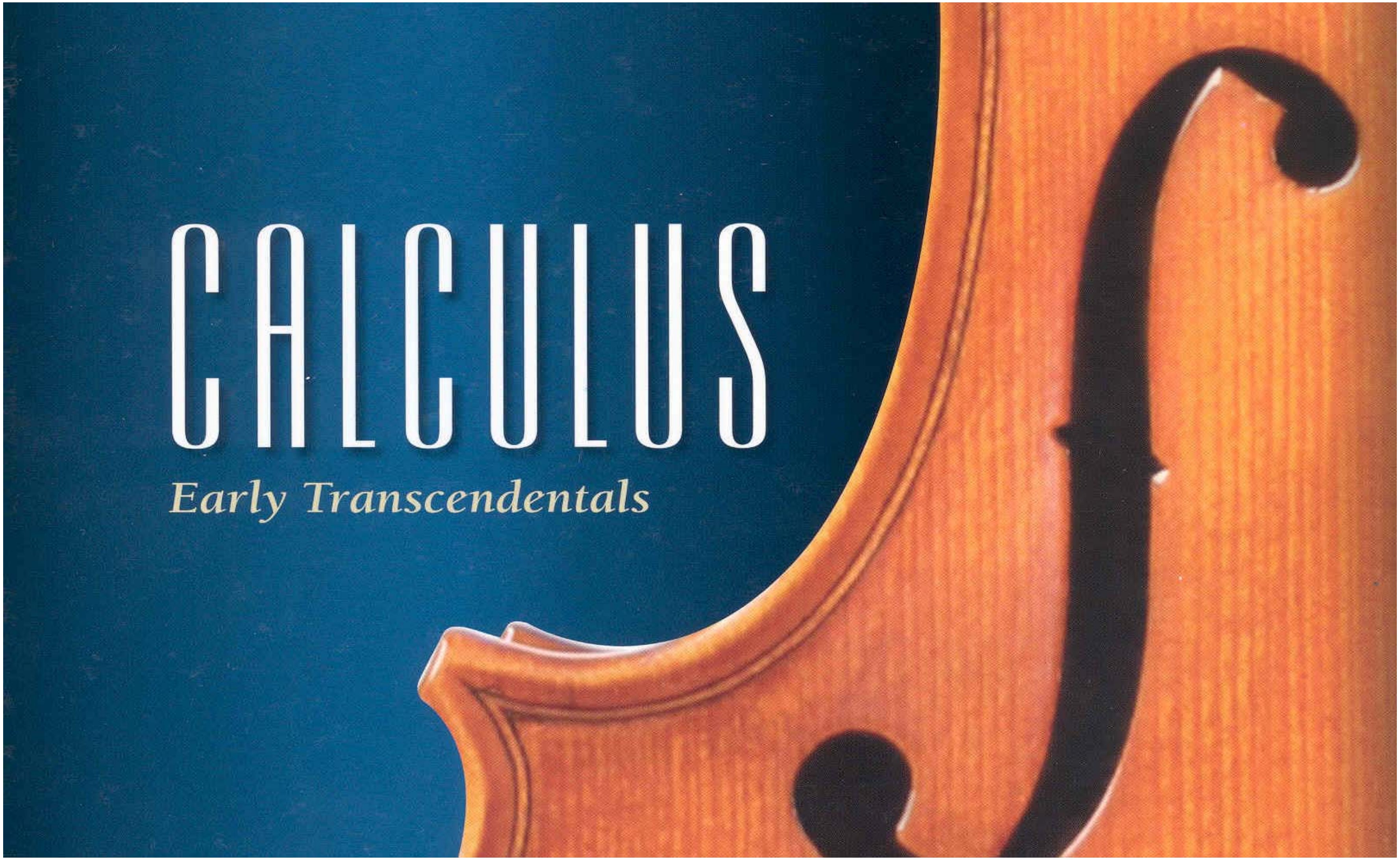


Tutorial Lab 5

4.5 SUMMARY OF CURVE SKETCHING

1–52 Use the guidelines of this section to sketch the curve.

10. $y = \dfrac{x}{(x-1)^2}$

Sol)

A. $D=\{x \mid x\neq 1\}=(-\infty,1)\cup(1,\infty)$

B. $x-$ intercept $=0$, y –intercept $=f(0)=0$

C. No symmetry

D. $\lim_{x\to\pm\infty} \dfrac{x}{(x-1)^2}=0$, so $y=0$ is a HA. $\lim_{x\to 1} \dfrac{x}{(x-1)^2}=\infty$, so $x=1$ is a VA.

E. $f'(x)=\dfrac{(x-1)^2(1)-x(2)(x-1)}{(x-1)^4}=\dfrac{-x-1}{(x-1)^3}$.

This is negative on $(-\infty,-1)$ and $(1,\infty)$ and positive on $(-1,1)$

so $f(x)$ is decreasing on $(-\infty,-1)$ and $(1,\infty)$ and increasing on $(-1,1)$.

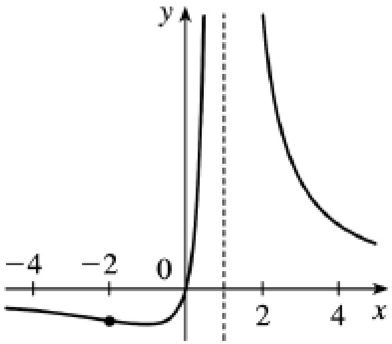
F. Local minimum value $f(-1) = -\dfrac{1}{4}$, no local maximum.

G. $f''(x) = \dfrac{(x-1)^3(-1)+(x+1)(3)(x-1)^2}{(x-1)^6} = \dfrac{2(x+2)}{(x-1)^4}$.

This is negative on $(-\infty, -2)$, and positive on $(-2,1)$ and $(1,\infty)$.

So f is CD on $(-\infty, -2)$ and CU on $(-2,1)$ and $(1,\infty)$. IP at $\left(-2, -\dfrac{2}{9}\right)$

H.



22. $y = \sqrt{x^2 + x} - x$

Sol) $y = f(x) = \sqrt{x^2 + x} - x = \sqrt{x(x+1)} - x$

A. $D = (-\infty, -1] \cup [0, \infty)$

B. y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0$

$\Rightarrow \quad \sqrt{x^2 + x} = x \quad \Rightarrow \quad x^2 + x = x^2 \quad \Rightarrow \quad x = 0$

C. No symmetry

D. $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \left(\sqrt{x^2 + x} - x\right) \frac{\sqrt{x^2 + x} + x}{\sqrt{x^2 + x} + x}$

$= \lim_{x \to \infty} \frac{x^2 + x - x^2}{\sqrt{x^2 + x} + x} \quad = \lim_{x \to \infty} \frac{x/x}{\left(\sqrt{x^2 + x} + x\right)/x}$

$= \lim_{x \to \infty} \frac{1}{\sqrt{1 + 1/x} + 1} = \frac{1}{2}$, so $y = \frac{1}{2}$ is a HA. No VA

E. $f'(x) = \frac{1}{2}(x^2+x)^{-1/2}(2x+1) - 1 = \dfrac{2x+1}{2\sqrt{x^2+x}} - 1 > 0$

$$\Leftrightarrow \quad 2x+1 > 2\sqrt{x^2+x} \quad \Leftrightarrow \quad x + \tfrac{1}{2} > \sqrt{\left(x+\tfrac{1}{2}\right)^2 - \tfrac{1}{4}}.$$

Keep in mind that the domain excludes the interval $(-1, 0)$.

When $x + \frac{1}{2}$ is positive (for $x \geq 0$),

the last inequality is *true* since the value of the radical is less than $x + \frac{1}{2}$.

When $x + \frac{1}{2}$ is negative (for $x \leq -1$),

the last inequality is *false* since the value of the radical is positive.

So f is increasing on $(0, \infty)$ and decreasing on $(-\infty, -1)$.

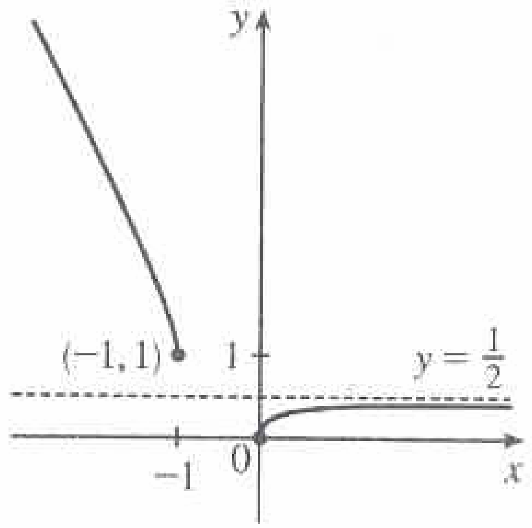
F. No local extrema

G. $f''(x) = \dfrac{2(x^2+x)^{1/2}(2) - (2x+1)\cdot 2\cdot \frac{1}{2}(x^2+x)^{-1/2}(2x+1)}{\left(2\sqrt{x^2+x}\right)^2}$

$$= \frac{(x^2+x)^{-1/2}[4(x^2+x)-(2x+1)^2]}{4(x^2+x)} = \frac{-1}{4(x^2+x)^{3/2}}.$$

$f''(x) < 0$ when it is defined, so f is CD on $(-\infty, -1)$ and $(0, \infty)$. No IP

H.



39. $y = e^{\sin x}$

Sol) $y = f(x) = e^{\sin x}$

A. $D = \mathbb{R}$

B. y-intercept: $f(0) = e^0 = 1$; x-intercepts: none, since $e^{\sin x} > 0$

C. f is periodic with period 2π, so we determine **E–G** for $0 \le x \le 2\pi$.

D. No asymptote

E. $f'(x) = e^{\sin x} \cos x$.

$f'(x) > 0 \Leftrightarrow \cos x > 0 \Rightarrow x$ is in $\left(0, \frac{\pi}{2}\right)$ or $\left(\frac{3\pi}{2}, 2\pi\right)$ [f is increasing]

and $f'(x) < 0 \Rightarrow x$ is in $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ [f is decreasing].

F. Local maximum value $f\left(\frac{\pi}{2}\right) = e$ and local minimum value $f\left(\frac{3\pi}{2}\right) = e^{-1}$

G. $f''(x) = e^{\sin x}(-\sin x) + \cos x\,(e^{\sin x}\cos x) = e^{\sin x}(\cos^2 x - \sin x).$

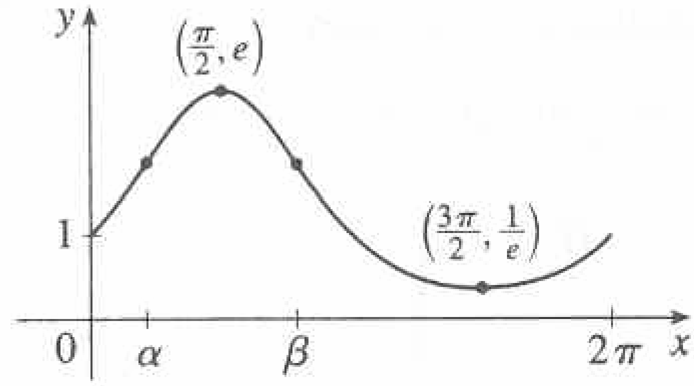
$$f''(x) = 0 \quad\Leftrightarrow\quad \cos^2 x - \sin x = 0 \;\Leftrightarrow\; 1 - \sin^2 x - \sin x = 0$$
$$\Leftrightarrow\quad \sin^2 x + \sin x - 1 = 0$$

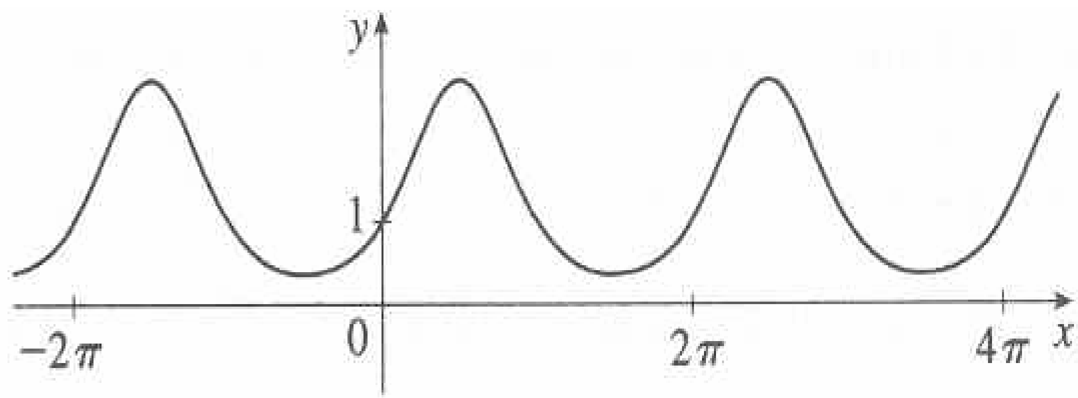
$\Rightarrow\quad \sin x = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow\quad \alpha = \sin^{-1}\left(\frac{-1+\sqrt{5}}{2}\right) \approx 0.67$ and $\beta = \pi - \alpha \approx 2.48.$

$f''(x) < 0$ on (α, β) [f is CD] and $f''(x) > 0$ on $(0, \alpha)$ and $(\beta, 2\pi)$ [f is CU].

The inflection points occur when $x = \alpha, \beta$.

H.





13–14 Sketch the graph by hand using asymptotes and intercepts, but not derivatives. Then use your sketch as a guide to producing graphs (with a graphing device) that display the major features of the curve. Use these graphs to estimate the maximum and minimum values.

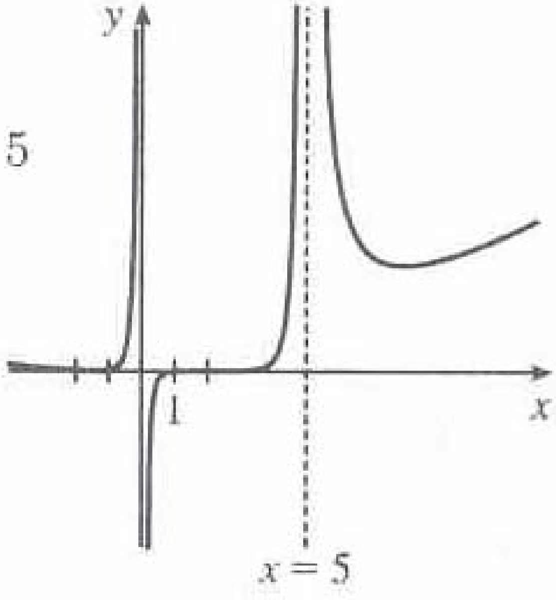
14. $f(x) = \dfrac{(2x+3)^2(x-2)^5}{x^3(x-5)^2}$

Sol) $f(x) = \dfrac{(2x+3)^2(x-2)^5}{x^3(x-5)^2}$ has VAs at $x = 0$ and $x = 5$

since $\lim_{x\to 0^-} f(x) = \infty$, $\lim_{x\to 0^+} f(x) = -\infty$,

and $\lim_{x\to 5} f(x) = \infty$.

No HA since $\lim_{x\to \pm\infty} f(x) = \infty$.



Since f is undefined at $x = 0$, it has no y-intercept.

$f(x) = 0 \quad \Leftrightarrow \quad (2x+3)^2\,(x-2)^5 = 0 \;\Leftrightarrow\; \quad x = -\frac{3}{2}$ or $x = 2$,

so f has x-intercepts at $-\frac{3}{2}$ and 2.

Note, however, that the graph of f is only tangent to

the x-axis and does not cross it at $x = -\frac{3}{2}$,

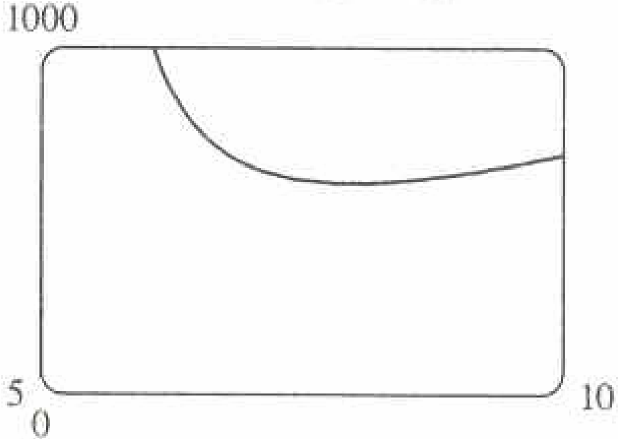
since f is positive as $x \to \left(-\frac{3}{2}\right)^-$ and as $x \to \left(-\frac{3}{2}\right)^+$.

There is a local minimum value of $f\left(-\frac{3}{2}\right) = 0$.

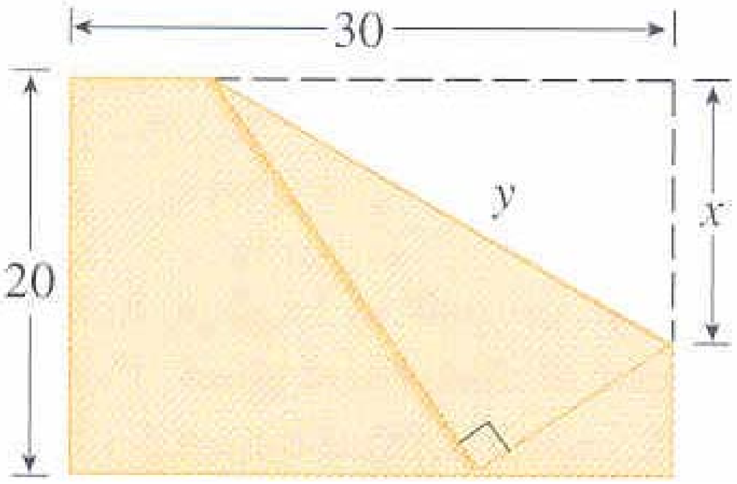
The only "mystery" feature is the local minimum to the right of the VA $x = 5$.

From the graph,

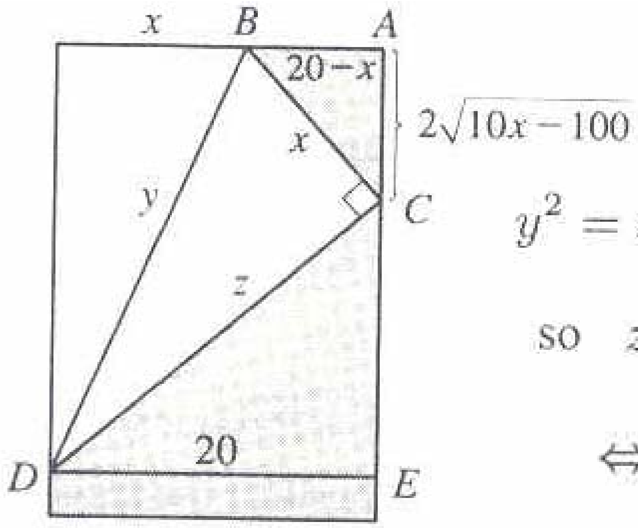
we see that $f(7.98) \approx 609$ is local minimum value.



65. The upper right-hand corner of a piece of paper, 30 cm by 20 cm, as in the figure, is folded over to the bottom edge. How would you fold it so as to minimize the length of the fold? In other words, how would you choose x to minimize y?



Sol)



$y^2 = x^2 + z^2$, but triangles CDE and BCA are similar,

so $z/20 = x/(2\sqrt{10x - 100})$

$\Leftrightarrow z^2 = 100x^2/(10x - 100)$

Thus, we minimize

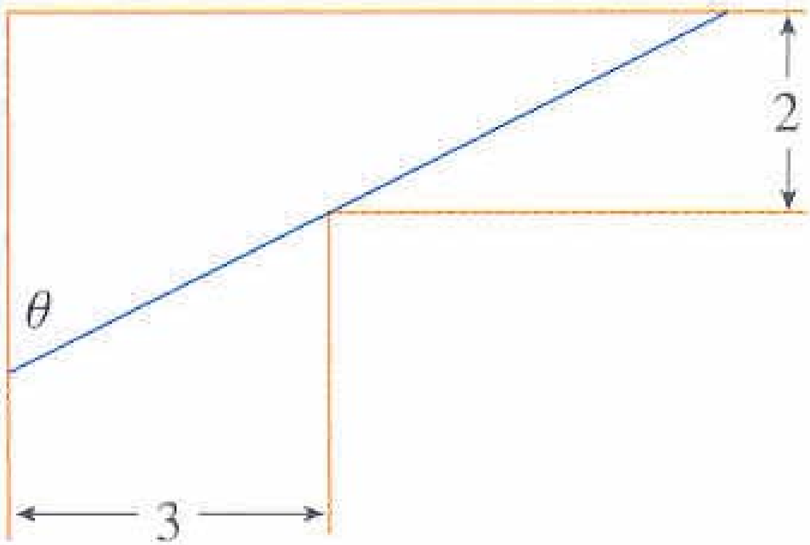
$$f(x) = y^2 = x^2 + 100x^2/(10x - 100) = x^3/(x - 10), \quad 10 < x \leq 20.$$

$$f'(x) = \frac{(x-10)(3x^2) - x^3}{(x-10)^2} = \frac{x^2[3(x-10) - x]}{(x-10)^2} = \frac{2x^2(x-15)}{(x-10)^2} = 0 \text{ when } x = 15.$$

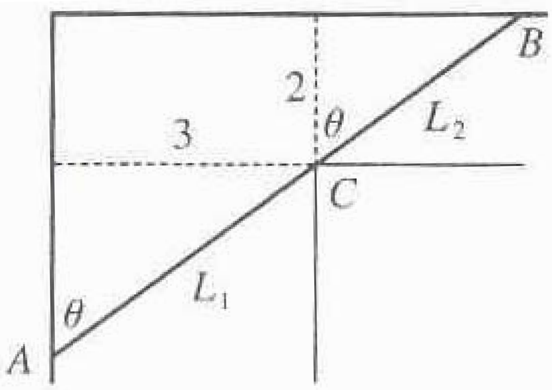
$f'(x) < 0$ when $x < 15$, $f'(x) > 0$ when $x > 15$,

so the minimum occurs when $x = 15$ cm.

66. A steel pipe is being carried down a hallway 3 m wide. At the end of the hall there is a right-angled turn into a narrower hallway 2 m wide. What is the length of the longest pipe that can be carried horizontally around the corner?



Sol) Paradoxically, we solve this maximum problem by solving a minimum problem.



Let L be the length of the line ACB going from wall to wall touching the inner corner C.

As $\theta \to 0$ or $\theta \to \frac{\pi}{2}$, we have $L \to \infty$ and there will be an angle that makes L a minimum.

A pipe of this length will just fit around the corner.

From the diagram, $L = L_1 + L_2 = 3\csc\theta + 2\sec\theta$

$\Rightarrow dL/d\theta = -3\csc\theta\ \cot\theta + 2\sec\theta\ \tan\theta = 0$

when $2\sec\theta\ \tan\theta = 3\csc\theta\ \cot\theta \Leftrightarrow \tan^3\theta = \frac{3}{2} = 1.5 \Leftrightarrow \tan\theta = \sqrt[3]{1.5}$.

Then $\sec^2\theta = 1 + \left(\frac{3}{2}\right)^{2/3}$ and $\csc^2\theta = 1 + \left(\frac{3}{2}\right)^{-2/3}$

$\Rightarrow$ longest pipe length $L = 3\left[1 + \left(\frac{3}{2}\right)^{-2/3}\right]^{1/2} + 2\left[1 + \left(\frac{3}{2}\right)^{2/3}\right]^{1/2} \approx 7.02$ m.

Or, use $\theta = \tan^{-1}\left(\sqrt[3]{1.5}\right) \approx 0.853 \quad \Rightarrow \quad L = 3\csc\theta + 2\sec\theta \approx 7.02$ m.

17–20 Express the limit as a definite integral on the given interval.

17. $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \ln(1 + x_i^2)\,\Delta x, \quad [2, 6]$

Sol) On $[2, 6]$, $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \ln(1 + x_i^2)\,\Delta x = \int_2^6 x \ln(1 + x^2)\,dx$.

19. $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{2x_i^* + (x_i^*)^2}\,\Delta x, \quad [1, 8]$

Sol) On $[1,8]$, $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{2x_i^* + \left(x_i^*\right)^2}\,\Delta x = \int_1^8 \sqrt{2x + x^2}\,dx$.

30. $\int_1^{10} (x - 4\ln x)\, dx$

Sol) $\Delta x = \frac{10-1}{n} = \frac{9}{n}$ and $x_i = 1 + i\,\Delta x = 1 + \frac{9i}{n}$, so

$$\int_1^{10} (x - 4\ln x)\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{9i}{n}\right) - 4\ln\left(1+\frac{9i}{n}\right) \right] \cdot \frac{9}{n}.$$

48. If $\int_1^5 f(x)\,dx = 12$ and $\int_4^5 f(x)\,dx = 3.6$, find $\int_1^4 f(x)\,dx$.

Sol) $\int_1^4 f(x)\,dx = \int_1^5 f(x)\,dx - \int_4^5 f(x)\,dx = 12 - 3.6 = 8.4$

52–54 Use the properties of integrals to verify the inequality without evaluating the integrals.

52. $\int_0^1 \sqrt{1 + x^2}\, dx \leqslant \int_0^1 \sqrt{1 + x}\, dx$

Sol) $x^2 \leq x$ on $[0, 1]$, so $\sqrt{1+x^2} \leq \sqrt{1+x}$ on $[0, 1]$.

Hence, $\int_0^1 \sqrt{1+x^2}\, dx \leq \int_0^1 \sqrt{1+x}\, dx$ [Property 7].

54. $\dfrac{\sqrt{2}\,\pi}{24} \leqslant \int_{\pi/6}^{\pi/4} \cos x\, dx \leqslant \dfrac{\sqrt{3}\,\pi}{24}$

Sol) If $\frac{\pi}{6} \leq x \leq \frac{\pi}{4}$, then $\cos\frac{\pi}{6} \geq \cos x \geq \cos\frac{\pi}{4}$ and $\frac{\sqrt{2}}{2} \leq \cos x \leq \frac{\sqrt{3}}{2}$,

so $\dfrac{\sqrt{2}}{2}\left(\dfrac{\pi}{4} - \dfrac{\pi}{6}\right) \leq \displaystyle\int_{\pi/6}^{\pi/4} \cos x\, dx \leq \dfrac{\sqrt{3}}{2}\left(\dfrac{\pi}{4} - \dfrac{\pi}{6}\right)$ [Property 8];

that is, $\dfrac{\sqrt{2}\,\pi}{24} \leq \displaystyle\int_{\pi/6}^{\pi/4} \cos x\, dx \leq \dfrac{\sqrt{3}\,\pi}{24}$.

65. If f is continuous on $[a, b]$, show that

$$\left| \int_a^b f(x)\,dx \right| \leqslant \int_a^b |f(x)|\,dx$$

[*Hint:* $-|f(x)| \leqslant f(x) \leqslant |f(x)|$.]

Sol) Since $-|f(x)| \leq f(x) \leq |f(x)|$, it follows from Property 7 that

$$-\int_a^b |f(x)|\,dx \leq \int_a^b f(x)\,dx \leq \int_a^b |f(x)|\,dx \Rightarrow \left| \int_a^b f(x)\,dx \right| \leq \int_a^b |f(x)|\,dx$$

Note that the definite integral is a real number,

and so the following property applies:

$-a \leq b \leq a \Rightarrow |b| \leq a$ for all real numbers b and nonnegative numbers a.

66. Use the result of Exercise 65 to show that

$$\left| \int_0^{2\pi} f(x) \sin 2x \, dx \right| \leqslant \int_0^{2\pi} | f(x) | \, dx$$

Sol)

$$\left| \int_0^{2\pi} f(x)\sin 2x \, dx \right| \leq \int_0^{2\pi} |f(x)\sin 2x| \, dx$$

$$= \int_0^{2\pi} |f(x)| \, |\sin 2x| \, dx \leq \int_0^{2\pi} |f(x)| \, dx \text{ by Property 7 },$$

$$\text{since} \quad |\sin 2x| \leq 1 \Rightarrow |f(x)| \, |\sin 2x| \leq |f(x)| .$$

69–70 Express the limit as a definite integral.

69. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$ [*Hint:* Consider $f(x) = x^4$.]

Sol) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5} = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \left(\frac{i}{n} \right)^4 = \int_0^1 x^4\, dx$

7–18 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

9. $g(y) = \int_2^y t^2 \sin t \, dt$

Sol) $f(t) = t^2 \sin t$ and $g(y) = \int_2^y t^2 \sin t \, dt$, so by FTC1, $g'(y) = f(y) = y^2 \sin y$.

11. $F(x) = \int_x^\pi \sqrt{1 + \sec t} \, dt$

$\left[\textit{Hint:} \int_x^\pi \sqrt{1 + \sec t} \, dt = -\int_\pi^x \sqrt{1 + \sec t} \, dt \right]$

Sol) $$F(x) = \int_x^\pi \sqrt{1 + \sec t} \, dt = -\int_\pi^x \sqrt{1 + \sec t} \, dt$$

$$\Rightarrow \quad F'(x) = -\frac{d}{dx} \int_\pi^x \sqrt{1 + \sec t} \, dt = -\sqrt{1 + \sec x}$$

15. $y = \int_0^{\tan x} \sqrt{t + \sqrt{t}}\, dt$

Sol) Let $u = \tan x$. Then $\frac{du}{dx} = \sec^2 x$. Also, $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$,

so $$y' = \frac{d}{dx}\int_0^{\tan x} \sqrt{t + \sqrt{t}}\, dt = \frac{d}{du}\int_0^{u} \sqrt{t + \sqrt{t}}\, dt \cdot \frac{du}{dx}$$

$$= \sqrt{u + \sqrt{u}}\,\frac{du}{dx} = \sqrt{\tan x + \sqrt{\tan x}}\,\sec^2 x.$$

19–42 Evaluate the integral.

30. $\int_0^2 (y-1)(2y+1)\,dy$

Sol) $\int_0^2 (y-1)(2y+1)\,dy = \int_0^2 (2y^2 - y - 1)\,dy$

$= \left[\frac{2}{3}y^3 - \frac{1}{2}y^2 - y\right]_0^2 = \left(\frac{16}{3} - 2 - 2\right) - 0 = \frac{4}{3}$

31. $\int_0^{\pi/4} \sec^2 t\,dt$

Sol) $\int_0^{\pi/4} \sec^2 t\,dt = [\tan t]_0^{\pi/4} = \tan\frac{\pi}{4} - \tan 0 = 1 - 0 = 1$

42. $\int_{-2}^{2} f(x)\,dx$ where $f(x) = \begin{cases} 2 & \text{if } -2 \leqslant x \leqslant 0 \\ 4 - x^2 & \text{if } 0 < x \leqslant 2 \end{cases}$

Sol)

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} 2\,dx + \int_{0}^{2} (4 - x^2)\,dx$$

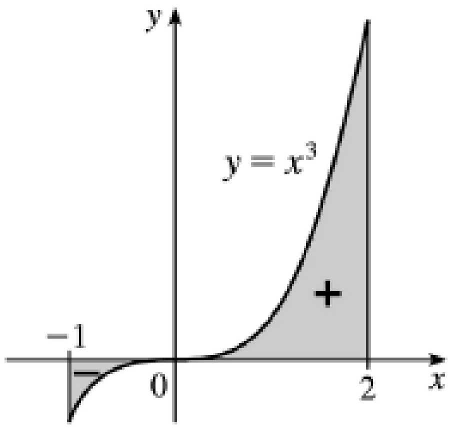
$$= \left[2x\right]_{-2}^{0} + \left[4x - \tfrac{1}{3}x^3\right]_{0}^{2} = [0 - (-4)] + \left(\tfrac{16}{3} - 0\right) = \tfrac{28}{3}$$

Note that f is integrable by Theorem 3 in Section 5.2.

51–52 Evaluate the integral and interpret it as a difference of areas. Illustrate with a sketch.

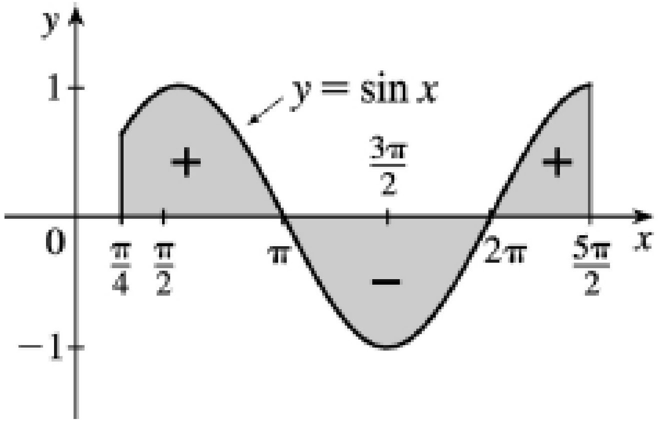
51. $\int_{-1}^{2} x^3\, dx$

Sol) $\int_{-1}^{2} x^3\, dx = \left[\frac{1}{4} x^4 \right]_{-1}^{2} = 4 - \frac{1}{4} = \frac{15}{4} = 3.75$



52. $\int_{\pi/4}^{5\pi/2} \sin x\, dx$

Sol) $\int_{\pi/4}^{5\pi/2} \sin x\, dx = [-\cos x]_{\pi/4}^{5\pi/2} = 0 + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$



53–56 Find the derivative of the function.

53. $g(x) = \int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1}\, du$

$\left[\textit{Hint: } \int_{2x}^{3x} f(u)\, du = \int_{2x}^{0} f(u)\, du + \int_{0}^{3x} f(u)\, du \right]$

Sol) $g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\, du = \int_{2x}^{0} \frac{u^2-1}{u^2+1}\, du + \int_{0}^{3x} \frac{u^2-1}{u^2+1}\, du = -\int_{0}^{2x} \frac{u^2-1}{u^2+1}\, du + \int_{0}^{3x} \frac{u^2-1}{u^2+1}\, du$

$$\Rightarrow g'(x) = -\frac{(2x)^2-1}{(2x)^2+1} \cdot \frac{d}{dx}(2x) + \frac{(3x)^2-1}{(3x)^2+1} \cdot \frac{d}{dx}(3x) = -2 \cdot \frac{4x^2-1}{4x^2+1} + 3 \cdot \frac{9x^2-1}{9x^2+1}$$

73. Find a function f and a number a such that

$$6 + \int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x} \qquad \text{for all } x > 0$$

Sol) Using FTC1, we differentiate both sides of $6+\int_a^x \frac{f(t)}{t^2}\,dt=2\sqrt{x}$

to get $\frac{f(x)}{x^2}=2\,\frac{1}{2\sqrt{x}} \Rightarrow f(x)=x^{3/2}$.

To find a , we substitute $x=a$ in the original equation

to obtain $6+\int_a^a \frac{f(t)}{t^2}\,dt=2\sqrt{a} \Rightarrow 6+0=2\sqrt{a} \Rightarrow 3=\sqrt{a} \Rightarrow a=9$.

76. A high-tech company purchases a new computing system whose initial value is V. The system will depreciate at the rate $f = f(t)$ and will accumulate maintenance costs at the rate $g = g(t)$, where t is the time measured in months. The company wants to determine the optimal time to replace the system.

(a) Let

$$C(t) = \frac{1}{t} \int_0^t [f(s) + g(s)]\, ds$$

Show that the critical numbers of C occur at the numbers t where $C(t) = f(t) + g(t)$.

Sol) **(a)** $C(t)=\frac{1}{t}\int_0^t [f(s)+g(s)]\,ds$.

Using FTC1 and the Product Rule, we have

$$C'(t)=\frac{1}{t}[f(t)+g(t)]-\frac{1}{t^2}\int_0^t [f(s)+g(s)]\,ds .$$

Set $C'(t)=0$: $\frac{1}{t}[f(t)+g(t)]-\frac{1}{t^2}\int_0^t [f(s)+g(s)]\,ds=0$

$$\Rightarrow [f(t)+g(t)]-\frac{1}{t}\int_0^t [f(s)+g(s)]\,ds=0$$

$$\Rightarrow [f(t)+g(t)]-C(t)=0 \Rightarrow C(t)=f(t)+g(t) .$$

(b) Suppose that

$$f(t) = \begin{cases} \dfrac{V}{15} - \dfrac{V}{450}t & \text{if } 0 < t \leqslant 30 \\ 0 & \text{if } t > 30 \end{cases}$$

and
$$g(t) = \frac{Vt^2}{12{,}900} \qquad t > 0$$

Determine the length of time T for the total depreciation $D(t) = \int_0^t f(s)\,ds$ to equal the initial value V.

Sol) **(b)** For $0 \leq t \leq 30$, we have $D(t) = \int_0^t \left(\frac{V}{15} - \frac{V}{450} s \right) ds$

$$= \left[\frac{V}{15} s - \frac{V}{900} s^2 \right]_0^t = \frac{V}{15} t - \frac{V}{900} t^2 .$$

So $D(t) = V \Rightarrow \frac{V}{15} t - \frac{V}{900} t^2 = V \Rightarrow 60t - t^2 = 900 \Rightarrow t^2 - 60t + 900 = 0$

$\Rightarrow (t-30)^2 = 0 \Rightarrow t = 30$. So the length of time T is 30 months.

(c) Determine the absolute minimum of C on $(0, T]$.

(d) Sketch the graphs of C and $f + g$ in the same coordinate system, and verify the result in part (a) in this case.

Sol)

(c) $$C(t) = \frac{1}{t}\int_0^t \left(\frac{V}{15} - \frac{V}{450}s + \frac{V}{12{,}900}s^2 \right) ds = \frac{1}{t}\left[\frac{V}{15}s - \frac{V}{900}s^2 + \frac{V}{38{,}700}s^3 \right]_0^t$$

$$= \frac{1}{t}\left(\frac{V}{15}t - \frac{V}{900}t^2 + \frac{V}{38.700}t^3 \right) = \frac{V}{15} - \frac{V}{900}t + \frac{V}{38.700}t^2$$

$$\Rightarrow \; C'(t) = -\frac{V}{900} + \frac{V}{19{,}350}t = 0 \text{ when } \frac{1}{19{,}350}t = \frac{1}{900} \Rightarrow t = 21.5.$$

$$C(21.5) = \frac{V}{15} - \frac{V}{900}(21.5) + \frac{V}{38{,}700}(21.5)^2 \approx 0.05472V, \; C(0) = \frac{V}{15} \approx 0.06667V,$$

$$\text{and } \; C(30) = \frac{V}{15} - \frac{V}{900}(30) + \frac{V}{38.700}(30)^2 \approx 0.05659V$$

so the absolute minimum is $C(21.5) \approx 0.05472V$.

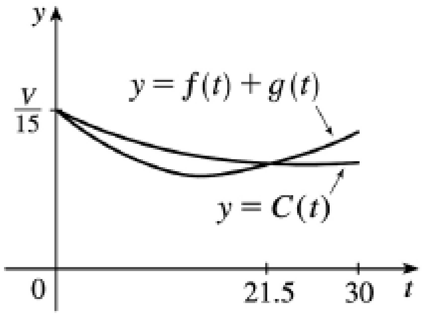
(d) As in part (c), we have $C(t)=\frac{V}{15}-\frac{V}{900}t+\frac{V}{38{,}700}t^2$,

so $C(t)=f(t)+g(t) \Leftrightarrow \frac{V}{15}-\frac{V}{900}t+\frac{V}{38{,}700}t^2=\frac{V}{15}-\frac{V}{450}t+\frac{V}{12{,}900}t^2$

$$\Leftrightarrow t^2\left(\frac{1}{12{,}900}-\frac{1}{38{,}700}\right)=t\left(\frac{1}{450}-\frac{1}{900}\right)$$

$$\Leftrightarrow \quad t=\frac{1/900}{2/38{,}700}=\frac{43}{2}=21.5\ .$$

This is the value of t that we obtained as the critical number of C in part (c), so we have verified the result of (a) in this case.



5–18 Find the general indefinite integral.

6. $\int (\sqrt{x^3} + \sqrt[3]{x^2})\, dx = \int (x^{3/2} + x^{2/3})\, dx = \dfrac{x^{5/2}}{5/2} + \dfrac{x^{5/3}}{5/3} + C = \frac{2}{5}x^{5/2} + \frac{3}{5}x^{5/3} + C$

12. $\int \left(x^2 + 1 + \dfrac{1}{x^2 + 1} \right) dx = \dfrac{x^3}{3} + x + \tan^{-1} x + C$

14. $\int (\csc^2 t - 2e^t)\, dt = -\cot t - 2e^t + C$

21–44 Evaluate the integral.

26. $\int_0^4 (2v+5)(3v-1)\,dv = \int_0^4 (6v^2+13v-5)\,dv$

$= \left[6\cdot \frac{1}{3} v^3 + 13\cdot \frac{1}{2} v^2 - 5v \right]_0^4 = \left[2v^3 + \frac{13}{2} v^2 - 5v \right]_0^4$

$= (128+104-20)-0 = 212$

28. $\int_0^9 \sqrt{2t}\,dt = \int_0^9 \sqrt{2}\, t^{1/2}\,dt = \left[\sqrt{2}\cdot \frac{2}{3} t^{3/2} \right]_0^9 = \sqrt{2}\cdot \frac{2}{3}\cdot 27 - 0 = 18\sqrt{2}$

34. $\int_1^9 \frac{3x-2}{\sqrt{x}}\,dx = \int_1^9 (3x^{1/2} - 2x^{-1/2})\,dx = \left[3\cdot \frac{2}{3} x^{3/2} - 2\cdot 2x^{1/2} \right]_1^9 = \left[2x^{3/2} - 4x^{1/2} \right]_1^9$

$= (54-12)-(2-4) = 44$

40. $\displaystyle\int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x}\, dx$

$$= \int_{-10}^{10} \frac{2e^x}{\frac{e^x - e^{-x}}{2} + \frac{e^x + e^{-x}}{2}}\, d$$

$$= \int_{-10}^{10} \frac{2e^x}{e^x}\, dx = \int_{-10}^{10} 2\, dx = \left[2x\right]_{-10}^{10} = 20 - (-20) = 40$$

62. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \leqslant t \leqslant 50$. Find the amount of water that flows from the tank during the first 10 minutes.

Sol) By the Net Change Theorem, the amount of water that flows from the tank is

$$\int_0^{10} r(t)dt = \int_0^{10} (200-4t)\,dt = \left[200t - 2t^2\right]_0^{10} = (2000-200)-0 = 1800 \text{ liters.}$$