The answer is **C**

NR #2)

$$a^{5x} = \left(\log_c c^a\right)^{3x+8}$$

$$a^{5x} = \left(a \log_c c\right)^{3x+8} \qquad \text{Power Law}$$

$$a^{5x} = \left(a\right)^{3x+8} \qquad \log_c c = 1$$

$$5x = 3x + 8 \qquad \text{Common Base}$$

$$2x = 8$$

$$x = 4$$

The answer is **4.00**.

9) The point $(0, a)$ can be transformed to the point $(a, 0)$ by drawing the inverse graph.
The answer is **A.**

10) Use the formula $A = A_0 (b)^{\frac{t}{P}}$

A = the future score S
A_0 = the initial score 50000
t = elapsed days d
b = rate. Decreasing percentage; subtract this decimal from 1. (0.973)
P = the percentage loss is per day, so the period is 1.

Plug these into the formula to get $S = 50000(0.973)^d$
The answer is **D.**

11)

$$dB = 10\log\left(10^{12} \bullet I\right)$$

$$\frac{dB}{10} = \log\left(10^{12} \bullet I\right)$$

$$10^{\frac{dB}{10}} = 10^{12} \bullet I$$

$$I = \frac{10^{\frac{dB}{10}}}{10^{12}}$$

$$I = 10^{\frac{dB}{10} - 12}$$

Common denominator: $\frac{dB}{10} - 12 = \frac{dB}{10} - \frac{120}{10} = \frac{dB - 120}{10}$

$$I = 10^{\frac{dB-120}{10}}$$

The answer is **C**

12)

$$I = 10^{\frac{dB-120}{10}}$$

$$I = 10^{\frac{150-120}{10}}$$

$$I = 10^{\frac{30}{10}}$$

$$I = 10^3$$

$$I = 1000$$

The answer is **C**.

NR #3)

$$\log_b\left(\frac{1}{b^{-100}}\right)$$

$$\log_b\left(b^{100}\right)$$

$$100\log_b b$$

$$= 100$$

The answer is **100**

13)

Rewrite as: $y = 3^x + 4$

Swap *x* & *y* to get: $x = 3^y + 4$

Bring 4 to the left side: $x - 4 = 3^y$

Convert to log form (remember *a base is always a base*)

$\log_3(x-4) = y$

Rewrite as $f^{-1}(x) = \log_3(x-4)$

The answer is **B.**

14)

$x = (b)^{-y}$

$\log x = \log b^{-y}$ Solve for y by taking the log of both sides

$\log x = -y \log b$

$y = -\dfrac{\log x}{\log b}$

$y = -\log_b x$ Change of base in reverse

The negative in front indicates a reflection in the x-axis.
The answer is **C.**

NR 4)

$T(t) = T_0 e^{-kt}$

$65 = 82(2.718)^{-0.043t}$

$1.261 = (2.718)^{-0.043t}$ $\rightarrow$ Solve by graphing & point of intersection. Keep exponent in brackets!

$t = 5.40$ minutes

15) Graph $f(x) = 4^x$ and $g(x) = \log_4 x = \dfrac{\log x}{\log 4}$ in your calculator, and notice the reflection line is $y = x$

The answer is **C**.

16) Rewrite $y = g(3x - 12) + 2$ as $y = g\left[3(x-4)\right] + 2$ to see that the graph has been shifted 4 units right. Since a logarithm graph has a vertical asymptote along the *y*-axis, the asymptote is shifted 4 units right to make the line $x = 4$. Thus, the domain is $x > 4$
The answer is **C**.

17) If you are solving two equations graphically, the *x*-value of the point of intersection is what you require. B is the incorrect procedure.
The answer is **B**.

18)

$$4^{-2y} = x$$

$$\log 4^{-2y} = \log x$$

$$-2y \log 4 = \log x$$

$$y = -\frac{\log x}{2 \log 4}$$

Graphing this expression *(remember to keep the denominator in brackets)* gives graph **D.**

19)

$$f(x) = 7a^{2+x} - b$$

$$0 = 7a^{2+x} - b$$

$$b = 7a^{2+x}$$

$$\frac{b}{7} = a^{2+x}$$

$$\log\left(\frac{b}{7}\right) = \log a^{2+x}$$

$$\log b - \log 7 = (2 + x)\log a$$

$$\frac{\log b - \log 7}{\log a} = 2 + x$$

$$x = \frac{\log b - \log 7}{\log a} - 2$$

The answer is **A**.

20)

$\log_{27}(81a) = b$

$27^b = 81a$

$a = \dfrac{27^b}{81}$

$a = \dfrac{\left(3^3\right)^b}{3^4}$

$a = \dfrac{3^{3b}}{3^4}$

$a = 3^{3b-4}$

The answer is **B**.

NR #5) Determine the exponential regression equation for the data. Don't use the actual years for the regression - use elapsed time. For example, 2001 can be replaced with 1 since one full year has passed.

Year	1	2	3	4	5
Profit	\$28 000	\$32 000	\$40 000	\$49 000	\$60 000

$y = 22409.56(1.215)^x$

Determine the profit in 2010 by plugging in x = 10, since you want the tenth year.
The resulting value is \$157567.21, or 158 thousand dollars.

Answer = **158**

NR #6) Graph $y = 22409.56(1.215)^x$ and $y = 800000$. The *x*-value of the point of intersection gives the value 18.33. Since the 18th year is under \$800000, the first time the business will be over \$800000 is the 19th year.

Answer = **2019**

21) Graph both equations and find the *x*-value of the point of intersection. This occurs at 25.25 years, so the second business will overtake the first business during the 25th year.

The answer is **B**

22)

$a^{\frac{5}{4}} = 2b$

$\left(a^{\frac{5}{4}}\right)^{\frac{4}{5}} = (2b)^{\frac{4}{5}}$

$a = (2b)^{\frac{4}{5}}$

The answer is **B**

23)

$\log_x(6-x)$

Rewrite as $\frac{\log(6-x)}{\log x}$

The numerator is defined for $x < 6$
The denominator is defined for $x > 0$

The entire graph is defined between 0 and 6, with the exception of $x = 1$ since that makes the denominator zero.
The answer is **C**

24)

$y = b\log_c ax$

$0 = b\log_c ax$

$0 = \log_c ax$

$c^0 = ax$

$1 = ax$

$x = \frac{1}{a}$

The answer is **B**

NR #7) Group like terms

$2\log x + 3\log x = 8$

$5\log x = 8$

$\log x = \frac{8}{5}$

$10^{\frac{8}{5}} = x$

$x = 39.8$

The answer is **39.8**

25)

$\left(\frac{1}{3}\right)3^{234}$

$\frac{3^{234}}{3}$

$= 3^{234-1}$

$= 3^{233}$

The answer is **C.**

26)

$\log_a x + y = \log_a z$

$y = \log_a z - \log_a x$

$y = \log_a\left(\frac{z}{x}\right)$

The answer is **B.**

27)

$A = A_0(b)^{\frac{t}{P}}$

$A = 60(2)^{\frac{33}{20}}$

$A = 188.30$

The answer is **C.**

28)

$P = 100000(1.03)^t$

$\log P = \log\left[100000 \bullet (1.03)^t\right]$

$\log P = \log 100000 + \log(1.03)^t$

$\log P = 5 + t\log 1.03$

$\log P - 5 = t\log 1.03$

$t = \frac{\log P - 5}{\log 1.03}$

The answer is **D.**

29)

$\log(2-x)+\log(2+x)=\log 3$

$\log(2-x)(2+x)=\log 3$

$(2-x)(2+x)=3$

$4-x^2=3$

$1=x^2$

$x=\pm 1$

The answer is **C.**

30)

$\log_6\left(\frac{1}{36}x\right)$

$\log_6\left(\frac{x}{36}\right)$

$\log_6 x-\log_6 36$

$=120-2$

$=118$

The answer is **D.**

31)

$\left(a^{\log_b c}\right)\left(a^{\log_b c}\right)$

$=a^{\log_b c+\log_b c}$

$=a^{2\log_b c}$

$=\left(a^2\right)^{\log_b c}$

The answer is **B.**

32) The graph has been moved down by three units. There is a horizontal asymptote at the line y = -3, and the graph is above this line. The range is y > -3

The answer is **B.**

33)

$\log(x+2)+\log(x-1)=1$

$\log(x+2)(x-1)=1$

$10^1=(x+2)(x-1)$

$10=x^2+x-2$

$0=x^2+x-12$

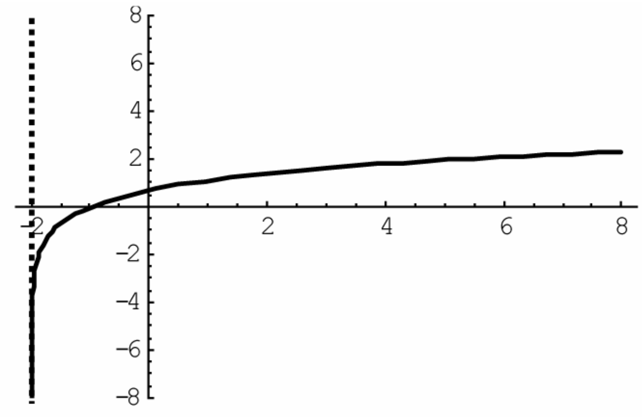
$0=(x+4)(x-3)$

$x=-4,3$ Reject -4

x = 3

The answer is **B**.

Written Response 1:



Domain	$x > -2$
Range	$y \varepsilon R$
Equation of Asymptote	$x = -2$
***x*-intercept**	(-1, 0)
***y*-intercept**	(0, 0.30)
***y*-value when *x* = 2**	0.60

- $\log(x+2)$
  Horizontal translation of 2 units left.

- The domain of a logarithmic expression can be found by setting what is in the brackets greater than zero.
  For the expression $a\log(bx+c)+d$

$$bx+c>0$$

$$bx>-c$$

$$x>-\frac{c}{b}$$

Written Response 2:

- **Solving Graphically**

  Graph $y_1 = 27 \bullet 81^{x-2}$

  $y_2 = 243^{-2x}$

A window setting that will let you see the point of intersection clearly is
x: [-1 , 1 , 0.1]
y: [-0.1 , 0.1 , 0.01]

The answer is $x = 0.357$

- **Solve Using a Common Base**

$27 \bullet 81^{x-2} = 243^{-2x}$

$(3^3) \bullet (3^4)^{x-2} = (3^5)^{-2x}$

$3^3 \bullet 3^{4x-8} = 3^{-10x}$

$3^{4x-5} = 3^{-10x}$

$4x - 5 = -10x$

$14x = 5$

$x = \dfrac{5}{14}$

$x = 0.357$

- **Solve Using Logarithms**

$27 \bullet 81^{x-2} = 243^{-2x}$

$\log 27 \bullet 81^{x-2} = \log 243^{-2x}$

$\log 27 + \log 81^{x-2} = \log 243^{-2x}$

$\log 27 + (x-2)\log 81 = -2x \log 243$

$\log 27 + x\log 81 - 2\log 81 = -2x \log 243$

$x\log 81 + 2x\log 243 = 2\log 81 - \log 27$

$x(\log 81 + 2\log 243) = 2\log 81 - \log 27$

$x = \dfrac{2\log 81 - \log 27}{\log 81 + 2\log 243}$

$x = 0.357$

- Graph $y_1 = \dfrac{\log x}{\log 3}$

  $y_2 = 4$

Use a window of
x: [0, 500, 100]
y: [-10, 10, 1]

Answer: $x = 256$

Written Response 3:

- **Solve for P**

$$A = A_0\left(b\right)^{\frac{t}{P}}$$

$$\frac{A}{A_0} = b^{\frac{t}{P}}$$

$$\log\left(\frac{A}{A_0}\right) = \log b^{\frac{t}{P}}$$

$$\log A - \log A_0 = \frac{t}{P}\log b$$

$$P\left(\log A - \log A_0\right) = t\log b$$

$$t = \frac{P\left(\log A - \log A_0\right)}{\log b}$$

- **Plug in your values and solve by graphing.** ***(Or use the equation derived above.)***

$$A = A_0\left(b\right)^{\frac{t}{P}}$$

$$93000 = 60000\left(2\right)^{\frac{3}{P}}$$

$$1.55 = 2^{\frac{3}{P}}$$

$$P = 4.745 \text{ hours}$$

- **If the population of the town doubles, the initial amount is A_0 and the final amount is $2A_0$. Simplify and solve by graphing.**

$$A = A_0\left(b\right)^{\frac{t}{P}}$$

$$2A_0 = A_0\left(3\right)^{\frac{t}{8}}$$

$$2 = \left(3\right)^{\frac{t}{8}}$$

$$t = 5.05 \text{ years}$$

- **If the light intensity is 64% of the initial amount, the initial amount is A_0 and the final amount is $0.64A_0$. Simplify and solve by graphing.**

$$A = A_0\left(b\right)^{\frac{t}{P}}$$

$$0.64A_0 = A_0\left(\frac{3}{4}\right)^{t}$$

$$0.64 = \left(\frac{3}{4}\right)^{t}$$

$$t = 1.55 \text{ m}$$

- **Subtract the rate from 1 since it is a decreasing percent. Simplify and solve by graphing.**

$$A = A_0\left(b\right)^{\frac{t}{P}}$$

$$\frac{1}{2}A_0 = A_0\left(0.957\right)^{t}$$

$$\frac{1}{2} = \left(0.957\right)^{t}$$

$$t = 15.77 \text{ years}$$