# 수학 1
# Tutorial Lab 1

(제 1강 ~ 3강)

Sec 1.1 # 19, 20, 22,36, 62, 64

Sec 1.2 # 1,2,20

Sec 1.3 # 4, 32, 37, 55, 59

Sec 1.5 # 18,29

Sec 1.6 # 17, 18, 19, 23, 28, 31, 59, 62

Sec 2.2 # 7, 9, 25, 29, 31

Sec 2.6 # 14, 17, 22, 25, 34, 57

**19.** The number $N$ (in millions) of cellular phone subscribers worldwide is shown in the table. (Midyear estimates are given.)

| $t$ | 1990 | 1992 | 1994 | 1996 | 1998 | 2000 |
|---|---|---|---|---|---|---|
| $N$ | 11 | 26 | 60 | 160 | 340 | 650 |

(a) Use the data to sketch a rough graph of $N$ as a function of $t$.

(b) Use your graph to estimate the number of cell-phone subscribers at midyear in 1995 and 1999.

**22.** A spherical balloon with radius $r$ centimeters has volume $V(r) = \frac{4}{3}\pi r^3$. Find a function that represents the amount of air required to inflate the balloon from a radius of $r$ centimeters to a radius of $r + 1$ centimeters.

(sol) A volume of spherical balloon with radius $r+1$ is

$$V(r+1) = \frac{4}{3}\pi(r+1)^3 = \frac{4}{3}\pi(r^3+3r^2+3r+1).$$

We wish to find the amount of air needed to inflate the balloon from a radius of $r$ to $r+1$.

Hence, we need to find the difference

$$V(r+1) - V(r) = \frac{4}{3}\pi(r^3+3r^2+3r+1) - \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi(3r^2+3r+1)$$

**33–44** Find the domain and sketch the graph of the function.

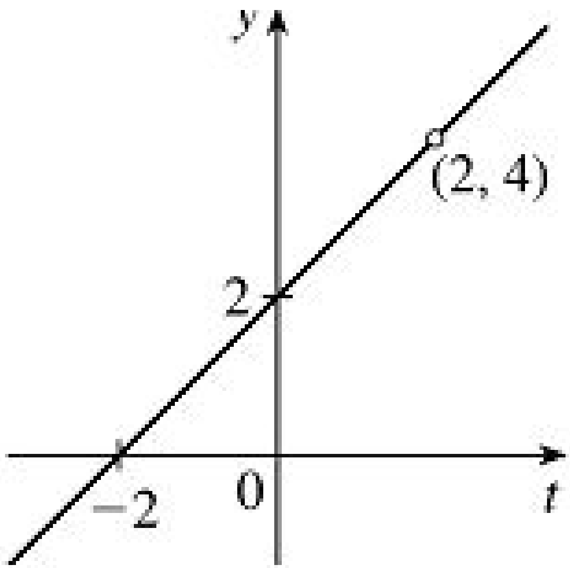
**36.** $H(t) = \dfrac{4 - t^2}{2 - t}$

(sol) $H(t) = \dfrac{4-t^2}{2-t} = \dfrac{(2+t)(2-t)}{2-t}$,

so for $t \neq 2$, $H(t) = 2 + t$.

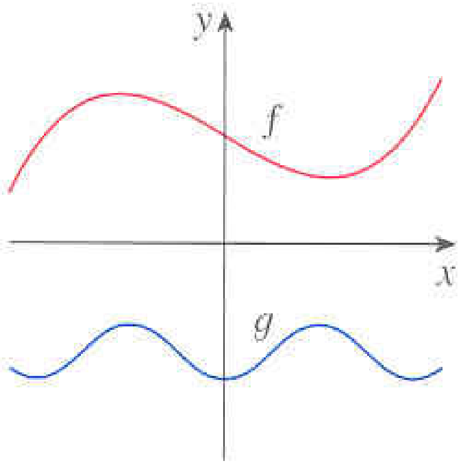
The domain is $\{t \mid t \neq 2\}$.

So the graph of is the same as the graph of the ft $f(t) = t + 2$ except for the hole at $(2,4)$.



**61–62** Graphs of $f$ and $g$ are shown. Decide whether each function is even, odd, or neither. Explain your reasoning.

**62.**

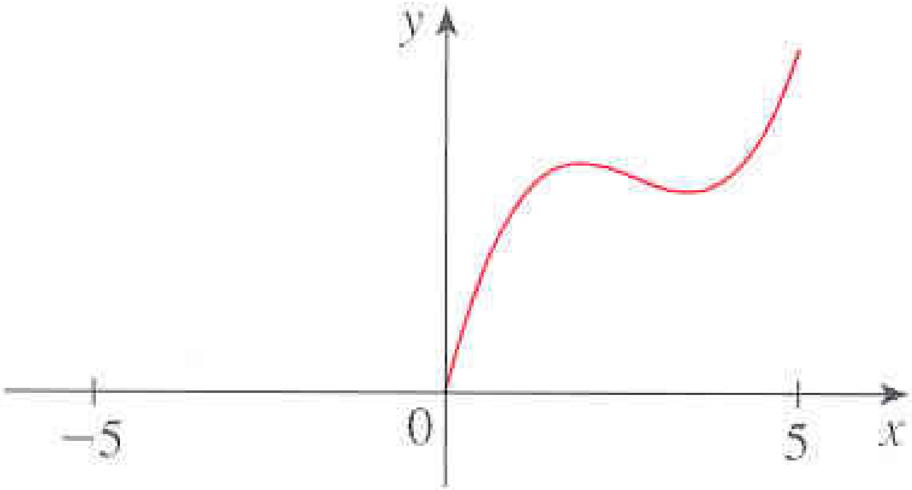



(sol) f is not an even function since it is not symmetric with respect to the y − axis. f is not an odd function since it is not symmetric about the origin. Hence, f is neither even nor odd.
g is an even function because its graph is symmetric with respect to the y − axis.

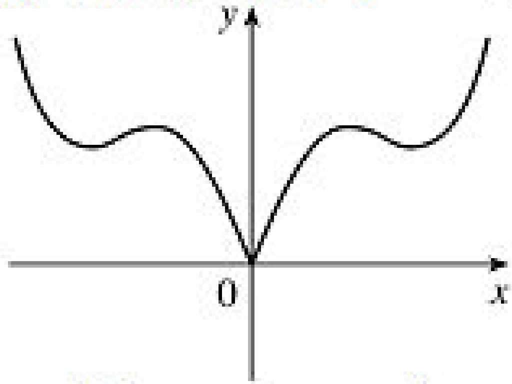
**64.** A function $f$ has domain $[-5, 5]$ and a portion of its graph is shown.

(a) Complete the graph of $f$ if it is known that $f$ is even.

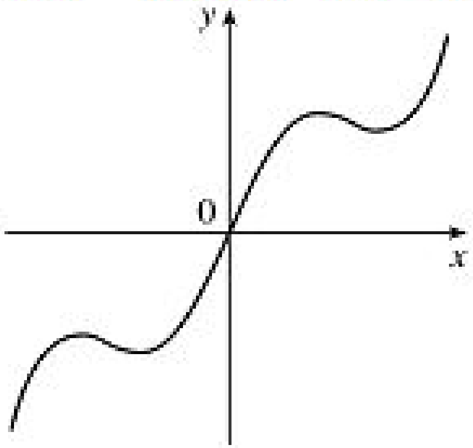
(b) Complete the graph of $f$ if it is known that $f$ is odd.



(sol)(a) If f is even, we get the rest of the graph by reflecting about the y − axis.




(b) If f is odd, we get the rest of the graph by rotating 180° about the origin.



**1–2** Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.

(sol) **(a)** $f(x) = -\sqrt[5]{x}$ is a root function with n=5.

**(b)** $g(x) = \sqrt{1-x^2}$ is an algebraic function because it is a root of a polynomial.

**(c)** $h(x) = x^9 + x^4$ is a polynomial of degree 9.

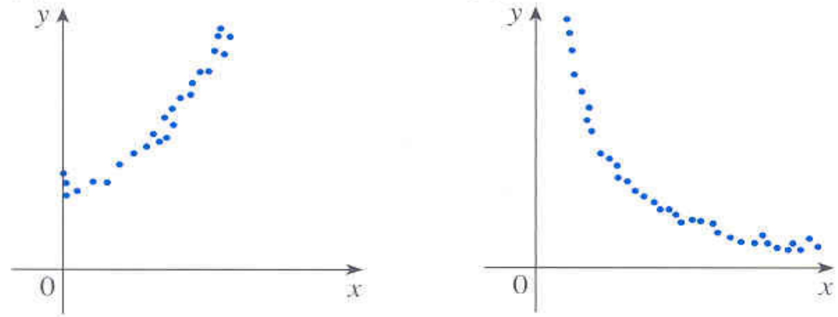
**(d)** $r(x) = \dfrac{x^2+1}{x^3+x}$ is a rational function because it is a ratio of polynomials.

**(e)** $s(x) = \tan 2x$ is a trigonometric function.

**(f)** $t(x) = \log_{10} x$ is a logarithmic function.

**19–20** For each scatter plot, decide what type of function you might choose as a model for the data. Explain your choices.

**20.** (a) (b)




**(sol) (a)** The data appear to be increasing exponentially.

A model of the form $f(x) = a \cdot b^x$ or $f(x) = a \cdot b^x + c$ seems appropriate.

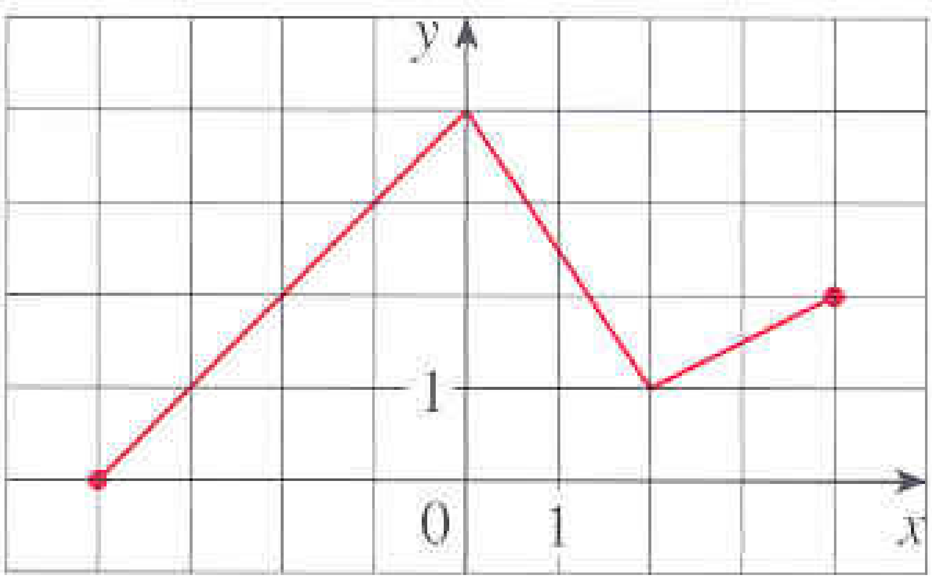
**(b)** The data appear to be decreasing similarly to the values of the reciprocal function.

A model of the form $f(x) = \dfrac{a}{x}$ seems appropriate.

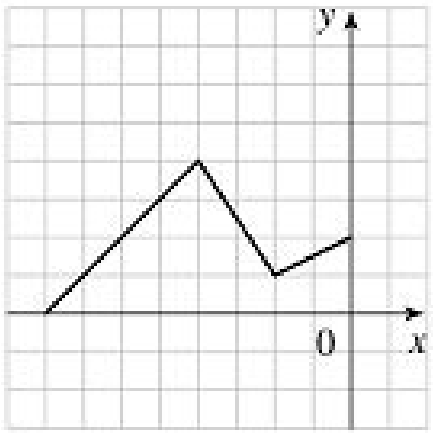
**4.** The graph of $f$ is given. Draw the graphs of the following functions.

(a) $y = f(x + 4)$ (b) $y = f(x) + 4$

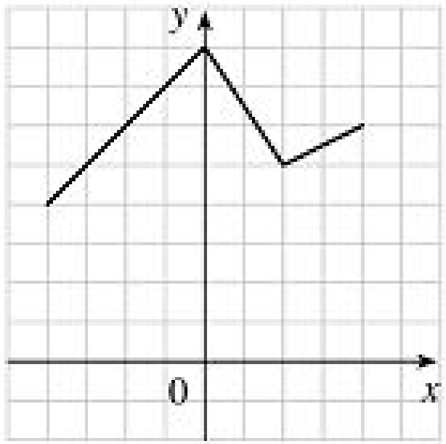
(c) $y = 2f(x)$ (d) $y = -\frac{1}{2}f(x) + 3$



(sol) **(a)** To graph $y=f(x+4)$ we shift the graph of f, 4 units to the left.



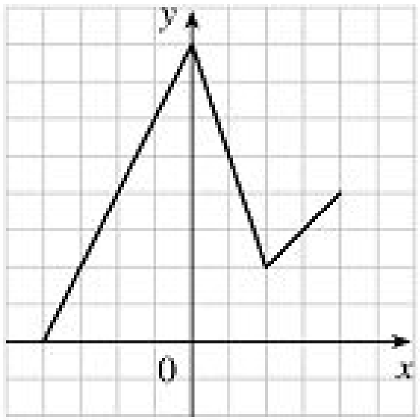

**(b)** To graph $y=f(x)+4$ we shift the graph of f, 4 units upward.



**(c)** To graph $y = 2f(x)$

we stretch the graph of f

vertically by a factor of 2.




**(d)** To graph $y = \frac{1}{2}f(x) + 3$,

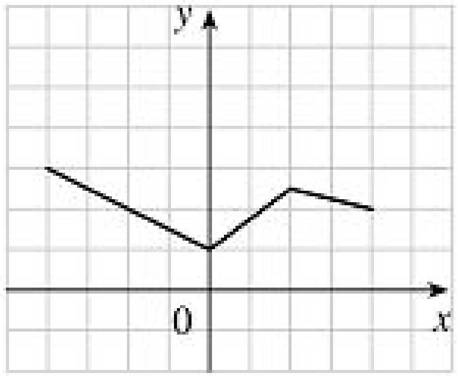
we shrink the graph of f

vertically by a factor of 2,

then reflect the resulting graph

about the x-axis, then shift

the resulting graph 3 units upward.



**31–36** Find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$ and their domains.

**32.** $f(x) = x - 2, \quad g(x) = x^2 + 3x + 4$

(sol) $D = R$ for both $f$ and $g$, and hence for their composites.

(a) $(f \circ g)(x) = f(g(x)) = f(x^2 + 3x + 4) = x^2 + 3x + 4 - 2 = x^2 + 3x + 2$

(b) $(g \circ f)(x) = g(f(x)) = g(x-2) = (x-2)^2 + 3(x-2) + 4 = x^2 - x + 2$

(c) $(f \circ f)(x) = f(f(x)) = f(x-2) = (x-2) - 2 = x - 4$

(d) $(g \circ g)(x) = g(g(x)) = g(x^2 + 3x + 4) = (x^2 + 3x + 4)^2 + 3(x^2 + 3x + 4) + 4$

$= x^4 + 6x^3 + 20x^2 + 33x + 32$

**37–40** Find $f \circ g \circ h$.

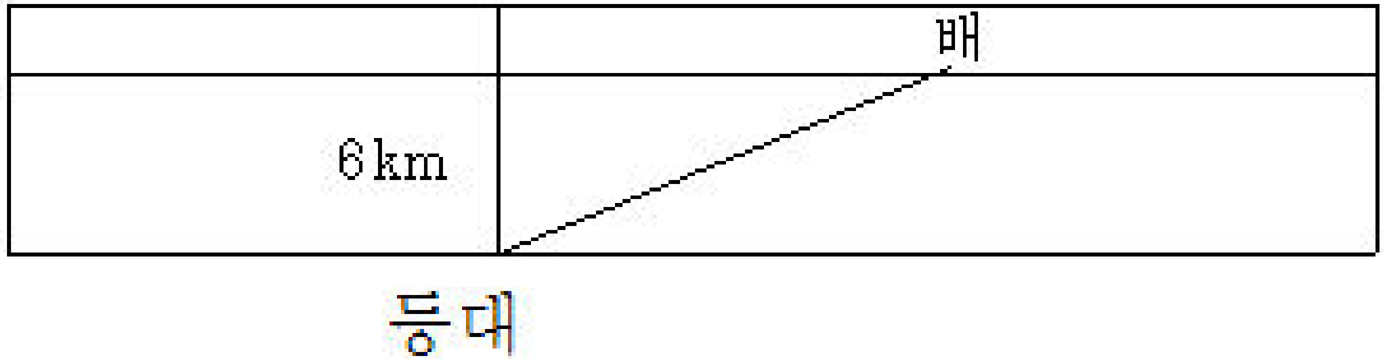
**37.** $f(x) = x + 1, \quad g(x) = 2x, \quad h(x) = x - 1$

$$\text{(sol)}\ (f \circ g \circ h)(x) = f(g(h(x)))$$
$$= f(g(x-1))$$
$$= f(2(x-1))$$
$$= 2(x-1)+1$$
$$= 2x-1$$

**55.** A ship is moving at a speed of 30 km/h parallel to a straight shoreline. The ship is 6 km from shore and it passes a lighthouse at noon.

(a) Express the distance $s$ between the lighthouse and the ship as a function of $d$, the distance the ship has traveled since noon; that is, find $f$ so that $s = f(d)$.

(sol)




$s^2 = d^2 + 36 \qquad \therefore s = \sqrt{d^2 + 36}$

(b) Express $d$ as a function of $t$, the time elapsed since noon; that is, find $g$ so that $d = g(t)$.

(sol) velocity = distance / time

distance = (velocity)(time)

$\therefore d = 30t$

(c) Find $f \circ g$. What does this function represent?

(sol) $f \circ g = \sqrt{(30t)^2 + 36} = \sqrt{900t^2 + 36}$

$\therefore s = \sqrt{900t^2 + 36}$

**59.** Let $f$ and $g$ be linear functions with equations $f(x) = m_1x + b_1$ and $g(x) = m_2x + b_2$. Is $f \circ g$ also a linear function? If so, what is the slope of its graph?

(sol) $(f \circ g)(x) = (f(g(x))$

$$= m_1(m_2x + b_2) + b_1$$

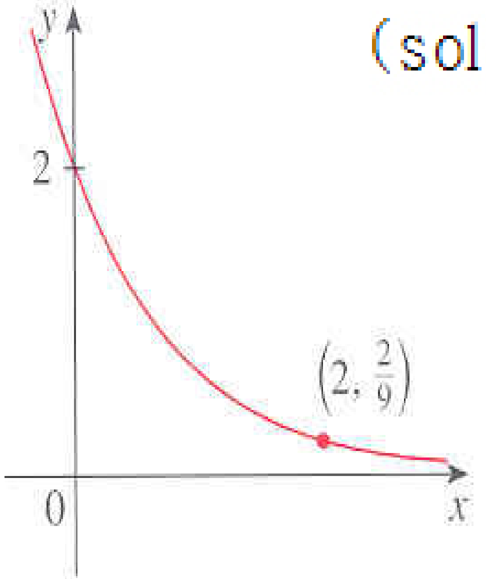
$$= m_1m_2x + m_1b_2 + b_1$$

$\therefore f \circ g$ is also a linear function

slope : $m_1m_2$

**17–18** Find the exponential function $f(x) = Ca^x$ whose graph is given.

**18.**




(sol) Given the y-intercept (0,2), we have $y = Ca^x = 2a^x$.

Using the point $(2, \frac{2}{9})$ gives us

$\frac{2}{9} = 2a^2 \Rightarrow \frac{1}{9} = a^2 \Rightarrow a = \frac{1}{3}$. [since a>0].

The function is $f(x) = 2(\frac{1}{3})^x$ or $f(x) = 2(3)^{-x}$.

**19.** If $f(x) = 5^x$, show that

$$\frac{f(x+h) - f(x)}{h} = 5^x \left( \frac{5^h - 1}{h} \right)$$

(pf) If $f(x) = 5^x$, then $\frac{f(x+h)-f(x)}{h} = \frac{5^{x+h} - 5^h}{h} = \frac{5^x(5^h - 1)}{h}$.

**17.** If $g(x) = 3 + x + e^x$, find $g^{-1}(4)$.

(pf) First, we must determine such that $g(x) = 4$.

By inspection, we see that if $x = 0$, then $g(x) = 4$.

Since $g$ is 1−1($g$ is an increasing function),

it has an inverse, and $g^{-1}(4) = 0$.

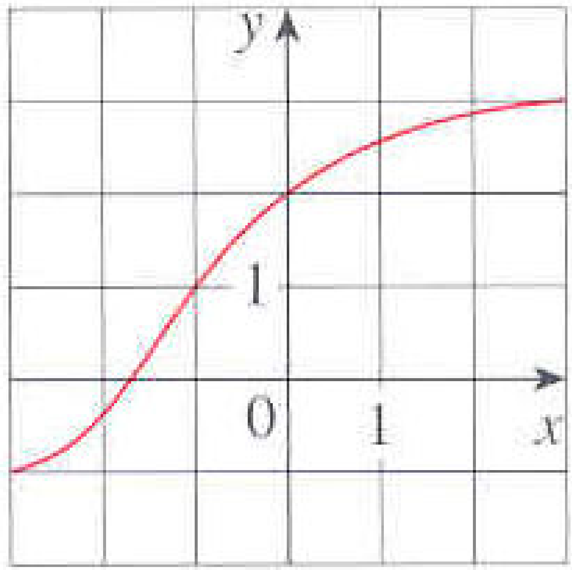
**18.** The graph of $f$ is given.

(a) Why is $f$ one-to-one?

(b) What are the domain and range of $f^{-1}$

(c) What is the value of $f^{-1}(2)$?

(d) Estimate the value of $f^{-1}(0)$.




**(sol)** **(a)** $f$ is 1-1 because it passes the Horizontal Line Test.

**(b)** Domain of $f = [-3,3] =$ Range of $f^{-1}$.

Range of $f = [-1,3] =$ Domain of $f^{-1}$.

**(c)** Since $f(3) = 2$, $f^{-1}(2) = 3$.

**(d)** Since $f(-1.7) \fallingdotseq 0$, $f^{-1}(0) \fallingdotseq -1.7$

**19.** The formula $C = \frac{5}{9}(F - 32)$, where $F \geqslant -459.67$, expresses the Celsius temperature $C$ as a function of the Fahrenheit temperature $F$. Find a formula for the inverse function and interpret it. What is the domain of the inverse function?

(sol) We solve $C = \frac{5}{9}(F-32)$ for $F$ : $\frac{9}{5}C = F - 32 \Rightarrow F = \frac{5}{9}C + 32$.

This gives us a formula for the inverse function, that is, the Fahrenheit temperature $F$ as a function of the Celsius temperature $C$.

$F \geq -459.67 \Rightarrow \frac{9}{5}C + 32 \geq -459.67 \Rightarrow \frac{9}{5}C \geq -491.67 \Rightarrow C \geq -273.15$,

the domain of the inverse function.

**21–26** Find a formula for the inverse of the function.

**23.** $f(x) = e^{x^3}$

(sol) $f(x) = e^{x^3} \Rightarrow y = e^{x^3} \Rightarrow \ln y = x^3 \Rightarrow x = \sqrt[3]{\ln y}$.

Interchange $x$ and $y$ : $y = \sqrt[3]{\ln x}$ .

So $f^{-1}(x) = \sqrt[3]{\ln y}$.

**27–28** Find an explicit formula for $f^{-1}$ and use it to graph $f^{-1}$, $f$, and the line $y = x$ on the same screen. To check your work, see whether the graphs of $f$ and $f^{-1}$ are reflections about the line.

**28.** $f(x) = 2 - e^x$

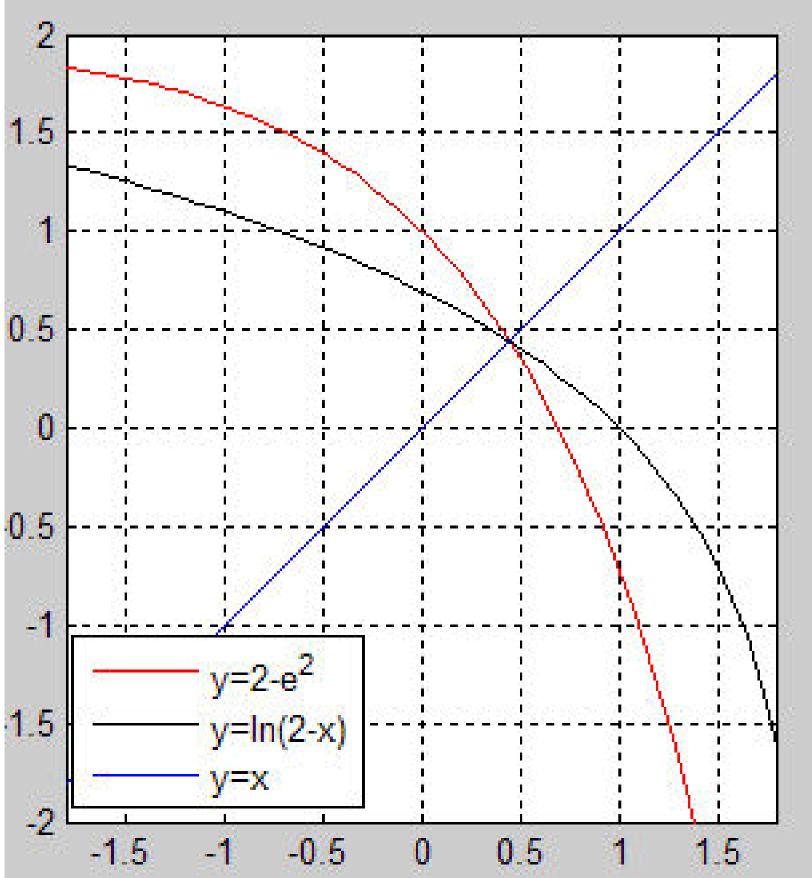
(sol) $y = 2 - e^x \Rightarrow 2 - y = e^x$

$$\Rightarrow x = \ln(2 - y)$$

Interchange $x$ and $y$ :

$$y = \ln(2 - x) \;.$$

So $f^{-1}(x) = \ln(2 - x)$.



**31.** (a) How is the logarithmic function $y = \log_a x$ defined?
(b) What is the domain of this function?
(c) What is the range of this function?
(d) Sketch the general shape of the graph of the function $y = \log_a x$ if $a > 1$.

(sol) **(a)** It is defined as the inverse of the exponential function with base $a$, that is, $\log_a x = y \Rightarrow a^y = x$.

**(b)** $(0, \infty)$

**(c)** **R**

**(d)** See Figure 11.

**59–64** Find the exact value of each expression.

**59.** (a) $\sin^{-1}(\sqrt{3}/2)$ (b) $\cos^{-1}(-1)$

(sol) **(a)** $\sin^{-1}(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$

since $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\frac{\pi}{3}$ is in $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

**(b)** $\cos^{-1}(-1) = \pi$ since $\cos\pi = -1$ and $\pi$ is in $[0, \pi]$.

**62.** (a) $\cot^{-1}(-\sqrt{3})$ (b) $\arccos(-\frac{1}{2})$

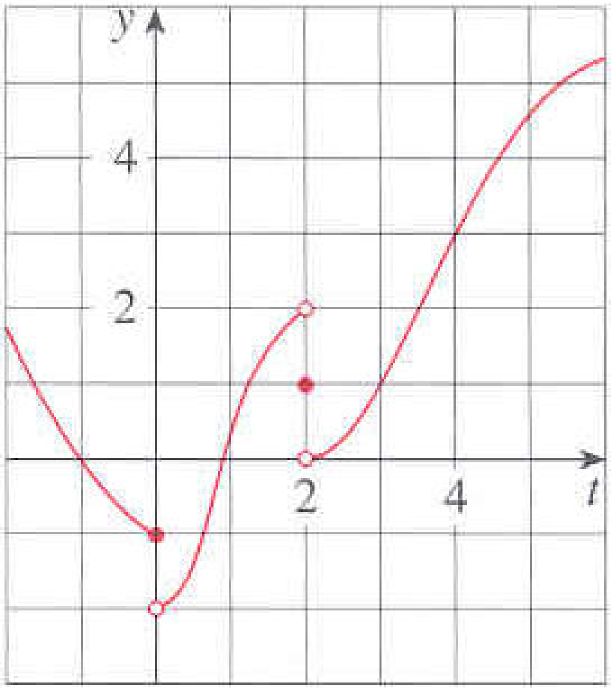
(sol) (a) $\cot^{-1}(-\sqrt{3}) = \alpha \Rightarrow \cot(\alpha) = -\sqrt{3} \Rightarrow \tan^{-1}\alpha = -\frac{1}{\sqrt{3}}$

$\therefore \alpha = \frac{5}{6}\pi$

(b) $\cos^{-1}(-\frac{1}{2}) = \beta \Rightarrow \cos(\beta) = -\frac{1}{2}$

$\therefore \beta = \frac{2}{3}\pi$

**7.** For the function $g$ whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.




(sol) (a) $\lim_{t \to 0^-} g(t) = -1$

(b) $\lim_{t \to 0^+} g(t) = -2$

(c) $\lim_{t \to 0} g(t)$ does not exist.

(d) $\lim_{t \to 2^-} g(t) = 2$

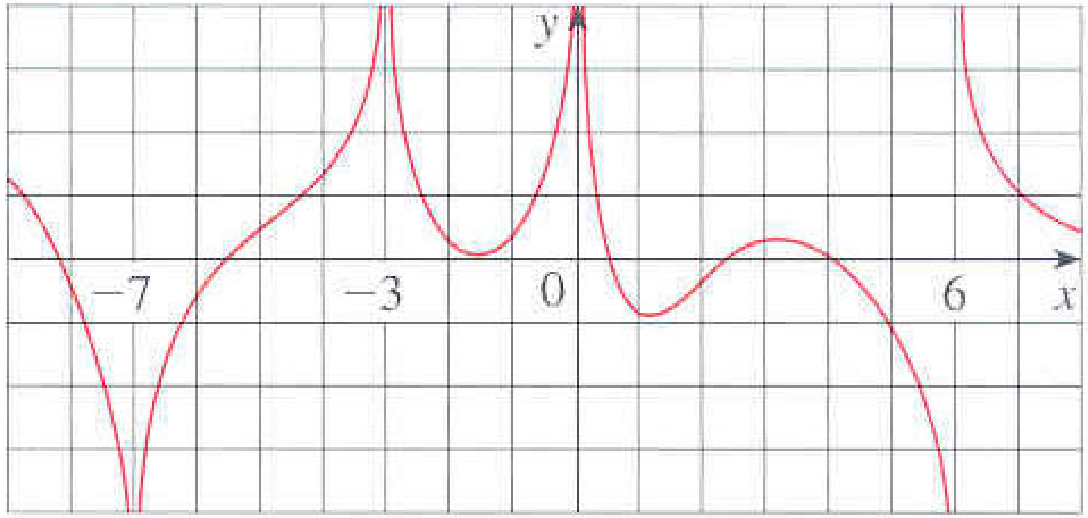
(e) $\lim_{t \to 2^+} g(t) = 0$

(f) $\lim_{t \to 2} g(t)$ does not exist.

(g) $g(2) = 1$

(h) $\lim_{t \to 4} g(t) = 3$

**9.** For the function $f$ whose graph is shown, state the following.




(sol) **(a)** $\lim_{x \to -7} f(x) = -\infty$ **(b)** $\lim_{x \to -3} f(x) = \infty$

**(c)** $\lim_{x \to 0} f(x) = \infty$ **(d)** $\lim_{x \to 6^-} f(x) = -\infty$

**(e)** $\lim_{x \to 6^+} f(x) = \infty$

**(f)** The equations of the vertical asymptotes are

$x = -7$, $x = -3$, $x = 0$ and $x = 6$.

**25–32** Determine the infinite limit.

**25.** $\lim_{x \to 5^+} \frac{6}{x-5}$

(sol) $\lim_{x \to 5^+} \frac{6}{x-5} = \infty$

since $(x-5) \to 0$ as $x \to 5^+$ and $\frac{6}{x-5} > 0$ for $x > 5$.

**29.** $\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)}$

(sol) $\lim_{x \to -2^+} \frac{x-1}{x^2(x+2)} = -\infty$

since $(x+2) \to 0$ as $x \to -2^+$ and $\frac{x-1}{x^2(x+2)} < 0$ for $-2 < x < 0$.

**31.** $\lim_{x \to (-\pi/2)^-} \sec x$

(sol) $\lim_{x \to (-\pi/2)^-} \sec x = \lim_{x \to (-\pi/2)^-} \frac{1}{\cos x} = -\infty$

since $\cos x \to 0$ as $x \to (-\pi/2)^-$ and $\cos x < 0$ for $-\pi < x \leftarrow \pi/2$.

**13–14** Evaluate the limit and justify each step by indicating the appropriate properties of limits.

**14.** $$\lim_{x\to\infty}\sqrt{\frac{12x^3-5x+2}{1+4x^2+3x^3}} = \sqrt{\lim_{x\to\infty}\frac{12x^3-5x+2}{1+4x^2+3x^3}} \quad \text{[ Limit Law 11]}$$

$$= \sqrt{\lim_{x\to\infty}\frac{12-5/x^2+2/x^3}{1/x^3+4/x+3}} \quad \text{[ divide by } x^3 \text{ ]}$$

$$= \sqrt{\frac{\lim_{x\to\infty}(12-5/x^2+2/x^3)}{\lim_{x\to\infty}(1/x^3+4/x+3)}} \quad \text{[ Limit Law 5]}$$

$$= \sqrt{\frac{\lim_{x\to\infty}12-\lim_{x\to\infty}(5/x^2)+\lim_{x\to\infty}(2/x^3)}{\lim_{x\to\infty}(1/x^3)+\lim_{x\to\infty}(4/x)+\lim_{x\to\infty}3}} \quad \text{[ Limit Laws 1 and 2]}$$

$$= \sqrt{\frac{12 - 5\lim_{x\to\infty}(1/x^2) + 2\lim_{x\to\infty}(1/x^3)}{\lim_{x\to\infty}(1/x^3) + 4\lim_{x\to\infty}(1/x) + 3}}$$

[ Limit Laws 7 and 3 ]

$$= \sqrt{\frac{12 - 5(0) + 2(0)}{0 + 4(0) + 3}}$$

[Theorem 5 of Section 2.5]

$$= \sqrt{\frac{12}{3}} = \sqrt{4} = 2$$

**15–36** Find the limit.

17. $$\lim_{x\to-\infty}\frac{1-x-x^2}{2x^2-7}=\lim_{x\to-\infty}\frac{(1-x-x^2)/x^2}{(2x^2-7)/x^2}=\frac{\lim_{x\to-\infty}(1/x^2-1/x-1)}{\lim_{x\to-\infty}(2-7/x^2)}$$

$$=\frac{\lim_{x\to-\infty}(1/x^2)-\lim_{x\to-\infty}(1/x)-\lim_{x\to-\infty}1}{\lim_{x\to-\infty}2-7\lim_{x\to-\infty}(1/x^2)}=\frac{0-0-1}{2-7(0)}=-\frac{1}{2}$$

22. $$\lim_{x\to\infty}\frac{x+2}{\sqrt{9x^2+1}}=\lim_{x\to\infty}\frac{(x+2)/x}{\sqrt{9x^2+1}/\sqrt{x^2}}=\lim_{x\to\infty}\frac{1+2/x}{\sqrt{9+1/x^2}}=\frac{1+0}{\sqrt{9+0}}=\frac{1}{3}$$

25. $\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right) = \lim_{x\to\infty}\frac{\left(\sqrt{9x^2+x}-3x\right)\left(\sqrt{9x^2+x}+3x\right)}{\sqrt{9x^2+x}+3x} = \lim_{x\to\infty}\frac{\left(\sqrt{9x^2+x}\right)^2-(3x)^2}{\sqrt{9x^2+x}+3x}$

$$= \lim_{x\to\infty}\frac{\left(9x^2+x\right)-9x^2}{\sqrt{9x^2+x}+3x} = \lim_{x\to\infty}\frac{x}{\sqrt{9x^2+x}+3x}\cdot\frac{1/x}{1/x}$$

$$= \lim_{x\to\infty}\frac{x/x}{\sqrt{9x^2/x^2+x/x^2}+3x/x} = \lim_{x\to\infty}\frac{1}{\sqrt{9+1/x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

34. $\lim_{x\to\infty}\tan^{-1}(x^2-x^4)=\lim_{x\to\infty}\tan^{-1}(x^2(1-x^2))$.

If we let $t=x^2(1-x^2)$, we know that $t\to-\infty$ as $x\to\infty$,

since $x^2\to\infty$ and $1-x^2\to-\infty$.

So $\lim_{x\to\infty}\tan^{-1}(x^2(1-x^2))=\lim_{t\to-\infty}\tan^{-1}t=\frac{\pi}{2}$

**57.** Find $\lim_{x\to\infty} f(x)$ if, for all $x > 1$,

$$\frac{10e^x - 21}{2e^x} < f(x) < \frac{5\sqrt{x}}{\sqrt{x-1}}$$

(sol) $$\lim_{x\to\infty}\left(\frac{10e^x - 21}{2e^x}\right) = \lim_{x\to\infty}\left(5 - \frac{21}{2e^x}\right) = 5$$

$$\lim_{x\to\infty}\frac{5\sqrt{x}}{\sqrt{x-1}} = \lim_{x\to\infty}\frac{5\sqrt{1}}{\sqrt{1-\frac{1}{x}}} = 5$$

By squeeze theorem, $\lim_{x\to\infty} f(x) = 5$