



## **AP<sup>®</sup> Calculus AB 2006 Scoring Guidelines**

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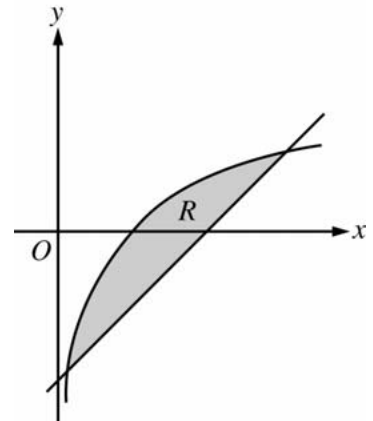
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**Question 1**

Let  $R$  be the shaded region bounded by the graph of  $y = \ln x$  and the line  $y = x - 2$ , as shown above.

- (a) Find the area of  $R$ .  
 (b) Find the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = -3$ .  
 (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when  $R$  is rotated about the  $y$ -axis.



$\ln(x) = x - 2$  when  $x = 0.15859$  and  $3.14619$ .  
 Let  $S = 0.15859$  and  $T = 3.14619$

(a) Area of  $R = \int_S^T (\ln(x) - (x - 2)) dx = 1.949$

3 : { 1 : integrand  
 1 : limits  
 1 : answer

(b) Volume =  $\pi \int_S^T ((\ln(x) + 3)^2 - (x - 2 + 3)^2) dx$   
 = 34.198 or 34.199

3 : { 2 : integrand  
 1 : limits, constant, and answer

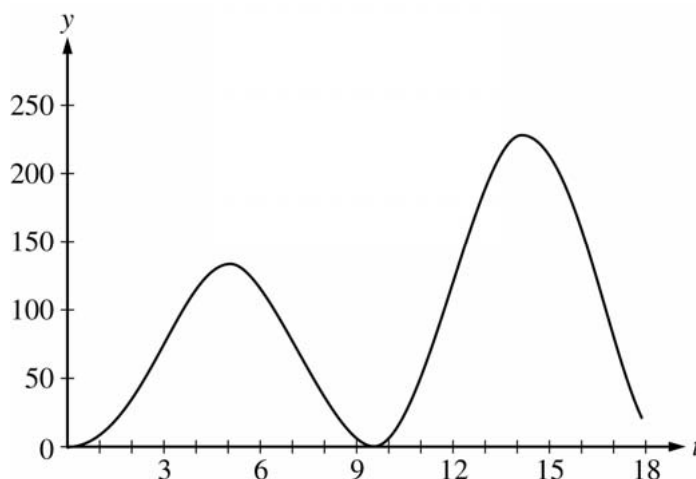
(c) Volume =  $\pi \int_{S-2}^{T-2} ((y + 2)^2 - (e^y)^2) dy$

3 : { 2 : integrand  
 1 : limits and constant

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**Question 2**

At an intersection in Thomasville, Oregon, cars turn left at the rate  $L(t) = 60\sqrt{t} \sin^2\left(\frac{t}{3}\right)$  cars per hour over the time interval  $0 \leq t \leq 18$  hours. The graph of  $y = L(t)$  is shown above.



- (a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval  $0 \leq t \leq 18$  hours.
- (b) Traffic engineers will consider turn restrictions when  $L(t) \geq 150$  cars per hour. Find all values of  $t$  for which  $L(t) \geq 150$  and compute the average value of  $L$  over this time interval. Indicate units of measure.
- (c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

(a)  $\int_0^{18} L(t) dt \approx 1658$  cars

- (b)  $L(t) = 150$  when  $t = 12.42831, 16.12166$   
 Let  $R = 12.42831$  and  $S = 16.12166$   
 $L(t) \geq 150$  for  $t$  in the interval  $[R, S]$

$$\frac{1}{S - R} \int_R^S L(t) dt = 199.426 \text{ cars per hour}$$

- (c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400.

$$\int_{13}^{15} L(t) dt = 431.931 > 400$$

OR

The number of cars turning left will be greater than 400 on a two-hour interval if  $L(t) \geq 200$  on that interval.

$$L(t) \geq 200 \text{ on any two-hour subinterval of } [13.25304, 15.32386].$$

Yes, a traffic signal is required.

$$2 : \begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$$

$$3 : \begin{cases} 1 : t\text{-interval when } L(t) \geq 150 \\ 1 : \text{average value integral} \\ 1 : \text{answer with units} \end{cases}$$

$$4 : \begin{cases} 1 : \text{considers 400 cars} \\ 1 : \text{valid interval } [h, h + 2] \\ 1 : \text{value of } \int_h^{h+2} L(t) dt \\ 1 : \text{answer and explanation} \end{cases}$$

OR

$$4 : \begin{cases} 1 : \text{considers 200 cars per hour} \\ 1 : \text{solves } L(t) \geq 200 \\ 1 : \text{discusses 2 hour interval} \\ 1 : \text{answer and explanation} \end{cases}$$

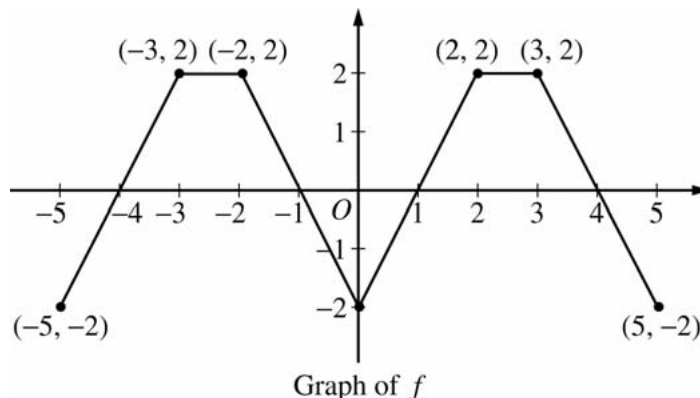
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**Question 3**

The graph of the function  $f$  shown above consists of six line segments. Let  $g$  be the function given by

$$g(x) = \int_0^x f(t) dt.$$

- (a) Find  $g(4)$ ,  $g'(4)$ , and  $g''(4)$ .
- (b) Does  $g$  have a relative minimum, a relative maximum, or neither at  $x = 1$ ? Justify your answer.



- (c) Suppose that  $f$  is defined for all real numbers  $x$  and is periodic with a period of length 5. The graph above shows two periods of  $f$ . Given that  $g(5) = 2$ , find  $g(10)$  and write an equation for the line tangent to the graph of  $g$  at  $x = 108$ .

(a)  $g(4) = \int_0^4 f(t) dt = 3$

$$g'(4) = f(4) = 0$$

$$g''(4) = f'(4) = -2$$

- (b)  $g$  has a relative minimum at  $x = 1$  because  $g' = f$  changes from negative to positive at  $x = 1$ .

- (c)  $g(0) = 0$  and the function values of  $g$  increase by 2 for every increase of 5 in  $x$ .

$$g(10) = 2g(5) = 4$$

$$\begin{aligned} g(108) &= \int_0^{105} f(t) dt + \int_{105}^{108} f(t) dt \\ &= 21g(5) + g(3) = 44 \end{aligned}$$

$$g'(108) = f(108) = f(3) = 2$$

An equation for the line tangent to the graph of  $g$  at  $x = 108$  is  $y - 44 = 2(x - 108)$ .

$$3 : \begin{cases} 1 : g(4) \\ 1 : g'(4) \\ 1 : g''(4) \end{cases}$$

$$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$$

$$4 : \begin{cases} 1 : g(10) \\ 3 : \begin{cases} 1 : g(108) \\ 1 : g'(108) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$$

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**Question 4**

$t$ (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket  $A$  has positive velocity  $v(t)$  after being launched upward from an initial height of 0 feet at time  $t = 0$  seconds. The velocity of the rocket is recorded for selected values of  $t$  over the interval  $0 \leq t \leq 80$  seconds, as shown in the table above.

(a) Find the average acceleration of rocket  $A$  over the time interval  $0 \leq t \leq 80$  seconds. Indicate units of measure.

(b) Using correct units, explain the meaning of  $\int_{10}^{70} v(t) dt$  in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate  $\int_{10}^{70} v(t) dt$ .

(c) Rocket  $B$  is launched upward with an acceleration of  $a(t) = \frac{3}{\sqrt{t+1}}$  feet per second per second. At time  $t = 0$  seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 feet per second. Which of the two rockets is traveling faster at time  $t = 80$  seconds? Explain your answer.

(a) Average acceleration of rocket  $A$  is

$$\frac{v(80) - v(0)}{80 - 0} = \frac{49 - 5}{80} = \frac{11}{20} \text{ ft/sec}^2$$

(b) Since the velocity is positive,  $\int_{10}^{70} v(t) dt$  represents the distance, in feet, traveled by rocket  $A$  from  $t = 10$  seconds to  $t = 70$  seconds.

A midpoint Riemann sum is

$$20[v(20) + v(40) + v(60)]$$

$$= 20[22 + 35 + 44] = 2020 \text{ ft}$$

(c) Let  $v_B(t)$  be the velocity of rocket  $B$  at time  $t$ .

$$v_B(t) = \int \frac{3}{\sqrt{t+1}} dt = 6\sqrt{t+1} + C$$

$$2 = v_B(0) = 6 + C$$

$$v_B(t) = 6\sqrt{t+1} - 4$$

$$v_B(80) = 50 > 49 = v(80)$$

Rocket  $B$  is traveling faster at time  $t = 80$  seconds.

Units of  $\text{ft/sec}^2$  in (a) and  $\text{ft}$  in (b)

1 : answer

3 :  $\left\{ \begin{array}{l} 1 : \text{explanation} \\ 1 : \text{uses } v(20), v(40), v(60) \\ 1 : \text{value} \end{array} \right.$

4 :  $\left\{ \begin{array}{l} 1 : 6\sqrt{t+1} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{finds } v_B(80), \text{ compares to } v(80), \\ \text{and draws a conclusion} \end{array} \right.$

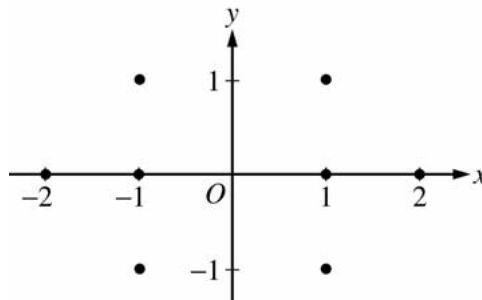
1 : units in (a) and (b)

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**Question 5**

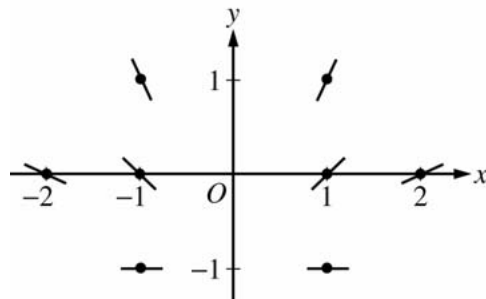
Consider the differential equation  $\frac{dy}{dx} = \frac{1+y}{x}$ , where  $x \neq 0$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.  
(Note: Use the axes provided in the pink exam booklet.)



- (b) Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(-1) = 1$  and state its domain.

(a)



2 : sign of slope at each point and relative steepness of slope lines in rows and columns

(b)  $\frac{1}{1+y} dy = \frac{1}{x} dx$

$$\ln|1+y| = \ln|x| + K$$

$$|1+y| = e^{\ln|x|+K}$$

$$1+y = C|x|$$

$$2 = C$$

$$1+y = 2|x|$$

$$y = 2|x| - 1 \text{ and } x < 0$$

or

$$y = -2x - 1 \text{ and } x < 0$$

6 :  $\left\{ \begin{array}{l} 1 : \text{separates variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{array} \right.$

7 :  $\left\{ \begin{array}{l} \text{Note: max } 3/6 [1-2-0-0-0] \text{ if no} \\ \text{constant of integration} \\ \text{Note: } 0/6 \text{ if no separation of variables} \\ 1 : \text{domain} \end{array} \right.$

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**Question 6**

The twice-differentiable function  $f$  is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2, \quad f'(0) = -4, \quad \text{and} \quad f''(0) = 3.$$

- (a) The function  $g$  is given by  $g(x) = e^{ax} + f(x)$  for all real numbers, where  $a$  is a constant. Find  $g'(0)$  and  $g''(0)$  in terms of  $a$ . Show the work that leads to your answers.
- (b) The function  $h$  is given by  $h(x) = \cos(kx)f(x)$  for all real numbers, where  $k$  is a constant. Find  $h'(x)$  and write an equation for the line tangent to the graph of  $h$  at  $x = 0$ .

(a)  $g'(x) = ae^{ax} + f'(x)$   
 $g'(0) = a - 4$

$g''(x) = a^2e^{ax} + f''(x)$   
 $g''(0) = a^2 + 3$

$$4 : \begin{cases} 1 : g'(x) \\ 1 : g'(0) \\ 1 : g''(x) \\ 1 : g''(0) \end{cases}$$

(b)  $h'(x) = f'(x)\cos(kx) - k\sin(kx)f(x)$   
 $h'(0) = f'(0)\cos(0) - k\sin(0)f(0) = f'(0) = -4$   
 $h(0) = \cos(0)f(0) = 2$   
The equation of the tangent line is  $y = -4x + 2$ .

$$5 : \begin{cases} 2 : h'(x) \\ 3 : \begin{cases} 1 : h'(0) \\ 1 : h(0) \\ 1 : \text{equation of tangent line} \end{cases} \end{cases}$$