Logic and Computer Design Fundamentals

# Chapter 1 – Digital Systems and Information

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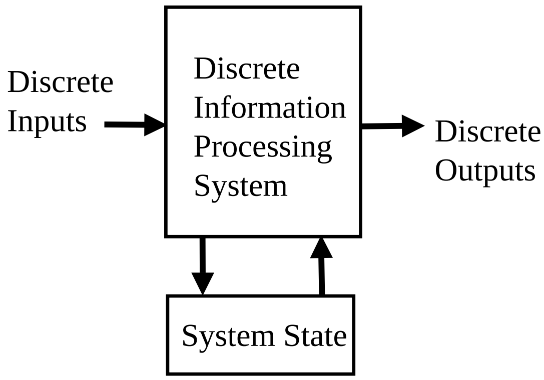
(Hyperlinks are active in View Show mode)

## Overview

- **Digital Systems, Computers, and Beyond**
- **Information Representation**
- **Number Systems** [binary, octal and hexadecimal]
- **Arithmetic Operations**
- **Base Conversion**
- **Decimal Codes** [BCD (binary coded decimal)]
- **Alphanumeric Codes**
- **Parity Bit**
- **Gray Codes**

## DIGITAL & COMPUTER SYSTEMS - Digital System

- **Takes a set of discrete information <u>inputs</u> and discrete internal information <u>(system state)</u> and generates a set of discrete information <u>outputs</u>.**








## Types of Digital Systems

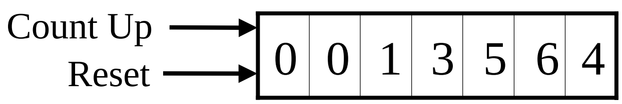
- **No state present**
  - **Combinational Logic System**
  - **Output = Function(Input)**
- **State present**
  - **State updated at discrete times**

    **=> Synchronous Sequential System**
  - **State updated at any time**

    **=>Asynchronous Sequential System**
  - **State = Function (State, Input)**
  - **Output = Function (State) or Function (State, Input)**

## Digital System Example:

**A Digital Counter (e. g., odometer):**



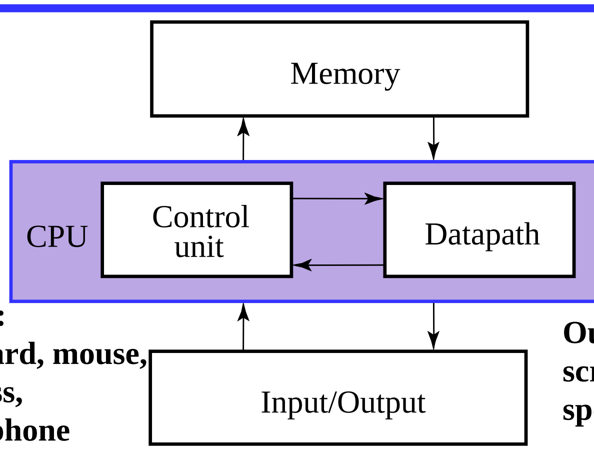

**Inputs:** **Count Up, Reset**
**Outputs:** **Visual Display**
**State:** **"Value" of stored digits**

**Synchronous or Asynchronous?**





## Digital Computer Example




**Inputs: keyboard, mouse, wireless, microphone**

**Outputs: LCD screen, wireless, speakers**

**Synchronous or Asynchronous?**

## And Beyond – Embedded Systems

- Computers as integral parts of other products
- Examples of embedded computers
  - Microcomputers
  - Microcontrollers
  - Digital signal processors



## Embedded Systems

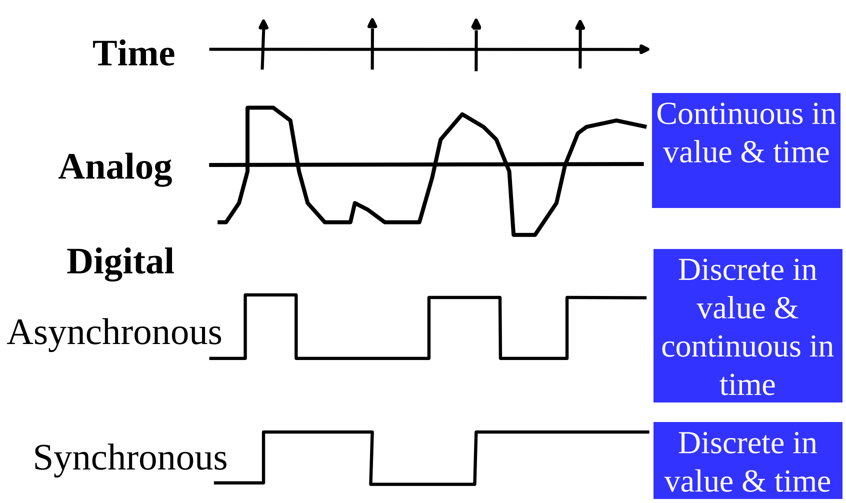
- Examples of Embedded Systems Applications
  - Cell phones
  - Automobiles
  - Video games
  - Copiers
  - Dishwashers
  - Flat Panel TVs
  - Global Positioning Systems

# INFORMATION REPRESENTATION - Signals

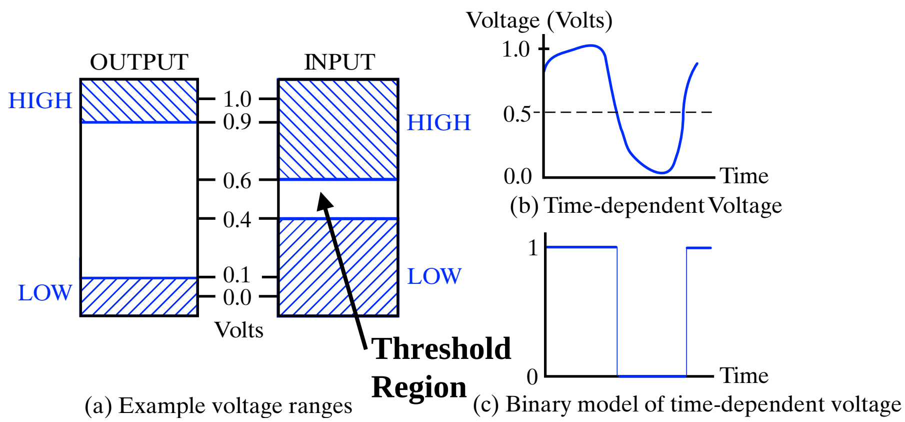
- Information variables represented by physical quantities.
- For digital systems, the variables take on discrete values.
- Two level, or binary values are the most prevalent values in digital systems.
- Binary values are represented abstractly by:
  - digits 0 and 1
  - words (symbols) False (F) and True (T)
  - words (symbols) Low (L) and High (H)
  - and words On and Off.
- Binary values are represented by values or ranges of values of physical quantities



# Signal Examples Over Time



## Signal Example – Physical Quantity: Voltage








## Binary Values: Other Physical Quantities

- **What are other physical quantities represent 0 and 1?**
  - **CPU** **Voltage**
  - **Disk** **Magnetic Field Direction**
  - **CD** **Surface Pits/Light**
  - **Dynamic RAM** **Electrical Charge**

## Number Systems – Representation

- Positive radix, positional number systems
- A number with *radix* $\boldsymbol{r}$ is represented by a string of digits:

  $$A_{n-1}A_{n-2}\ \ldots\ A_1A_0\ .\ A_{-1}\,A_{-2}\ \ldots\ A_{-m+1}\,A_{-m}$$

  in which $\mathbf{0} \le A_i < \boldsymbol{r}$ and **.** is the *radix point*.
- The string of digits represents the power series:

$$(\text{Number})_r = \left(\sum_{i=0}^{i=n-1} A_i \cdot r^i\right) + \left(\sum_{j=-m}^{j=-1} A_j \cdot r^j\right)$$

**(Integer Portion) + (Fraction Portion)**





## Number Systems – Examples

| | | General | Decimal | Binary |
|---|---|---|---|---|
| Radix (Base) | | r | 10 | 2 |
| Digits | | 0 => r - 1 | 0 => 9 | 0 => 1 |
| Powers of Radix | 0 | $r^0$ | 1 | 1 |
| | 1 | $r^1$ | 10 | 2 |
| | 2 | $r^2$ | 100 | 4 |
| | 3 | $r^3$ | 1000 | 8 |
| | 4 | $r^4$ | 10,000 | 16 |
| | 5 | $r^5$ | 100,000 | 32 |
| | -1 | $r^{-1}$ | 0.1 | 0.5 |
| | -2 | $r^{-2}$ | 0.01 | 0.25 |
| | -3 | $r^{-3}$ | 0.001 | 0.125 |
| | -4 | $r^{-4}$ | 0.0001 | 0.0625 |
| | -5 | $r^{-5}$ | 0.00001 | 0.03125 |

## Special Powers of 2

- $2^{10}$ (1024) is Kilo, denoted "K"
- $2^{20}$ (1,048,576) is Mega, denoted "M"
- $2^{30}$ (1,073, 741,824)is Giga, denoted "G"
- $2^{40}$ (1,099,511,627,776 ) is Tera, denoted “T"





## ARITHMETIC OPERATIONS - Binary Arithmetic

- Single Bit Addition with Carry
- Multiple Bit Addition
- Single Bit Subtraction with Borrow
- Multiple Bit Subtraction
- Multiplication
- BCD Addition

## Single Bit Binary Addition with Carry

**Given two binary digits (X,Y), a carry in (Z) we get the following sum (S) and carry (C):**

**Carry in (Z) of 0:**

| | | | | |
|---|---|---|---|---|
| Z | 0 | 0 | 0 | 0 |
| X | 0 | 0 | 1 | 1 |
| + Y | + 0 | + 1 | + 0 | + 1 |
| C S | 0 0 | 0 1 | 0 1 | 1 0 |

**Carry in (Z) of 1:**

| | | | | |
|---|---|---|---|---|
| Z | 1 | 1 | 1 | 1 |
| X | 0 | 0 | 1 | 1 |
| + Y | + 0 | + 1 | + 0 | + 1 |
| C S | 0 1 | 1 0 | 1 0 | 1 1 |





## Multiple Bit Binary Addition

- **Extending this to two multiple bit examples:**

| | | |
|---|---|---|
| **Carries** | **0** | **0** |
| **Augend** | **01100** | **10110** |
| **Addend** | **+10001** | **+10111** |
| **Sum** | | |

- **Note: The 0 is the default Carry-In to the least significant bit.**

## Single Bit Binary Subtraction with Borrow

- **Given two binary digits (X,Y), a borrow in (Z) we get the following difference (S) and borrow (B):**
- **Borrow in (Z) of 0:**

| | | | | |
|---|---|---|---|---|
| **Z** | **0** | **0** | **0** | **0** |
| **X** | **0** | **0** | **1** | **1** |
| **- Y** | **-0** | **-1** | **-0** | **-1** |
| **BS** | **0 0** | **1 1** | **0 1** | **0 0** |

- **Borrow in (Z) of 1:**

| | | | | |
|---|---|---|---|---|
| **Z** | **1** | **1** | **1** | **1** |
| **X** | **0** | **0** | **1** | **1** |
| **- Y** | **-0** | **-1** | **-0** | **-1** |
| **BS** | **11** | **1 0** | **0 0** | **1 1** |




## Multiple Bit Binary Subtraction

- **Extending this to two multiple bit examples:**

| | | |
|---|---|---|
| **Borrows** | **0** | **0** |
| **Minuend** | **10110** | **10110** |
| **Subtrahend** | **- 10010** | **- 10011** |
| **Difference** | | |

- **Notes: The 0 is a Borrow-In to the least significant bit. If the Subtrahend > the Minuend, interchange and append a – to the result.**

## Binary Multiplication

**The binary multiplication table is simple:**

**$0 * 0 = 0$ | $1 * 0 = 0$ | $0 * 1 = 0$ | $1 * 1 = 1$**

**Extending multiplication to <u>multiple digits</u>:**

| | |
|---|---|
| **Multiplicand** | **1011** |
| **Multiplier** | **x 101** |
| **Partial Products** | **1011** |
| | **0000 -** |
| | **1011 - -** |
| **Product** | **110111** |





## BASE CONVERSION - Positive Powers of 2

- Useful for Base Conversion

| Exponent | Value |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |
| 6 | 64 |
| 7 | 128 |
| 8 | 256 |
| 9 | 512 |
| 10 | 1024 |

| Exponent | Value |
|---|---|
| 11 | 2,048 |
| 12 | 4,096 |
| 13 | 8,192 |
| 14 | 16,384 |
| 15 | 32,768 |
| 16 | 65,536 |
| 17 | 131,072 |
| 18 | 262,144 |
| 19 | 524,288 |
| 20 | 1,048,576 |
| 21 | 2,097,152 |

## Converting Binary to Decimal

- **To convert to decimal, use decimal arithmetic to form Σ (digit × respective power of 2).**
- **Example:Convert $11010_2$ to $N_{10}$:**





## Converting Decimal to Binary

- **Method 1**
  - **Subtract the largest power of 2 (see slide 14) that gives a positive remainder and record the power.**
  - **Repeat, subtracting from the prior remainder and recording the power, until the remainder is zero.**
  - **Place 1's in the positions in the binary result corresponding to the powers recorded; in all other positions place 0's.**
- **Example: Convert $625_{10}$ to $N_2$**

## Commonly Occurring Bases

| Name | Radix | Digits |
|---|---|---|
| Binary | 2 | 0,1 |
| Octal | 8 | 0,1,2,3,4,5,6,7 |
| Decimal | 10 | 0,1,2,3,4,5,6,7,8,9 |
| Hexadecimal | 16 | 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F |

- The six letters (in addition to the 10 integers) in hexadecimal represent:





## Numbers in Different Bases

- Good idea to memorize!

| Decimal (Base 10) | Binary (Base 2) | Octal (Base 8) | Hexadecimal (Base 16) |
|---|---|---|---|
| 00 | 00000 | 00 | 00 |
| 01 | 00001 | 01 | 01 |
| 02 | 00010 | 02 | 02 |
| 03 | 00011 | 03 | 03 |
| 04 | 00100 | 04 | 04 |
| 05 | 00101 | 05 | 05 |
| 06 | 00110 | 06 | 06 |
| 07 | 00111 | 07 | 07 |
| 08 | 01000 | 10 | 08 |
| 09 | 01001 | 11 | 09 |
| 10 | 01010 | 12 | 0A |
| 11 | 01011 | 13 | 0B |
| 12 | 01100 | 14 | 0C |
| 13 | 01101 | 15 | 0D |
| 14 | 01110 | 16 | 0E |
| 15 | 01111 | 17 | 0F |
| 16 | 10000 | 20 | 10 |

## Conversion Between Bases

- **Method 2**
- **To convert from one base to another:**

  **1) Convert the Integer Part**

  **2) Convert the Fraction Part**

  **3) Join the two results with a radix point**





## Conversion Details

- **To Convert the Integral Part:**

  **Repeatedly divide the number by the new radix and save the remainders. The digits for the new radix are the remainders in *reverse order* of their computation. If the new radix is > 10, then convert all remainders > 10 to digits A, B, …**

- **To Convert the Fractional Part:**

  **Repeatedly multiply the fraction by the new radix and save the integer digits that result. The digits for the new radix are the integer digits in *order* of their computation. If the new radix is > 10, then convert all integers > 10 to digits A, B, …**

## Example: Convert $46.6875_{10}$ To Base 2

- **Convert 46 to Base 2**

- **Convert 0.6875 to Base 2:**

- **Join the results together with the radix point:**





## Additional Issue - Fractional Part

- **Note that in this conversion, the fractional part can become 0 as a result of the repeated multiplications.**
- **In general, it may take many bits to get this to happen or it may never happen.**
- **Example Problem: Convert $0.65_{10}$ to $N_2$**
  - **0.65 = 0.1010011001001 …**
  - **The fractional part begins repeating every 4 steps yielding repeating 1001 forever!**
- **Solution: Specify number of bits to right of radix point and round or truncate to this number.**

## Checking the Conversion

- To convert back, sum the digits times their respective powers of r.
- From the prior conversion of $46.6875_{10}$

$101110_2 = 1 \cdot 32 + 0 \cdot 16 + 1 \cdot 8 + 1 \cdot 4 + 1 \cdot 2 + 0 \cdot 1$
$= 32 + 8 + 4 + 2$
$= 46$

$0.1011_2 = 1/2 + 1/8 + 1/16$
$= 0.5000 + 0.1250 + 0.0625$
$= 0.6875$





## Why Do Repeated Division and Multiplication Work?

- Divide the integer portion of the power series on slide 11 by radix r. The remainder of this division is $A_0$, represented by the term $A_0/r$.
- Discard the remainder and repeat, obtaining remainders $A_1$, …
- Multiply the fractional portion of the power series on slide 11 by radix r. The integer part of the product is $A_{-1}$.
- Discard the integer part and repeat, obtaining integer parts $A_{-2}$, …
- This demonstrates the algorithm for any radix r >1.

## Octal (Hexadecimal) to Binary and Back

- **Octal (Hexadecimal) to Binary:**
  - **Restate the octal (hexadecimal) as three (four) binary digits starting at the radix point and going both ways.**
- **Binary to Octal (Hexadecimal):**
  - **Group the binary digits into three (four) bit groups starting at the radix point and going both ways, padding with zeros as needed in the fractional part.**
  - **Convert each group of three bits to an octal (hexadecimal) digit.**





## Octal to Hexadecimal via Binary

- **Convert octal to binary.**
- **Use groups of <u>four bits</u> and convert as above to hexadecimal digits.**
- **Example: Octal to Binary to Hexadecimal**

$$6\ \ 3\ \ 5\ .\ 1\ \ 7\ \ 7_8$$

- **Why do these conversions work?**

## A Final Conversion Note

- You can use arithmetic in other bases if you are careful:
- Example: Convert $101110_2$ to Base 10 using binary arithmetic:

Step 1 101110 / 1010 = 100 r 0110

Step 2 100 / 1010 = 0 r 0100

Converted Digits are $0100_2$ | $0110_2$

or 4 $6_{10}$





## Binary Numbers and Binary Coding

- Flexibility of representation
  - Within constraints below, can assign any binary combination (called a code word) to any data as long as data is uniquely encoded.
- Information Types
  - Numeric
    - Must represent range of data needed
    - Very desirable to represent data such that simple, straightforward computation for common arithmetic operations permitted
    - Tight relation to binary numbers
  - Non-numeric
    - Greater flexibility since arithmetic operations not applied.
    - Not tied to binary numbers

## Non-numeric Binary Codes

- Given $n$ binary digits (called bits), a binary code is a mapping from a set of represented elements to a subset of the $2^n$ binary numbers.
- Example: A binary code for the seven colors of the rainbow
- Code 100 is not used

| Color | Binary Number |
|---|---|
| Red | 000 |
| Orange | 001 |
| Yellow | 010 |
| Green | 011 |
| Blue | 101 |
| Indigo | 110 |
| Violet | 111 |





## Number of Bits Required

- Given M elements to be represented by a binary code, the minimum number of bits, $n$, needed, satisfies the following relationships:

  $2^n \geq M > 2^{(n-1)}$

  $n = \lceil \log_2 M \rceil$ where $\lceil x \rceil$, called the *ceiling function*, is the integer greater than or equal to $x$.
- Example: How many bits are required to represent decimal digits with a binary code?

## Number of Elements Represented

- Given $n$ digits in radix $r$, there are $r^n$ distinct elements that can be represented.
- But, you can represent m elements, m < $r^n$
- Examples:
  - You can represent 4 elements in radix $r = 2$ with $n = 2$ digits: (00, 01, 10, 11).
  - You can represent 4 elements in radix $r = 2$ with $n = 4$ digits: (0001, 0010, 0100, 1000).
  - This second code is called a "one hot" code.





## DECIMAL CODES - Binary Codes for Decimal Digits

- There are over 8,000 ways that you can chose 10 elements from the 16 binary numbers of 4 bits. A few are useful:

| Decimal | 8,4,2,1 | Excess3 | 8,4,-2,-1 | Gray |
|---|---|---|---|---|
| 0 | 0000 | 0011 | 0000 | 0000 |
| 1 | 0001 | 0100 | 0111 | 0100 |
| 2 | 0010 | 0101 | 0110 | 0101 |
| 3 | 0011 | 0110 | 0101 | 0111 |
| 4 | 0100 | 0111 | 0100 | 0110 |
| 5 | 0101 | 1000 | 1011 | 0010 |
| 6 | 0110 | 1001 | 1010 | 0011 |
| 7 | 0111 | 1010 | 1001 | 0001 |
| 8 | 1000 | 1011 | 1000 | 1001 |
| 9 | 1001 | 1100 | 1111 | 1000 |

## Binary Coded Decimal (BCD)

- **The BCD code is the 8,4,2,1 code.**
- **8, 4, 2, and 1 are weights**
- **BCD is a *weighted* code**
- **This code is the simplest, most intuitive binary code for decimal digits and uses the same powers of 2 as a binary number, but only encodes the first ten values from 0 to 9.**
- **Example: 1001 (9) = 1000 (8) + 0001 (1)**
- **How many "invalid" code words are there?**
- **What are the "invalid" code words?**



## Excess 3 Code and 8, 4, –2, –1 Code

| Decimal | Excess 3 | 8, 4, –2, –1 |
|---|---|---|
| 0 | 0011 | 0000 |
| 1 | 0100 | 0111 |
| 2 | 0101 | 0110 |
| 3 | 0110 | 0101 |
| 4 | 0111 | 0100 |
| 5 | 1000 | 1011 |
| 6 | 1001 | 1010 |
| 7 | 1010 | 1001 |
| 8 | 1011 | 1000 |
| 9 | 1100 | 1111 |

- **What interesting property is common to these two codes?**

## Warning: Conversion or Coding?

- Do <u>NOT</u> mix up <u>conversion</u> of a decimal number to a binary number with <u>coding</u> a decimal number with a BINARY CODE.
- $13_{10} = 1101_2$ (This is <u>conversion</u>)
- 13 ⇔ 0001|0011 (This is <u>coding</u>)





## BCD Arithmetic

- Given a BCD code, we use binary arithmetic to add the digits:

| | | |
|---|---|---|
| 8 | 1000 | Eight |
| <u>+5</u> | <u>+0101</u> | Plus 5 |
| 13 | 1101 | is 13 (> 9) |

- Note that the result is MORE THAN 9, so must be represented by two digits!
- To correct the digit, subtract 10 by adding 6 modulo 16.

| | | |
|---|---|---|
| 8 | 1000 | Eight |
| <u>+5</u> | <u>+0101</u> | Plus 5 |
| 13 | 1101 | is 13 (> 9) |
| | <u>+0110</u> | so add 6 |
| | carry = 1 0011 | leaving 3 + cy |
| | 0001 \| 0011 | Final answer (two digits) |

- If the digit sum is > 9, add one to the next significant digit

## BCD Addition Example

- **Add $2905_{BCD}$ to $1897_{BCD}$ showing carries and digit corrections.**

$$
\begin{array}{rcccc}
 & & & & 0 \\
 & 0001 & 1000 & 1001 & 0111 \\
+ & \underline{0010} & \underline{1001} & \underline{0000} & \underline{0101} \\
 & & & & \\
 & \underline{\quad\quad} & \underline{\quad\quad} & \underline{\quad\quad} & \underline{\quad\quad}
\end{array}
$$





## ALPHANUMERIC CODES - ASCII Character Codes

- **American Standard Code for Information Interchange (Refer to Table 1-4 in the text)**
- **This code is a popular code used to represent information sent as character-based data. It uses 7-bits to represent:**
  - **94 Graphic printing characters.**
  - **34 Non-printing characters**
- **Some non-printing characters are used for text format (e.g. BS = Backspace, CR = carriage return)**
- **Other non-printing characters are used for record marking and flow control (e.g. STX and ETX start and end text areas).**

## ASCII Properties

**ASCII has some interesting properties:**

- **Digits 0 to 9 span Hexadecimal values $30_{16}$ to $39_{16}$.**
- **Upper case A-Z span $41_{16}$ to $5A_{16}$ .**
- **Lower case a-z span $61_{16}$ to $7A_{16}$.**
  - **Lower to upper case translation (and vice versa) occurs by flipping bit 6.**
- **Delete (DEL) is all bits set, a carryover from when punched paper tape was used to store messages.**
- **Punching all holes in a row erased a mistake!**



## PARITY BIT Error-Detection Codes

- **Redundancy (e.g. extra information), in the form of extra bits, can be incorporated into binary code words to detect and correct errors.**
- **A simple form of redundancy is parity, an extra bit appended onto the code word to make the number of 1's odd or even. Parity can detect all single-bit errors and some multiple-bit errors.**
- **A code word has even parity if the number of 1's in the code word is even.**
- **A code word has odd parity if the number of 1's in the code word is odd.**

## 4-Bit Parity Code Example

- Fill in the even and odd parity bits:

| Even Parity Message - Parity | Odd Parity Message - Parity |
|---|---|
| 000 - | 000 - |
| 001 - | 001 - |
| 010 - | 010 - |
| 011 - | 011 - |
| 100 - | 100 - |
| 101 - | 101 - |
| 110 - | 110 - |
| 111 - | 111 - |

- The codeword "1111" has <u>even parity</u> and the codeword "1110" has <u>odd parity</u>. Both can be used to represent 3-bit data.





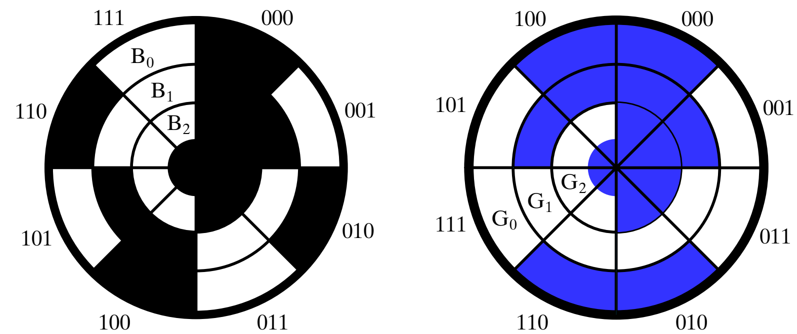
## GRAY CODE – Decimal

| Decimal | 8,4,2,1 | Gray |
|---|---|---|
| 0 | 0000 | 0000 |
| 1 | 0001 | 0100 |
| 2 | 0010 | 0101 |
| 3 | 0011 | 0111 |
| 4 | 0100 | 0110 |
| 5 | 0101 | 0010 |
| 6 | 0110 | 0011 |
| 7 | 0111 | 0001 |
| 8 | 1000 | 1001 |
| 9 | 1001 | 1000 |

- What special property does the Gray code have in relation to adjacent decimal digits?

## Optical Shaft Encoder

- **Does this special Gray code property have any value?**
- **An Example: Optical Shaft Encoder**




(a) Binary Code for Positions 0 through 7 (b) Gray Code for Positions 0 through 7



## Shaft Encoder (Continued)

- **How does the shaft encoder work?**
- **For the binary code, what codes may be produced if the shaft position lies between codes for 3 and 4 (011 and 100)?**
- **Is this a problem?**

## Shaft Encoder (Continued)

- For the Gray code, what codes may be produced if the shaft position lies between codes for 3 and 4 (010 and 110)?
- Is this a problem?
- Does the Gray code function correctly for these borderline shaft positions for all cases encountered in octal counting?



## UNICODE

- UNICODE extends ASCII to 65,536 universal characters codes
  - For encoding characters in world languages
  - Available in many modern applications
  - 2 byte (16-bit) code words
  - See Reading Supplement – Unicode on the Companion Website http://www.prenhall.com/mano

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