

Pre - Calculus Math 40S:

Exponential and Logarithmic Functions

$$y = \log_2\left(\frac{A}{B}\right)$$

Lesson 3

Solving Logarithm Equations

Pre - Calculus
Math 40S

EXPLAINED!

By
Barry
Mabillard

Logarithms Lesson 3

Part I - Raising Reciprocals

EQUATIONS WITH X AS THE BASE:

Example 1: Solve for x in: $x^3 = 2$

$$x^3 = 2$$

$$(x^3)^{\frac{1}{3}} = (2)^{\frac{1}{3}}$$

$$x = (2)^{\frac{1}{3}}$$

$$x = 1.26$$

Example 2: $x^{\frac{1}{4}} = 6$

$$x^{\frac{1}{4}} = 6$$

$$\left(x^{\frac{1}{4}}\right)^4 = (6)^4$$

$$x = 1296$$

Example 3: $x^{-\frac{3}{5}} = 8$

$$x^{-\frac{3}{5}} = 8$$

$$\left(x^{-\frac{3}{5}}\right)^{-\frac{5}{3}} = (8)^{-\frac{5}{3}}$$

$$x = \frac{1}{32}$$

Example 4: $(3x)^{\frac{2}{3}} = 2$

$$(3x)^{\frac{2}{3}} = 2$$

$$\left((3x)^{\frac{2}{3}}\right)^{\frac{3}{2}} = (2)^{\frac{3}{2}}$$

$$3x = 2.8284$$

$$x = 0.9428$$

Reciprocal Exponents

To get x by itself, raise both sides to the reciprocal exponent.

Example 5: $x^2 = \frac{1}{3}$

$$x^2 = \frac{1}{3}$$

$$(x^2)^{\frac{1}{2}} = \left(\frac{1}{3}\right)^{\frac{1}{2}}$$

$$x = 0.577$$

Example 6: If $\log_{(x+2)} 5 = 10$, then what is x?

$$\log_{(x+2)} 5 = 10$$

$$(x+2)^{10} = 5$$

$$\left[(x+2)^{10}\right]^{\frac{1}{10}} = (5)^{\frac{1}{10}}$$

$$x+2 = 1.175$$

$$x = -0.825$$

Example 7: $\log_x \left(\frac{125}{27}\right) = -\frac{3}{2}$

$$\log_x \left(\frac{125}{27}\right) = -\frac{3}{2}$$

$$x^{-\frac{3}{2}} = \frac{125}{27}$$

$$\left(x^{-\frac{3}{2}}\right)^{-\frac{2}{3}} = \left(\frac{125}{27}\right)^{-\frac{2}{3}}$$

$$x = \frac{9}{25}$$

When you see the terms expand and condense, you can leave a logarithm in the final answer. When you see the word solve, you must get x by itself.

Logarithms Lesson 3

Part I - Raising Reciprocals

QUESTIONS: SOLVE FOR X

1) $x^5 = 4$

6) If $\log_{(x+3)} 4 = 19$, then what is x?

2) $x^{\frac{1}{3}} = 7$

7) $\log_x \left(\frac{16}{64} \right) = -\frac{2}{3}$

3) $x^{\frac{5}{7}} = 6$

4) $(2x)^{\frac{3}{2}} = 3$

5) $x^{-6} = \frac{1}{8}$

ANSWERS:

- 1) 1.32
- 2) 343
- 3) 0.0814
- 4) 1.04
- 5) 1.41
- 6) -1.92
- 7) 8

Logarithms Lesson 3

Part III - Common Bases

EQUATIONS WITH COMMON BASES:

Example 1: $2^x = 8^{3x-1}$

$$2^x = 8^{3x-1}$$

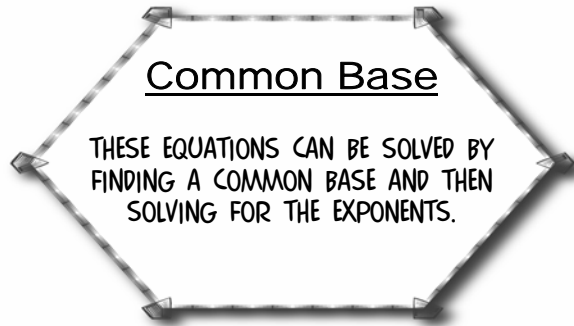
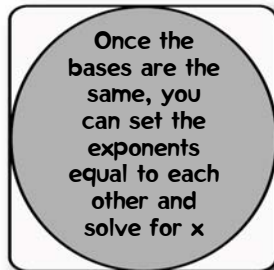
$$2^x = (2^3)^{3x-1}$$

$$2^x = 2^{9x-3}$$

$$x = 9x - 3$$

$$-8x = -3$$

$$x = \frac{3}{8}$$



Example 2: $9^{2x-4} = 27^x$

$$9^{2x-4} = 27^x$$

$$(3^2)^{2x-4} = (3^3)^x$$

$$3^{4x-8} = 3^{3x}$$

$$4x - 8 = 3x$$

$$x = 8$$

Example 3: $\left(\frac{9}{4}\right)^{4x} = \left(\frac{8}{27}\right)^{x-2}$

$$\left(\frac{9}{4}\right)^{4x} = \left(\frac{8}{27}\right)^{x-2}$$

$$\left(\left(\frac{3}{2}\right)^2\right)^{4x} = \left(\left(\frac{2}{3}\right)^3\right)^{x-2}$$

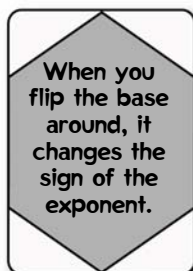
$$\left(\frac{3}{2}\right)^{8x} = \left(\frac{2}{3}\right)^{3x-6}$$

$$\left(\frac{2}{3}\right)^{-8x} = \left(\frac{2}{3}\right)^{3x-6}$$

$$-8x = 3x - 6$$

$$-11x = -6$$

$$x = \frac{6}{11}$$



Example 4:

$$16^{3x} = 64^x \cdot 8^{3x-1}$$

$$16^{3x} = 64^x \cdot 8^{3x-1}$$

$$(2^4)^{3x} = (2^6)^x \cdot (2^3)^{3x-1}$$

$$2^{12x} = 2^{6x} \cdot 2^{9x-3}$$

$$2^{12x} = 2^{6x+9x-3}$$

$$12x = 15x - 3$$

$$-3x = -3$$

$$x = 1$$

Example 5: If

$$X = 16^m \cdot 8^{2m-1},$$

then $\log_2 X$ is:

$$X = 16^m \cdot 8^{2m-1}$$

$$X = (2^4)^m \cdot (2^3)^{2m-1}$$

$$X = 2^{4m} \cdot 2^{6m-3}$$

$$X = 2^{4m+6m-3}$$

$$X = 2^{10m-3}$$

$$\log_2 X = \log_2 2^{10m-3}$$

$$= (10m - 3)\log_2 2$$

$$= 10m - 3$$

Example 6:

$$\frac{1}{3}(27)^{x-1} = \sqrt[4]{9^x}$$

$$\frac{1}{3}(27)^{x-1} = \sqrt[4]{9^x}$$

$$\frac{1}{3}(3^3)^{x-1} = \sqrt[4]{(3^2)^x}$$

$$\frac{1}{3}3^{3x-3} = \sqrt[4]{3^{2x}}$$

$$\frac{3^{3x-3}}{3} = 3^{\frac{2x}{4}}$$

$$3^{(3x-3)-1} = 3^{\frac{x}{2}}$$

$$3x - 4 = \frac{x}{2}$$

$$6x - 8 = x$$

$$5x = 8$$

$$x = \frac{8}{5}$$

Logarithms Lesson 3

Part III - Common Bases

QUESTIONS: SOLVE FOR X

1) $3^{3x} = 9^{2x-1}$

4) $9^{6x} = 81^{8x} \cdot 27^{2x-5}$

2) $16^{3x-6} = 8^x$

5) If $A = 32^m \times 64^{3m-1}$, then $\log_2 A =$

3) $\left(\frac{16}{81}\right)^{2x} = \left(\frac{8}{27}\right)^{3x-7}$

6) $\frac{1}{2}(8)^{x-6} = \sqrt[4]{4^x}$

ANSWERS:

1) 2

2) $\frac{8}{3}$

3) 21

4) 0.577

5) $23m-6$

6) 7.6

Logarithms Lesson 3

Part III - No Common Bases

EQUATIONS WITHOUT COMMON BASES:

Example 1:

$$3^x = 4$$

$$3^x = 4$$

$$\log 3^x = \log 4$$

$$x \log 3 = \log 4$$

$$x = \frac{\log 4}{\log 3}$$

$$x = 1.26$$

Example 2:

$$2 \cdot 5^{x+2} = 7$$

$$2 \cdot 5^{x+2} = 7$$

$$5^{x+2} = \frac{7}{2}$$

$$\log 5^{x+2} = \log \frac{7}{2}$$

$$(x+2) \log 5 = \log \frac{7}{2}$$

$$x \log 5 + 2 \log 5 = \log \frac{7}{2}$$

$$x \log 5 = \log \frac{7}{2} - 2 \log 5$$

$$x = \frac{\log \frac{7}{2} - 2 \log 5}{\log 5}$$

$$x = -1.22$$

Example 3:

$$2 \cdot 3^{x+2} = 6$$

$$2 \cdot 3^{x+2} = 6$$

$$3^{x+2} = \frac{6}{2}$$

$$3^{x+2} = 3$$

$$x+2 = 1$$

$$x = -1$$

Example 4:

$$5^x = -2$$

$$5^x = -2$$

$$\log 5^x = \log(-2)$$

No Solution

Example 5:

$$\left(\frac{2}{5}\right)^{x-3} = \frac{1}{3}$$

$$\left(\frac{2}{5}\right)^{x-3} = \frac{1}{3}$$

$$\log\left(\frac{2}{5}\right)^{x-3} = \log \frac{1}{3}$$

$$(x-3) \log\left(\frac{2}{5}\right) = \log \frac{1}{3}$$

$$x \log\left(\frac{2}{5}\right) - 3 \log\left(\frac{2}{5}\right) = \log \frac{1}{3}$$

$$x \log\left(\frac{2}{5}\right) = \log \frac{1}{3} + 3 \log\left(\frac{2}{5}\right)$$

$$x = \frac{\log \frac{1}{3} + 3 \log\left(\frac{2}{5}\right)}{\log\left(\frac{2}{5}\right)}$$

$$x = 4.199$$

Example 6:

$$a^{x+3} = b^{-1}$$

$$a^{x+3} = b^{-1}$$

$$\log a^{x+3} = \log b^{-1}$$

$$(x+3) \log a = -\log b$$

$$x \log a + 3 \log a = -\log b$$

$$x = \frac{-\log b - 3 \log a}{\log a}$$

Log Both Sides

EQUATIONS WITHOUT COMMON BASES CAN BE SOLVED BY TAKING THE LOG OF EACH SIDE, THEN SIMPLIFYING.

Example 7: If

$\log x + \log x + \log x = 6$,
solve for x

$$\log x + \log x + \log x = 6$$

$$3 \log x = 6$$

$$\log x = 2$$

$$10^2 = x$$

$$x = 100$$

Example 8:

$$\log_4(x^2 + 1) - \log_4 6 = \log_4 5$$

$$\log_4(x^2 + 1) - \log_4 6 = \log_4 5$$

$$\log_4 \frac{x^2 + 1}{6} = \log_4 5$$

$$\frac{x^2 + 1}{6} = 5$$

$$x^2 + 1 = 30$$

$$x^2 = 29$$

$$x = \pm \sqrt{29}$$

When you arrive at a step where you have a log on one side and a log on the other (with no extra terms) you can cancel the logs and keep going.

Logarithms Lesson 3

Part III - No Common Bases

QUESTIONS: SOLVE FOR X

1) $4^x = 5$

5) $\left(\frac{4}{5}\right)^{2x-3} = \frac{1}{4}$

2) $3 \cdot 7^{x+5} = 8$

6) $a^{x+2} = b^{-5}$

3) $3 \cdot 4^{x+6} = 2$

7) If $\log_x a + \log_x a = 8$, then $x =$

4) $4^x = -7$

8) $\log_3(x^2 + 5) - \log_3 8 = \log_3 3$

ANSWERS:

1) 1.16

2) -4.50

3) -6.29

4) Undefined

5) 4.61

6) $\frac{-5\log b - 2\log a}{\log a}$

7) $x = a^{\frac{1}{4}}$

8) ± 4.36

Logarithms Lesson 3

Part IV - Solving By Grouping

SOLVE BY GROUPING:

Example 1:

$$6^{5x} = 3^{2x-1}$$

$$6^{5x} = 3^{2x-1}$$

$$\log 6^{5x} = \log 3^{2x-1}$$

$$5x \log 6 = (2x-1) \log 3$$

$$5x \log 6 = 2x \log 3 - \log 3$$

$$5x \log 6 - 2x \log 3 = -\log 3$$

$$x(5 \log 6 - 2 \log 3) = -\log 3$$

$$x = \frac{-\log 3}{5 \log 6 - 2 \log 3}$$

$$x = -0.1625$$

Example 2:

$$2^{x+3} = 3^{2x-1}$$

$$2^{x+3} = 3^{2x-1}$$

$$\log 2^{x+3} = \log 3^{2x-1}$$

$$(x+3) \log 2 = (2x-1) \log 3$$

$$x \log 2 + 3 \log 2 = 2x \log 3 - \log 3$$

$$x \log 2 - 2x \log 3 = -\log 3 - 3 \log 2$$

$$x(\log 2 - 2 \log 3) = -\log 3 - 3 \log 2$$

$$x = \frac{-\log 3 - 3 \log 2}{\log 2 - 2 \log 3}$$

$$x = 2.11$$

Factoring x

THESE EQUATIONS WILL REQUIRE YOU TO BRING THE TERMS INVOLVING X TO ONE SIDE, FACTOR THEM, AND THEN SOLVE.

Example 3:

$$\frac{4^{2x-1}}{3} = 5^x$$

$$\frac{4^{2x-1}}{3} = 5^x$$

$$\log \frac{4^{2x-1}}{3} = \log 5^x$$

$$\log 4^{2x-1} - \log 3 = \log 5^x$$

$$(2x-1) \log 4 - \log 3 = x \log 5$$

$$2x \log 4 - \log 4 - \log 3 = x \log 5$$

$$2x \log 4 - x \log 5 = \log 4 + \log 3$$

$$x(2 \log 4 - \log 5) = \log 4 + \log 3$$

$$x = \frac{\log 4 + \log 3}{2 \log 4 - \log 5}$$

$$x = 2.14$$

Example 4:

$$2 \cdot 3^{x+3} = 6^{3x}$$

$$2 \cdot 3^{x+3} = 6^{3x}$$

$$\log(2 \cdot 3^{x+3}) = \log 6^{3x}$$

$$\log 2 + \log 3^{x+3} = \log 6^{3x}$$

$$\log 2 + (x+3) \log 3 = 3x \log 6$$

$$\log 2 + x \log 3 + 3 \log 3 = 3x \log 6$$

$$x \log 3 - 3x \log 6 = -\log 2 - 3 \log 3$$

$$x(\log 3 - 3 \log 6) = -\log 2 - 3 \log 3$$

$$x = \frac{-\log 2 - 3 \log 3}{\log 3 - 3 \log 6}$$

$$x = 0.9327$$

Example 5: $3 \cdot 4^{2x+3} = 7^{4x-2}$

$$3 \cdot 4^{2x+3} = 7^{4x-2}$$

$$\log(3 \cdot 4^{2x+3}) = \log 7^{4x-2}$$

$$\log 3 + \log 4^{2x+3} = \log 7^{4x-2}$$

$$\log 3 + (2x+3) \log 4 = (4x-2) \log 7$$

$$\log 3 + 2x \log 4 + 3 \log 4 = 4x \log 7 - 2 \log 7$$

$$2x \log 4 - 4x \log 7 = -2 \log 7 - \log 3 - 3 \log 4$$

$$x(2 \log 4 - 4 \log 7) = -2 \log 7 - \log 3 - 3 \log 4$$

$$x = \frac{-2 \log 7 - \log 3 - 3 \log 4}{2 \log 4 - 4 \log 7}$$

$$x = 1.826$$

Example 6:

$$\log_3(x) - \log_3 2 = \log_3 7$$

$$\log_3(x) - \log_3 2 = \log_3 7$$

$$\log_3\left(\frac{x}{2}\right) = \log_3 7$$

$$\frac{x}{2} = 7$$

$$x = 14$$

Logarithms Lesson 3

Part IV - Solving By Grouping

QUESTIONS: SOLVE FOR X

1: $6^{5x} = 3^{2x-1}$

3: $\frac{5^{3x-1}}{2} = 6^x$

5: $2 \cdot 3^{3x+2} = 5^{4x-2}$

2: $3^{2x+1} = 4^{5x-1}$

4: $3 \cdot 5^{2x+6} = 4$

6: $\log_2(x) - \log_2 4 = \log_2 6$

ANSWERS:

- 1) -0.16
- 2) 0.52
- 3) 0.76
- 4) -2.91
- 5) 1.94
- 6) 24

Logarithms Lesson 3

Part V - Solving Using Log Rules

SOLVING USING LOG RULES:

Example 1: $3\log x + 5 = 8$

$$3\log x + 5 = 8$$

$$3\log x = 3$$

$$\log x = 1$$

$$10^1 = x$$

$$x = 10$$

Example 2: $3\log_2 x = 12$

$$3\log_2 x = 12$$

$$\log_2 x = 4$$

$$2^4 = x$$

$$x = 16$$

Example 3: $\log_3(x - 2) = \log_3(3x + 2)$

$$\log_3(x - 2) = \log_3(3x + 2)$$

$$x - 2 = 3x + 2$$

$$-2x = 4$$

$$x = -2 \quad \text{Reject - 2 since it would make the original logarithm undefined}$$

Example 4: $2\log_5 3 = \log_5(x + 1)$

$$2\log_5 3 = \log_5(x + 1)$$

$$\log_5 3^2 = \log_5(x + 1)$$

$$9 = x + 1$$

$$x = 8$$

Example 5: $2\log_2 x + \log_2 16 = \log_2 9$

$$2\log_2 x + \log_2 16 = \log_2 9$$

$$\log_2 x^2 + \log_2 16 = \log_2 9$$

$$\log_2 16x^2 = \log_2 9$$

$$16x^2 = 9$$

$$x^2 = \frac{9}{16}$$

$$x = \frac{3}{4} \quad \text{Reject } -\frac{3}{4} \text{ since it would make the original logarithm undefined}$$

Example 6: $5\log_3 x = 2\log_3 b$

$$5\log_3 x = 2\log_3 b$$

$$\log_3 x^5 = \log_3 b^2$$

$$x^5 = b^2$$

$$(x^5)^{\frac{1}{5}} = (b^2)^{\frac{1}{5}}$$

$$x = b^{\frac{2}{5}}$$

Example 7:

$$\log_2(x + 1) + \log_2(x - 2) = 3$$

$$\log_2(x + 1) + \log_2(x - 2) = 3$$

$$\log_2(x + 1)(x - 2) = 3$$

$$2^3 = (x + 1)(x - 2)$$

$$8 = x^2 - x - 2$$

$$0 = x^2 - x - 10 \quad \text{Solve by graphing and finding the } x - \text{ intercepts}$$

$$x = 3.70 \quad \text{Reject } -2.70 \text{ since it would make the original logarithm undefined}$$

Example 8: $\log_3(x - 1) = 2 - \log_3(x + 2)$

$$\log_3(x - 1) = 2 - \log_3(x + 2)$$

$$\log_3(x - 1) + \log_3(x + 2) = 2$$

$$\log_3(x - 1)(x + 2) = 2$$

$$3^2 = (x - 1)(x + 2)$$

$$9 = x^2 + x - 2$$

$$0 = x^2 + x - 11$$

$$x = 2.85 \quad \text{Reject } -3.85 \text{ since it would make the original logarithm undefined}$$

Example 9: $\log_2 x = 3 - \log_2(x + 2)$

$$\log_2 x = 3 - \log_2(x + 2)$$

$$\log_2 x + \log_2(x + 2) = 3$$

$$\log_2 x(x + 2) = 3$$

$$8 = x(x + 2)$$

$$8 = x^2 + 2x$$

$$0 = x^2 + 2x - 8$$

$$x = 2 \quad \text{Reject } -4$$

Logarithms Lesson 3

Part V - Solving Using Log Rules

Example 10: $\log x - \log(x+5) = 1$

$$\log x - \log(x+5) = 1$$

$$\log\left(\frac{x}{x+5}\right) = 1$$

$$10^1 = \frac{x}{x+5}$$

$$10 = \frac{x}{x+5}$$

$$10x + 50 = x$$

$$9x = -50$$

$$x = -5.56 \quad \text{Reject, so no solution.}$$

Example 11: $\log_3(2x+1) - \log_3(x-1) = 1$

$$\log_3(2x+1) - \log_3(x-1) = 1$$

$$\log_3\left(\frac{2x+1}{x-1}\right) = 1$$

$$3 = \frac{2x+1}{x-1}$$

$$3x - 3 = 2x + 1$$

$$x = 4$$

Example 12: $\log x^2 + \log 3 = \log 2x$

$$\log x^2 + \log 3 = \log 2x$$

$$\log 3x^2 = \log 2x$$

$$3x^2 = 2x$$

$$3x^2 - 2x = 0$$

$$x(3x - 2) = 0$$

$$x = \frac{2}{3} \quad \text{Reject 0}$$

Example 13: $\log_{(x-1)} 25 = 2$

$$\log_{(x-1)} 25 = 2$$

$$(x-1)^2 = 25$$

$$\sqrt{(x-1)^2} = \sqrt{25}$$

$$x-1 = \pm 5$$

$$x = 6 \quad \text{Reject - 4}$$

Example 14:

$$2\log(x-3) = \log 4 + \log(6-x)$$

$$2\log(x-3) = \log 4 + \log(6-x)$$

$$\log(x-3)^2 = \log 4(6-x)$$

$$(x-3)^2 = 4(6-x)$$

$$x^2 - 6x + 9 = 24 - 4x$$

$$x^2 - 2x - 15 = 0$$

(Factor, or graph and find x - intercepts)

$$x = 5 \quad \text{Reject - 3}$$

Example 15:

$$(\log x)^2 - 4\log x - 5 = 0$$

$$(\log x)^2 - 4\log x - 5 = 0$$

$$a^2 - 4a - 5 = 0$$

$$(a-5)(a+1) = 0$$

$$(\log x - 5)(\log x + 1) = 0$$

$$\log x - 5 = 0$$

$$\log x = 5$$

$$100000 = x$$

$$\log x + 1 = 0$$

$$\log x = -1$$

$$x = 0.1$$

Replace logx with "a" to see the quadratic equation better. Once factored, put logx back in to complete the question.

Logarithms Lesson 3

Part V - Solving Using Log Rules

QUESTIONS: SOLVE FOR X

1) $5\log x + 4 = 2$

5) $3\log_2 x + \log_2 25 = \log_2 16$

9) $\log_3(x-3) = 4 - \log_3(x+3)$

2) $4\log_2 x = 24$

6) $4\log_3 x = 8\log_3 b$

10) $\log_2 x = 5 - \log_2(x+3)$

3) $\log_3(x-1) = \log_3(2x+2)$

7) $\log_2(x-2) = 4$

11) $\log x - \log(x+4) = 1$

4) $2\log_5 5 = \log_5(2x+1)$

8) $\log(x+2) + \log(x-1) = 1$

Logarithms Lesson 3

Part V - Solving Using Log Rules

12) $\log_3(5x+1) - \log_3(3x-1) = 1$ 15) $2\log(x-2) = \log 5 + \log(5-x)$ 17) $(\log x)^2 + \log x^3 = -2$

13) $\log x^3 + \log 4 = \log 3x$

16) $3\log x - \log x = 2$

18) $(\log x)^4 - 16 = 0$

14) $\log_{(x-3)} 36 = 3$

ANSWERS:

1) 0.398

7) 18

13) 0.866

2) 64

8) 3

14) 6.3

3) No Solution

9) 9.49

15) 4.11

4) 12

10) 4.35

16) 10

5) 0.862

11) No Solution

17) 0.01, 0.1

6) b^2

12) 1

18) 0.01, 100

Logarithms Lesson 3

Part VII – Diploma Style

DIPLOMA STYLE EQUATION QUESTIONS

Example 1: The graph of $y = \log_a x$ goes through the point (3, 7).
What is the value of a?

$$y = \log_a x$$

$$7 = \log_a 3$$

$$a^7 = 3$$

$$a = 3^{\frac{1}{7}}$$

$$a = 1.17$$

Example 2: Determine the x-intercept of $y = \log_2(x+4)$

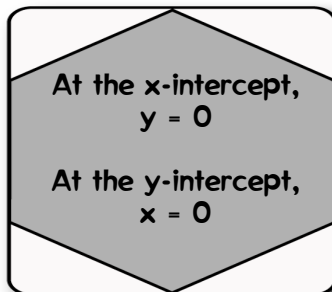
$$y = \log_2(x+4)$$

$$0 = \log_2(x+4)$$

$$2^0 = x+4$$

$$1 = x+4$$

$$x = -3$$



Example 3: The y-intercept of $y = \log_2(x+4)$ is

$$y = \log_2(0+4)$$

$$y = \log_2 4$$

$$y = 2$$

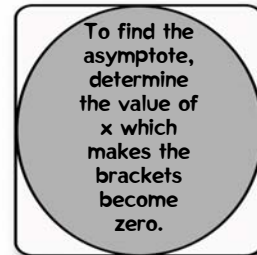
Example 4: Determine the equation of an asymptote of $y = \log_3(3x-8)+4$

$$y = \log_3(3x-8)+4$$

$$3x-8=0$$

$$3x=8$$

$$x = \frac{8}{3}$$



Example 5: An exponential function is defined as: $f(x) = 5(1.2)^{x-3}$.

If $f(k) = 83$, then the value of k is?

$$f(x) = 5(1.2)^{x-3}$$

$$83 = 5(1.2)^{k-3}$$

$$16.6 = (1.2)^{k-3}$$

$$\log 16.6 = \log(1.2)^{k-3}$$

$$\log 16.6 = (k-3)\log 1.2$$

$$\log 16.6 = k\log 1.2 - 3\log 1.2$$

$$\log 16.6 + 3\log 1.2 = k\log 1.2$$

$$k = \frac{\log 16.6 + 3\log 1.2}{\log 1.2}$$

$$k = 18.41$$

Logarithms Lesson 3

Part VII – Diploma Style

Example 6: Given the functions $f(x) = \log_4(x-2) - 2$ & $g(x) = \log_4(3x+1)$, determine the x-value at the point of intersection.

$$\begin{aligned} f(x) &= g(x) \\ \log_4(x-2) - 2 &= \log_4(3x+1) \\ \log_4(x-2) - \log_4(3x+1) &= 2 \\ \log_4\left(\frac{x-2}{3x+1}\right) &= 2 \\ 16 &= \frac{x-2}{3x+1} \\ 48x+16 &= x-2 \\ 47x &= -18 \\ x &= -0.383 \text{ (Reject)} \\ \text{No Solution Exists} \end{aligned}$$

To find the point of intersection of two functions, set them equal to each other and solve for x.

Example 7: The point (27, 3) lies on the graph of $y = \log_b x$. If the point (4, k) lies on the graph of $y = b^x$, then what is the value of k?

$$\begin{aligned} y &= \log_b x \\ 3 &= \log_b 27 & y &= b^x \\ b^3 &= 27 & k &= 3^4 \\ b &= 3 & k &= 81 \end{aligned}$$

First solve for the b-value, then use it to find k.

Example 8: Determine the equation of the inverse of $f(x) = \log_5 x$

$$\begin{aligned} y &= \log_5 x \\ x &= \log_5 y \\ 10^x &= 5y \\ f^{-1}(x) &= \frac{10^x}{5} \end{aligned}$$

Example 10: The ordered pair (3, 81) lies on the inverse of $y = \log_a x$. Find the value of a.

$$\begin{aligned} y &= \log_a x \\ x &= \log_a y \\ 3 &= \log_a 81 \\ a^3 &= 81 \\ a &= 4.33 \end{aligned}$$

Example 9: The ordered pair (k, 1.23) lies on the inverse of $y = 2^x$.

The value of k is:

$$\begin{aligned} y &= 2^x \\ x &= 2^y \\ k &= 2^{1.23} \end{aligned}$$

Example 11: Determine the y-intercept of the function

$$\begin{aligned} y &= a^{x-3} \\ y &= a^{x-3} \\ y &= a^{0-3} \\ y &= a^{-3} \\ y &= \frac{1}{a^3} \end{aligned}$$

Logarithms Lesson 3

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QUESTIONS:

- 1) The graph of $y = \log_a x$ goes through the point (3, 6). What is the value of a ?
- 2) Determine the x-intercept of $y = \log_4(x+7)$
- 3) The y-intercept of $y = \log_5(x-2)$ is
- 4) Determine the equation of an asymptote of $y = \log_5(2x - 6) + 3$
- 5) An exponential function is defined as $f(x) = 3(2.4)^{x-2}$. If $f(k) = 65$, then the value of k is?
- 6) The graphs of $f(x) = \log_3(3x-3)$ & $g(x) = \log_3(2x - 1)$ intersect at a point. What are the coordinates of that point?
- 7) The point (64, 4) lies on the graph of $y = \log_b x$. If the point (5, k) lies on the graph of $y = b^x$, then what is the value of k ?
- 8) Determine the equation of the inverse of $f(x) = \log_3 x$
- 9) The ordered pair (k , 4.11) lies on the inverse of $y = 3^x$.
The value of k is:

ANSWERS:

- 10) The ordered pair (4, 64) lies on the inverse of $y = \log_a x$.
Find the value of a .
 - 11) The graph of $y = b^{x-4}$ has a y –intercept of
- 1) 1.2
 - 2) -6
 - 3) Undefined
 - 4) $x = 3$
 - 5) 5.51
 - 6) (2, 1)
 - 7) 181.02
 - 8) $y = 3^x$
 - 9) 91.4
 - 10) 2.83
 - 11) $\frac{1}{b^4}$