Pre - Calculus Mathematics 40S

Standards Test Practice Exam - ANSWERS

Probability

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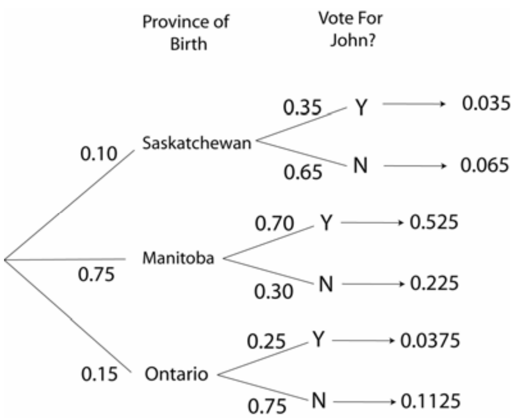
1. In a Manitoba school, 10% of the students were born in Saskatchewan, 75% were born in Manitoba, and the rest were born in Ontario. John decides to run for student president. The results of the election are as follows:

35% of the students born in Saskatchewan voted for John.
70% of the students born in Manitoba voted for John.
25% of the students born in Ontario voted for John.

a) What is the probability a student born in Manitoba did not vote for John?



Following the probability tree, the probability a student born in Manitoba did NOT vote for John is 0.225.

b) If a student voted for John, what is the probability the student was born in Manitoba?

$$P(Manitoba\,|\,John) = \frac{P(Manitoba\ and\ John)}{P(John)}$$

$$P(Manitoba\,|\,John) = \frac{0.525}{0.035 + 0.525 + 0.0375}$$

$$P(Manitoba\,|\,John) = \mathbf{0.8787}$$

2. The probability of rolling a five on a six-sided die is $\frac{1}{6}$. Therefore, the probability of rolling three consecutive five's is $\frac{1}{216}$. If a six-sided die is rolled three times, what is the probability of not rolling three consecutive five's?

If the probability of rolling three consecutive five's is $\frac{1}{216}$*,*
then the complement will give the probability of *<u>not</u>*
rolling three consecutive five's.

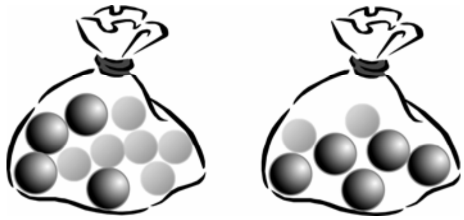
The answer is: $1-\frac{1}{216} = \frac{216}{216} - \frac{1}{216} = \mathbf{\frac{215}{216}}$

3. A box contains 3 orange, 2 blue, and 3 purple marbles. If a marble is randomly selected from the box, determine the probability it is not purple.

This is a basic probability question.
There are 5 non-purple marbles out of 8 marbles.

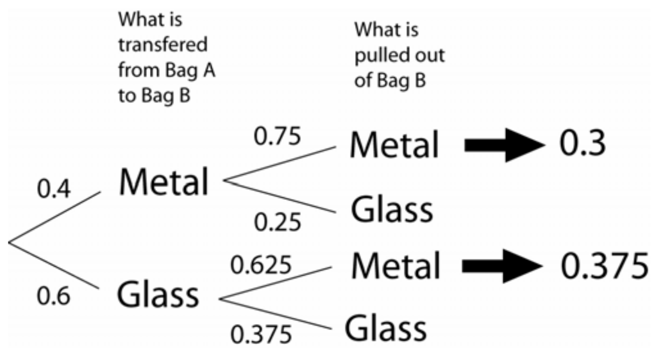
$$P(\text{not purple}) = \frac{5}{8}$$

4. Bag A contains four metal balls and six glass balls, and Bag B contains five metal balls and two glass balls.



A ball is randomly selected from Bag A and placed in Bag B. A ball is then pulled at random out of Bag B. Determine the probability that the ball from Bag B is metal.

First draw a tree diagram to determine the probabilities



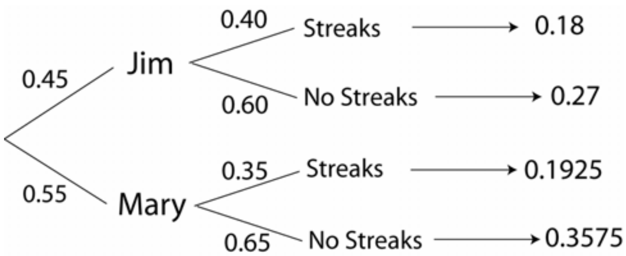
** To get the first set of probabilities, look at the first bag. There are 4 metal balls out of 10. The probability of drawing a metal ball is 0.4. Using the complement, the probability or getting a glass ball is 0.6*

If a metal ball was transferred, it follows that there are 6 metal balls out of 8 in Bag B. The probability of pulling a metal ball is then 6/8 = 0.75. Likewise, there are 2 out of 8 glass balls, so the probability of pulling a glass ball is now 2/8 = 0.25.

If a glass ball was transferred, it follows that there are 5 metal balls out of 8 in Bag B. The probability of pulling a metal ball from Bag B is 5/8 = 0.625. Likewise, there are 3 out of 8 glass balls, so the probability of pulling a glass ball is 3/8 = 0.375.

The probability a metal ball is pulled from Bag B is 0.3 + 0.375 = ***0.675***

5. Jim and Mary share the job of washing the windows in their house. Jim washes the windows 45% of the time, and Mary washes the windows 55% of the time.
The probability of streaks being left on the window is 40% when Jim cleans the windows, and 35% when Mary cleans the windows. A visitor to the house notices streaks on the window. The probability Mary washed the windows that day is



$$P(Mary\,|\,Streaks) = \frac{P(Mary \text{ and } Streaks)}{P(Streaks)}$$

$$P(Mary\,|\,Streaks) = \frac{0.1925}{0.18 + 0.1925}$$

$$\boldsymbol{P(Mary\,|\,Streaks) = 0.5168}$$

6. The probability Kristen brings a soft drink to school is 0.3. The probability she brings a chocolate bar to school is 0.35. If the events are independent, what is the probability she brings both a soft drink *and* a candy bar to school?

To determine "and" probabilities for independent events, multiply them together.

$$P(\text{soft drink and chocolate bar}) = 0.3 \times 0.35 = \mathbf{0.105}$$

7. A student writes letters of the alphabet on some cards and places those cards in two different bags. The letters A, C, E, G, I are in Bag 1, and B, C, D, F, H, I are in Bag 2. A card is randomly chosen from each bag. Determine the probability
a) Both cards are the same letter

$$P(\text{Both } C) = \frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$$

$$P(\text{Both } I) = \frac{1}{5} \times \frac{1}{6} = \frac{1}{30}$$

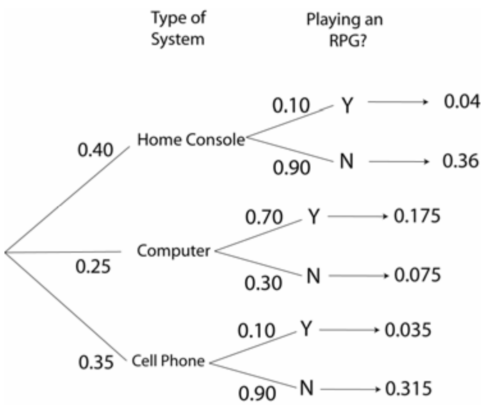
$$P(\text{Both the same letter}) = \frac{1}{30} + \frac{1}{30} = \frac{2}{30} = \boldsymbol{\frac{1}{15}}$$

b) The cards are different letters

$$\text{Use the complement: } 1 - \frac{1}{15} = \frac{15}{15} - \frac{1}{15} = \boldsymbol{\frac{14}{15}}$$

8. Dave plays video games 40% of the time on his home console system, 25% of the time on the computer, and 35% of the time on his cell phone. If he is playing on the computer, there is a 70% chance he is playing an RPG (*role-playing game*). If he is playing on his console or cell phone, there is a 10% chance he is playing an RPG.

a) What is the probability Dave chooses to use his home console and then selects an RPG to play?



Following the probability tree, the probability Dave chooses to use his home console then plays an RPG is 0.40 $\times$ 0.10 = 0.04

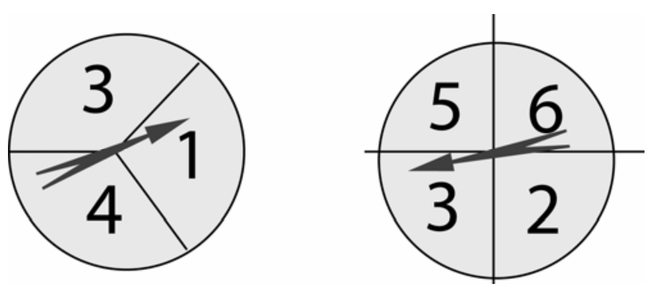
b) A friend comes over and finds Dave not playing an RPG. What is the probability he is on the computer?

$$P(Computer \mid No\ RPG) = \frac{P(Computer\ and\ No\ RPG)}{P(No\ RPG)}$$

$$P(Computer \mid No\ RPG) = \frac{0.075}{0.36 + 0.075 + 0.315}$$

$$\boldsymbol{P(Computer \mid No\ RPG) = 0.1}$$

9. A number is randomly picked using Spinner 1, and another number is randomly picked using Spinner 2.



List the ordered pairs of the sample space resulting in a sum greater than seven.
[3, 5], [3, 6], [4, 5], [4, 6]

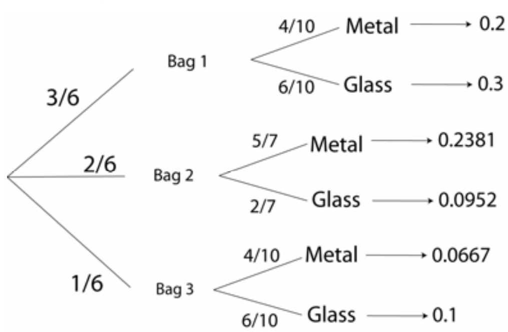
10. The probability that the first light bulb on a string of Christmas lights blinks is 0.4. The probability the second light bulb blinks is 0.65. If the probabilities are independent, determine the probability neither bulb blinks.

The probability the first bulb does not blink is 0.6
The probability the second bulb does not blink is 0.35.

Multiply them together to determine the probability neither bulb blinks: $\boldsymbol{0.6 \times 0.35 = 0.21}$

11. Bags 1 & 3 contains four metal balls (*darker*) and six glass balls (*lighter*).
Bag 2 contains five metal balls and two glass balls.

In a game, a person rolls a die to determine which bag to pull a ball out of. If the die rolls a 1, 2, or 3, the ball is pulled from Bag 1. If the die comes up 4 or 5, the ball is pulled from Bag 2. If the die comes up 6, the ball is pulled from Bag 3. The probability of selecting a metal ball is



The probability is
0.2 + 0.2381 + 0.0667 = ***0.505***

12. The probability a student has to perform in a violin recital next Wednesday is 0.7. The probability the student has a hockey game that same night is 0.6. The events are independent.

a) Determine the probability the student will have to attend both events next Wednesday.

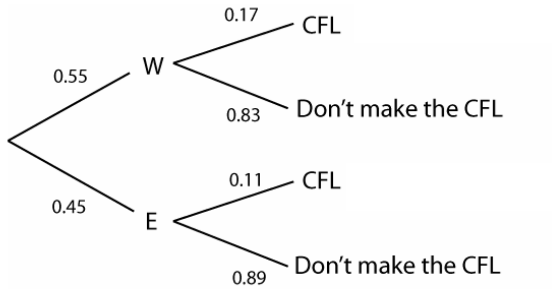
Since the events are independent, the probability for the "and" case can be obtained through multiplication. $0.7 \times 0.6 = 0.42$

b) Determine the probability the student will have to attend one event <u>or</u> the other next Wednesday.

$$P(\text{violin or hockey}) = P(\text{violin}) + P(\text{hockey}) - P(\text{violin and hockey})$$

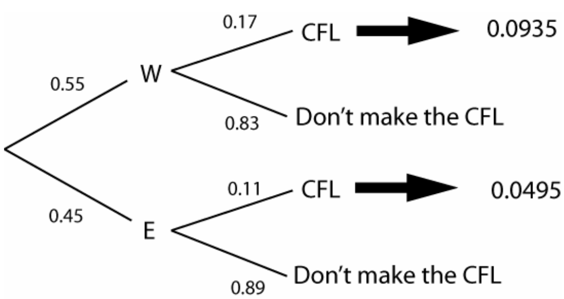
$$P(\text{violin or hockey}) = 0.7 + 0.6 - 0.42 = \mathbf{0.88}$$

13. In a junior football league, 55% of the players come from Western Canada and 45% are from Eastern Canada. From this league, 17% of the Western players and 11% of the Eastern players will go on to the CFL. The following diagram contains the results:



If a randomly chosen CFL player who came from the junior league is selected, the probability he came from Eastern Canada is:

Finish the tree diagram:



$P(\textit{Eastern Canada \& CFL}) = 0.0495$

$P(CFL) = 0.0935 + 0.04950 = 0.143$

To answer this question, we must use Bayes' formula since we want the probability the player is from Eastern Canada ***given that*** *he is in the CFL.*

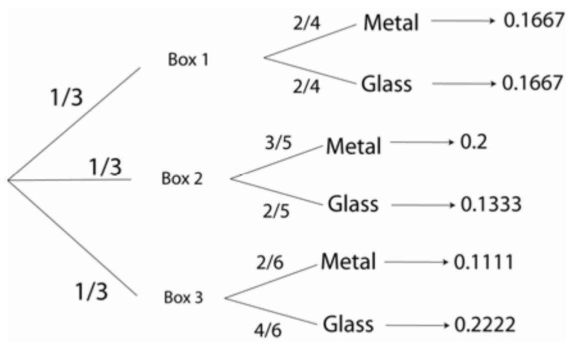
$$P(\textit{Eastern Canada}|CFL) = \frac{P(\textit{Eastern Canada \& CFL})}{P(CFL)}$$

$$P(\textit{Eastern Canada}|CFL) = \frac{0.0495}{0.143} = \boldsymbol{\frac{9}{26}}$$

14. A student randomly selects a marble from of the boxes below.

Box 1	**Box 2**	**Box 3**
2 metal	3 metal	2 metal
2 glass	2 glass	4 glass

Given that a metal marble is selected, determine the probability it came from Box 3.

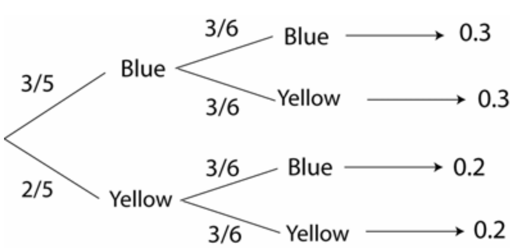


$$P(Box\ 3\,|\,Metal) = \frac{P(Box\ 3\ and\ Metal)}{P(Metal)}$$

$$P(Box\ 3\,|\,Metal) = \frac{0.1111}{0.1667 + 0.2 + 0.1111}$$

$$P(Box\ 3\,|\,Metal) = \mathbf{0.2325}$$

15. Box A contains 3 blue and 2 yellow balls, and Box B contains 3 blue and 3 yellow balls. A ball is pulled from Box A, then a ball is pulled from Box B. The probability both balls are the same color is



The probability both balls are the same color is:
P(both blue) + P(both yellow)
= 0.3 + 0.2
= ***0.5***

16. Seven people are randomly selected from a group of 10 men and 11 women to form a committee. The probability exactly 5 males are on the committee is

Exactly five males / two females can be chosen in ${}_{10}C_5 \times {}_{11}C_2$ *ways*

Any seven members can be chosen in ${}_{21}C_7$ *ways*

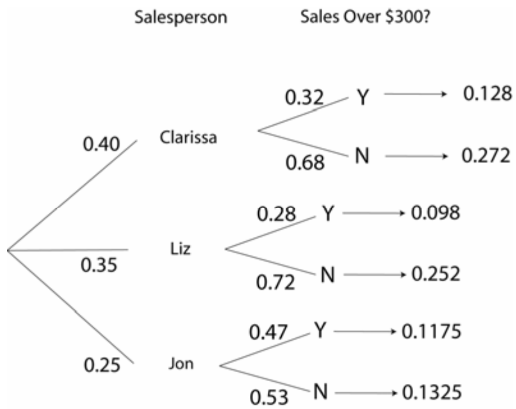
The probability is $\dfrac{{}_{10}C_5 \times {}_{11}C_2}{{}_{21}C_7} = \mathbf{0.12}$

17. The probability Chelsea wears a blue coat is 0.32. The probability Chelsea goes to the movies is 0.4. Determine the probability Chelsea goes to the movies but does not wear her blue coat.

Multiply the probability Chelsea goes to the movies (0.4) by the probability she does <u>not</u> wear her blue coat (0.68).

The answer is $0.4 \times 0.68 = \mathbf{0.272}$

18. Clarissa, Liz, and Jon sell luggage. Clarissa sells 40% of the luggage, Liz sells 35% of the luggage, and Jon sells 25% of the luggage. Of the luggage Clarissa sells, 32% have a sticker price over $300. Of the luggage Liz sells, 28% have a sticker price over $300. Of the luggage Jon sells, 47% has a sticker value over $300.
If a piece of luggage over $300 is sold, what is the probability it was sold by Clarissa?



$$P(Clarissa|Over\ \$300) = \frac{P(Clarissa\ and\ Over\ \$300)}{P(Over\ \$300)}$$

$$P(Clarissa|Over\ \$300) = \frac{0.128}{0.128 + 0.098 + 0.1175}$$

$$P(Clarissa|Over\ \$300) = \mathbf{0.3726}$$

19. If four coins are tossed, determine the probability all four will come up heads.

The probability is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \mathbf{\frac{1}{16}}$

20. The probability Steve scores a goal is $\frac{1}{3}$. The probability Adam scores a goal is $\frac{3}{7}$. If Steve and Adam each take one shot at the net, what is the probability they both miss?

Use the complements of each probability: $\frac{2}{3} \times \frac{4}{7} = \mathbf{\frac{8}{21}}$

22. A unique tetrahedral die has one side marked 1, two sides marked 2, and one side marked 3.

a) What is the sample space for this die?

The sample space is ***1, 2, 2, 3***

b) If the die is thrown twice, determine the probability the sum is even.

Create a chart:

	1	2	2	3
1	2	3	3	4
2	3	4	4	5
2	3	4	4	5
3	4	5	5	6

The probability of obtaining an even number is $\frac{8}{16} = \boldsymbol{\frac{1}{2}}$

23. A student council consisting of eight people is to be randomly chosen from a group of 12 students. Brittany, Elisha, and Gwen are three of the twelve students. Determine the probability that Brittany, Elisha, and Gwen are on the student council.

The number of ways Brittany, Elisha, and Gwen are on the council is ${}_9C_5 = 126$*. We use 9 since there are three fewer people in the selection pool, and we use 5 because there are only five spaces left to fill now that the three girls are on the council.*

The number of ways to select any 8 people is ${}_{12}C_8 = 495$

The probability Brittany, Elisha, and Gwen are on the student council is $\frac{126}{495} = \boldsymbol{\frac{14}{55}}$

24. There are 7 men and 9 women available for selection to a committee. Two of the men and one of the women are good friends. If the committee requires three men and 4 women, what is the probability that all three friends will be on the same committee?

Start with the fact the two specific men and one specific woman must be on the committee.

The remaining men can be chosen in $_5C_1 = 5$ *.*
The remaining women can be chosen in $_8C_3 = 56$ *.*
The total number of ways to form a committee with the three friends is $5 \times 56 = 280$

The total number of ways to from the committee without restrictions is
$_7C_3 \times {_9C_4} = 4410$

The probability is $\dfrac{280}{4410} = \mathbf{0.0635}$