Pre - Calculus Math 40S:

Exponential and Logarithmic Functions

$y = \log_2\left(\frac{A}{B}\right)$

Lesson 2

Laws of Logarithms

Pre - Calculus Math 40S

EXPLAINED!

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Logarithms Lesson 2

Part I - Logarithmic to Exponential Form

CONVERTING FROM LOGARITHMIC TO EXPONENTIAL FORM:

Example 1: Convert $\log_2 x = y$ to exponential form:

$\log_2 x = y$

$2^y = x$

Example 2: Given $3 = \log_5 x$ solve for x.

$\log_5 x = 3$

$\log_5 x = 3$

$5^3 = x$

$x = 125$

Put the 3 on the other side so you can do the seven rule.

Example 3: Convert $3\log b = a$ to exponential form

$3\log b = a$

$\log_{10} b = \frac{a}{3}$

$10^{\frac{a}{3}} = b$

Whenever you have a log written without a base, it actually has a base of 10.

Example 4: Solve for x in $\log_3(2x) = y$

$\log_3 2x = y$

$3^y = 2x$

$x = \frac{3^y}{2}$

7 Rule

THE "SEVEN" RULE IS AN EASY WAY OF REMEMBERING THE CONVERSION. JUST DRAW A SEVEN AS SHOWN BELOW, AND IT WILL GIVE THE EXPONENTIAL FORM!

$\log_a b = c$

$\log_a b = c$

$a^c = b$

QUESTIONS:

Convert each of the following logarithms to exponential form:

1) $\log_3 a = b$ **2)** $5 = \log_m n$

3) $\log_4 y = x$ **4)** $3 = \log_2 b$

Solve for x in each of the following:

5) $\log_2(x-1) = 3$ **6)** $\log_3(2x+4) = 2$

ANSWERS:

1) $3^b = a$ **2)** $m^5 = n$ **3)** $4^x = y$

4) $b = 8$ **5)** $x = 9$ **6)** $x = 2.5$

CONVERTING FROM EXPONENTIAL TO LOGARITHMIC FORM

Example 1: Convert $y = x^2$ to logarithmic form

First write out the logarithm with the base:

$\log_x \square = \square$

Now fill in the rest so the "seven" rule will give you back what you started with.

$\log_x y = 2$

Example 2: Convert $a = 10x^4$ to logarithmic form

First get the x^4 by itself.

$x^4 = \frac{a}{10}$

Now set up the logarithm

$\log_x \square = \square$

Then fill it in so the "seven" rule works.

$\log_x \frac{a}{10} = 4$

Example 3: Convert $\sqrt{y} = 3x$ to logarithmic form

Rewrite as: $y^{\frac{1}{2}} = 3x$

$\log_y 3x = \frac{1}{2}$

Example 4: Convert $10^{x-y} = \frac{a}{b}$ to logarithmic form:

$\log_{10} \square = \square$

$\log_{10} \frac{a}{b} = x - y$

Example 5: The logarithmic form of $a = \frac{b^m}{b^n}$ is:

First simplify using exponent rules:

$a = b^{m-n}$

$\log_b \square = \square$

$\log_b a = m - n$

A Base is Always a Base

REMEMBER *"A BASE IS ALWAYS A BASE"* WHEN DOING THIS CONVERSION.

IF YOU HAVE $A^C = B$, THE *A*, BEING THE BASE, WILL ALSO BE THE BASE OF YOUR LOGARITHM! PLACE THIS FIRST.

$\log_a \square = \square$

NOW FILL IN THE REST SO WHEN THE SEVEN RULE IS APPLIED, YOU GET BACK THE EXPONENTIAL FUNCTION

$\log_a b \leftarrow c$

QUESTIONS:

Convert each of the following from exponential to logarithmic form

1) $y = x^3$ **2)** $4 = 3a^2$

3) $m^5 = n$ **4)** $2x^6 = 8$

5) $b = \frac{a^5}{a^3}$ **6)** $2\sqrt{x} = 5$

ANSWERS:

1) $\log_x y = 3$ **2)** $\log_a \frac{4}{3} = 2$ **3)** $\log_m n = 5$

4) $\log_x 4 = 6$ **5)** $\log_a b = 2$ **6)** $\log_x \left(\frac{5}{2}\right) = \frac{1}{2}$

Logarithms Lesson 2
Part III - Change of Base

CHANGE OF BASE:

Example 1: Evaluate $\log_2 3$

$$\log_2 3 = \frac{\log 3}{\log 2} = 1.585$$

Example 2: Evaluate $\log_2 \frac{2}{3}$

$$\log_2 \frac{2}{3} = \frac{\log \frac{2}{3}}{\log 2} = -0.585$$

Example 3: Evaluate $\log 5$

$\log 5 = 0.699$

Already base 10. Change of base not needed.

Example 4: Expand $\log_{2x}(y+z)$

$$\log_{2x}(y+z) = \frac{\log(y+z)}{\log 2x}$$

We can't expand log(y + z) any further since logs are not distributive!

$\log(y+z) \neq \log y + \log z$

Example 5: Express $\frac{\log 4}{\log 7}$ as a single logarithm

$$\frac{\log 4}{\log 7} = \log_7 4$$

Change of Base

THE ONLY LOGS YOU CAN DO IN YOUR CALCULATOR ARE BASE 10 LOGS. CHANGE OF BASE LETS YOU DO ANY LOGARITHM IN YOUR CALCULATOR!

$$\log_a b = \frac{\log_{10} b}{\log_{10} a}$$

BY WRITING A LOG AS A FRACTION, YOU AUTOMATICALLY CONVERT IT TO BASE 10 LOGS, SO NOW YOU CAN TYPE IT INTO YOUR CALCULATOR.

Example 6: Express $(\log_a x)(\log_x b)$ as a single logarithm

$$(\log_a x)(\log_x b) = \left(\frac{\cancel{\log x}}{\log a}\right)\left(\frac{\log b}{\cancel{\log x}}\right) = \frac{\log b}{\log a} = \log_a b$$

Example 7: Evaluate the expression: $3^{\log_2 8}$

$$3^{\frac{\log 8}{\log 2}} = 3^3 = 27$$

Logarithms Lesson 2
Part III - Change of Base

QUESTIONS:

Evaluate using change of base :

1) $\log_4 5 =$

2) $\log_5 \frac{8}{9} =$

3) $\log 3 =$

Expand each of the following

4) $\log_{4x}(y - 2z)$

5) $\log_{(a+b)}(x + y)$

Express each as a single logarithm

6) $\frac{\log 5}{\log 8}$

7) $\frac{\log(a - 2b)}{\log c}$

8) $(\log_m n)(\log_n m)$

9) $(\log_a b)(\log_b c)(\log_c d)(\log_d x)$

10) Evaluate : $5^{\log_2 3}$

ANSWERS:

1) 1.16

2) - 0.073

3) 0.48

4) $\frac{\log(y - 2z)}{\log 4x}$

5) $\frac{\log(x + y)}{\log(a + b)}$

6) $\log_8 5$

7) $\log_c(a - 2b)$

8) 1

9) $\log_a x$

10) 12.82

MULTIPLICATION LAW OF LOGARITHMS

Multiplication Law:

WHEN NUMBERS/VARIABLES ARE BEING MULTIPLIED INSIDE A LOGARITHM, THEY CAN BE EXPANDED BY ADDING SEPARATE LOGARITHMS.

$$\log_a(bc) = \log_a b + \log_a c$$

Example 1: Expand $\log(xy)$

$\log(xy) = \log x + \log y$

Example 2: Expand $\log(3a \bullet 2b)$

$\log(3a \bullet 2b)$

$= \log(3a) + \log(2b)$

$= \log 3 + \log a + \log 2 + \log b$

Example 3: Expand: $\log(x+y)$

$\log(x+y) = \log(x+y)$ REMEMBER: A LOG CAN'T BE MULTIPLIED THROUGH THE BRACKETS!

Example 4: Condense $\log 3 + \log 4$

$\log 3 + \log 4 = \log(3 \bullet 4) = \log 12$

Example 5: Condense $\log(x+1) + \log(x-2)$

$\log(x+1) + \log(x-2)$

$= \log(x+1)(x-2)$

$= \log\left[x^2 - x - 2\right]$

Example 6: Condense $a\log x + a\log y$

$a\log x + a\log y$

$a(\log x + \log y)$

$a\log(xy)$

Example 7: Solve for y in the equation: $2 = \log_a x + \log_a y$

$2 = \log_a x + \log_a y$

$2 = \log_a(xy)$

$a^2 = xy$

$y = \frac{a^2}{x}$

Logarithms Lesson 2
Part IV - Multiplication Law

QUESTIONS:

Expand each of the following

1) $\log(abc)$

2) $2\log(4x)$

3) $3\log(x + y)$

Condense each of the following

4) $\log 2 + \log 6$

5) $\log(x + 3) + \log x$

6) $a\log(xy) + a\log(xz)$

7) $\log(2x + 1) + \log(3x - 2)$

Solve for x

8) $4 = \log_b x + \log_b y$

9) $7 = \log_m x + \log_m x$

ANSWERS:

1) $\log a + \log b + \log c$

2) $2\log 4 + 2\log x$

3) $3\log(x + y)$

4) $\log 12$

5) $\log(x^2 + 3x)$

6) $a\log\left(x^2 yz\right)$

7) $\log(6x^2 - x - 2)$

8) $x = \dfrac{b^4}{y}$

9) $x = \sqrt{m^7}$

DIVISION LAW OF LOGARITHMS

Division Law:

WHEN NUMBERS/VARIABLES ARE BEING DIVIDED INSIDE A LOGARITHM, THEY CAN BE EXPANDED BY SUBTRACTING SEPARATE LOGARITHMS.

$$\log_a\left(\frac{b}{c}\right)=\log_a b-\log_a c$$

Example 1: Expand $\log\left(\frac{x}{y}\right)$

$\log\left(\frac{x}{y}\right)=\log x-\log y$

Example 2: Expand $\log(3a-2b)$

$\log(3a-2b)=\log(3a-2b)$ *DIVISION RULE DOES NOT APPLY HERE.

Example 3: Expand $-5\log\left(\frac{x}{3}\right)$

$-5\log\left(\frac{x}{3}\right)$

$=-5[\log x-\log 3]$

$=-5\log x+5\log 3$

Example 4: Expand: $\log(x-y)$

$\log(x-y)=\log(x-y)$ *DIVISION RULE DOES NOT APPLY HERE.

Example 5: Condense $\log 12-\log 4$

$\log 12-\log 4$

$=\log\left(\frac{12}{4}\right)$

$=\log 3$

Example 6: Condense $\log(x-1)-\log(x+2)$

$\log(x-1)-\log(x+2)$

$=\log\left(\frac{x-1}{x+2}\right)$

Example 7: Condense $a\log x-a\log y$

$a\log x-a\log y$

$a(\log x-\log y)$

$a\log\frac{x}{y}$

QUESTIONS:

Expand the following

1) $\log\frac{a}{b}$

2) $-3\log\left(\frac{a}{2}\right)$

3) $\log\left(x-y\right)$

4) $\log\left(\sqrt{x}-2\right)$

Condense the following

5) $\log 16 - \log 8$

6) $\log(x+2) - \log(x-1)$

7) $\frac{\log x}{\log y}$

8) $3\log 27 - 3\log 3$

9) $\log(8a^2b^4) - \log(4ab^2)$

10) $\log(2a^3b^{-2}) - \log(8a^{-5}b^6)$

ANSWERS:

1) $\log a - \log b$

2) $-3\log a + 3\log 2$

3) $\log(x-y)$

4) $\log(\sqrt{x}-2)$

5) $\log 2$

6) $\log\left(\frac{x+2}{x-1}\right)$

7) $\log_y x$ (Change of Base!)

8) $3\log 9$

9) $\log\left(2ab^2\right)$

10) $\log\left(\frac{a^8}{4b^8}\right)$

Logarithms Lesson 2
Part VI - Power Law

POWER LAW OF LOGARITHMS

Power Law:

WHEN THERE IS AN EXPONENT INSIDE A LOGARITHM, IT CAN BE TAKEN OUT IN FRONT OF THE LOGARITHM.

$\log_a b^c = c\log_a b$

Example 1: Simplify $\log x^2$

$\log x^2 = 2\log x$

Example 2: Simplify $\log x^2 + \log x^4$

$\log x^2 + \log x^4 = 2\log x + 4\log x = 6\log x$

Example 3: Expand $(\log x)^2$

$(\log x)^2 = (\log x)^2$

Power law does not apply when the entire log is raised to an exponent.

Example 4: Condense $3\log(xy)$

$3\log(xy)$

$= \log(xy)^3$

$= \log(x^3y^3)$

Example 5: Condense $2\log(x-1)$

$2\log(x-1)$

$= \log(x-1)^2$

You can also write as: $\log(x^2-2x+1)$

Example 6: Condense: $4\log a - x$

$4\log a - x = \log a^4 - x$

Logarithms Lesson 2
Part VI - Power Law

QUESTIONS:

1) Expand: $\log a^3$

2) Expand: $\log a^3 + \log a^7$

3) Expand: $(\log a)^3$

4) Condense: $5\log(ab)$

5) Condense: $2\log(a - b)$

6) Condense: $3\log\left(\frac{a}{b}\right) - 7$

7) Simplify: $(2\log 10)^2$

8) Solve for x: $\log_3 x^2 = 6$

ANSWERS:

1) $3\log a$

2) $10\log a$

3) $(\log a)^3$

4) $\log a^5 b^5$

5) $\log(a-b)^2$

6) $\log\left(\frac{a}{b}\right)^3 - 7$

7) 4

8) $x = 27$

OTHER LAWS OF LOGARITHMS

Other Laws:

1) $\log_a x$ is undefined for $x \leq 0$.
2) $\log_a 1 = 0$
3) $\log_a a = 1$
4) $a^{\log_a x} = x$
5) $\log_a a^x = x$

Example 1: Evaluate $\log_{3x} 3x$

$\log_{3x} 3x = 1$

Example 2: Evaluate $\log_{3x} 0$

$\log_{3x} 0 = \text{Undefined}$

Example 3: Evaluate $\log_{3x} 1$

$\log_{3x} 1 = 0$

Example 4: Evaluate $\log_{3x}(-3)$

$\log_{3x}(-3) = \text{Undefined}$

Example 5: Evaluate $\log_3 3^4$

$\log_3 3^4 = 4\log_3 3 = 4(1) = 4$

Example 6: Evaluate $\log_{x-1}(x-1)^2$

$\log_{x-1}(x-1)^2 = 2\log_{x-1}(x-1) = 2(1) = 2$

Example 7: Evaluate $3^{\log_3 x}$

$3^{\log_3 x} = x$

Example 8: Evaluate $4 \bullet 2^{\log_2 6}$

$4 \bullet 2^{\log_2 6} = 4 \bullet 6 = 24$

Example 9: Simplify the expression $\log_5 25^k$

$\log_5 25^k$

$= \log_5 (5^2)^k$

$= \log_5 5^{2k}$

$= 2k\log_5 5$

$= 2k(1)$

$= 2k$

Example 10: Simplify the expression $\log_a \left(\sqrt{a}\right)^x$

$\log_a \left(\sqrt{a}\right)^x$

$= \log_a \left(a^{\frac{1}{2}}\right)^x$

$= \log_a a^{\frac{x}{2}}$

$= \frac{x}{2}\log_a a$

$= \frac{x}{2}(1)$

$= \frac{x}{2}$

Logarithms Lesson 2
Part VII – Other Laws

QUESTIONS:

1) Evaluate : $\log_{4xy} 4xy$

2) Evaluate : $\log_3 1$

3) Evaluate : $\log_b 0$

4) Evaluate : $-2\log_{(a+b)}(a+b)$

5) Evaluate : $\log_{(a-b)}(a-b)^{2x}$

6) Evaluate : $5^{\log_5 3x}$

7) Simplify : $\log_3 81^k$

8) Simplify : $\log_{\frac{1}{2}}\left(\sqrt{\frac{1}{2}}\right)^{3k}$

9) Solve for x : $\log_5 125^x = 9$

10) Solve for x : $\log_4 256^{2x} = 32$

ANSWERS:

1) 1

2) 0

3) *Undefined*

4) -2

5) $2x$

6) $3x$

7) $4k$

8) $\frac{3k}{2}$

9) $x = 3$

10) $x = 4$

DIPLOMA STYLE LOGARITHM QUESTIONS

Example 1: Given that $\log_3 4 = x$, evaluate $\log_3 16$

$\log_3 16$

$\log_3(4 \bullet 4)$

$\log_3 4 + \log_3 4$

$x + x$

$2x$

Example 2: If $\log_m a = 3$ and $\log_m b = 4$, evaluate $\left(\log_m\left(\frac{1}{ab}\right)\right)$

$\left(\log_m\left(\frac{1}{ab}\right)\right)$

$\log_m(1) - \log_m(ab)$

$0 - [\log_m a + \log_m b]$

$-[3+4]$

-7

Example 3: If $x = \frac{y}{z^2}$, determine an expression for log x

$\log x$

$\log\left(\frac{y}{z^2}\right)$

$\log y - \log z^2$

$\log y - 2\log z$

Example 4: If $\log x = 3$, evaluate $\log 10x^2$

$\log 10x^2$

$\log 10 + \log x^2$

$1 + 2\log x$

$1 + 2(3)$

7

Example 5: If $\log_2 \mathcal{A} = \mathcal{B}$, then $\log_4 \mathcal{A} = ?$

$\log_2 \mathcal{A} = \mathcal{B} \rightarrow \mathcal{A} = 2^{\mathcal{B}}$

$\log_4 \mathcal{A} = \log_4 2^{\mathcal{B}} = \mathcal{B}\log_4 2 = \mathcal{B}\left(\frac{1}{2}\right) = \frac{\mathcal{B}}{2}$

Example 6: If $\log_a b = 0.92$, then the value of $\log_a\left(\frac{a}{b}\right)$ is:

$\log_a\left(\frac{a}{b}\right) = \log_a a - \log_a b$

$= 1 - 0.92$

$= 0.08$

Example 7: If $\log_3 x = 20$, then the value of $\log_3\left(\frac{1}{3}x\right)$ is:

$\log_3\left(\frac{1}{3}x\right) = \log_3\left(\frac{x}{3}\right)$

$= \log_3 x - \log_3 3$

$= 20 - 1$

$= 19$

Part VIII - Diploma Style

Example 8: If $\log x = 3.2$ and $\log y = -0.9$, then $\frac{x}{y} =$

$\log_{10}x = 3.2 \rightarrow x = 10^{3.2} \rightarrow x = 1584.89$

$\log_{10}y = -0.9 \rightarrow y = 10^{-0.9} \rightarrow y = 0.1259$

$\frac{x}{y} = \frac{1584.89}{0.1259} = 12589.25$

Example 9: If $\log_m 9 = 2$ and $\log_8 n = 2$, then $\log_2(mn) = ?$

$\log_m 9 = 2 \rightarrow m^2 = 9 \rightarrow m = 3$

$\log_8 n = 2 \rightarrow 8^2 = n \rightarrow n = 64$

$\log_2(mn) = \log_2(3 \bullet 64) = \log_2 192 = 7.58$

Example 10: If $x = y^2z$, then find an expression for logz

$x = y^2z \rightarrow z = \frac{x}{y^2}$

$\log z = \log\left(\frac{x}{y^2}\right)$

$= \log x - \log y^2$

$= \log x - 2\log y$

Example 11: If $\log_b \mathcal{A} = \mathcal{M}$, then $\log_b \frac{1}{\mathcal{A}^2} = ?$

$\log_b \frac{1}{\mathcal{A}^2} = \log_b 1 - \log_b \mathcal{A}^2$

$= 0 - 2\log_b \mathcal{A}$

$= -2\mathcal{M}$

Example 12: $\log_3(27a) = ?$

$\log_3(27a)$

$= \log_3 27 + \log_3 a$

$= 3 + \log_3 a$

Example 13: If $10^a = 4$, then $10^{1+2a} = ?$

$10^{1+2a} = 10 \bullet 10^a \bullet 10^a$

$= 10 \bullet 4 \bullet 4$

$= 160$

Logarithms Lesson 2
Part VIII - Diploma Style

QUESTIONS:

1) Given that $\log_2 3 = a$, evaluate $\log_2 81$

2) If $\log_a b = 4$, evaluate $\log_a \left(\frac{a}{b}\right)$

3) If $ab = 5$, evaluate $\log_5 a^2 b^2$

4) If $a = \frac{b^3}{c^2}$, expand $\log a$

5) Given $\log x = 2$, evaluate $\log 100x^4$

6) If $\log_{36} x = A$, then $\log_6 x = ?$

7) If $\log a = 0.6$, and $\log b = 0.4$, evaluate $\log\left(\frac{a^2}{b}\right)$

8) If $\log_4 x = 15$, then $\log_4 \left(\frac{4}{x^{-2}}\right) =$

9) If $\log x = 4.3$ and $\log y = -0.8$, then $\frac{y}{x} = ?$

10) If $\log_a 32 = 5$, and $\log_4 b = 2.5$, then $\log_4 (ab) = ?$

11) If $a = bc^2$, find an expression for $\log b$

12) If $\log_a x = b$, then $\log_a \left(\frac{a}{x^3}\right) =$

13) If $5^x = b$, evaluate $25^x \cdot 125^{3x}$

14) If $3^x = a$, evaluate $\log_3 a^4$

15) Given $\frac{y^2}{xz^3} = 3$, evaluate $\log_3 \left(\frac{1}{x}\right) + 2\log_3 y - \log_3 z^3$

ANSWERS:

1) $4a$	6) $2A$	11) $\log a - 2\log c$
2) -3	7) 0.8	12) $1 - 3b$
3) 2	8) 31	13) b^{11}
4) $3\log b - 2\log c$	9) $10^{-5.1}$	14) $4x$
5) 10	10) 3	15) 1