Pre - Calculus Mathematics 40S

Standards Test Practice Exam - ANSWERS

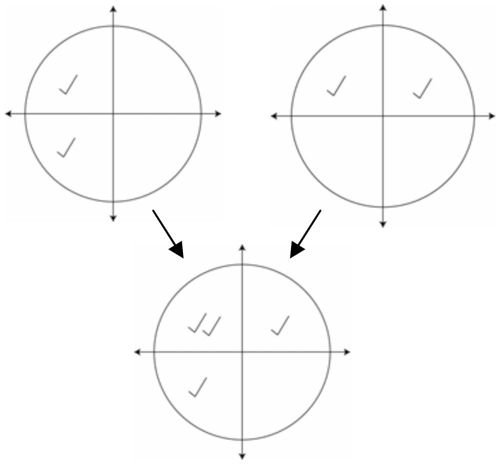
TRIGONOMETRY II

Copyright © 2006, Barry Mabillard. www.math40s.com

1. If $\cos\alpha = -\dfrac{2}{5}$, where $\sin\alpha > 0$, and $\sin\beta = \dfrac{4}{9}$, where $\tan\beta > 0$, determine the exact value of $\cos(\alpha+\beta)$

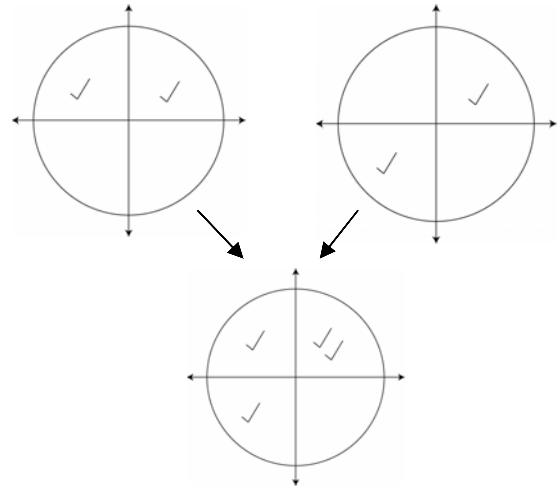
First determine the quadrant α is in:

$\cos\alpha < 0$ $\qquad$ $\sin\alpha > 0$

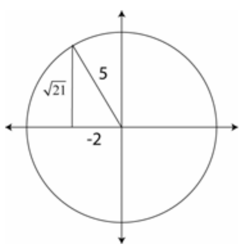


Then determine the quadrant β is in:

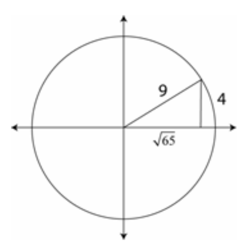
$\sin\beta > 0$ $\qquad$ $\tan\beta > 0$



Now solve the triangles for both α and β; use Pythagoras to find the unknown sides.



$$a^2+b^2=c^2$$
$$(-2)^2+b^2=5^2$$
$$4+b^2=25$$
$$b^2=21$$
$$b=\sqrt{21}$$



$$a^2+b^2=c^2$$
$$(4)^2+b^2=9^2$$
$$16+b^2=81$$
$$b^2=65$$
$$b=\sqrt{65}$$

At this point, state all required trigonometric ratios:

$\cos\alpha = -\dfrac{2}{5}$ $\qquad$ $\cos\beta = \dfrac{\sqrt{65}}{9}$

$\sin\alpha = \dfrac{\sqrt{21}}{5}$ $\qquad$ $\sin\beta = \dfrac{4}{9}$

Finally, evaluate $\cos(\alpha+\beta)$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \left(-\frac{2}{5}\right)\left(\frac{\sqrt{65}}{9}\right) - \left(\frac{\sqrt{21}}{5}\right)\left(\frac{4}{9}\right)$$
$$\cos(\alpha+\beta) = \frac{2\sqrt{65}}{45} - \frac{4\sqrt{21}}{45}$$
$$\cos(\alpha+\beta) = \mathbf{\frac{2\sqrt{65}-4\sqrt{21}}{45}}$$

2. Express $1-\sin^2(\alpha+\beta)$ using only cosine.

Start with the identity: $\sin^2\theta+\cos^2\theta=1$

Rearrange to get: $\cos^2\theta=1-\sin^2\theta$

It follows that: $\cos^2(\alpha+\beta)=1-\sin^2(\alpha+\beta)$

The answer is $\cos^2(\alpha+\beta)$

3. If $\cos 3\theta=\frac{\sqrt{3}}{2}$, where 3θ is an ***acute*** angle, determine the exact value of θ.

If you evaluate cosx = $\frac{\sqrt{3}}{2}$*, you get* $x=\frac{\pi}{6},\cancel{\frac{11\pi}{6}}$.

You do not want to use any other angles since only $\frac{\pi}{6}$ *is acute (<90°).*

Divide by 3 to get the angle θ*:* $\frac{\pi}{6}\div 3=\frac{\pi}{6}\times\frac{1}{3}=\boldsymbol{\frac{\pi}{18}}$

**Alternatively, you could graph each side of* $\cos 3\theta=\frac{\sqrt{3}}{2}$ *in your calculator using degree mode, then convert the answer to a radian fraction.*

4. Determine the exact value of $\tan(15^o)$

Rewrite as $\tan(45^o-30^o)$ *and use the formula* $\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

$$\tan(45^o-30^0)=\frac{\tan 45^o-\tan 30^0}{1+\tan 45^o\tan 30^0}$$

$$=\frac{1-\frac{1}{\sqrt{3}}}{1+(1)\left(\frac{1}{\sqrt{3}}\right)}=\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}$$

$$=\frac{\frac{\sqrt{3}}{\sqrt{3}}-\frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}}+\frac{1}{\sqrt{3}}}$$ *Get a common denominator for top & bottom*

$$=\frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$

$$=\frac{\sqrt{3}-1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}+1}$$ *Divide fractions by multiplying the reciprocal*

$$=\boldsymbol{\frac{\sqrt{3}-1}{\sqrt{3}+1}}$$

Evaluate the required tan ratios before doing the calculation on the left.

$$\tan 30^o=\frac{\sin 30^o}{\cos 30^o}=\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\frac{1}{2}\times\frac{2}{\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$\tan 45^o=\frac{\sin 45^o}{\cos 45^o}=\frac{\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}}=1$$

5. Prove the identity: $\frac{1-\cot^2 x}{1+\cot^2 x}=\sin^2 x-\cos^2 x$

$$\frac{1-\cot^2 x}{1+\cot^2 x}$$
$$=\frac{1-\cot^2 x}{\csc^2 x}$$
$$=\left(1-\cot^2 x\right)\sin^2 x$$
$$=\left(1-\frac{\cos^2 x}{\sin^2 x}\right)\sin^2 x$$
$$\boldsymbol{=\sin^2 x-\cos^2 x}$$

6. Solve for *x*, where $0\le x\le 2\pi$: $\csc^2 x-\csc x=2$

Rewrite as $\csc^2 x-\csc x-2=0$

Then factor: $(\csc x-2)(\csc x+1)=0$

Now equate each set of brackets to zero and solve for x.

$$\csc x-2=0$$
$$\csc x=2$$
$$\sin x=\frac{1}{2}$$
$$x=\frac{\pi}{6},\frac{5\pi}{6}$$

$$\csc x+1=0$$
$$\csc x=-1$$
$$\sin x=-1$$
$$x=\frac{3\pi}{2}$$

The answer is $\boldsymbol{x=\frac{\pi}{6},\frac{5\pi}{6},\frac{3\pi}{2}}$

7. Solve for *x*: $2\cos^2 x+\cos x-1=0$

Factor to obtain: $(2\cos x-1)(\cos x+1)=0$

Now equate each set of brackets to zero and solve for x.

$$2\cos x-1=0$$
$$2\cos x=1$$
$$\cos x=\frac{1}{2}$$
$$x=\frac{\pi}{3},\frac{5\pi}{3}$$

$$\cos x+1=0$$
$$\cos x=-1$$
$$x=\pi$$

The answer is $\boldsymbol{x=\frac{\pi}{3},\pi,\frac{5\pi}{3}}$

8. Simplify: $\dfrac{\sin x \cot x + \cos x}{\sin x}$

$$= \frac{\sin x\left(\frac{\cos x}{\sin x}\right) + \cos x}{\sin x}$$

$$= \frac{\cos x + \cos x}{\sin x}$$

$$= \frac{2\cos x}{\sin x}$$

$$= \mathbf{2cotx}$$

9. Solve $(2\cos\theta + 1)(\tan\theta - 1) = 0$ for θ in the interval $\frac{\pi}{2} \le x \le 2\pi$

Equate each set of brackets to zero and solve for θ.

$2\cos\theta + 1 = 0$

$2\cos\theta = -1$

$\cos\theta = -\dfrac{1}{2}$

$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$

$\tan\theta - 1 = 0$

$\tan\theta = 1$

$\theta = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$

$\boldsymbol{\theta = \dfrac{2\pi}{3}, \dfrac{5\pi}{4}, \dfrac{4\pi}{3}}$ *(Omit $\frac{\pi}{4}$ since it lies outside the specified domain.)*

10. Determine the exact value of

a) $\cos\dfrac{11\pi}{12}$

First convert $\frac{11\pi}{12}$ to degrees. This will make the remainder of the calculation easier than working in radian fractions. $\dfrac{11\pi}{12} \times \dfrac{180^\circ}{\pi} = 165^\circ$

Write as $\cos(120^\circ + 45^\circ)$, then apply the cosine sum formula

$\cos(120^\circ + 45^\circ) = \cos 120^\circ \cos 45^\circ - \sin 120^\circ \cos 45^\circ$

$$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \boldsymbol{\frac{-\sqrt{2} - \sqrt{6}}{4}}$$

b) $\sec\dfrac{11\pi}{12}$

$$\sec 165^\circ = \frac{1}{\cos 165^\circ}$$

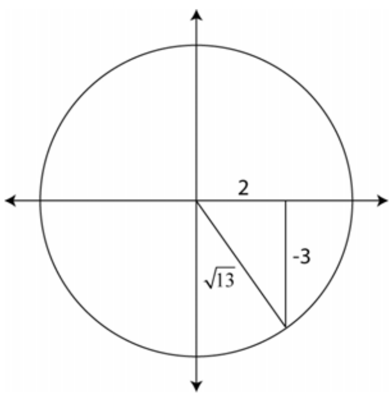
This is simply the reciprocal of the answer you found in part a)

The answer is $\boldsymbol{\dfrac{4}{-\sqrt{2} - \sqrt{6}}}$

11. Prove the identity $\dfrac{\cot\theta\sec^2\theta}{\cot^2\theta+1}=\tan\theta$

$$\frac{cot\theta sec^2\theta}{cot^2\theta+1}$$
$$=\frac{cot\theta sec^2\theta}{csc^2\theta}$$
$$=cot\theta\bullet\frac{\frac{1}{cos^2\theta}}{\frac{1}{sin^2\theta}}$$
$$=\frac{cos\theta}{sin\theta}\left(\frac{1}{cos^2\theta}\right)\left(sin^2\theta\right)$$
$$=\frac{sin\theta}{cos\theta}$$
$$=\boldsymbol{tan\theta}$$

12. If $\tan\theta=-\dfrac{3}{2}$ and $\dfrac{3\pi}{2}<\theta<2\pi$, state the exact value of $\cos 2\theta$



Draw the triangle and find the hypotenuse using Pythagoras:

$$a^2+b^2=c^2$$
$$(2)^2+(-3)^2=c^2$$
$$4+9=c^2$$
$$13=c^2$$
$$c=\sqrt{13}$$

From the diagram, $cos\theta=\dfrac{2}{\sqrt{13}}$

Now use the formula: $cos2\theta=2cos^2\theta-1$

$$cos2\theta=2\left(\frac{2}{\sqrt{13}}\right)^2-1$$
$$cos2\theta=2\left(\frac{4}{13}\right)-1$$
$$cos2\theta=\frac{8}{13}-1$$
$$\boldsymbol{cos2\theta=-\frac{5}{13}}$$

13. Solve for θ over $[0,2\pi]$: $\sin 2\theta = \frac{\sqrt{3}}{2}$

To solve this equation algebraically, you need to determine the solutions within 2 rotations of the unit circle.

Solve the equation $\sin x = \frac{\sqrt{3}}{2}$ *to get the angles* $\frac{\pi}{3}$ *and* $\frac{2\pi}{3}$*, both of which are within the first rotation.*

Add the period to each one to get the co-terminal angles in the second rotation.

$$\frac{\pi}{3}+2\pi=\frac{\pi}{3}+\frac{6\pi}{3}=\frac{7\pi}{3}$$

$$\frac{2\pi}{3}+2\pi=\frac{2\pi}{3}+\frac{6\pi}{3}=\frac{8\pi}{3}$$

Finally, take all your solutions $\frac{\pi}{3},\frac{2\pi}{3},\frac{7\pi}{3},\frac{8\pi}{3}$ *and divide by 2.*

The answer is $\boldsymbol{\frac{\pi}{6},\frac{2\pi}{6},\frac{7\pi}{6},\frac{8\pi}{6}}$

14. Solve the following equation for $x \in R$: $2\cos 2\theta + 1 = 0$

Get $\cos 2\theta$ *by itself before trying to determine the angles:*

$$2\cos 2\theta = -1$$

$$\cos 2\theta = \frac{-1}{2}$$

To solve this equation algebraically, you need to determine the solutions within 2 rotations of the unit circle.

Solve the equation $\cos x = -\frac{1}{2}$ *to get the angles* $\frac{2\pi}{3}$ *and* $\frac{4\pi}{3}$*, both of which are within the first rotation.*

Add the period to each one to get the co-terminal angles in the second rotation.

$$\frac{2\pi}{3}+2\pi=\frac{2\pi}{3}+\frac{6\pi}{3}=\frac{8\pi}{3}$$

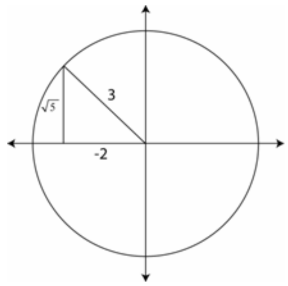
$$\frac{4\pi}{3}+2\pi=\frac{4\pi}{3}+\frac{6\pi}{3}=\frac{10\pi}{3}\pi$$

Finally, take all your solutions $\frac{2\pi}{3},\frac{4\pi}{3},\frac{8\pi}{3},\frac{10\pi}{3}$ *and divide by 2.*

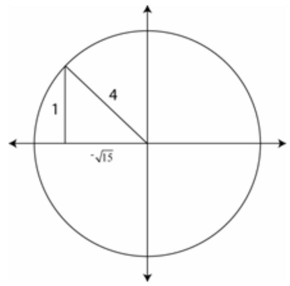
The answer is $\boldsymbol{\frac{2\pi}{6},\frac{4\pi}{6},\frac{8\pi}{6},\frac{10\pi}{6}}$

15. If α and β are second quadrant angles, and $\cos\alpha = -\dfrac{2}{3}$ and $\sin\beta = \dfrac{1}{4}$, determine the exact value of $\sin(\alpha - \beta)$

Solve the triangles for both α and β; use Pythagoras to find the unknown sides.



$$a^2 + b^2 = c^2$$
$$(-2)^2 + b^2 = 3^2$$
$$4 + b^2 = 9$$
$$b^2 = 5$$
$$b = \sqrt{5}$$



$$a^2 + b^2 = c^2$$
$$1^2 + b^2 = 4^2$$
$$1 + b^2 = 16$$
$$b^2 = 15$$
$$b = \sqrt{15}$$

Use a negative since the b-value lies on the negative x-axis.

At this point, state all required trigonometric ratios:

$$\cos\alpha = -\frac{2}{3} \qquad \cos\beta = \frac{-\sqrt{15}}{4}$$
$$\sin\alpha = \frac{\sqrt{5}}{3} \qquad \sin\beta = \frac{1}{4}$$

Finally, evaluate $\sin(\alpha - \beta)$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$\sin(\alpha-\beta) = \left(\frac{\sqrt{5}}{3}\right)\left(\frac{-\sqrt{15}}{4}\right) - \left(-\frac{2}{3}\right)\left(\frac{1}{4}\right)$$
$$\sin(\alpha-\beta) = \frac{-\sqrt{75}}{12} + \frac{2}{12}$$
$$\sin(\alpha-\beta) = \mathbf{\frac{2-\sqrt{75}}{12}}$$

16. Solve for x over the interval $[0, 2\pi]$ for $\sin^2 x = \sin x$

Set the equation to zero: $\sin^2 x - \sin x = 0$
Then factor: $\sin x(\sin x - 1) = 0$

Now set each factor equal to zero and solve for the angles

$\sin x = 0$
$x = 0, \pi, 2\pi$

$\sin x - 1 = 0$
$\sin x = 1$
$x = \dfrac{\pi}{2}$

The solution is

$$\mathbf{x = 0, \frac{\pi}{2}, \pi, 2\pi}$$

17. Prove $\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta}=2\csc\theta$

$$\left(\frac{1}{1+\cos\theta}\right)\left(\frac{1-\cos\theta}{1-\cos\theta}\right)+\left(\frac{1}{1-\cos\theta}\right)\left(\frac{1+\cos\theta}{1+\cos\theta}\right)$$
$$=\left(\frac{1}{1+\cos\theta}\right)\left(\frac{1-\cos\theta}{1-\cos\theta}\right)+\left(\frac{1}{1-\cos\theta}\right)\left(\frac{1+\cos\theta}{1+\cos\theta}\right)$$
$$=\frac{1-\cos\theta}{1-\cos^2\theta}+\frac{1+\cos\theta}{1-\cos^2\theta}$$
$$=\frac{1-\cos\theta+1+\cos\theta}{1-\cos^2\theta}$$
$$=\frac{2}{1-\cos^2\theta}$$
$$=\frac{2}{\sin^2\theta}$$
$$=\boldsymbol{2\csc\theta}$$

18. Solve for x over the interval $[0,2\pi]$: $1+\tan^2 x=3$.

State the solutions as radians to three decimal places

First isolate tanx :

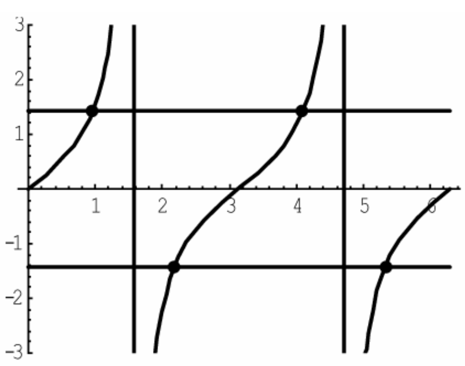
$$1+\tan^2 x = 3$$
$$\tan^2 x = 2$$
$$\tan x = \pm\sqrt{2}$$

Now solve in your calculator by graphing

$Y_1=\tan x$

$Y_2=+\sqrt{2}$

$Y_3=-\sqrt{2}$



The answer is $\boldsymbol{x = 0.9555, 2.186, 4.097, 5.328}$

19.

a) Prove $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$

$$\frac{sin2x}{1+cos2x}$$
$$= \frac{2sinxcosx}{1+\left(2cos^2x - 1\right)}$$
$$= \frac{2sinxcosx}{2cos^2x}$$
$$= \frac{sinx}{cosx}$$
$$= \boldsymbol{tanx}$$

b) State a value of x where $\dfrac{\sin 2x}{1+\cos 2x}$ is undefined

The expression is undefined when $1+cos2x = 0$

$$1+cos2x = 0$$
$$cos2x = -1$$

First solve $cosx = -1$ over two rotations:

The solution to this is $x = \pi, 3\pi$

Now divide by the 2 to get the solutions to $cos2x = -1$

The final answer is $\boldsymbol{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

(You could also solve this by graphing $Y_1 = cos2x$ & $Y_2 = -1$ in your calculator and finding the points of intersection)

20. Find the exact value of $2\cos^2\left(\dfrac{\pi}{8}\right) - 1$

Use the identity $cos2a = 2cos^2a - 1$

$$cos2a = 2cos^2\left(\frac{\pi}{8}\right) - 1$$

What goes here is double what goes here.

$$\boxed{2\times\frac{\pi}{8} = \frac{\pi}{4}}$$

We now have the equation $cos\dfrac{\pi}{4} = 2cos^2\left(\dfrac{\pi}{8}\right) - 1$

The answer is: $\boldsymbol{cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}}$

21. Simplify $\dfrac{\left(\sin x + \dfrac{\cos^2 x}{\sin x}\right)}{\csc x}$

$$\frac{\left(sinx + \dfrac{cos^2x}{sinx}\right)}{cscx}$$

$$= \left(sinx + \frac{cos^2x}{sinx}\right) \times sinx$$

$$= sin^2x + cos^2x$$

$$\mathbf{= 1}$$

22. Find the exact value of:

a) $\cos\left(\dfrac{7\pi}{12}\right)$

First express the angle in terms of degrees: $\dfrac{7\pi}{12} \times \dfrac{180^o}{\pi} = 105^o$

$$cos\left(105^o\right) = cos\left(60^o + 45^o\right)$$

$$cos\left(60^o + 45^o\right) = cos60^o cos45^o - sin60^o sin45^o$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\mathbf{= \frac{\sqrt{2} - \sqrt{6}}{4}}$$

b) $\sec\left(\dfrac{7\pi}{12}\right)$

This is simply the reciprocal of the answer you found in part ***a)***

$$\mathbf{sec\left(\frac{7\pi}{12}\right) = \frac{4}{\sqrt{2} - \sqrt{6}}}$$

23. Express $\tan^2\theta$ using only $\sin\theta$

Start with the identity $tan^2\theta = \dfrac{sin^2\theta}{cos^2\theta}$

Rewrite the identity $sin^2\theta + cos^2\theta = 1$ *as* $cos^2\theta = 1 - sin^2\theta$

Finally, $\boldsymbol{\dfrac{sin^2\theta}{cos^2\theta} = \dfrac{sin^2\theta}{1 - sin^2\theta}}$

24. Prove: $\dfrac{\cos x}{\sin x + 1} + \dfrac{\sin x + 1}{\cos x} = 2\sec x$

$$\left(\frac{cosx}{sinx+1}\right)\left(\frac{cosx}{cosx}\right) + \left(\frac{sinx+1}{cosx}\right)\left(\frac{sinx+1}{sinx+1}\right)$$

$$\frac{cos^2x}{cosx\,(sinx+1)} + \frac{sin^2x + 2sinx + 1}{cosx\,(sinx+1)}$$

$$\frac{cos^2x + sin^2x + 2sinx + 1}{cosx\,(sinx+1)}$$

$$\frac{1 + 2sinx + 1}{cosx\,(sinx+1)}$$

$$\frac{2 + 2sinx}{cosx\,(sinx+1)}$$

$$\frac{2\,(1 + sinx)}{cosx\,(sinx+1)}$$

$$= \frac{2}{cosx}$$

$$\boldsymbol{= 2secx}$$

25. Solve for θ in the interval $[0, 2\pi]$: $2\sin^2\theta - 3\sin\theta + 1 = 0$

Factor to obtain: $(2sin\theta - 1)(sin\theta - 1) = 0$

Now solve for the angles:

$2sin\theta - 1 = 0$

$2sin\theta = 1$

$sin\theta = \dfrac{1}{2}$

$\theta = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

$sin\theta - 1 = 0$

$sin\theta = 1$

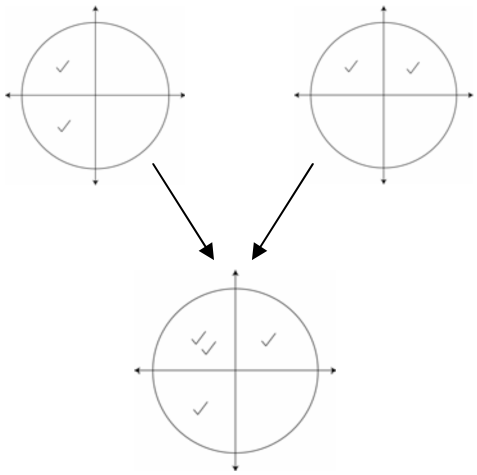
$\theta = \dfrac{\pi}{2}$

The answer is $\boldsymbol{\theta = \dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}}$

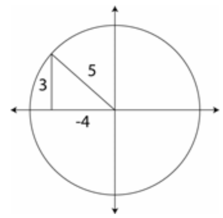
26. If $\cos\theta = -\frac{4}{5}$ and $\csc\theta > 0$, determine the exact value of:

a) $\tan 2\theta$

First determine which quadrant the angle is found in:



Now fill in the triangle and use Pythagoras to determine the unknown side:



$$a^2 + b^2 = c^2$$
$$(-4)^2 + b^2 = 5^2$$
$$16 + b^2 = 25$$
$$b^2 = 9$$
$$b = 3$$

From the triangle,

$$\sin\theta = \frac{3}{5}$$
$$\cos\theta = -\frac{4}{5}$$
$$\tan\theta = -\frac{3}{4}$$

Use the formula $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$$\tan 2\theta = \frac{2\left(-\frac{3}{4}\right)}{1-\left(-\frac{3}{4}\right)^2}$$
$$= \frac{-\frac{6}{4}}{1-\frac{9}{16}}$$
$$= \frac{-\frac{6}{4}}{\frac{16}{16}-\frac{9}{16}}$$
$$= \frac{-\frac{6}{4}}{\frac{7}{16}}$$
$$= -\frac{6}{4} \times \frac{16}{7}$$
$$\boldsymbol{= -\frac{24}{7}}$$

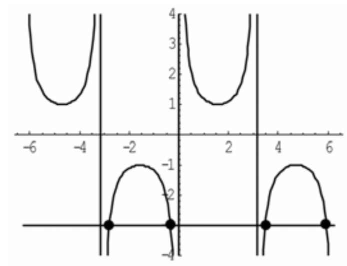
b) $\sin 2\theta$

Use the formula $\sin 2\theta = 2\sin\theta\cos\theta$

$$\sin 2\theta = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right)$$
$$\sin 2\theta = \boldsymbol{-\frac{24}{25}}$$

27. Solve the equation $\csc\theta = -3$, where $\theta \in R$. State your solution to three decimal places.

Solve by graphing:



The question specifies the domain as $\theta \in R$, so a general solution is required.

The (positive) solutions are 3.481 and 5.943
Since the solutions repeat themselves every period, write the general solution as:

$$\boldsymbol{\theta = \begin{Bmatrix} 3.481 + 2k\pi,\ k \in I \\ 5.943 + 2k\pi,\ k \in I \end{Bmatrix}}$$

28. Show that $\sin 8x$ is equivalent to $2\sin 4x \cos 4x$

Start with the identity $sin(2x) = 2sinxcosx$

What goes here is
double what goes here.

In the equation $sin8x = 2sin4xcos4x$

the $4x$ here
becomes the
$8x$ here

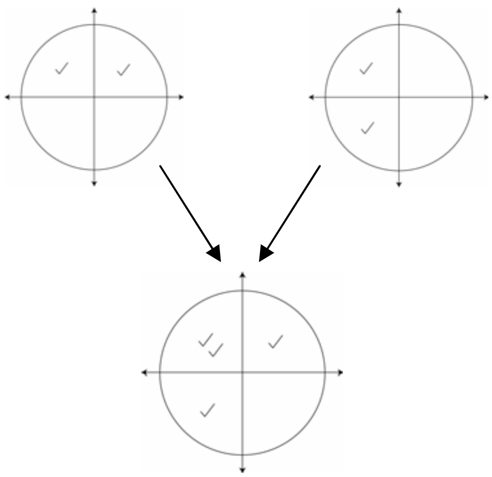
29. Express $\cot\theta \sec\theta$ as a single trigonometric function

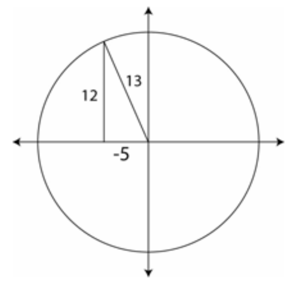
$$cot\theta sec\theta$$
$$= \frac{cos\theta}{sin\theta} \times \frac{1}{cos\theta}$$
$$= \frac{1}{sin\theta}$$
$$\boldsymbol{= csc\theta}$$

30. If $\sin\theta = \dfrac{12}{13}$ and $\cos\theta < 0$, find the exact value of

a) $\tan\theta$

First determine which quadrant the angle is found in:





$$a^2 + b^2 = c^2$$
$$(12)^2 + b^2 = 13^2$$
$$144 + b^2 = 169$$
$$b^2 = 65$$
$$b = 5$$

From the triangle, the trig rations are:

$$\sin\theta = \frac{12}{13}$$
$$\cos\theta = -\frac{5}{13}$$
$$\tan\theta = -\frac{12}{5}$$

The answer is $\boldsymbol{\tan\theta = -\dfrac{12}{5}}$

b) $\cos\left(\theta - \dfrac{\pi}{4}\right)$

Expand $\cos\left(\theta - \dfrac{\pi}{4}\right)$ *using the subtraction law for cosine:*

$$\cos\left(\theta - \frac{\pi}{4}\right) = \cos\theta\cos\frac{\pi}{4} + \sin\theta\sin\frac{\pi}{4}$$

*Now use the values found in part **a)** from the triangle, and values from the unit circle:*

$$\cos\theta\cos\frac{\pi}{4} + \sin\theta\sin\frac{\pi}{4} = \left(-\frac{5}{13}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{12}{13}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= -\frac{5\sqrt{2}}{26} + \frac{12\sqrt{2}}{26}$$

$$\boldsymbol{= \frac{12\sqrt{2} - 5\sqrt{2}}{26}}$$

31. Find the exact values of x in the interval $[0, 2\pi]$: $\cos^2 x + \cos x + 1 = \sin^2 x$

$$\cos^2 x + \cos x + 1 = \sin^2 x$$
$$\cos^2 x + \cos x = \sin^2 x - 1$$
$$\cos^2 x + \cos x = -\cos^2 x$$
$$2\cos^2 x + \cos x = 0$$
$$\cos x\,(2\cos x + 1) = 0$$

Now solve for x:

$$\cos x = 0$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

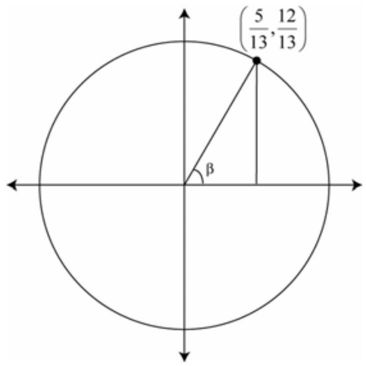
$$2\cos x + 1 = 0$$
$$2\cos x = -1$$
$$\cos x = -\frac{1}{2}$$
$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

The full solution set is $\boldsymbol{x = \frac{\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{3\pi}{2}}$

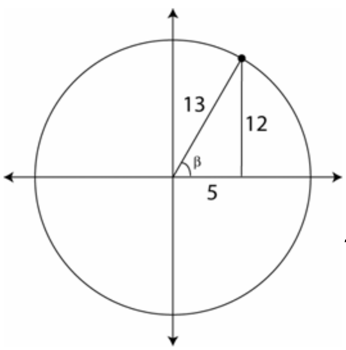
32. Prove the identity: $\dfrac{\cos x - \cos^3 x}{\sin^3 x} = \cot x$

$$\frac{\cos x - \cos^3 x}{\sin^3 x}$$
$$= \frac{\cos x\,(1 - \cos^2 x)}{\sin^3 x}$$
$$= \frac{\cos x \sin^2 x}{\sin^3 x}$$
$$= \frac{\cos x}{\sin x}$$
$$\boldsymbol{= \cot x}$$

33. Given the point shown on the circle, determine the value of $\tan 2\beta$



Recall that the x-coordinate is cosθ , and the y-coordinate is sinθ .



$$\cos\beta = \frac{5}{13} = \frac{adj}{hyp}$$

$$\sin\beta = \frac{12}{13} = \frac{opp}{hyp}$$

Therefore, $\tan\beta = \frac{12}{5}$

Now use the identity $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$$\tan 2\theta = \frac{2\left(\frac{12}{5}\right)}{1-\left(\frac{12}{5}\right)^2}$$

$$\tan 2\theta = \frac{\frac{24}{5}}{1-\frac{144}{25}}$$

$$\tan 2\theta = \frac{\frac{24}{5}}{\frac{25}{25}-\frac{144}{25}}$$

$$\tan 2\theta = \frac{\frac{24}{5}}{-\frac{119}{25}}$$

$$\tan 2\theta = \frac{24}{5} \times -\frac{25}{119}$$

$$\boldsymbol{\tan 2\theta = -\frac{120}{119}}$$