Proceedings of the 2002 IEEE/RSJ
Intl. Conference on Intelligent Robots and Systems
EPFL, Lausanne, Switzerland • October 2002

Balance Control Analysis of Humanoid Robot based on ZMP Feedback Control

Napoleon[1], Shigeki Nakaura[2] and Mitsuji Sampei[2]

1. *Mechanical and Control Engineering, Tokyo Institute of Technology, Tokyo, Japan leo@sc.ctrl.titech.ac.jp*

2. *Mechanical and Control Engineering, Tokyo Institute of Technology, Tokyo, Japan {nakaura, sampei}@ctrl.titech.ac.jp*

Abstract

Balance control analysis of humanoid robot based on Zero Moment Point(ZMP) feedback control is presented. ZMP is mostly used as standard evaluation of stability of humanoid robot, and balance control is conducted by controlling ZMP position so that it is always in convex hull of the foot-support area. To simplify design of controller, it is mainly used one mass inverted pendulum model which represents lower body of humanoid robot model. However this model causes system become non-minimum phase and performance limitation of system is occured, because of the presence of Waterbed effect in frequency domain and unavoidable undershoot in time domain. This paper proposes ZMP feedback control using two masses inverted pendulum model which represents lower and upper body of humanoid robot and creates minimum phase system. The design of controller based on proposed model using Linear Quadratic by considering output is described and confirmed using simulation.

1 Introduction

Zero Moment Point(ZMP) is mostly used as standard evaluation of stability of humanoid robot and firstly introduced by Vukobratovic [1]. ZMP is defined as the point on the floor at which the moment $\boldsymbol{T} : (T_x, T_y, T_z)$ generated by the reaction force and the reaction torque satisfies $T_x = 0$ and $T_y = 0$. If ZMP is in convex hull of the foot-support area then humanoid robot can stand or walk without falling down. Thus to mantain the balance of humanoid robot, it is usually conducted by controlling the position of ZMP so that it is always in convex hull of the foot-support area.

Most of the research to mantain the balance of humanoid robot is conducted by planning the desired trajectory of ZMP such that it is always in convex hull of the foot-support area while humanoid robot stands or walks, then the controller is designed so that the actual value of ZMP realizes the desired trajectory. The planning of desired trajectory of ZMP can be carried out both of offline or online.

To simplify the design of the controller and to minimize the calculation time during real time application to mantain the balance, it is mainly used one mass model of inverted pendulum which represents the lower body of humanoid robot model. Thus, the balance control of humanoid robot is conducted by moving the waist of humanoid robot so that the actual position of ZMP tracks the desired position of ZMP, while the upper body of robot is not move too much. However using one mass model inverted pendulum model to design the controller causes the system become non-minimum phase and the limitation in the performance of the system is occured as will be described in this paper. Thus one mass model of inverted pendulum is not appropriate and to overcome this problem, this paper proposes the balance control of humanoid robot based on ZMP feedback control using two masses model of inverted pendulum which will create minimum phase system. Based on the proposed model, the controller is designed using Linear Quadratic optimal control and confirmed by computer simulation.

2 Preliminaries

In this section, the concept of Sensitivity function and undershoot in control theory is described. This concept is used to show the limitation of the performance in feedback control.

General block diagram of control system can be shown in Fig. 1. From this block diagram, the transfer function from reference r, disturbance d, sensor noize n to error e can be described as follows,

$$E(s) = \frac{1}{1+C(s)P(s)}R(s) - \frac{1}{1+C(s)P(s)}D(s) + \frac{C(s)P(s)}{1+C(s)P(s)}N(s).$$

From the expression above, Sensitivity function of

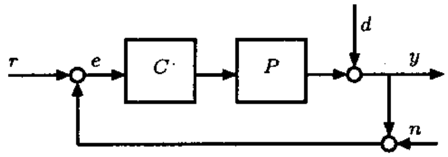

Figure 1: General block diagram of control system

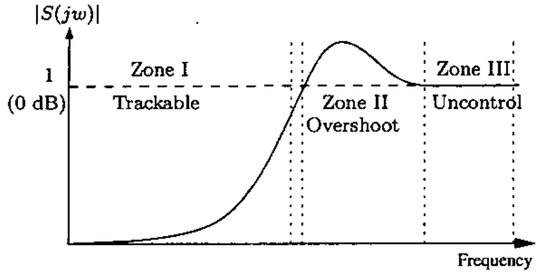

Figure 2: Characteristics of Sensitivity function

feedback system is defined as,

$$S(s) := \frac{1}{1 + C(s)P(s)} := \frac{1}{1 + L(s)}.$$

Sensitivity function maps the reference and disturbance to the error of the system, and has relation with closed-loop performance and robustness properties of the system. In above equation, $L(s) := C(s)P(s)$ is open loop transfer function.

The characteristics of Sensitivity function $S(s)$ is shown in Fig. 2. From the figure, the characteristic can be divided to 3 zones in frequency domain. In zone I, $||S(j\omega)|| \ll 1$ comprises, the error decreases about the reference which results the output of the system approaches the reference. In this zone, the reference to the system can be tracked quickly without problem. In zone II, $||S(j\omega)|| > 1$ comprises, the error increases about the reference which results the output of the system is far from the reference. Thus, overshoot will be occurred since the presence of feedback. On the other hand, in zone III, $||S(j\omega)|| = 1$ comprises, the reference will directly appear in the error which corresponding to system being uncontrolled. Thus to make the system can track the reference quickly, it is needed to make the Sensitivity function $S(j\omega) \ll 1$ as possible.

2.1 Waterbed Effect for Non-Minimum Phase System

By means of constructing the controller, the form of Sensitivity function $S(s)$ can be restructed, however in the case of non-minimum phase system, i.e.

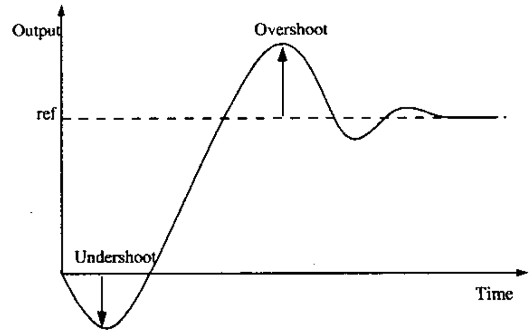

Figure 3: Undershoot and Overshoot

open loop transfer function of system has unstable zero, decreasing the magnitude of Sensitivity function in one area will increase the magnitude of it in other area. This is known as waterbed effect which is shown in the following theorem.

Theorem 1 *[3]*
Suppose that plant P has a zero at z with Re(z) > 0. Then there exist positive constants c_1 and c_2, depending only on ω_1, ω_2, and z, such that

$$c_1 \log M_1 + c_2 log M_2 \geq log|S_{ap}(z)^{-1}| \geq 0. \quad (1)$$

where

$$\begin{aligned} z &:= \rho_o + j\omega_o &&: \text{Unstable zero,} \\ M_1 &:= \max_{\omega_1 \leq \omega \leq \omega_2} |S(j\omega)| &&= ||S(j\omega)||_{\omega_{12}}, \\ M_2 &:= \max_{\omega} |S(j\omega)| &&= ||S(j\omega)||_{\infty}. \end{aligned}$$

□

In general, Sensitivity function $S(s)$ can be written as the product of all-pass and minimum phase transfer function as,

$$S(s) = S_{ap}(s)S_{mp}(s), \quad (2)$$

where $S_{ap}(s)$ is all-pass transfer function and $S_{mp}(s)$ is minimum phase transfer function. The all-pass transfer function consists of unstable zeros of Sensitivity function $S(s)$, has magnitude 1 at all frequencies and can be written in the form,

$$S_{ap}(s) = \frac{(s - s_o)(s - s_1)\cdots}{(s + \bar{s}_o)(s + \bar{s}_1)\cdots}.$$

2.2 Unavoidable Undershoot for Non-Minimum Phase System

In time domain, the response of the system generally can be shown in Fig. 3. Suppose that transfer function from reference to output of the system is described as follows,

$$H(s) = \frac{N(s)}{D(s)} = \frac{k(s + z_1)(s + z_2)\cdots(s + z_m)}{(s + p_1)(s + p_2)\cdots(s + p_n)}. \quad (3)$$

where it is assumed that $k > 0$ and $n > m$.

Using final value theorem and initial value theorem of Laplace transform, the steady state and initial output of the system can be found as the following,

$$\begin{aligned}
y(\infty) &= \lim_{s\to 0} s\frac{N(s)}{D(s)}\frac{1}{s} = \lim_{s\to 0}\frac{N(s)}{D(s)}, \\
&= \frac{kz_1z_2\cdots z_m}{p_1p_2\cdots p_n}, \\
\dot{y}(0^+) &= \lim_{s\to\infty} s\frac{sN(s)}{D(s)}\frac{1}{s} = \lim_{s\to\infty}\frac{sN(s)}{D(s)}, \\
&= 0, \\
\vdots & \quad \vdots \\
y^{n-m}(0^+) &= \lim_{s\to\infty} s\frac{s^{n-m}N(s)}{D(s)}\frac{1}{s} = \lim_{s\to\infty}\frac{s^{n-m}N(s)}{D(s)}, \\
&= k.
\end{aligned}$$

If all the poles and zeros are stable, which results $p_1p_2\cdots p_n > 0$ and $z_1z_2\cdots z_n > 0$, then for positive reference value, the initial velocity, acceleration and so on are zero or positive as shown below. In this case, the happening of undershoot in the system can not be determined.

$$\begin{aligned}
y(\infty) &= \frac{kz_1z_2\cdots z_m}{p_1p_2\cdots p_n} > 0, \\
\dot{y}(0^+) &= 0, \\
&\vdots \\
y^{n-m}(0^+) &= k > 0.
\end{aligned}$$

If all the poles are stable and odd number of zeros are unstable, which results $p_1p_2\cdots p_n > 0$ and $z_1z_2\cdots z_n < 0$, then for negative reference value, the initial velocity, acceleration and so on are zero or positive as shown below. In this case, undershoot of the system is inevitably occured.

$$\begin{aligned}
y(\infty) &= \frac{kz_1z_2\cdots z_m}{p_1p_2\cdots p_n} < 0, \\
\dot{y}(0^+) &= 0, \\
&\vdots \\
y^{n-m}(0^+) &= k > 0.
\end{aligned}$$

If all the poles are stable and even number of zeros are unstable, which results $p_1p_2\cdots p_n > 0$ and $z_1z_2\cdots z_n > 0$, then for positive reference value, the initial velocity, acceleration and so on are zero or positive as shown below. In this case, the happening of undershoot for the system can not be determined.

$$\begin{aligned}
y(\infty) &= \frac{kz_1z_2\cdots z_m}{p_1p_2\cdots p_n} > 0, \\
\dot{y}(0^+) &= 0, \\
&\vdots \\
y^{n-m}(0^+) &= k > 0.
\end{aligned}$$

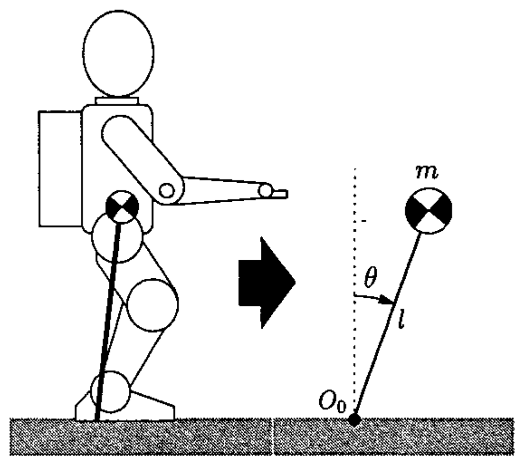

Figure 4: One mass model of ZMP feedback control

From the above results, for non-minimum phase system with odd number of unstable zeros, undershoot of the system is always occurred and can not be avoided.

3 Performance Limitation in One Mass Inverted Pendulum Model

To simplify the design of the controller, it is mainly used one mass model of inverted pendulum which represents the lower body of humanoid robot model. The balance control is conducted by moving the waist of the robot so that the actual position of ZMP tracks the desired position of ZMP. In this case, effect of the motion of upper body of humanoid robot is assumed to be neglectible. This model can be shown in Fig. 4, linearizing the equation of motion around origin results the following state space representation,

$$\begin{aligned}
\frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix} &= \begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}u, \\
p &= \begin{bmatrix} l & 0 \end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix} + \begin{bmatrix} -l^2/g \end{bmatrix}u.
\end{aligned} \tag{4}$$

In this model, the input to the system is angular acceleration of link and the output of the system is the position of ZMP p. The actual input to link is torque instead of acceleration, however since the actual model of one mass inverted pendulum can be linearized using feedback linearization, considering the input to the system as acceleration is similar as considering torque to actual system.

The transfer function of this system can be described as,

$$P(s) = \frac{gl - l^2s^2}{gs^2}. \tag{5}$$

This sytem has two zeros and one of the zeros is unstable, thus it is non-minimum phase system.

3.1 Waterbed Effect for One Mass Inverted Pendulum Model

Since this system is non-minimum phase system, the form of Sensitivity function can not be freely constructed since minimizing the magnitude of Sensitivity function in one frequency range will make the magnitude of it become very large in the different frequency range. This effect is called waterbed effect, using Theorem 1 which is described in the previous section, the resulted limitation can be shown as follows. Suppose that the Sensitivity function is minimized between frequency range 0 and ω_{max}, and the maximum magnitude of Sensitivity function in this range is $||S(j\omega)||_{\omega_{12}}$ while the maximum magnitude of Sensitivity function in overall frequency range is $||S(j\omega)||_{\infty}$. Then the relation between ω_{max}, $||S(j\omega)||_{\omega_{12}}$ and $||S(j\omega)||_{\infty}$ can be found by the following relation,

$$||S(j\omega)||_{\infty} \geq \left\{ \frac{1}{||S(j\omega)||_{\omega_{12}}} \right\}^{\frac{\arctan(\omega_{max}\sqrt{\frac{l}{g}})}{\frac{\pi}{2} - \arctan(\omega_{max}\sqrt{\frac{l}{g}})}}. \tag{6}$$

Since $||S(j\omega)||_{\omega_{12}} \ll 1$, the maximum magnitude of Sensitivity function over all frequency ranges $||S(j\omega)||_{\infty}$ increase, thus it will make the response of the system in other frequency ranges become worst. This relation is shown in Fig. 5. Making the trackable frequency range between 0 and 10 [rad/s], and $||S(j\omega)||_{\omega_{12}} = -1.0$ [dB] will make the maximum magnitude of $||S(j\omega)||_{\infty}$ in other frequency range become bigger than 4 [dB]. The more trackable frequency range is increased or $||S(j\omega)||_{\omega_{12}}$ is minimized, the bigger $||S(j\omega)||_{\infty}$ will become.

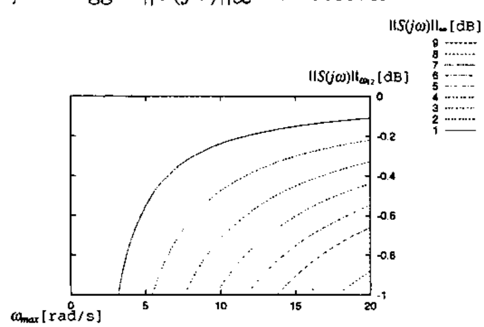

Figure 5: Relation between ω_{max}, $||S(j\omega)||_{\omega_{12}}$ and $||S(j\omega)||_{\infty}$ for one mass model

3.2 Unavoidable Undershoot for One Mass Inverted Pendulum Model

One mass inverted pendulum model which is used to control ZMP of robot has two zeros which one of this zeros is unstable. Thus from the result in the previous section, undershoot in this system is inevitably occured no matter what kind of feedback controller is used.

The physical interpretation of this undershoot can be described as follows. Suppose that robot in the rest position, to move the position of ZMP into ahead of robot need to actuate robot links to move ahead, however at this time the position of ZMP moves to the backward.

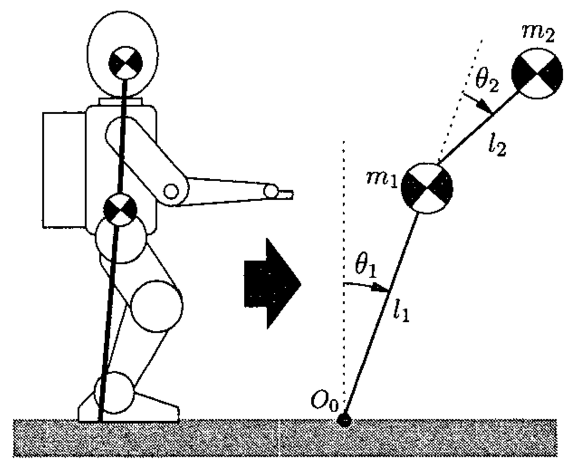

Figure 6: Two masses model of ZMP feedback control

4 ZMP Feedback Control using Two Masses Inverted Pendulum Model

Since limitation in the control performance using one mass inverted pendulum model to conduct ZMP feedback control is occured, this paper proposes two masses inverted pendulum model which is shown in Fig. 6. This model can tipify the lower and the upper body of humanoid robot. Linearizing the equation of motion around origin results the following state space representation of the system,

$$\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) \left\{ \begin{array}{lcl} \frac{d}{dt}\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} & = & \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u, \\ p & = & \begin{bmatrix} C_1 & 0 \end{bmatrix}\begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + Du, \end{array} \right. \tag{7}$$

where, $\theta := [\theta_1, \theta_2]^T$, $u := [u_1, u_2]^T$ and p are state, input and output of the system, and define $q := [\theta, \dot{\theta}]^T$. Similar to one mass inverted pendulum model, the input to the system is angular acceleration and the output of the system is position of ZMP, and matrices parameter in the above equation have the following form,

$$C_1 = \left[\frac{(m_1 + m_2)l_1 + m_2 l_2}{m_1 + m_2} \quad \frac{m_2 l_2}{m_1 + m_2} \right],$$

$$D = \begin{bmatrix} -\frac{m_1 l_1^2 + m_2(l_1+l_2)^2}{(m_1+m_2)g} & -\frac{m_2(l_1+l_2)l_2}{(m_1+m_2)g} \end{bmatrix}.$$

4.1 Zero Analysis

The limitation in the performance of feedback control is happened because of the existence of unstable zero in the open loop transfer function of the system. Thus, it is needed to confirm that the proposed model of ZMP feedback control does not cause the system become non-minimum phase system. It can be examined by analysis of zero of the system. However since the proposed model consists of two inputs, the concept of zero in multi input multi output (MIMO) system is different from single input single ouput(SISO) system. The zero of the system actually can be found by transforming system's transfer function to Smith-McMillan form.

Since the controller can be selected arbitrary, provided the controller does not have unstable zero, the zero analysis of open loop transfer function can be conducted by considering the input output transfer function of the plant only. The input output transfer function of the proposed model can be described as follows,

$$P(s) = \begin{bmatrix} \frac{d_1 s^2 + c_1}{s^2} & \frac{d_2 s^2 + c_2}{s^2} \end{bmatrix}. \tag{8}$$

This transfer function has similar Smith-McMillan form of transfer function which can be described as [4],

$$P(s) \sim M(s) = \begin{bmatrix} \frac{\delta}{s^2} & 0 \end{bmatrix}, \tag{9}$$

where, $\sim$ means similar and δ is defined as,

$$\begin{cases} \frac{1}{(m_1+m_2)g} & \text{if} \quad m_1 \neq m_2, \quad l_2 \neq 0, \\ \frac{(m_1+m_2)gl_1 - m_1 l_1^2 s^2}{(m_1+m_2)g} & \text{if} \quad m_1 \neq m_2, \quad l_2 = 0, \\ \frac{1}{2g} & \text{if} \quad m_1 = m_2, \quad l_2 \neq 0, \\ \frac{gl_1 - l_1^2 s^2}{g} & \text{if} \quad m_1 = m_2, \quad l_2 = 0. \end{cases}$$

From the above result, it can be concluded that provided that $l_2 \neq 0$, (unstable) zero of the system does not exist, thus waterbed effect and undershoot limitation does not occured. The case when $l_2 = 0$ means that the model is represented by one mass inverted pendulum model, and the limitation in the performance is occured as described in the previous section.

Since the purpose of ZMP feedback control is to make the actual position of ZMP can track the desired position of ZMP, the problem can be formulated as servo system. Thus suppose that the desired position of ZMP is p^{ref}, then by adopting new state into original state space representation of the system, the following extended system will make the actual position of ZMP can track constant reference of ZMP p^{ref},

$$\begin{cases} \frac{d}{dt}\begin{bmatrix} q \\ z \\ v \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ -C & 0 & 0 \\ \mathbf{1}_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ z \\ v \end{bmatrix} + \begin{bmatrix} B \\ -D \\ 0 \end{bmatrix} u, \\ y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} q \\ z \\ v \end{bmatrix}, \end{cases} \tag{10}$$

where, $z := \int (p^{ref} - p)\, dt$ is the integration of error between desired value and actual value of ZMP and used to track the desired position of ZMP. And $v := \int \theta_2\, dt$ is the integration of second link angle and used to make the posture of the robot become straight in steady state. Here, define $\mathbf{1}_2 := \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$ and $x^T := \begin{bmatrix} q^T & z & v \end{bmatrix}$.

4.2 Controller Design

Since there is feedthrough term in the system, to control the output of the system, it is needed to consider the output as well as the state and the input to increase the performance of the output. Thus in designing the controller, beside using the general Linear Quadratic controller with state and input as objective function, this paper proposes the designing of the controller using the following objective function,

$$J = \int_0^\infty \{p(t)^T W p(t) + x(t)^T \tilde{Q} x(t) + u(t)^T \tilde{R} u(t)\} dt, \tag{11}$$

where, p is the position of ZMP which is output of the system. And from the definition of output $p := Cx + Du$, the objective function can be rewritten as,

$$J = \int_0^\infty \left(x(t)^T Q x(t) + 2x(t)^T S u(t) + u(t)^T R u(t) \right) dt,$$

where, the matrices parameter used above can described as,

$$Q = \begin{bmatrix} C^T W C & 0 \\ 0 & 0 \end{bmatrix} + \tilde{Q}, \qquad R = D^T W D + \tilde{R},$$

$$S = \begin{bmatrix} C^T W D \\ 0 \end{bmatrix}.$$

To find optimal solution by solving Riccati equation, it is needed to transform the above objective function to the form of standard Linear Quadratic objective function. And it can be done by introducing the following transformation,

$$x^T Q x + 2x^T S u + u^T R u = x^T (Q - SR^{-1}S^T) x + (u + R^{-1}S^T x)^T R (u + R^{-1} S^T x).$$

Substituting this transformation to objective function above and define $u' = u + R^{-1}S^T x$ as new input, objective function become standard form of Linear Quadratic objective function as follows,

$$J = \int_0^\infty \Big(x(t)^T (Q - SR^{-1}S^T) x(t) + (u(t) + R^{-1}S^T x(t))^T R (u(t) + R^{-1}S^T x(t)) \Big) dt.$$

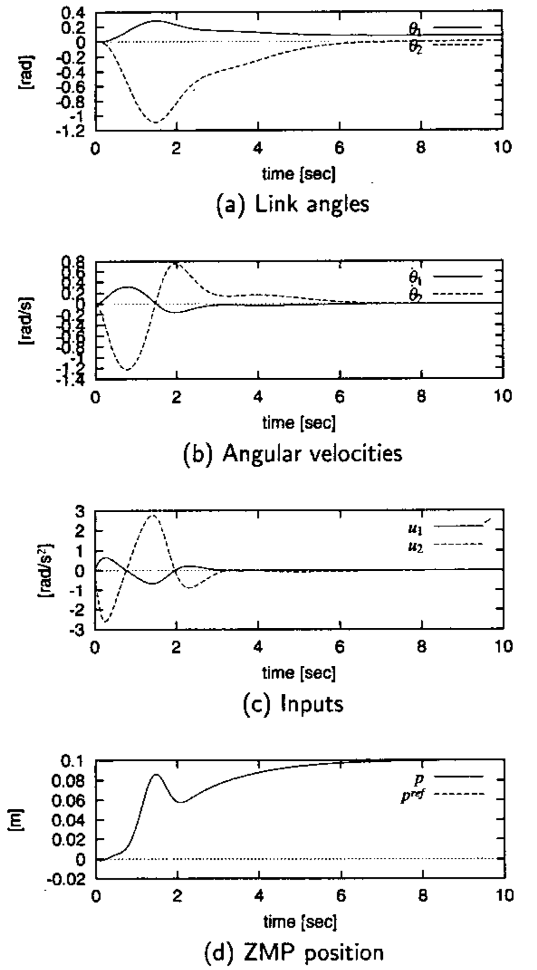

Figure 7: Simulation results of 2 masses inverted pendulum model

The condition that the optimal solution exist is given by the following relation,

$$Q - SR^{-1}S^T \geq 0. \tag{12}$$

4.3 Simulation

Using the proposed model and controller which considering the output objective, the simulation is conducted using weighted matrices of objective function as, $\boldsymbol{W} = \mathrm{diag}\,[1000.0]$, $\tilde{\boldsymbol{Q}} = \mathrm{diag}\,[1.0, 1.0, 0.01, 0.01, 5000.0, 1.0]$ and $\tilde{\boldsymbol{R}} = \mathrm{diag}\,[1.0, 0.01]$. Robot parameters used in simulation are $m_1 = 30.0$ $[kg]$, $m_2 = 40.0$ $[kg]$, $l_1 = 1.0$ $[m]$, $l_2 = 0.5$ $[m]$, $= 9.81$ $[m/sec^2]$. In simulation the output is supposed to track 0.1 $[m]$ as desired position of ZMP and simulation results are shown in Fig. 7.

From simulation results, It can be seen that the proposed control method can make the system to track the desired position of ZMP quickly.And it can be concluded to track desired position of ZMP, the second link which is the upper body of robot, moves in a large way so that the input of the first link can be suppresed as shown in Fig. 7(a) and (c). The motion of second link can be suppressed by changing the weighting matrix if the upper body is not desired to be moved in a large way. In the transient response, the output goes to the opposite direction as shown in Fig. 7(d) and it is caused by the alteration in direction of the input.

5 Conclusion

This paper described performance limitation of ZMP feedback control using one mass inverted pendulum model. Thus, this kind of model is not appropriate to use to design the controller which tracks desired position of ZMP. The limitation in the performance is occured because of the presence of Waterbed effect in frequency domain and undershoot in time domain. To overcome this problem, this paper proposed two mass inverted pendulum model be used to design controller which tracks desired position of ZMP, and the controller design based on Linear Quadratic optimal control by considering output evaluation is described and confirmed by simulation.

The implementation of the proposed model and control method to real application of humanoid robot is considered to be the future works of this research.

Acknowledgments

A part of this research is supported by Humanoid and Human Friendly Robotics System Project of Ministry of Economy, Trade and Industry Japan. The authors wish to thank S. Kajita at National Institute of Advanced Industrial Science and Technology(AIST) Japan for his suggestions on this work.

References

[1] M. Vukobratovic, B. Borovac, D. Surla, D. Stokic: "Biped Locomotion: Dynamics, Stability, Control and Application", Springer Verlag, 1975.

[2] A. Goswami: "Postural Stability of Biped Robots and the Foot-Rotation Indicator (FRI) Point", The International Journal of Robotics Research, Vol. 18, No. 6, pp. 523–533, 1999.

[3] J.C. Doyle, B.A. Francis, A.R. Tannenbaum: "Feedback Control Theory", MacMillan Publishing Company, 1992.

[4] T. Kailath:"Linear System", Prentice-Hall, 1980.

[5] J. Chen: "Sensitivity Integral Relations and Design Trade-Offs in Linear Multivariable Feedback Systems", IEEE Transaction on Automatic Control, Vol. 40, No. 10, pp. 1700–1716, 1995.

[6] B.D.O. Anderson, J.B. Moore: "Linear Optimal Control", Prentice-Hall Inc., 1971.