Introduction to Smith Charts

Dr. Russell P. Jedlicka
Klipsch School of Electrical and Computer Engineering
New Mexico State University
Las Cruces, NM 88003

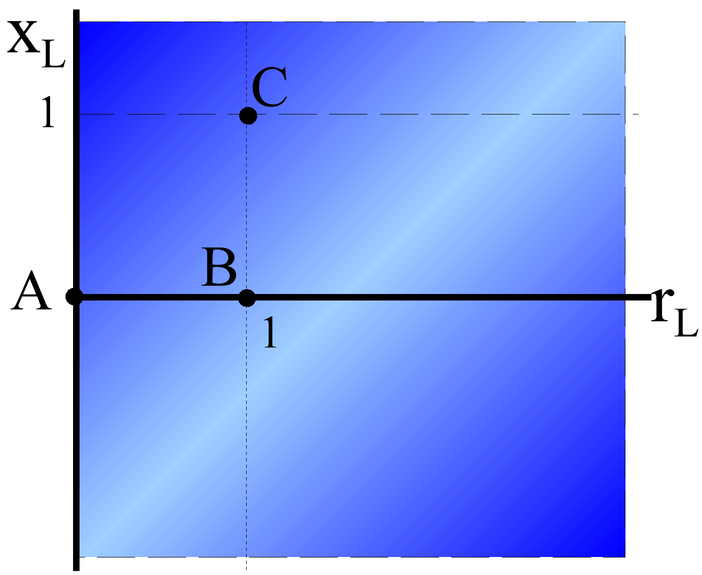
September 2002

Smith Chart Summary

A Smith Chart is a conformal mapping between the **normalized** complex impedance plane (z = r + j x) and the complex reflection coefficient plane. First, the normalized impedance is given as

$$z_L = \frac{Z_L}{Z_o} = \frac{R_L + jX_L}{Z_o}$$

Consider the right-hand portion of the normalized complex impedance plane. All values of impedance such that R ≥ 0 are represented by points in the plane. The impedance of all ***passive*** devices will be represented by points in the right-half plane.



The complex
reflection coefficient may be written as a magnitude and a phase or as real and imaginary parts.

$$\Gamma_L = |\Gamma_L| e^{\angle \Gamma_L} = \Gamma_{L_r} + j\Gamma_{L_i}$$

Remember that $0 \le |\Gamma_L| \le 1$ and the $\angle \Gamma_L$ is measured with respect to the positive real Γ_r axis. The reflection coefficient in terms of the load, Z_L, terminating a line, Z_o is defined as

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

Rearranging the above equation we get

$$Z_L = Z_o \frac{1+\Gamma_L}{1-\Gamma_L}$$

So we get the conformal mapping by dividing through by Z_o (remember Γ_L is complex).

$$z_L = r_L + j x_L = \frac{1+\Gamma_L}{1-\Gamma_L}$$

Substituting in the complex expression for Γ_L and equating real and imaginary parts we find the two equations which represent circles in the complex reflection coefficient plane.

$$\left(\Gamma_{L_r} - \frac{r_L}{1+r_L}\right)^2 + \left(\Gamma_{L_i} - 0\right)^2 = \left(\frac{1}{1+r_L}\right)^2$$

$$\left(\Gamma_{L_r} - 1\right)^2 + \left(\Gamma_{L_i} - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

The first circle is centered at

$$\left(\frac{r_L}{1+r_L}, 0\right)$$

whose location is always inside the unit circle in the complex reflection coefficient plane. The corresponding radius is

$$\frac{1}{1+r_L}$$

So it is observed that this circle will always be fully contained within the unit circle. The radius can never by greater than unity. The second circle is centered at

$$\left(1, \frac{1}{x_L}\right)$$

whose location is always outside the unit circle in the complex reflection coefficient plane. Note that the center of these circles will always be to the right of the unit circle. The corresponding radius is $\left|\frac{1}{x_L}\right|$. The radius can vary between 0 and infinity.

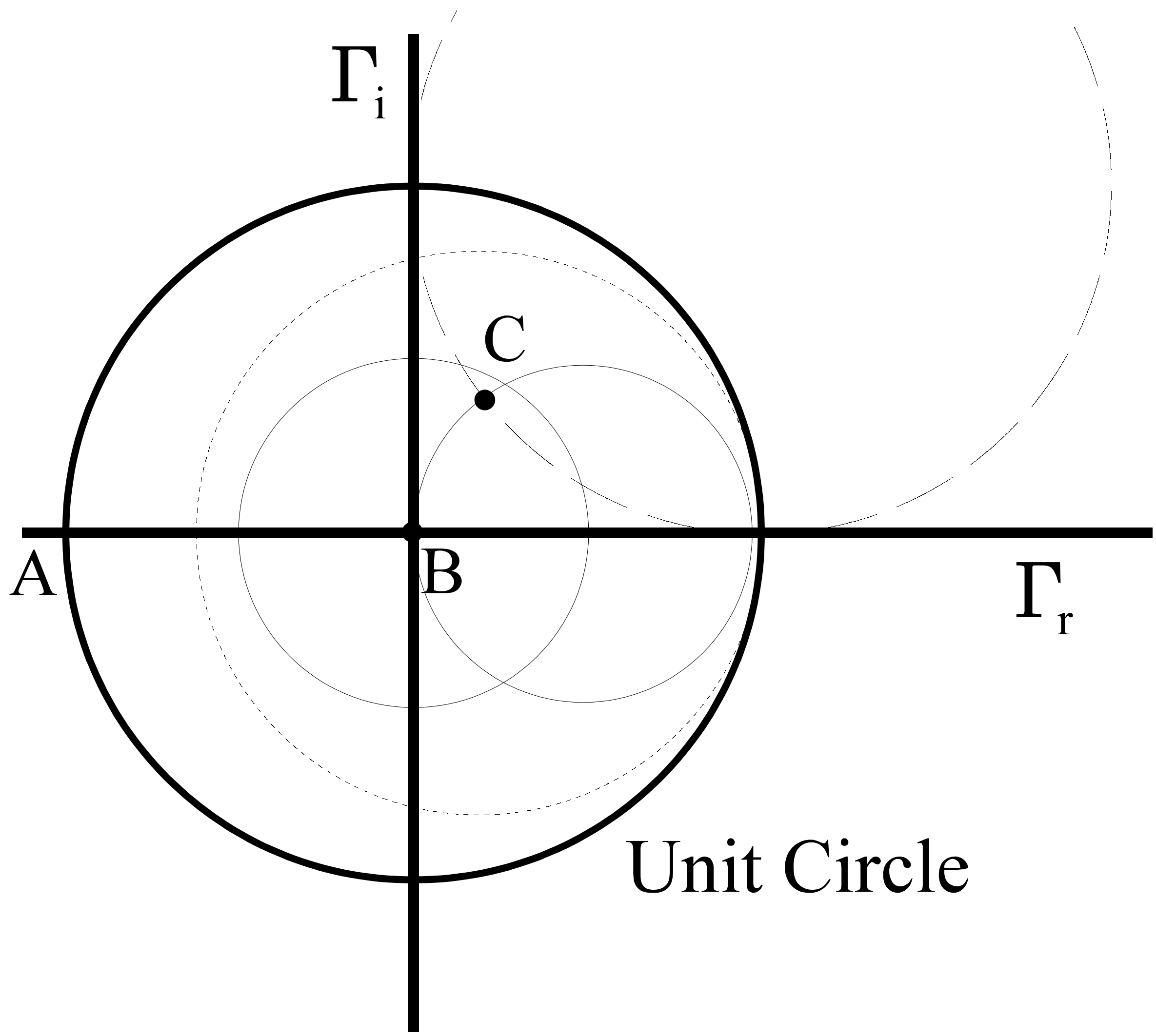
The first circles, those centered on the real axis, represent lines of constant real part of the load impedance (r_L is constant, x_L varies). The circles whose centers reside outside the unit circle represent lines of constant imaginary part of the load impedance (x_L is constant, r_L varies).

Circles centered at the match point ($Z_L = Z_o$, or $\Gamma_L = 0$) are equidistance from the origin ($|\Gamma_L|$ = constant) and are called constant VSWR circles. This related quantity is defined as

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

Note that the VSWR can vary between $1 \leq VSWR \leq \infty$.

The phase of the reflection coefficient is given by the angle from the right-hand horizontal axis. We have $-180° \leq \angle\Gamma_L \leq +180°$. Above the horizontal axis is positive, below negative.



When we place these other circles on the REFLECTION COEFFICIENT PLANE we call it a **SMITH CHART**. It can also be referred to as an:

impedance chart

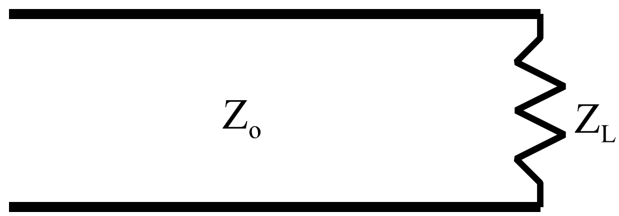
admittance chart

immittance chart (impedance *and* admittance)

INSERT SMITH CHART GRAPHIC HERE

Normalized Impedance

The transmission line of characteristic impedance, Z_o, is terminated in a load impedance, Z_L.



If the characteristic impedance is $Z_o = 50 + j\,0\ \Omega$ and the load impedance is $Z_L = 100 + j\,100\ \Omega$, the normalized load impedance is

$$z_L = \frac{100 + j100}{50} = 2 + j2 \qquad (\text{dimensionless})$$

Now enter this on the Smith Chart as shown in below.

INSERT SMITH CHART GRAPHIC HERE

VSWR Circles

To construct the VSWR circle for a lossless line, place the compass center at the center of the Smith Chart ($Z_L = Z_o$, or $\Gamma_L = 0$) and construct a circle through z_L.

To read off the VSWR on the line, note the intercept of the VSWR circle with the right-half zero reactance line (it is the horizontal line that bisects the top and bottom of the chart.

For the previous example, this is shown in Figure 2 where we read off the VSWR as

$$VSWR = 4.3$$

Let's check this against the **calculated VSWR**. First, compute the reflection coefficient.

$$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{(100 + j100) - 50}{(100 + j100) + 50}$$

Taking the magnitude we find $|\Gamma| = 0.62$. Then the VSWR is

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 4.265$$

This tells us that the graphically obtained value (VSWR = 4.3) is accurate to about one significant figure.

INSERT SMITH CHART GRAPHIC HERE

Load Admittance

The load admittance is the reciprocal of the load impedance.

$$Y_L = \frac{1}{Z_L}$$

For the previous example, we find

$$Y_L = \frac{1}{100 + j100} = 0.005 - j0.005 \quad Siemens$$

Normalized Load Admittance

The normalized load admittance is the reciprocal of the normalized load impedance which can be expressed as

$$y_L = \frac{Y_L}{Y_o} = \frac{\frac{1}{100 + j100}}{\frac{1}{500}} = 0.25 - j0.25 \qquad (\text{dim}\,ensionless)$$

We can find this more easily by noting that y_L is on the constant VSWR circle at a point diametrically opposed from z_L. This is shown in Figure 3.

INSERT SMITH CHART GRAPHIC HERE

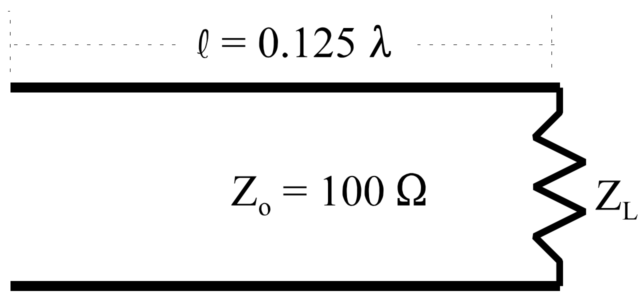
Input Impedance

The input impedance to the line of characteristic impedance, Z_o, a length ℓ from the load, Z_L, is found via the equation.

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\left(\frac{2\pi}{\lambda}\ell\right)}{Z_o + jZ_L \tan\left(\frac{2\pi}{\lambda}\ell\right)}$$

Where λ is the wavelength *on the transmission line*.

Consider the following example with $Z_L = 100 + j\,100\ \Omega$.



$$Z_{in} = 100 \frac{(100 + j100) + j100 \tan\frac{\pi}{4}}{100 + j(100 + j100)\tan\frac{\pi}{4}} = 100\frac{1 + j2}{j1} = 200 - j100 \quad \Omega$$

Now consider this process on the Smith Chart.

1. Enter the normalized load impedance, $z_L = 1 + j\,1$
2. Construct the constant VSWR circle.
3. Travel clockwise (*transform towards the generator*) on the circle by a distance equivalent to the line length, starting at z_L. Remember the distance on the Smith Chart is in terms of wavelength. Specifically, wavelength on the transmission line, which is not necessarily the free space wavelength. One full revolution is $\lambda/2$. The outer scale is calibrated in wavelengths.
4. Read off z_{in}. In this case, $z_{in} = 2 - j\,1$.
5. Compute $Z_{in} = z_{in} * Z_o = (2 - j\,1) * 100 = 200 - j\,100\ \Omega$.

INSERT SMITH CHART GRAPHIC HERE

Input Admittance

Use the Smith Chart to find the input admittance. Consider the example above.

1. Enter the normalized load admittance on the VSWR circle. This is accomplished by entering the normalized load impedance, drawing the VSWR circle and marking the point diametrically opposed.

2. Travel clockwise (*towards the generator*) from y_L on the VSWR circle by a distance equivalent to the line length.

3. Read off $y_{in} = 0.44 + j\,0.2$

4. Find $Y_{in} = y_{in} * Y_o = (0.44 + j\,0.2) / (100) = 0.004 + j\,0.002$ Seimens.

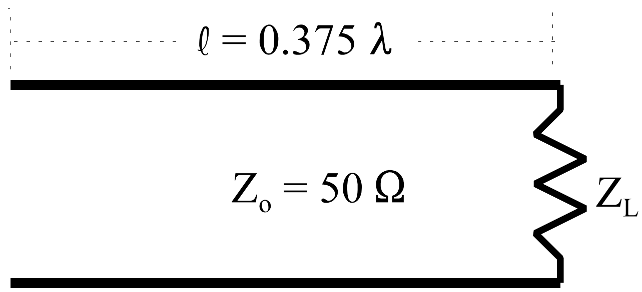
Let's compare the result from the Smith Chart on the next page to the analytical result. From the example above we found $Z_{in} = 200 - j\,100\ \Omega$.

$$Y_{in} = \frac{1}{Z_{in}} = \frac{1}{200 - j100} = 0.004 - j0.002 \qquad Siemens$$

INSERT SMITH CHART GRAPHIC HERE

Drill Problem # 1

Consider a load impedance, $Z_L = 100 - j\,25\ \Omega$, connected to a transmission line of characteristic impedance $Z_o = 50\ \Omega$. The line is $\ell = 3/8\ \lambda$ long.



Find the following quantities using the Smith Chart on the next page.

a) z_L = _______________

b) VSWR = _______________

c) y_L = _______________

d) z_{in} = _______________

e) Z_{in} = _______________

f) y_{in} = _______________

g) Y_{in} = _______________

h) Check the answer in part e) using the analytical result.

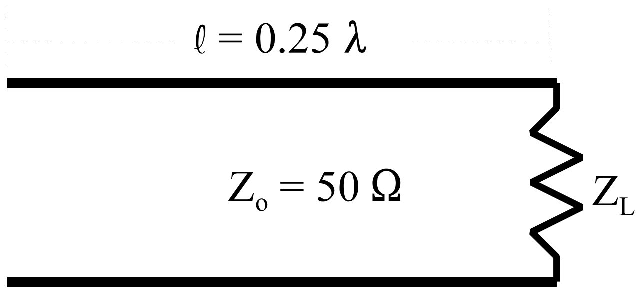
\

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\left(\frac{2\pi}{\lambda}\ell\right)}{Z_o + jZ_L \tan\left(\frac{2\pi}{\lambda}\ell\right)}$$

INSERT SMITH CHART GRAPHIC HERE

Drill Problem # 2

Consider a load impedance, $Z_L = 0 - j\,0\ \Omega$, connected to a transmission line of characteristic impedance $Z_o = 50\ \Omega$. The line is $\ell = 1/4\ \lambda$ long.



Find the following quantities using the Smith Chart on the next page.

a) z_L = _______________

b) VSWR = _______________

c) y_L = _______________

d) z_{in} = _______________

e) Z_{in} = _______________

f) y_{in} = _______________

g) Y_{in} = _______________

h) Check the answer in part e) using the analytical result.

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan\left(\frac{2\pi}{\lambda}\ell\right)}{Z_o + jZ_L \tan\left(\frac{2\pi}{\lambda}\ell\right)}$$

INSERT SMITH CHART GRAPHIC HERE

Drill Problem #3

Use the Smith Chart on the next page to find the input impedance to the parallel connected lines $Z_{o1} = Z_{o2} = 70.7\ \Omega$. Furthermore, $Z_{L1} = Z_{L2} = 50 + j\,0\ \Omega$. The line lengths are $\ell_1 = \ell_2 = \lambda/4$. The characteristic impedance of the main line is $Z_o = 50\ \Omega$ to which the two parallel lines are connected, what is the VSWR on this main line?

Drill Problem #4

Use the Smith Chart to find the input impedance to the parallel connected lines $Z_{o1} = 50\ \Omega$ and $Z_{o2} = 70.7\ \Omega$. Furthermore, $Z_{L1} = 50 + j\,0\ \Omega$ and $Z_{L2} = 100 + j\,0\ \Omega$. The line lengths are $\ell_1 = \lambda/2$ and $\ell_2 = \lambda/4$. The characteristic impedance of the main line is $Z_o = 50\ \Omega$ to which the two parallel lines are connected, what is the VSWR on this main line?

INSERT SMITH CHART GRAPHIC HERE

INSERT SMITH CHART GRAPHIC HERE