Pre - Calculus Mathematics 40S

STANDARDS TEST PRACTICE EXAM - ANSWERS

Logarithms

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1. An investment earns interest at a rate of 4% compounded quarterly. How long, in years, will it take for the investment to double?

Start with the compound interest formula: $A = P\left(1+\frac{r}{n}\right)^{nt}$.

The rate is 4%, which must be expressed as a decimal $\rightarrow 0.04$

Use n = 4 since the compounding is quarterly.

The principal is P, and the final amount is double $\rightarrow$ *2P*

$$2P = P\left(1+\frac{0.04}{4}\right)^{4t}$$
$$2 = (1.01)^{4t}$$
$$\log 2 = \log (1.01)^{4t}$$
$$\log 2 = 4t\log(1.01)$$
$$t = \frac{\log 2}{4\log 1.01}$$
$$t = 17.4$$

The investment will take ***17.4*** *years to double.*

2. Solve the equation $2^{x+1} = 3\left(4^x\right)$ algebraically

$$2^{x+1} = 3\left(4^x\right)$$
$$\log 2^{x+1} = \log\left[3\left(4^x\right)\right]$$
$$\log 2^{x+1} = \log 3 + \log 4^x$$
$$(x+1)\log 2 = \log 3 + x\log 4$$
$$x\log 2 + \log 2 = \log 3 + x\log 4$$
$$x\log 2 - x\log 4 = \log 3 - \log 2$$
$$x(\log 2 - \log 4) = \log 3 - \log 2$$
$$x = \frac{\log 3 - \log 2}{\log 2 - \log 4}$$
$$\mathbf{x = -0.585}$$

3. If $\log_a 4 = p$ and $\log_a 7 = q$, determine an expression for $\log_a 28$ in terms of p and q

Start by rewriting $\log_a 28$ *as* $\log_a(4 \times 7)$

Then expand using the log rule for multiplication:

$$\log_a(4 \times 7) = \log_a 4 + \log_a 7$$

Finally, rewrite $\log_a 4 + \log_a 7$ *as* ***p + q***.

4. Given $f(x)=e^x$, determine the function after a reflection across the line $y = x$

From the lessons, you must remember that $y = e^x$ and $y = \ln(x)$ are inverse functions.

The inverse is $\boldsymbol{f^{-1}(x) = \ln(x)}$

5. Solve for x: $\log_2(\log_{16} x) = -2$

$$\log_2(\log_{16} x) = -2$$
$$2^{-2} = \log_{16} x$$
$$\frac{1}{4} = \log_{16} x$$
$$16^{\frac{1}{4}} = x$$
$$\boldsymbol{x = 2}$$

6. Solve for x: $\left(\frac{1}{2}\right)^{2x-1} = 4^x$

$$\left(\frac{1}{2}\right)^{2x-1} = 4^x$$
$$\left(2^{-1}\right)^{2x-1} = 4^x$$
$$2^{-2x+1} = 4^x$$
$$2^{-2x+1} = \left(2^2\right)^x$$
$$2^{-2x+1} = 2^{2x}$$
$$-2x + 1 = 2x$$
$$1 = 4x$$
$$\boldsymbol{x = \frac{1}{4}}$$

7. Solve algebraically: $3^{2x-1} = 5^{x+4}$

$$\log 3^{2x-1} = \log 5^{x+4}$$
$$(2x - 1)\log 3 = (x + 4)\log 5$$
$$2x\log 3 - \log 3 = x\log 5 + 4\log 5$$
$$2x\log 3 - x\log 5 = 4\log 5 + \log 3$$
$$x(2\log 3 - \log 5) = 4\log 5 + \log 3$$
$$x = \frac{4\log 5 + \log 3}{2\log 3 - \log 5}$$
$$\boldsymbol{x = 12.82}$$

8. In April 1994, the population of a small town in Manitoba was estimated at 2500 people. The population can be represented by the equation $A = Pe^{rt}$, where r is the annual rate of increase, and t is the time in years. Determine the annual rate of increase if there were 3900 people in April 1999. State your answer to three decimal places.

Plug in 2500 for P, 3900 for A, and 5 for t

$$A = Pe^{rt}$$
$$3900 = (2500)e^{r(5)}$$
$$\frac{3900}{2500} = \frac{(2500)e^{r(5)}}{2500}$$
$$1.56 = e^{5r}$$
$$\ln 1.56 = \ln e^{5r}$$
$$\ln 1.56 = 5r \ln e$$
$$r = \frac{\ln 1.56}{5 \ln e}$$
$$r = 0.0889$$

The annual rate of increase is ***8.89 %***

9. If $\log_a x = 16$, find the value of $\log_a \sqrt{x}$

Start by simplifying $log_a \sqrt{x}$

$$log_a \sqrt{x} = log_a x^{\frac{1}{2}} = \frac{1}{2} log_a x$$

Now replace $log_a x$ *with 16.*

$$\frac{1}{2} log_a x = \frac{1}{2}(16) = \mathbf{8}$$

10. If $f(x) = \log x$, determine the equation of the inverse function

From the notes, recall that $y = logx$ *is the inverse graph of* $y = 10^x$

The inverse function is $\mathbf{f^{-1}(x) = 10^x}$

11. Solve for x: $\log_4(\log_9 x) = \frac{1}{2}$

$$log_4(log_9 x) = \frac{1}{2}$$
$$4^{\frac{1}{2}} = log_9 x$$
$$2 = log_9 x$$
$$9^2 = x$$
$$\mathbf{x = 81}$$

12. Determine the x-intercepts in the following equation: $y = \log(5-4x) - 2\log x$

Solve for x-intercepts by setting y = 0, then solving for x.

$$0 = \log(5-4x) - 2\log x$$
$$2\log x = \log(5-4x)$$
$$\log x^2 = \log(5-4x)$$
$$x^2 = 5-4x$$
$$x^2 + 4x - 5 = 0$$
$$(x+5)(x-1) = 0$$

$\mathbf{x = 1}$ *Reject -5, since it makes logx undefined*

13. Solve for x algebraically: $2^{x+4} = 3^{2x+1}$

$$2^{x+4} = 3^{2x+1}$$
$$\log 2^{x+4} = \log 3^{2x+1}$$
$$(x+4)\log 2 = (2x+1)\log 3$$
$$x\log 2 + 4\log 2 = 2x\log 3 + \log 3$$
$$x\log 2 - 2x\log 3 = \log 3 - 4\log 2$$
$$x(\log 2 - 2\log 3) = \log 3 - 4\log 2$$
$$x = \frac{\log 3 - 4\log 2}{\log 2 - 2\log 3}$$
$$\mathbf{x = 1.113}$$

14. If $\log_a 4 = 1.2619$ and $\log_a 5 = 1.4650$, show that $\log_a 20 = 2.7269$

Start by rewriting $\log_a 20$ *as* $\log_a(4 \times 5)$

Then expand using the log rule for multiplication:

$$\log_a(4 \times 5) = \log_a 4 + \log_a 5$$

Plug in the numbers given in the question:

$$\log_a 4 + \log_a 5 = 1.2619 + 1.4650 = \mathbf{2.7269}$$

15. A new automobile costs \$32,000. The value of the auto after *t* years is given by $V = 32000(0.8)^t$

a) Determine the value after 9 years

$$V = 32000(0.8)^t$$
$$V = 32000(0.8)^9$$
$$\mathbf{V = 4294.97}$$

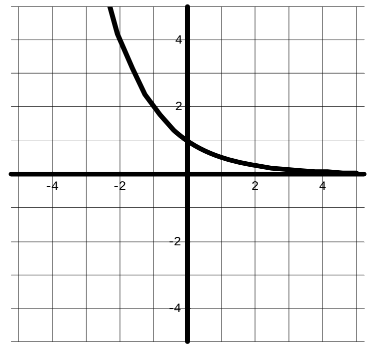
b) How many years will it take for the value to decrease to one-eighth the initial value?
One-eighth \$32,000 is \$4000

$$4000 = 32000(0.8)^t$$
$$\frac{4000}{32000} = \frac{32000(0.8)^t}{32000}$$
$$0.125 = 0.8^t$$
$$\log 0.125 = \log 0.8^t$$
$$\log 0.125 = t\log 0.8$$
$$t = \frac{\log 0.125}{\log 0.8}$$
$$\mathbf{t = 9.32}$$

16. Solve for x $\log_2(x+4) + \log_2(x-3) = 3$

$$\log_2(x+4) + \log_2(x-3) = 3$$
$$\log_2(x+4)(x-3) = 3$$
$$2^3 = (x+4)(x-3)$$
$$8 = x^2 + x - 12$$
$$0 = x^2 + x - 20$$
$$0 = (x+5)(x-4)$$
$$\mathbf{x = 4} \qquad \text{Reject } -5 \text{ since it makes } \log_2(x+4) \text{ undefined}$$

17. State the range of $f(x) = 2^{-x}$



Graph the function in your calculator
The range is y > 0.

18. Solve for *x*: $\left(\frac{1}{3}\right)^{2x} = 27^{x-5}$

$$\left(3^{-1}\right)^{2x} = 27^{x-5}$$
$$3^{-2x} = \left(3^{3}\right)^{x-5}$$
$$3^{-2x} = 3^{3x-15}$$
$$-2x = 3x - 15$$
$$15 = 5x$$
$$\mathbf{x = 3}$$

19. Solve the equation: $2^{3x+1} = 9^{x+3}$. Express your answers to three decimal places

$$2^{3x+1} = 9^{x+3}$$
$$\log 2^{3x+1} = \log 9^{x+3}$$
$$(3x+1)\log 2 = (x+3)\log 9$$
$$3x\log 2 + \log 2 = x\log 9 + 3\log 9$$
$$3x\log 2 - x\log 9 = 3\log 9 - \log 2$$
$$x(3\log 2 - \log 9) = 3\log 9 - \log 2$$
$$x = \frac{3\log 9 - \log 2}{3\log 2 - \log 9}$$
$$\mathbf{x = -50.080}$$

20. Determine the domain of the function $y = \ln(x-3)$

The number inside the natural logarithm must be positive, or else it's undefined. Therefore, x – 3 must be bigger than 0

x – 3 > 0

The domain is ***x > 3***

21. Solve for *x*: $2^{4x-1} = 4^{x+1}$

$$\log 2^{4x-1} = \log 4^{x+1}$$
$$(4x-1)\log 2 = (x+1)\log 4$$
$$4x\log 2 - \log 2 = x\log 4 + \log 4$$
$$4x\log 2 - x\log 4 = \log 4 + \log 2$$
$$x(4\log 2 - \log 4) = \log 4 + \log 2$$
$$x = \frac{\log 4 + \log 2}{4\log 2 - \log 4}$$
$$\boldsymbol{x = 1.5}$$

22. Given $f(x) = \log(x+100)+5$, evaluate $f(0)$

$$f(0) = \log(0+100)+5$$
$$f(0) = \log 100 + 5$$
$$f(0) = 2+5$$
$$\boldsymbol{f(0) = 7}$$

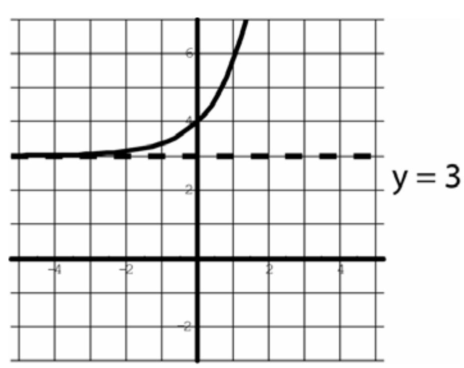
23. Evaluate $(\log_3(\log_2 8))$

$$(\log_3(\log_2 8))$$
$$\log_3(3)$$
$$\boldsymbol{= 1}$$

24. The point (64, 3) lies on the graph of $y = \log_b x$. The exact value of *b* is

$$y = \log_b x$$
$$3 = \log_b 64$$
$$b^3 = 64$$
$$(b^3)^{\frac{1}{3}} = 64^{\frac{1}{3}}$$
$$\boldsymbol{b = 4}$$

25. Sketch a clearly labeled graph of $y = e^x + 3$



26. Given $f\left(x\right)=\log_2\left(x-5\right)$, find an equation for $f^{-1}\left(x\right)$

$$\text{Rewrite as } y = \log_2\left(x - 5\right)$$

$$\text{Interchange } x \,\&\, y\text{: } x = \log_2\left(y - 5\right)$$

$$2^x = y - 5$$

$$y = 2^x + 5$$

$$\boldsymbol{f^{-1}\left(x\right) = 2^x + 5}$$

27. Solve the equation: $\log(x+2)+\log(x-1)=1$

$$\log(x + 2) + \log(x - 1) = 1$$

$$\log\left(x + 2\right)\left(x - 1\right) = 1$$

$$10^1 = \left(x + 2\right)\left(x - 1\right)$$

$$10 = x^2 + x - 2$$

$$0 = x^2 + x - 12$$

$$0 = \left(x + 4\right)\left(x - 3\right)$$

$$\boldsymbol{x = 3} \qquad \text{Reject } -4$$

28. Solve the following equation algebraically: $4\left(10^x\right)=6^{(2-x)}$

$$\log4\left(10^x\right) = \log6^{(2-x)}$$

$$\log4 + \log10^x = \left(2 - x\right)\log6$$

$$\log4 + x\log10 = 2\log6 - x\log6$$

$$x\log10 + x\log6 = 2\log6 - \log4$$

$$x\left(\log10 + \log6\right) = 2\log6 - \log4$$

$$x = \frac{2\log6 - \log4}{\log10 + \log6}$$

$$\boldsymbol{x = 0.5366}$$

29. The initial population of a city was 4000 and grew exponentially to 8000 in 7 years. The population growth can be modeled by the equation $A = Pe^{rt}$, where r is the annual rate of increase, and t is the time in years. Find the growth rate for this city to three decimal places.

$$A = Pe^{rt}$$
$$8000 = 4000e^{r(7)}$$
$$\frac{8000}{4000} = \frac{4000e^{r(7)}}{4000}$$
$$2 = e^{7r}$$
$$ln2 = ln(e)^{7r}$$
$$ln2 = 7rln(e)$$
$$r = \frac{ln2}{7ln(e)}$$
$$\mathbf{r = 0.099}$$

30. Evaluate $\ln e^5$

$$lne^5 = 5lne = 5(1) = \mathbf{5}$$

31. Solve for x in the equation $4^x = \frac{1}{64}$

$$4^x = \frac{1}{64}$$
$$4^x = \frac{1}{4^3}$$
$$4^x = 4^{-3}$$
$$\mathbf{x = -3}$$

32. Express $\frac{1}{3}\log M - 5\log N$ as a single logarithm

$$\frac{1}{3}logM - 5logN$$
$$= logM^{\frac{1}{3}} - logN^5$$
$$\mathbf{= log\left(\frac{M^{\frac{1}{3}}}{N^5}\right)}$$

33. The population of bacteria grows exponentially according to the equation $A = 4e^{rt}$, where P is the population at time t, r is the hourly rate of increase, and t is the time in hours. If there are 100 bacteria after 2 hours, what is the population after 8 hours?

$$A = 4e^{rt}$$
$$100 = 4e^{2r}$$
$$\frac{100}{4} = \frac{4e^{2r}}{4}$$
$$25 = e^{2r}$$
$$\ln 25 = \ln(e)^{2r}$$
$$\ln 25 = 2r\ln(e)$$
$$r = \frac{\ln 25}{2\ln(e)}$$
$$r = 1.6094$$

$$A = 4e^{rt}$$
$$A = 4e^{(1.6094)(8)}$$
$$\mathbf{A = 1562026}$$

34. Solve for x:

$$3\left(2^{x}\right) = 7^{1-x}$$
$$\log 3\left(2^{x}\right) = \log 7^{1-x}$$
$$\log 3 + \log 2^{x} = (1 - x)\log 7$$
$$\log 3 + x\log 2 = \log 7 - x\log 7$$
$$x\log 2 + x\log 7 = \log 7 - \log 3$$
$$x(\log 2 + \log 7) = \log 7 - \log 3$$
$$x = \frac{\log 7 - \log 3}{\log 2 + \log 7}$$
$$\mathbf{x = 0.3211}$$

35. Simplify $\log_a(4x) - \log_a(2x)$

$$\log_a(4x) - \log_a(2x)$$
$$\log_a\left(\frac{4x}{2x}\right)$$
$$\mathbf{\log_a 2}$$

36. The graph of $y = \ln x$ is obtained by reflecting the graph of $y = e^x$ over the line ____.

$$\boldsymbol{y = x}$$

37. Evaluate $\log_7\left(\frac{1}{7}\right)$

$$log_7\left(7^{-1}\right)$$
$$= -log_7 7$$
$$\boldsymbol{= -1}$$

38. Determine the numerical value of $\log_3 81 - \log_3 3$

$$log_3 81 - log_3 3$$
$$= log_3\left(\frac{81}{3}\right)$$
$$= log_3 27$$
$$\boldsymbol{3}$$

39. Solve for *x*: $\log_3(2x+1) - \log_3(x-1) - 1 = 0$

$$log_3(2x + 1) - log_3(x - 1) = 1$$
$$log_3 \frac{2x+1}{x-1} = 1$$
$$3^1 = \frac{2x+1}{x-1}$$
$$3\left(x - 1\right) = 2x + 1$$
$$3x - 3 = 2x + 1$$
$$\boldsymbol{x = 4}$$

40. Evaluate $\ln e^4 - 1$

$$\ln e^4 - 1$$
$$= 4 \ln e - 1$$
$$= 4\left(1\right) - 1$$
$$\boldsymbol{= 3}$$