번호	1	2			3	4	5	6	7	8	합계
점수											

과 목 명	담당교수명	반명	학 년	학번	성명	검인
수학 1						(인)

☞ 주의 사항

⊙ 일체의 부정행위를 금지합니다.

⊙ 풀이 과정을 제외한 모든 답은 답란에 적으시오.(7번 제외)

⊙ 개인 연습장과 계산기 사용을 금지 합니다.

⊙ 입실후 30분 이내 퇴실 금지합니다.

⊙ 1번 문제를 제외한 모든 문제의 풀이과정을 적으시오.

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1. (곽동제 선생님.)(each 2pts) Determine whether the statement is true(T) or false(F).

(1) If $f(x)>1$ for all x and $\lim_{x\to 0} f(x)$ exists, then .

(2) If $f(x)$ is continuous on $[a,b]$ and is differentiable on (a,b), then there exists a number $c\in(a,b)$ such that $\frac{f(b)-f(a)}{b-a}=f'(c)$.

(3) If $f(x)$ is continuous on $[a,b]$, then there exists a number $c\in(a,b)$ such that $\int_a^b f(x)dx = f(c)(b-a)$.

(4) There exists a function f such that $f(x)>0$, $f'(x)<0$, and $f''(x)>0$ for all x.

(5) $\lim_{x\to 0}\frac{x}{e^x}=1$.

	(1)	(2)	(3)	(4)	(5)
Ans.	F	T	T	T	F

2. Find the following limits.

(1) (6pts) $\lim_{x\to\frac{\pi}{4}}(1-\tan x)\sec 2x$.

(2) (6pts) $\lim_{x\to\infty} x^{\frac{\ln 2}{1+\ln x}}$

(3) (8pts) $\lim_{x\to\infty} f(x)$,

where $\frac{10e^x-21}{2e^x}<f(x)<\frac{5\sqrt{x}}{\sqrt{x-1}}$ for all $x>1$.

(1) $\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\cos 2x}=\lim_{x\to\frac{\pi}{4}}\frac{-\sec^2 x}{-2\sin 2x}=1$**(로피탈정리)**

(2) $a=\lim_{x\to\infty} x^{\frac{\ln 2}{1+\ln x}}$

$\ln a=\lim_{x\to\infty}(\frac{\ln 2}{1+\ln x})\ln x=\lim_{x\to\infty}\ln 2\frac{\frac{1}{x}}{\frac{1}{x}}=\ln 2$**(로피탈정리)**

$a=2$

(3)

$\lim_{x\to\infty}\frac{10e^x-21}{2e^x}\le\lim_{x\to\infty}f(x)\le\lim_{x\to\infty}\frac{5\sqrt{x}}{\sqrt{x-1}}$ **(극한성질)**

$\lim_{x\to\infty}\frac{10e^x-21}{2e^x}=5, \lim_{x\to\infty}\frac{5\sqrt{x}}{\sqrt{x-1}}=5$

$\lim_{x\to\infty}f(x)=5$**(Squeeze 정리)**

	(1)	(2)	(3)
Ans	1	2	5

3. (10pts) Evaluate $\cos^2(\tan^{-1}x) - \frac{d}{dx}(\tan^{-1}x)$.

$\tan^{-1}x = a \Leftrightarrow x = \tan a$

$\cos^2 a = \dfrac{1}{\sec^2 a} = \dfrac{1}{\tan^2 a + 1} = \dfrac{1}{1+x^2}$

$\cos^2(\tan^{-1}x) = \dfrac{1}{1+x^2}$

$\dfrac{d}{dx}(\tan^{-1}x) = \dfrac{1}{1+x^2}$

따라서 $\cos^2(\tan^{-1}x) + \dfrac{d}{dx}(\tan^{-1}x) = 0$

Ans.	0

4. (신선정 선생님) (10pts) Find the derivative($\frac{dy}{dx}$) of the function

$y = \tanh^{-1}(\sin x) + x^{\sin x}$.

$\dfrac{d}{dx}\tanh^{-1}(\sin x) = \dfrac{1}{1-\sin^2 x}\cos x = \dfrac{\cos x}{\cos^2 x} = \sec x$

$f(x) = x^{\sin x}$

$\ln f(x) = \sin x \ln x$

$\dfrac{f'(x)}{f(x)} = \cos x \ln x + \dfrac{\sin x}{x}$

$f'(x) = x^{\sin x}(\cos x \ln x + \dfrac{\sin x}{x})$

따라서, $\dfrac{dy}{dx} = \sec x + x^{\sin x}(\cos x \ln x + \dfrac{\sin x}{x})$

Ans.	$\dfrac{dy}{dx} = \sec x + x^{\sin x}(\cos x \ln x + \dfrac{\sin x}{x})$

5. (10pts) Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 3$.

$2\pi \int_0^1 (3-x)(x-x^2)dx = \dfrac{5}{6}\pi$

Ans.	$\dfrac{5}{6}\pi$

6. (10pts) Let $F(x) = \int_x^{x^2} f(t)dt$ and $f(t) = \int_1^{t^4} \dfrac{1}{1+u+u^2}du$.

Find $F''(-1)$.

$F'(x) = f(x^2)2x - f(x)$

$F''(x) = 4x^2 f'(x^2) + 2f(x^2) - f'(x)$

$f'(t) = \dfrac{4t^3}{1+t^4+t^8}$

$F''(-1) = 4f'(1) + 2f(1) - f'(-1)$

$f'(1) = \dfrac{4}{3},\ f'(-1) = -\dfrac{4}{3},\ f(1) = 0$

따라서, $F''(-1) = \dfrac{20}{3}$

Ans.	$\dfrac{20}{3}$

7. (15pts) Sketch the graph of $f(x)=\frac{x}{x^2+9}$.

① Domain = 모든실수

② $f(-x)=-f(x)$: odd function

③ $\lim_{x\to\infty}\frac{x}{x^2+9}=0$, $\lim_{x\to-\infty}\frac{x}{x^2+9}=0$

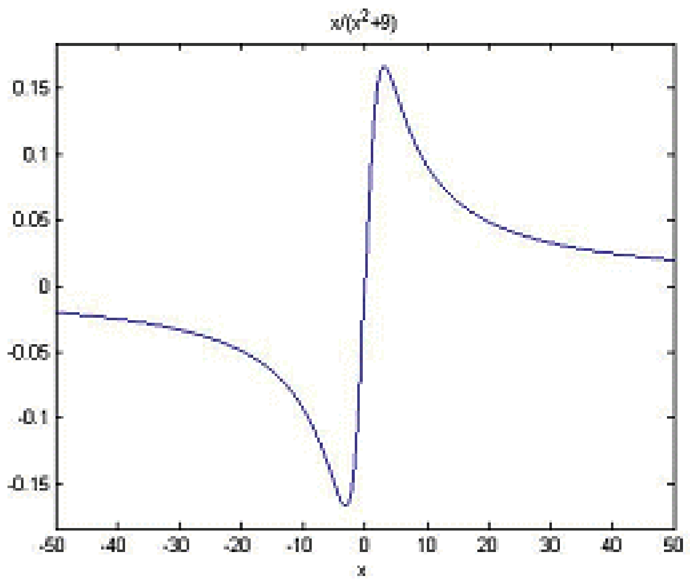
④ $f'(x)=\frac{-x^2+9}{(x^2+9)^2}=0$, $x=\pm 3$(critical numbers)

⑤ $f''(x)=\frac{2x(x^2-27)}{(x^2+9)^3}=0$, $x=0,\pm 3\sqrt{3}$

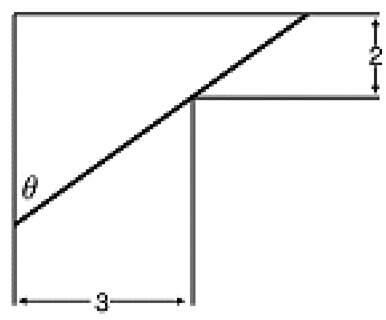
증감표 또는 위의 사실로부터,

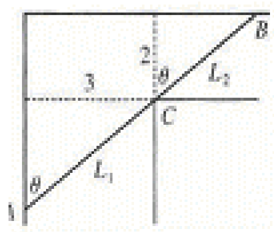
local min : $(-3,-\frac{1}{6})$, loca max : $(3,\frac{1}{6})$

inflection points : $(-3\sqrt{3},-\frac{\sqrt{3}}{12})$, $(3\sqrt{3},\frac{\sqrt{3}}{12})$



8. (15pts) A steel pipe is being carried down a hallway 3m wide. At the end of the hall there is a right-angled turn into a narrower hallway 2m wide. What is the length of the longest pipe that can be carried horizontally around the corner?





Let L be the length of the line ACB going from wall to wall touching the inner corner C. As $\theta\to 0$ or $\theta\to\frac{\pi}{2}$, we have $L\to\infty$ and there will be an angle that makes L a minimum. A pipe of this length will just fit around the corner.

From the diagram, $L=L_1+L_2=3\csc\theta+2\sec\theta$

$\Rightarrow dL/d\theta=-3\csc\theta\cot\theta+2\sec\theta\tan\theta=0$

when $2\sec\theta\tan\theta=3\csc\theta\cot\theta \Leftrightarrow \tan^3\theta=\frac{3}{2}=1.5 \Leftrightarrow \tan\theta=\sqrt[3]{1.5}$.

Then $\sec^2\theta=1+(\frac{3}{2})^{2/3}$ and $\csc^2\theta=1+(\frac{3}{2})^{-2/3}$

$\Rightarrow$ longest pipe length $L=3\left[1+(\frac{3}{2})^{-2/3}\right]^{1/2}+2\left[1+(\frac{3}{2})^{2/3}\right]^{1/2}\approx 7.02$ m.

Or, use $\theta=\tan^{-1}(\sqrt[3]{1.5})\approx 0.853 \Rightarrow L=3\csc\theta+2\sec\theta\approx 7.02$ m.

Ans	$3\sqrt{1+(\frac{3}{2})^{-2/3}}+2\sqrt{1+(\frac{3}{2})^{2/3}}$