Pure Math 30:

Exponential and Logarithmic Functions

$y = \log_2\left(\frac{A}{B}\right)$

Lesson 3

Algebraically Solving Logarithm Equations

Pure Math 30: EXPLAINED!

By Barry Mabillard

Part I – Raising Reciprocals

EQUATIONS WITH X AS THE BASE:

Reciprocal Exponents
To get x by itself, raise both sides to the reciprocal exponent.

Example 1: Solve for x in: $x^3 = 2$

$x^3 = 2$

$\left(x^3\right)^{\frac{1}{3}} = (2)^{\frac{1}{3}}$

$x = (2)^{\frac{1}{3}}$

$x = 1.26$

Example 2: $x^{\frac{1}{4}} = 6$

$x^{\frac{1}{4}} = 6$

$\left(x^{\frac{1}{4}}\right)^4 = (6)^4$

$x = 1296$

Example 3: $x^{-\frac{3}{5}} = 8$

$x^{-\frac{3}{5}} = 8$

$\left(x^{-\frac{3}{5}}\right)^{-\frac{5}{3}} = (8)^{-\frac{5}{3}}$

$x = \frac{1}{32}$

Example 4: $(3x)^{\frac{2}{3}} = 2$

$(3x)^{\frac{2}{3}} = 2$

$\left((3x)^{\frac{2}{3}}\right)^{\frac{3}{2}} = (2)^{\frac{3}{2}}$

$3x = 2.8284$

$x = 0.9428$

Example 5: $x^2 = \frac{1}{3}$

$x^2 = \frac{1}{3}$

$\left(x^2\right)^{\frac{1}{2}} = \left(\frac{1}{3}\right)^{\frac{1}{2}}$

$x = 0.577$

Example 6: If $\log_{(x+2)} 5 = 10$, then what is x?

$\log_{(x+2)} 5 = 10$

$(x+2)^{10} = 5$

$\left[(x+2)^{10}\right]^{\frac{1}{10}} = (5)^{\frac{1}{10}}$

$x + 2 = 1.175$

$x = -0.825$

Example 7: $\log_x\left(\frac{125}{27}\right) = -\frac{3}{2}$

$\log_x\left(\frac{125}{27}\right) = -\frac{3}{2}$

$x^{-\frac{3}{2}} = \frac{125}{27}$

$\left(x^{-\frac{3}{2}}\right)^{-\frac{2}{3}} = \left(\frac{125}{27}\right)^{-\frac{2}{3}}$

$x = \frac{9}{25}$

When you see the terms expand and condense, you can leave a logarithm in the final answer.
When you see the word solve, you must get x by itself.

QUESTIONS: SOLVE FOR X

1) $x^5 = 4$

2) $x^{\frac{1}{3}} = 7$

3) $x^{-\frac{5}{7}} = 6$

4) $(2x)^{\frac{3}{2}} = 3$

5) $x^{-6} = \frac{1}{8}$

6) If $\log_{(x+3)}4 = 19$, then what is x?

7) $\log_x\left(\frac{16}{64}\right) = -\frac{2}{3}$

ANSWERS:

1) 1.32
2) 343
3) 0.0814
4) 1.04
5) 1.41
6) -1.92
7) 8

Logarithms Lesson 3

Part III – Common Bases

EQUATIONS WITH COMMON BASES:

Example 1: $2^x = 8^{3x-1}$

$2^x = 8^{3x-1}$

$2^x = \left(2^3\right)^{3x-1}$

$2^x = 2^{9x-3}$

$x = 9x - 3$

$-8x = -3$

$x = \frac{3}{8}$

Once the bases are the same, you can set the exponents equal to each other and solve for x

Common Base

THESE EQUATIONS CAN BE SOLVED BY FINDING A COMMON BASE AND THEN SOLVING FOR THE EXPONENTS.

Example 2: $9^{2x-4} = 27^x$

$9^{2x-4} = 27^x$

$\left(3^2\right)^{2x-4} = \left(3^3\right)^x$

$3^{4x-8} = 3^{3x}$

$4x - 8 = 3x$

$x = 8$

Example 3: $\left(\frac{9}{4}\right)^{4x} = \left(\frac{8}{27}\right)^{x-2}$

$\left(\frac{9}{4}\right)^{4x} = \left(\frac{8}{27}\right)^{x-2}$

$\left(\left(\frac{3}{2}\right)^2\right)^{4x} = \left(\left(\frac{2}{3}\right)^3\right)^{x-2}$

$\left(\frac{3}{2}\right)^{8x} = \left(\frac{2}{3}\right)^{3x-6}$

$\left(\frac{2}{3}\right)^{-8x} = \left(\frac{2}{3}\right)^{3x-6}$

$-8x = 3x - 6$

$-11x = -6$

$x = \frac{6}{11}$

When you flip the base around, it changes the sign of the exponent.

Example 4: $16^{3x} = 64^x \bullet 8^{3x-1}$

$16^{3x} = 64^x \bullet 8^{3x-1}$

$\left(2^4\right)^{3x} = \left(2^6\right)^x \bullet \left(2^3\right)^{3x-1}$

$2^{12x} = 2^{6x} \bullet 2^{9x-3}$

$2^{12x} = 2^{6x+9x-3}$

$12x = 15x - 3$

$-3x = -3$

$x = 1$

Example 5: If $X = 16^m \bullet 8^{2m-1}$, then $\log_2 x$ is:

$X = 16^m \bullet 8^{2m-1}$

$X = \left(2^4\right)^m \bullet \left(2^3\right)^{2m-1}$

$X = 2^{4m} \bullet 2^{6m-3}$

$X = 2^{4m+6m-3}$

$X = 2^{10m-3}$

$\log_2 X = \log_2 2^{10m-3}$

$= (10m - 3)\log_2 2$

$= 10m - 3$

Example 6: $\frac{1}{3}(27)^{x-1} = \sqrt[4]{9^x}$

$\frac{1}{3}(27)^{x-1} = \sqrt[4]{9^x}$

$\frac{1}{3}(3^3)^{x-1} = \sqrt[4]{\left(3^2\right)^x}$

$\frac{1}{3}3^{3x-3} = \sqrt[4]{3^{2x}}$

$\frac{3^{3x-3}}{3} = 3^{\frac{2x}{4}}$

$3^{(3x-3)-1} = 3^{\frac{x}{2}}$

$3x - 4 = \frac{x}{2}$

$6x - 8 = x$

$5x = 8$

$x = \frac{8}{5}$

Part III – Common Bases

QUESTIONS: SOLVE FOR X

1) $3^{3x} = 9^{2x-1}$

4) $9^{6x} = 81^{8x} \bullet 27^{2x-5}$

2) $16^{3x-6} = 8^{x}$

5) If $A = 32^{m} \times 64^{3m-1}$, then $\log_2 A =$

3) $\left(\frac{16}{81}\right)^{2x} = \left(\frac{8}{27}\right)^{3x-7}$

6) $\frac{1}{2}(8)^{x-6} = \sqrt[4]{4^{x}}$

ANSWERS:

1) 2
2) $\frac{8}{3}$
3) 21
4) 0.577
5) 23m-6
6) 7.6

EQUATIONS WITHOUT COMMON BASES:

Log Both Sides

EQUATIONS WITHOUT COMMON BASES CAN BE SOLVED BY TAKING THE LOG OF EACH SIDE, THEN SIMPLIFYING.

Example 1:
$3^x = 4$

$$3^x = 4$$
$$\log 3^x = \log 4$$
$$x\log 3 = \log 4$$
$$x = \frac{\log 4}{\log 3}$$
$$x = 1.26$$

Example 2:
$2 \bullet 5^{x+2} = 7$

$$2 \bullet 5^{x+2} = 7$$
$$5^{x+2} = \frac{7}{2}$$
$$\log 5^{x+2} = \log\frac{7}{2}$$
$$(x+2)\log 5 = \log\frac{7}{2}$$
$$x\log 5 + 2\log 5 = \log\frac{7}{2}$$
$$x\log 5 = \log\frac{7}{2} - 2\log 5$$
$$x = \frac{\log\frac{7}{2} - 2\log 5}{\log 5}$$
$$x = -1.22$$

Example 3:
$2 \bullet 3^{x+2} = 6$

$$2 \bullet 3^{x+2} = 6$$
$$3^{x+2} = \frac{6}{2}$$
$$3^{x+2} = 3$$
$$x + 2 = 1$$
$$x = -1$$

Example 4:
$5^x = -2$

$$5^x = -2$$
$$\log 5^x = \log(-2)$$
No Solution

Example 5: $\left(\frac{2}{5}\right)^{x-3} = \frac{1}{3}$

$$\left(\frac{2}{5}\right)^{x-3} = \frac{1}{3}$$
$$\log\left(\frac{2}{5}\right)^{x-3} = \log\frac{1}{3}$$
$$(x-3)\log\left(\frac{2}{5}\right) = \log\frac{1}{3}$$
$$x\log\left(\frac{2}{5}\right) - 3\log\left(\frac{2}{5}\right) = \log\frac{1}{3}$$
$$x\log\left(\frac{2}{5}\right) = \log\frac{1}{3} + 3\log\left(\frac{2}{5}\right)$$
$$x = \frac{\log\frac{1}{3} + 3\log\left(\frac{2}{5}\right)}{\log\left(\frac{2}{5}\right)}$$
$$x = 4.199$$

Example 6:
$a^{x+3} = b^{-1}$

$$a^{x+3} = b^{-1}$$
$$\log a^{x+3} = \log b^{-1}$$
$$(x+3)\log a = -\log b$$
$$x\log a + 3\log a = -\log b$$
$$x = \frac{-\log b - 3\log a}{\log a}$$

Example 7: If
$\log x + \log x + \log x = 6$, solve for x

$$\log x + \log x + \log x = 6$$
$$3\log x = 6$$
$$\log x = 2$$
$$10^2 = x$$
$$x = 100$$

Example 8:
$\log_4(x^2+1) - \log_4 6 = \log_4 5$

$$\log_4(x^2+1) - \log_4 6 = \log_4 5$$
$$\log_4\frac{x^2+1}{6} = \log_4 5$$
$$\frac{x^2+1}{6} = 5$$
$$x^2 + 1 = 30$$
$$x^2 = 29$$
$$x = \pm\sqrt{29}$$

When you arrive at a step where you have a log on one side and a log on the other (with no extra terms) you can cancel the logs and keep going.

Logarithms Lesson 3

Part IIII – No Common Bases

QUESTIONS: SOLVE FOR X

1) $4^x = 5$

5) $\left(\frac{4}{5}\right)^{2x-3} = \frac{1}{4}$

2) $3 \bullet 7^{x+5} = 8$

6) $a^{x+2} = b^{-5}$

3) $3 \bullet 4^{x+6} = 2$

7) If $\log_x a + \log_x a = 8$, then x =

4) $4^x = -7$

8) $\log_3(x^2 + 5) - \log_3 8 = \log_3 3$

ANSWERS:

1) 1.16
2) -4.50
3) -6.29
4) Undefined
5) 4.61
6) $\frac{-5\log b - 2\log a}{\log a}$
7) $x = a^{\frac{1}{4}}$
8) ± 4.36

Logarithms Lesson 3

Part IV – Solving By Grouping

SOLVE BY GROUPING:

Factoring x

THESE EQUATIONS WILL REQUIRE YOU TO BRING THE TERMS INVOLVING X TO ONE SIDE, FACTOR THEM, AND THEN SOLVE.

Example 1:
$6^{5x} = 3^{2x-1}$

$6^{5x} = 3^{2x-1}$

$\log 6^{5x} = \log 3^{2x-1}$

$5x\log 6 = (2x-1)\log 3$

$5x\log 6 = 2x\log 3 - \log 3$

$5x\log 6 - 2x\log 3 = -\log 3$

$x(5\log 6 - 2\log 3) = -\log 3$

$x = \dfrac{-\log 3}{5\log 6 - 2\log 3}$

$x = -0.1625$

Example 2:
$2^{x+3} = 3^{2x-1}$

$2^{x+3} = 3^{2x-1}$

$\log 2^{x+3} = \log 3^{2x-1}$

$(x+3)\log 2 = (2x-1)\log 3$

$x\log 2 + 3\log 2 = 2x\log 3 - \log 3$

$x\log 2 - 2x\log 3 = -\log 3 - 3\log 2$

$x(\log 2 - 2\log 3) = -\log 3 - 3\log 2$

$x = \dfrac{-\log 3 - 3\log 2}{\log 2 - 2\log 3}$

$x = 2.11$

Example 3:
$\dfrac{4^{2x-1}}{3} = 5^x$

$\dfrac{4^{2x-1}}{3} = 5^x$

$\log\dfrac{4^{2x-1}}{3} = \log 5^x$

$\log 4^{2x-1} - \log 3 = \log 5^x$

$(2x-1)\log 4 - \log 3 = x\log 5$

$2x\log 4 - \log 4 - \log 3 = x\log 5$

$2x\log 4 - x\log 5 = \log 4 + \log 3$

$x(2\log 4 - \log 5) = \log 4 + \log 3$

$x = \dfrac{\log 4 + \log 3}{2\log 4 - \log 5}$

$x = 2.14$

Example 4:
$2 \bullet 3^{x+3} = 6^{3x}$

$2 \bullet 3^{x+3} = 6^{3x}$

$\log\left(2 \bullet 3^{x+3}\right) = \log 6^{3x}$

$\log 2 + \log 3^{x+3} = \log 6^{3x}$

$\log 2 + (x+3)\log 3 = 3x\log 6$

$\log 2 + x\log 3 + 3\log 3 = 3x\log 6$

$x\log 3 - 3x\log 6 = -\log 2 - 3\log 3$

$x(\log 3 - 3\log 6) = -\log 2 - 3\log 3$

$x = \dfrac{-\log 2 - 3\log 3}{\log 3 - 3\log 6}$

$x = 0.9327$

Example 5: $3 \bullet 4^{2x+3} = 7^{4x-2}$

$3 \bullet 4^{2x+3} = 7^{4x-2}$

$\log\left(3 \bullet 4^{2x+3}\right) = \log 7^{4x-2}$

$\log 3 + \log 4^{2x+3} = \log 7^{4x-2}$

$\log 3 + (2x+3)\log 4 = (4x-2)\log 7$

$\log 3 + 2x\log 4 + 3\log 4 = 4x\log 7 - 2\log 7$

$2x\log 4 - 4x\log 7 = -2\log 7 - \log 3 - 3\log 4$

$x(2\log 4 - 4\log 7) = -2\log 7 - \log 3 - 3\log 4$

$x = \dfrac{-2\log 7 - \log 3 - 3\log 4}{2\log 4 - 4\log 7}$

$x = 1.826$

Example 6:
$\log_3(x) - \log_3 2 = \log_3 7$

$\log_3(x) - \log_3 2 = \log_3 7$

$\log_3\left(\dfrac{x}{2}\right) = \log_3 7$

$\dfrac{x}{2} = 7$

$x = 14$

Logarithms Lesson 3

Part IV – Solving By Grouping

QUESTIONS: SOLVE FOR X

1: $6^{5x} = 3^{2x-1}$

2: $3^{2x+1} = 4^{5x-1}$

3: $\dfrac{5^{3x-1}}{2} = 6^{x}$

4: $3 \bullet 5^{2x+6} = 4$

5: $2 \bullet 3^{3x+2} = 5^{4x-2}$

6: $\log_2(x) - \log_2 4 = \log_2 6$

ANSWERS:

1) -0.16
2) 0.52
3) 0.76
4) -2.91
5) 1.94
6) 24

SOLVING USING LOG RULES:

Example 1: $3\log x + 5 = 8$

$3\log x + 5 = 8$

$3\log x = 3$

$\log x = 1$

$10^1 = x$

$x = 10$

Example 2: $3\log_2 x = 12$

$3\log_2 x = 12$

$\log_2 x = 4$

$2^4 = x$

$x = 16$

Example 3: $\log_3(x-2) = \log_3(3x+2)$

$\log_3(x-2) = \log_3(3x+2)$

$x - 2 = 3x + 2$

$-2x = 4$

$x = -2$ Reject -2 since it would make the original logarithm undefined

Example 4: $2\log_5 3 = \log_5(x+1)$

$2\log_5 3 = \log_5(x+1)$

$\log_5 3^2 = \log_5(x+1)$

$9 = x + 1$

$x = 8$

Example 5: $2\log_2 x + \log_2 16 = \log_2 9$

$2\log_2 x + \log_2 16 = \log_2 9$

$\log_2 x^2 + \log_2 16 = \log_2 9$

$\log_2 16x^2 = \log_2 9$

$16x^2 = 9$

$x^2 = \frac{9}{16}$

$x = \frac{3}{4}$ Reject $-\frac{3}{4}$ since it would make the original logarithm undefined

Example 6: $5\log_3 x = 2\log_3 b$

$5\log_3 x = 2\log_3 b$

$\log_3 x^5 = \log_3 b^2$

$x^5 = b^2$

$(x^5)^{\frac{1}{5}} = (b^2)^{\frac{1}{5}}$

$x = b^{\frac{2}{5}}$

Example 7:
$\log_2(x+1) + \log_2(x-2) = 3$

$\log_2(x+1) + \log_2(x-2) = 3$

$\log_2(x+1)(x-2) = 3$

$2^3 = (x+1)(x-2)$

$8 = x^2 - x - 2$

$0 = x^2 - x - 10$ Solve by graphing and finding the x - intercepts

$x = 3.70$ Reject -2.70 since it would make the original logarithm undefined

Example 8: $\log_3(x-1) = 2 - \log_3(x+2)$

$\log_3(x-1) = 2 - \log_3(x+2)$

$\log_3(x-1) + \log_3(x+2) = 2$

$\log_3(x-1)(x+2) = 2$

$3^2 = (x-1)(x+2)$

$9 = x^2 + x - 2$

$0 = x^2 + x - 11$

$x = 2.85$ Reject -3.85 since it would make the original logarithm undefined

Example 9: $\log_2 x = 3 - \log_2(x+2)$

$\log_2 x = 3 - \log_2(x+2)$

$\log_2 x + \log_2(x+2) = 3$

$\log_2 x(x+2) = 3$

$8 = x(x+2)$

$8 = x^2 + 2x$

$0 = x^2 + 2x - 8$

$x = 2$ Reject -4

Part V – Solving Using Log Rules

Example 10: $\log x - \log(x+5) = 1$

$$\log x - \log(x+5) = 1$$
$$\log\left(\frac{x}{x+5}\right) = 1$$
$$10^1 = \frac{x}{x+5}$$
$$10 = \frac{x}{x+5}$$
$$10x + 50 = x$$
$$9x = -50$$
$$x = -5.56$$ Reject, so no solution.

Example 11: $\log_3(2x+1) - \log_3(x-1) = 1$

$$\log_3(2x+1) - \log_3(x-1) = 1$$
$$\log_3\left(\frac{2x+1}{x-1}\right) = 1$$
$$3 = \frac{2x+1}{x-1}$$
$$3x - 3 = 2x + 1$$
$$x = 4$$

Example 12: $\log x^2 + \log 3 = \log 2x$

$$\log x^2 + \log 3 = \log 2x$$
$$\log 3x^2 = \log 2x$$
$$3x^2 = 2x$$
$$3x^2 - 2x = 0$$
$$x(3x-2) = 0$$
$$x = \frac{2}{3}$$ Reject 0

Example 13: $\log_{(x-1)} 25 = 2$

$$\log_{(x-1)} 25 = 2$$
$$(x-1)^2 = 25$$
$$\sqrt{(x-1)^2} = \sqrt{25}$$
$$x - 1 = \pm 5$$
$$x = 6$$ Reject - 4

Example 14:
$2\log(x-3) = \log 4 + \log(6-x)$

$$2\log(x-3) = \log 4 + \log(6-x)$$
$$\log(x-3)^2 = \log 4(6-x)$$
$$(x-3)^2 = 4(6-x)$$
$$x^2 - 6x + 9 = 24 - 4x$$
$$x^2 - 2x - 15 = 0$$
(Factor, or graph and find x - intercepts)
$$x = 5$$ Reject - 3

Example 15:
$(\log x)^2 - 4\log x - 5 = 0$

$$(\log x)^2 - 4\log x - 5 = 0$$
$$a^2 - 4a - 5 = 0$$
$$(a-5)(a+1) = 0$$
$$(\log x - 5)(\log x + 1) = 0$$

$$\log x - 5 = 0$$
$$\log x = 5$$
$$100000 = x$$

$$\log x + 1 = 0$$
$$\log x = -1$$
$$x = 0.1$$

Replace logx with "a" to see the quadratic equation better. Once factored, put logx back in to complete the question.

Logarithms Lesson 3

Part V – Solving Using Log Rules

QUESTIONS: SOLVE FOR X

1) $5\log x + 4 = 2$

2) $4\log_2 x = 24$

3) $\log_3(x-1) = \log_3(2x+2)$

4) $2\log_5 5 = \log_5(2x+1)$

5) $3\log_2 x + \log_2 25 = \log_2 16$

6) $4\log_3 x = 8\log_3 b$

7) $\log_2(x-2) = 4$

8) $\log(x+2) + \log(x-1) = 1$

9) $\log_3(x-3) = 4 - \log_3(x+3)$

10) $\log_2 x = 5 – \log_2(x+3)$

11) $\log x - \log(x+4) = 1$

Logarithms Lesson 3

Part V – Solving Using Log Rules

12) $\log_3(5x+1) - \log_3(3x-1) = 1$

15) $2\log(x-2) = \log 5 + \log(5-x)$

17) $(\log x)^2 + \log x^3 = -2$

13) $\log x^3 + \log 4 = \log 3x$

16) $3\log x - \log x = 2$

18) $(\log x)^4 - 16 = 0$

14) $\log_{(x-3)} 36 = 3$

ANSWERS:

1) 0.398
2) 64
3) No Solution
4) 12
5) 0.862
6) b^2
7) 18
8) 3
9) 9.49
10) 4.35
11) No Solution
12) 1
13) 0.866
14) 6.3
15) 4.11
16) 10
17) 0.01, 0.1
18) 0.01 , 100

DIPLOMA STYLE EQUATION QUESTIONS

Example 1: The graph of $y = \log_a x$ goes through the point (3, 7). What is the value of a?

$y = \log_a x$

$7 = \log_a 3$

$a^7 = 3$

$a = 3^{\frac{1}{7}}$

$a = 1.17$

Example 2: Determine the x-intercept of $y = \log_2(x+4)$

$y = \log_2(x+4)$

$0 = \log_2(x+4)$

$2^0 = x+4$

$1 = x+4$

$x = -3$

At the x-intercept, y = 0

At the y-intercept, x = 0

Example 3: The y-intercept of $y = \log_2(x+4)$ is

$y = \log_2(0+4)$

$y = \log_2 4$

$y = 2$

Example 4: Determine the equation of an asymptote of $y = \log_3(3x-8)+4$

$y = \log_3(3x-8)+4$

$3x - 8 = 0$

$3x = 8$

$x = \frac{8}{3}$

To find the asymptote, determine the value of x which makes the brackets become zero.

Example 5: An exponential function is defined as: $f(x) = 5(1.2)^{x-3}$.
If f(k) = 83, then the value of k is?

$f(x) = 5(1.2)^{x-3}$

$83 = 5(1.2)^{k-3}$

$16.6 = (1.2)^{k-3}$

$\log 16.6 = \log(1.2)^{k-3}$

$\log 16.6 = (k-3)\log 1.2$

$\log 16.6 = k\log 1.2 - 3\log 1.2$

$\log 16.6 + 3\log 1.2 = k\log 1.2$

$k = \frac{\log 16.6 + 3\log 1.2}{\log 1.2}$

$k = 18.41$

Logarithms Lesson 3

Part VI – Diploma Style

Example 6: **Given the functions $f(x) = \log_4(x-2)-2$ & $g(x) = \log_4(3x+1)$, determine the x-value at the point of intersection.**

$$f(x) = g(x)$$
$$\log_4(x-2)-2 = \log_4(3x+1)$$
$$\log_4(x-2)-\log_4(3x+1) = 2$$
$$\log_4\left(\frac{x-2}{3x+1}\right) = 2$$
$$16 = \frac{x-2}{3x+1}$$
$$48x+16 = x-2$$
$$47x = -18$$
$$x = -0.383 \quad \text{(Reject)}$$

No Solution Exists

To find the point of intersection of two functions, set them equal to each other and solve for x.

Example 7: **The point (27, 3) lies on the graph of $y = \log_b x$. If the point (4, k) lies on the graph of $y = b^x$, then what is the value of k?**

$$y = \log_b x$$
$$3 = \log_b 27$$
$$b^3 = 27$$
$$b = 3$$

$$y = b^x$$
$$k = 3^4$$
$$k = 81$$

First solve for the b-value, then use it to find k.

Example 8: **Determine the equation of the inverse of $f(x) = \log 5x$**

$$y = \log 5x$$
$$x = \log 5y$$
$$10^x = 5y$$
$$f^{-1}(x) = \frac{10^x}{5}$$

Example 9: **The ordered pair (k, 1.23) lies on the inverse of $y = 2^x$.**
The value of k is:

$$y = 2^x$$
$$x = 2^y$$
$$k = 2^{1.23}$$

Example 10: **The ordered pair (3, 81) lies on the inverse of $y = \log_a x$. Find the value of a.**

$$y = \log_a x$$
$$x = \log_a y$$
$$3 = \log_a 81$$
$$a^3 = 81$$
$$a = 4.33$$

Example 11: **Determine the y-intercept of the function $y = a^{x-3}$**

$$y = a^{x-3}$$
$$y = a^{0-3}$$
$$y = a^{-3}$$
$$y = \frac{1}{a^3}$$

Logarithms Lesson 3

Part VI – Diploma Style

QUESTIONS:

1) The graph of $y = \log_a x$ goes through the point (3, 6). What is the value of a?

2) Determine the x-intercept of $y = \log_4(x+7)$

3) The y-intercept of $y = \log_5(x-2)$ is

4) Determine the equation of an asymptote of $y = \log_5(2x - 6) + 3$

5) An exponential function is defined as $f(x) = 3(2.4)^{x-2}$. If f(k) = 65, then the value of k is?

6) The graphs of $f(x) = \log_3(3x-3)$ & $g(x) = \log_3(2x - 1)$ intersect at a point. What are the coordinates of that point?

7) The point (64, 4) lies on the graph of $y = \log_b x$. If the point (5, k) lies on the graph of $y = b^x$, then what is the value of k?

8) Determine the equation of the inverse of $f(x) = \log_3 x$

9) The ordered pair (k, 4.11) lies on the inverse of $y = 3^x$.
The value of k is:

10) The ordered pair (4, 64) lies on the inverse of $y = \log_a x$.
Find the value of a.

11) The graph of $y = b^{x-4}$ has a y –intercept of

ANSWERS:

1) 1.2
2) -6
3) Undefined
4) x =3
5) 5.51
6) (2, 1)
7) 181.02
8) $y = 3^x$
9) 91.4
10) 2.83
11) $\frac{1}{b^4}$