EGO Expert Group
2008.01.03 Issue 2

AP Calculus

Topics:

- **Motion along a line**
- **Motion along a curve**
- **AP FRQ 기출문제**

Motion along a line

직선 위를 움직이는 점(particle)의 위치가 시각 t 에 따른 함수로 주어지는 경우
즉, $s = f(t)$ 로 주어진 경우 위치(s), 속도(v), 속력(speed; |v|), 가속도(a) 간의 관계 및 움직이는 점의 운동을 어떻게 해석하는지 알아두도록 하자.

직선 위를 움직이는 점에 관련된 문제는 수식(Multiple choice questions)과 그래프(Free response questions)를 이용해 푸는 경우가 있으므로 모두 다룰 수 있도록 기억하자.

1) position(s), velocity(v), speed(|v|), acceleration(a) 간의 관계

$$s = f(t) \quad \underset{\text{적분}}{\overset{\text{미분}}{\rightleftarrows}} \quad \frac{ds}{dt} = f'(t) = v(t) \quad \underset{\text{적분}}{\overset{\text{미분}}{\rightleftarrows}} \quad \frac{d^2s}{dt^2} = f''(t) = \frac{dv}{dt} = v'(t) = a(t)$$

2) position 해석에서 주의할 점

- 초기위치가 '0'이 아닌 경우가 있으니 주의하자. 즉, $f(0) \neq 0$
  특히, $v(t)$, $a(t)$ 에서 적분을 이용해 $f(t)$ 를 구하는 경우에 실수하기 쉽다.
  이 경우에는 문제에서 주는 초기조건(s, v)을 이용한다.
- $v(t)$, $a(t)$ 에서 적분하는 경우 position(위치; 변위)과 travel distance 를 구분하자.
  position 은 $\int v(t)\,dt$ (정적분) 로 구하고, travel distance 는 $\int |v(t)|\,dt$ 로 구한다.

3) velocity, speed 해석에서 주의할 점

- $v > 0$ 이면 점은 오른쪽(+)으로 움직이고, $v < 0$ 이면 점은 왼쪽(+)으로 움직인다.
  이 때, 원점에서 멀어지고 가까워지는 것은 s, t 모두 고려해야 한다.
- 속력(speed)은 속도(velocity)의 크기이다. 즉, |v|
  $v > 0$, $v < 0$ 에 상관없이 절대값이 크면 속력(speed)이 크다.
- $v(t) = 0$, $v'(t) = a(t) \neq 0$ 인 시각 t 에서 운동 방향이 바뀐다.
  $a(t) > 0$ 이면 좌에서 우로, $a(t) < 0$ 이면 우에서 좌로 운동방향이 바뀐다.

점이 원점 좌측에 있을 때,
$v > 0$ 일 때 가까워짐
$v < 0$ 일 때 멀어짐

점이 원점 우측에 있을 때,
$v > 0$ 일 때 멀어짐
$v < 0$ 일 때 가까워짐

$v(t) = 0$, $a(t) = 0$ 인 시각 t 에서는 운동방향이 바뀌지 않는다.

velocity, acceleration
부호가 같으면
speed 증가, accelerating
부호가 다르면
speed 감소, decelerating

4) acceleration 해석에서 주의할 점

- $a > 0$ 이면 v 가 증가(increasing)하고, $a < 0$ 이면 v 가 감소(decreasing)한다.
  이것은 velocity 의 부호가 +/- 인 것이 아니라, 값이 증가, 감소함을 의미한다.
- $a > 0$, $v > 0$ 이면 velocity 도 증가하고 speed 도 증가한다.
  $a > 0$, $v < 0$ 이면 velocity 는 증가하고 speed 는 감소한다.
  $a < 0$, $v < 0$ 이면 velocity 는 감소하고 speed 는 증가한다.
  $a < 0$, $v > 0$ 이면 velocity 도 감소하고 speed 도 증가한다.

Example 1

Barron's AP Calculus
p.111 #26

Motion along a line

A particle moves along a line according to the law $s = 2t^3 - 9t^2 + 12t - 4$, where $t \geqq 0$. (a) Find all t for which the distance s is increasing. (b) Find all t for which the velocity is increasing. (c) Find all t for which the speed of the particle is increasing. (d) Find the speed when $t = \frac{3}{2}$. (e) Find the total distance traveled between $t = 0$ and $t = 4$.

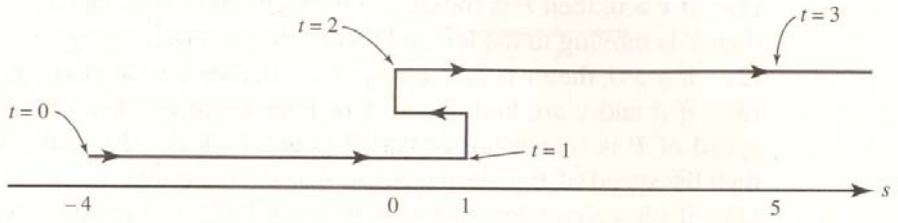
정답 :

(a) s increases when $t < 1$ or $t > 2$.

(b) v increases when $t > \frac{3}{2}$.

(c) The speed $|v|$ is increasing when v and a are both positive, that is, for $t > 2$, and when v and a are both negative, that is, for $1 < t < \frac{3}{2}$.

(d) The speed when $t = \frac{3}{2}$ equals $|v| = \left|-\frac{3}{2}\right| = \frac{3}{2}$.



(e) P's motion can be indicated as shown in Figure N4–15. P moves to the right if $t < 1$, reverses its direction at $t = 1$, moves to the left when $1 < t < 2$, reverses again at $t = 2$, and continues to the right for all $t > 2$. The position of P at certain times t are shown in the following table:

t:	0	1	2	4
s:	−4	1	0	28

Thus P travels a total of 34 units between times $t = 0$ and $t = 4$.

Example 2

A particle moves along the x-axis so that its velocity at time t is given by $v(t) = 6t^2 - 18t + 12$. (a) Find the total distance covered between $t = 0$ and $t = 4$. (b) Find the displacement of the particle from $t = 0$ to $t = 4$.

Barron's AP Calculus
p. 276 #2

Motion along a line

정답 : (a) 34 , (b) 32

Example 3

In Questions 1~4 , the position of a particle moving along a straight line is given by $s = t^3 - 6t^2 + 12t - 8$.

1. The distance s is increasing for

   (A) $t < 2$ **(B)** all t except $t = 2$ **(C)** $1 < t < 3$
   (D) $t < 1$ or $t > 3$ **(E)** $t > 2$

2. The minimum value of the speed is

   (A) 1 **(B)** 2 **(C)** 3 **(D)** 0 **(E)** none of these

3. The acceleration is positive

   (A) when $t > 2$ **(B)** for all t, $t \neq 2$ **(C)** when $t < 2$
   (D) for $1 < t < 3$ **(E)** for $1 < t < 2$

4. The speed of the particle is decreasing for

   (A) $t > 2$ **(B)** $t < 3$ **(C)** all t
   (D) $t < 1$ or $t > 2$ **(E)** none of these

Barron's AP Calculus
p. 127 #18~21

Motion along a line

정답 : 1. (B) 2. (D) 3. (A) 4. (E)

Example 4

A particle moves along a line in such a way that its position at time t is given by $s = t^3 - 6t^2 + 9t + 3$. Its direction of motion changes when

(A) $t = 1$ only **(B)** $t = 2$ only **(C)** $t = 3$ only
(D) $t = 1$ and $t = 3$ **(E)** $t = 1$, 2, and 3

Barron's AP Calculus
p. 286 #1

Motion along a line

정답 : (D)

Example 5

Barron's AP Calculus
p. 286 #3, 4

Motion along a line.

A particle moves along a line with velocity $v = 3t^2 - 6t$.

(A) The distance the particle covers from $t = 0$ to $t = 3$ equals

(B) The net change in the position of the particle is

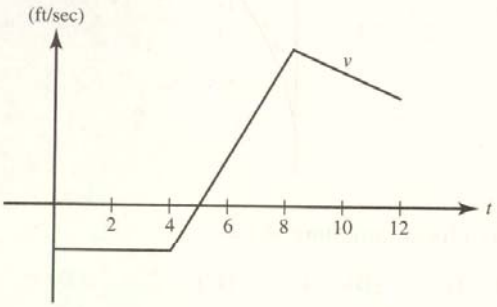
정답 : (A) 8 , (B) 0

Example 6

Barron's AP Calculus
p. 136 #82, 83

Motion along a line.

The graph for Questions 1 and 2 shows the velocity of an object moving along a straight line during the time interval $0 \leq t \leq 12$.



1. For what t does this object attain its maximum acceleration?

(A) $0 < t < 4$ **(B)** $4 < t < 8$ **(C)** $t = 5$ **(D)** $t = 8$ **(E)** $t = 12$

2. The object reverses direction at $t =$

(A) 4 only **(B)** 5 only **(C)** 8 only
(D) 5 and 8 **(E)** none of these

정답 : 1. (B) 2. (B)

Motion along a curve

점이 직선이 아닌 곡선(curve)위를 움직이는 경우, 즉 평면 위를 움직이는 경우는 점의 위치가 x, y 좌표의 순서쌍으로 나타난다는 점만 빼면 직선 위에서 움직이는 경우와 동일하다.

곡선 위를 움직이는 점의 경우 x, y 성분이 구분된다는 점만 빼면 동일하다. 즉,

$$\vec{R}(t)=(x,y) \quad \overset{\text{미분}}{\underset{\text{적분}}{\rightleftarrows}} \quad \vec{v}(t)=\left(\frac{dx}{dt},\frac{dy}{dt}\right) \quad \overset{\text{미분}}{\underset{\text{적분}}{\rightleftarrows}} \quad \vec{a}(t)=\left(\frac{d^2x}{dt^2},\frac{d^2y}{dt^2}\right)$$

이 때, $\vec{R}(t)$, $\vec{v}(t)$, $\vec{a}(t)$ 를 각각 position vector, velocity vector, acceleration vector 라 하며, position, velocity, speed, acceleration 에 대한 해석은 직선 위를 움직이는 점의 경우와 동일하다.

1) slope of $\vec{v}(t)$ 란 시각 t 일 때, 곡선 위의 점 $\vec{R}(t)=(x,y)$ 에서의 접선의 기울기 이다.

$$\text{Slope of } \vec{v}(t)=\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

2) speed 는 magnitude of $\vec{v}(t)$ 로 다음과 같이 정의된다.

$$\text{speed}=\left|\vec{v}(t)\right|=\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}$$

3) direction of acceleration

$$\text{direction of } \vec{a}(t)=\tan^{-1}\frac{\frac{d^2y}{dt^2}}{\frac{d^2x}{dt^2}}$$

4) magnitude of acceleration

$$\text{magnitude of } \vec{a}(t)=\left|\vec{a}(t)\right|=\sqrt{\left(\frac{d^2x}{dt^2}\right)^2+\left(\frac{d^2y}{dt^2}\right)^2}$$

참고) parametrically defined function
Barron's AP Calculus
P.55

참고) parametrically defined function
Barron's AP Calculus
P.185
참고) arc length
Barron's AP Calculus
P. 235

Barron's AP Calculus
p. 115 #28

Motion along a curve.

5) traveled distance

$$\int |\vec{v}(t)|\,dt = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Example 7

A particle moves according to the equations $x = 3\cos t$, $y = 2\sin t$. (a) Find a single equation in x and y for the path of the particle and sketch the curve. (b) Find the velocity and acceleration vectors at any time t, and show that $\mathbf{a} = -\mathbf{R}$ at all times. (c) Find $\mathbf{R}$, $\mathbf{v}$, and $\mathbf{a}$ when (1) $t_1 = \frac{\pi}{6}$, (2) $t_2 = \pi$, and draw them on the sketch. (d) Find the speed of the particle and the magnitude of its acceleration at each instant in (c). (e) When is the speed a maximum? A minimum?

정답 : (a) $\frac{x^2}{9} + \frac{y^2}{4} = 1$

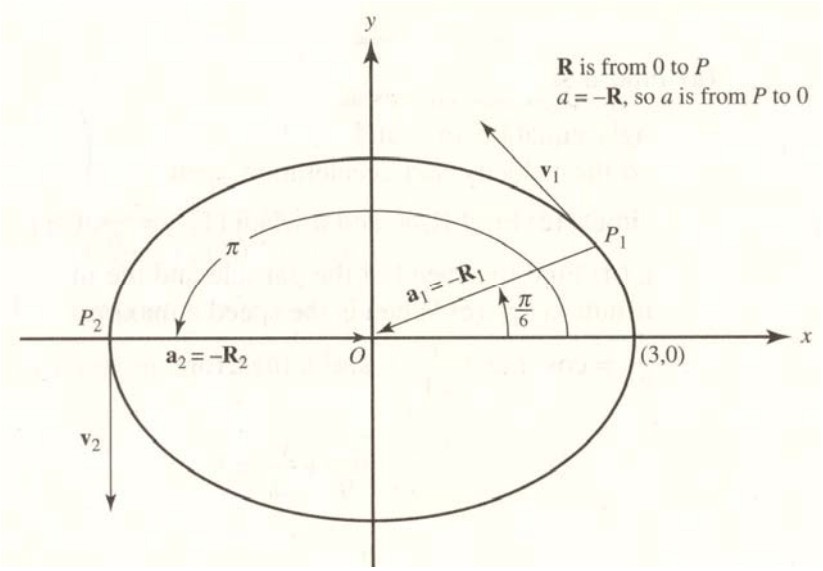
(b) $\vec{R} = 3\cos t\,\vec{i} + 2\sin t\,\vec{j}$, $\vec{v} = -3\sin t\,\vec{i} + 2\cos t\,\vec{j}$, $\vec{a} = -3\cos t\,\vec{i} - 2\sin t\,\vec{j} = -\vec{R}$

(c) (1) $\vec{R}_1 = \frac{3\sqrt{3}}{2}\vec{i} + \vec{j}$, $\vec{v}_1 = -\frac{3}{2}\vec{i} + \sqrt{3}\vec{j}$, $\vec{a}_1 = -\frac{3\sqrt{3}}{2}\vec{i} - \vec{j}$

(2) $\vec{R}_2 = -3\vec{i}$, $\vec{v}_2 = -2\vec{j}$, $\vec{a}_1 = 3\vec{i}$

(d) (1) $|\vec{v}_1| = \frac{\sqrt{21}}{2}$, $|\vec{a}_1| = \frac{\sqrt{31}}{2}$ (2) $|\vec{v}_2| = 2$, $|\vec{a}_2| = 3$

(e) maximum at $t = \frac{\pi}{2}$ or $t = \frac{3\pi}{2}$, minimum at $t = 0$ or $t = \pi$



Example 8

The motion of a particle satisfies the equations

$$\frac{d^2x}{dt^2} = 0, \quad \frac{d^2y}{dt^2} = -g\,.$$

Find parametric equations for the motion if the initial conditions are $x = 0$, $y = 0$, $\frac{dx}{dt} = v_0 \cos\alpha$, and $\frac{dy}{dt} = v_0 \sin\alpha$, where v_0 and α are constants.

Barron's AP Calculus
p. 278 #4

Motion along a curve

정답 : $x(t) = (v_0 \cos\alpha)t$, $y(t) = -\frac{1}{2}gt^2 + (v_0 \sin\alpha)t$

Example 9

A particle $P(x, y)$ moves along a curve so that its acceleration is given by

$$\mathbf{a} = -4\cos 2t\mathbf{i} - 2\sin t\mathbf{j} \quad \left(-\frac{\pi}{2} \leqq t \leqq \frac{\pi}{2}\right);$$

when $t = 0$, the particle is at (1, 0) with $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 2$. (a) Find the position vector $\mathbf{R}$ at any time t. (b) Find a Cartesian equation for the path of the particle, and identify the conic on which P moves.

Barron's AP Calculus
p. 278 #4

Motion along a curve

정답 : (a) $\vec{R} = \cos 2t\vec{\mathrm{i}} + 2\sin t\vec{\mathrm{j}}$ (b) $x = 1 - \frac{y^2}{2}$ $(-1 \le x \le 1,\ -2 \le y \le 2)$

Example 10

If a particle moves along a curve with constant speed, then

(A) the magnitude of its acceleration must equal zero
(B) the direction of acceleration must be constant
(C) the curve along which the particle moves must be a straight line
(D) its velocity and acceleration vectors must be perpendicular
(E) the curve along which the particle moves must be a circle

Barron's AP Calculus
p. 134 #73

Motion along a curve

정답 : (D)

Example 11

Barron's AP Calculus
p. 134 #74

Motion along a curve.

A particle is moving on the curve of $y = 2x - \ln x$ so that $\frac{dx}{dt} = -2$ at all times t. At the point $(1,2)$, $\frac{dy}{dt}$ is

(A) 4 **(B)** 2 **(C)** –4 **(D)** 1 **(E)** –2

정답 : (E)

Example 12

Barron's AP Calculus
p. 128 #27~30

Motion along a curve.

In Questions 1 – 4, $\mathbf{R} = 3\cos\frac{\pi}{3}t\mathbf{i} + 2\sin\frac{\pi}{3}t\mathbf{j}$ is the (position) vector $x\mathbf{i} + y\mathbf{j}$ from the origin to a moving point $P(x, y)$ at time t.

1. A single equation in x and y for the path of the point is

(A) $x^2 + y^2 = 13$ **(B)** $9x^2 + 4y^2 = 36$ **(C)** $2x^2 + 3y^2 = 13$
(D) $4x^2 + 9y^2 = 1$ **(E)** $4x^2 + 9y^2 = 36$

2. When $t = 3$, the speed of the particle is

(A) $\frac{2\pi}{3}$ **(B)** 2 **(C)** 3 **(D)** π **(E)** $\frac{\sqrt{13}}{3}\pi$

3. The magnitude of the acceleration when $t = 3$ is

(A) 2 **(B)** $\frac{\pi^2}{3}$ **(C)** 3 **(D)** $\frac{2\pi^2}{9}$ **(E)** π

4. At the point where $t = \frac{1}{2}$, the slope of the curve along which the particle moves is

(A) $-\frac{2\sqrt{3}}{9}$ **(B)** $-\frac{\sqrt{3}}{2}$ **(C)** $\frac{2}{\sqrt{3}}$

(D) $-\frac{2\sqrt{3}}{3}$ **(E)** none of these

정답 : 1. (E) , 2. (A), 3. (B), 4. (D)

Example 13

Barron's AP Calculus
p. 287 #8~10

Motion along a curve

For Questions 8–10 use the following information: The velocity **v** of a particle moving on a curve is given, at time t, by $\mathbf{v} = t\mathbf{i} - (1-t)\mathbf{j}$. When $t = 0$, the particle is at point (0,1).

*8. At time t the position vector **R** is

(A) $\frac{t^2}{2}\mathbf{i} - \frac{(1-t^2)}{2}\mathbf{j}$ (B) $\frac{t^2}{2}\mathbf{i} + \frac{(1-t)^2}{2}\mathbf{j}$

(C) $\frac{t^2}{2}\mathbf{i} + \frac{t^2-2t}{2}\mathbf{j}$ (D) $\frac{t^2}{2}\mathbf{i} + \frac{t^2-2t+2}{2}\mathbf{j}$

(E) $\frac{t^2}{2}\mathbf{i} + (1-t)^2\mathbf{j}$

*9. The acceleration vector at time $t = 2$ is

(A) $\mathbf{i} + \mathbf{j}$ (B) $\mathbf{i} - \mathbf{j}$ (C) $\mathbf{i} + 2\mathbf{j}$ (D) $2\mathbf{i} - \mathbf{j}$ (E) none of these

*10. The speed of the particle is at a minimum when t equals

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 1.5 (E) 2

정답 : 8. (D), 9. (A), 10. (B)

Example 14

Barron's AP Calculus
p. 287 #11

Motion along a curve

A particle moves along a curve in such a way that its position vector and velocity vector are perpendicular at all times. If the particle passes through the point (4, 3), then the equation of the curve is

(A) $x^2 + y^2 = 5$ (B) $x^2 + y^2 = 25$ (C) $x^2 + 2y^2 = 34$

(D) $x^2 - y^2 = 7$ (E) $2x^2 - y^2 = 23$

정답 : (C)

Example 15

Barron's AP Calculus
p. 287 #13

Motion along a curve

The velocity of a particle is given by $(3 - e^{-t})\mathbf{i} + (e^t - 1)\mathbf{j}$. When $t = 0$, the particle is at (1,2). The position vector **R** at time t is

(A) $(3t + e^{-t})\mathbf{i} + (e^t - t)\mathbf{j}$ (B) $(3t + e^{-t} + 1)\mathbf{i} + (e - t + 1)\mathbf{j}$

(C) $(3t + e^{-t})\mathbf{i} + (e^t - t + 1)\mathbf{j}$ (D) $(3t + e^{-t} + 1)\mathbf{i} + (e^t - t + 2)\mathbf{j}$

(E) none of these

정답 : (B)

2005 AP Calculus AB FREE-RESPONSE QUESTIONS (Form B)

A graphing calculator is required for some problems or parts of problems

3. A particle moves along the x-axis so that its velocity v at time t, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

   (a) Find the acceleration of the particle at time $t = 4$.

   (b) Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

   (c) Find the position of the particle at time $t = 2$.

   (d) Find the average speed of the particle over the interval $0 \le t \le 2$.

2004 AP Calculus AB FREE-RESPONSE QUESTIONS

A graphing calculator is required for some problems or parts of problems

3. A particle moves along the y-axis so that its velocity v at time $t \ge 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$. At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

   (a) Find the acceleration of the particle at time $t = 2$.

   (b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

   (c) Find the time $t \ge 0$ at which the particle reaches its highest point. Justify your answer.

   (d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.

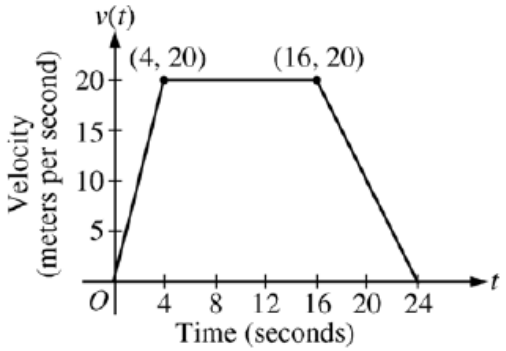
2003 AP Calculus AB FREE-RESPONSE QUESTIONS (Form B)

No Calculator is allowed for these problems.

4. A particle moves along the x-axis with velocity at time $t \geq 0$ given by $v(t) = -1 + e^{1-t}$.
   (a) Find the acceleration of the particle at time $t = 3$.
   (b) Is the speed of the particle increasing at time $t = 3$? Give a reason for your answer.
   (c) Find all values of t at which the particle changes direction. Justify your answer.
   (d) Find the total distance traveled by the particle over the time interval $0 \leq t \leq 3$.

2005 AP Calculus BC FREE-RESPONSE QUESTIONS

No Calculator is allowed for these problems.



5. A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.
   (a) Find $\int_0^{24} v(t)\,dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)\,dt$.
   (b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.
   (c) Let $a(t)$ be the car's acceleration at time t, in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.
   (d) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c, for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?

News letter 를 받고도 풀어보는지 안 풀어 보는지 심히 걱정이 됩니다.

다시 강조하지만 문제를 풀어보지 않고도 시험을 잘 볼 수는 없습니다.

꼭 좀 풀어보시길...

2008 년 새해 복 많이 받고 무사히 고 3 시절 보내기를 기원합니다.

We're on the Web!
See us at:
http://apcalculus.tistory.com

2008 Course Calendar

January	Talk to your AP teachers and/or AP Coordinator about taking the exams. Contact the disabilities (SSD) coordinator at your school if you will need testing accommodations.
February 22	Deadline for submitting complete disability documentation for students with disabilities whose SSD Eligibility Forms require Documentation Review.
March 1	Deadline for homeschooled students and students whose schools do not offer AP to contact AP Services for a list of local AP Coordinators at whose schools they could arrange to test.
March 7	Deadline for submitting complete student Eligibility Forms for students with disabilities using the School Verification Process.
March 15	Deadline for homeschooled students and students whose schools do not offer AP to contact AP Coordinators identified by AP Services.
May 5-9 and May 12-16	Exam dates (AP Calculus – May, 7)

AP Calculus 뉴스레터에 대해…

- 본 뉴스레터는 AP Calculus 수강생의 개념 재확인 및 기출문제 풀이를 위해 제작되었습니다.
- 기출문제 정답 및 풀이는 1 월 7 일 발송됩니다.
  (답이 미리 필요한 분은 메일이나 전화로 연락바랍니다.)
- 내용 및 문제에 대한 질문은 email 및 블로그를 이용해 주시기 바랍니다.

EGO Expert Group

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