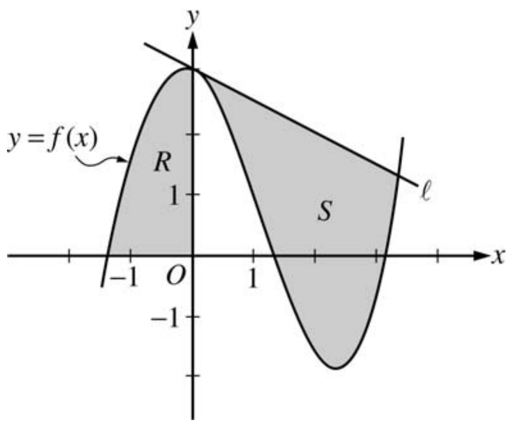
CollegeBoard
Advanced Placement Program

AP® Calculus BC
2006 Free-Response Questions
Form B

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f be the function given by $f(x) = \dfrac{x^3}{4} - \dfrac{x^2}{3} - \dfrac{x}{2} + 3\cos x$. Let R be the shaded region in the second quadrant bounded by the graph of f, and let S be the shaded region bounded by the graph of f and line ℓ, the line tangent to the graph of f at $x = 0$, as shown above.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -2$.

   (c) Write, but do not evaluate, an integral expression that can be used to find the area of S.

WRITE ALL WORK IN THE EXAM BOOKLET.

GO ON TO THE NEXT PAGE.

2. An object moving along a curve in the xy-plane is at position $(x(t),\ y(t))$ at time t, where

$$\frac{dx}{dt} = \tan\left(e^{-t}\right) \text{ and } \frac{dy}{dt} = \sec\left(e^{-t}\right)$$

for $t \geq 0$. At time $t = 1$, the object is at position $(2, -3)$.

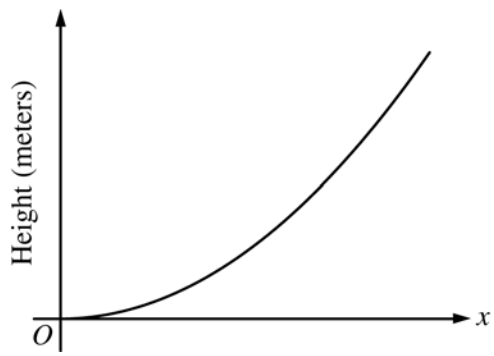
(a) Write an equation for the line tangent to the curve at position $(2, -3)$.

(b) Find the acceleration vector and the speed of the object at time $t = 1$.

(c) Find the total distance traveled by the object over the time interval $1 \leq t \leq 2$.

(d) Is there a time $t \geq 0$ at which the object is on the y-axis? Explain why or why not.

WRITE ALL WORK IN THE EXAM BOOKLET.



3. The figure above is the graph of a function of x, which models the height of a skateboard ramp. The function meets the following requirements.

   (i) At $x = 0$, the value of the function is 0, and the slope of the graph of the function is 0.

   (ii) At $x = 4$, the value of the function is 1, and the slope of the graph of the function is 1.

   (iii) Between $x = 0$ and $x = 4$, the function is increasing.

   (a) Let $f(x) = ax^2$, where a is a nonzero constant. Show that it is not possible to find a value for a so that f meets requirement (ii) above.

   (b) Let $g(x) = cx^3 - \dfrac{x^2}{16}$, where c is a nonzero constant. Find the value of c so that g meets requirement (ii) above. Show the work that leads to your answer.

   (c) Using the function g and your value of c from part (b), show that g does not meet requirement (iii) above.

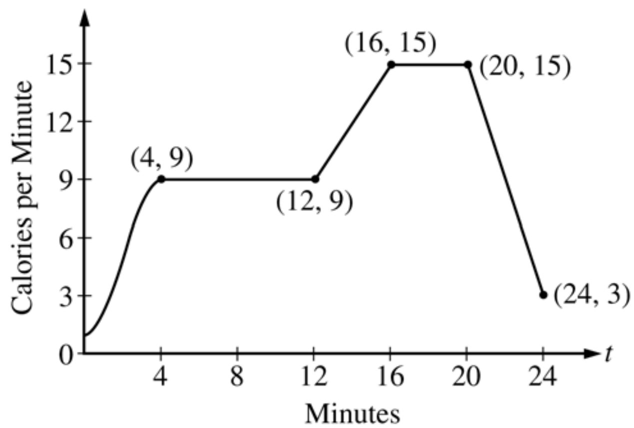
   (d) Let $h(x) = \dfrac{x^n}{k}$, where k is a nonzero constant and n is a positive integer. Find the values of k and n so that h meets requirement (ii) above. Show that h also meets requirements (i) and (iii) above.

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF PART A OF SECTION II

CALCULUS BC
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.



4. The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f. In the figure above, $f(t) = -\frac{1}{4}t^3 + \frac{3}{2}t^2 + 1$ for $0 \le t \le 4$ and f is piecewise linear for $4 \le t \le 24$.

   (a) Find $f'(22)$. Indicate units of measure.

   (b) For the time interval $0 \le t \le 24$, at what time t is f increasing at its greatest rate? Show the reasoning that supports your answer.

   (c) Find the total number of calories burned over the time interval $6 \le t \le 18$ minutes.

   (d) The setting on the machine is now changed so that the person burns $f(t) + c$ calories per minute. For this setting, find c so that an average of 15 calories per minute is burned during the time interval $6 \le t \le 18$.

WRITE ALL WORK IN THE EXAM BOOKLET.

5. Let f be a function with $f(4) = 1$ such that all points (x, y) on the graph of f satisfy the differential equation

$$\frac{dy}{dx} = 2y(3 - x).$$

Let g be a function with $g(4) = 1$ such that all points (x, y) on the graph of g satisfy the logistic differential equation

$$\frac{dy}{dx} = 2y(3 - y).$$

(a) Find $y = f(x)$.

(b) Given that $g(4) = 1$, find $\lim\limits_{x \to \infty} g(x)$ and $\lim\limits_{x \to \infty} g'(x)$. (It is not necessary to solve for $g(x)$ or to show how you arrived at your answers.)

(c) For what value of y does the graph of g have a point of inflection? Find the slope of the graph of g at the point of inflection. (It is not necessary to solve for $g(x)$.)

---

6. The function f is defined by $f(x) = \dfrac{1}{1 + x^3}$. The Maclaurin series for f is given by

$$1 - x^3 + x^6 - x^9 + \cdots + (-1)^n x^{3n} + \cdots,$$

which converges to $f(x)$ for $-1 < x < 1$.

(a) Find the first three nonzero terms and the general term for the Maclaurin series for $f'(x)$.

(b) Use your results from part (a) to find the sum of the infinite series $-\dfrac{3}{2^2} + \dfrac{6}{2^5} - \dfrac{9}{2^8} + \cdots + (-1)^n \dfrac{3n}{2^{3n-1}} + \cdots$.

(c) Find the first four nonzero terms and the general term for the Maclaurin series representing $\int_0^x f(t)\, dt$.

(d) Use the first three nonzero terms of the infinite series found in part (c) to approximate $\int_0^{1/2} f(t)\, dt$. What are the properties of the terms of the series representing $\int_0^{1/2} f(t)\, dt$ that guarantee that this approximation is within $\dfrac{1}{10{,}000}$ of the exact value of the integral?

WRITE ALL WORK IN THE EXAM BOOKLET.

END OF EXAM