

Please be reminded of the following rules:

Required Information. The front page must include your name, student number, your tutorial code (which will be assigned to you when tutorial rooms are announced), and the name of your teaching assistant. *Failure to put your name and/or your student number will result in a zero in your assignment. Failure to put the name of your TA or your tutorial code will result in a 20% reduction of your assignment mark.* A cover page is not required as long as the necessary information is on the top of the first page.

Paper Size and Requirements. Assignments must be submitted on letter-sized (8.5×11 inch) paper. *Using ripped notebook paper is unacceptable and will result in a zero in your assignment mark.* Assignments that are more than one page in length must be stapled in the top left corner. *Failure to staple such assignments will result in a 20% reduction of your assignment mark.* Do not use clear plastic binders.

Submitting your assignment. You must hand your assignment to your instructor before the beginning of lecture, or deposit the assignment into the MAT 137Y Assignment Box located inside SS 1071. *The penalty for late assignments is zero for the assignment, regardless of the excuse. Assignments handed in after 6:10 p.m. on Thursday will not be accepted for any reason, even if it is one minute late!*

Policy on Plagiarism on Assignments. It is very helpful to have other students with whom to study, and we encourage you to work together. However, **it is extremely important that problem set solutions be written up independently, otherwise this constitutes plagiarism! Don't copy other people's work, and don't let others copy your work!** The teaching assistants will enforce this rule very strictly, and will apply severe penalties to any one in violation. In particular, the Department of Mathematics reminds all students that plagiarism, cheating, and all forms of academic misconduct will not be tolerated. Students in violation of the *Code of Student Conduct* will be dealt with severely by the Department of Mathematics and the Faculty of Arts & Science.

Supplementary Problems. "SHE" refers to the textbook by Salas, Hille, and Etgen (10th Edition)

1. SHE 11.1: 1, 3, 5, 7, 13, 21, 23.

Required Problems. Hand in solutions to all the problems below.

1. Find the maximum value, minimum value, least upper bound, and greatest lower bound of the following sets, if they exist. Provide justification for all your answers. (For example, you need to prove why 1 is the least upper bound of the set $\{x : x \in (0, 1)\}$.)
 - (i) $\{x : x \in (0, 1)\}$.
 - (ii) $\left\{\frac{1}{3^n} : n = 0, 1, 2, \dots\right\}$.
 - (iii) $\left\{\frac{(-1)^n}{n} : n = 1, 2, 3, \dots\right\}$.
 - (iv) $\left\{x \in [0, 2\pi] : |\sin x| < \frac{1}{2}\right\}$.
 - (v) $\{x \in \mathbb{Q} : -\sqrt{2} < x < 2\}$.
 - (vi) $\{x \notin \mathbb{Q} : -\sqrt{2} < x \leq 2\}$.
2. SHE 11.1: 22, 24, 26.

3. A function is *decreasing* if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ and $x_1, x_2 \in \mathbb{R}$.
- (a) Show that the set $\{f(x) : x < a\}$ is bounded below.
 - (b) Prove that $\lim_{x \rightarrow a^-} f(x) = \text{glb}\{f(x) : x < a\}$.
(Hint: Show that for any $\varepsilon > 0$, there exists $\delta > 0$ such that $f(a - \delta) < c + \varepsilon$.)
4. Suppose A and B are bounded sets of real numbers.
- (a) Prove that $A \cup B$ and $A \cap B$ are bounded above.
 - (b) Prove that $\text{lub}(A \cup B) = \max(\text{lub } A, \text{lub } B)$.
 - (c) Prove that $\text{lub}(A \cap B) \leq \min(\text{lub } A, \text{lub } B)$ if $A \cap B \neq \emptyset$.
 - (d) Show that the inequality in part (c) is not equality in general.

Note: This is the last assignment that is due before the first term test. The first term test covers all material in the course outline up to and including the material on least upper bounds and the Intermediate Value Theorem. More information about the term test will be available on the course website.